High Speed, Precision Motion Strategies for Lightweight Structures

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SUMMARY

The period covered by this report saw one Ph.D. proposal submitted and approved, one paper submitted for conference presentation and two papers accepted for presentation at the forthcoming NASA Conference on Space Telerobotics.[1-4] These documents contain the detailed results of the research summarized here. Several presentations made during this period that have been described in the previous progress report. In addition, adaptive control experiments on the RALF (Robotic Arm, Large and Flexible) have been performed and will be documented in a Ph.D. thesis nearing completion. A Master's thesis nearing completion has provided a joint controller design for the SAM (Small Articulated Manipulator) to be mounted on RALF.

The recent availability of RALF for experiments enabled Bau-San Yuan to apply the theoretical results of his Ph.D. Thesis to control the research arm. The algorithm he used can be described as a robust decentralized adaptive control based on a bounded uncertainty approach. Decentralization allows the algorithm to be of reduced complexity. The design considers explicitly the flexible dynamics of only the first link outboard of the joint. The coupling to the other link's flexible dynamics is treated as an uncertainty in the dynamic equations. The form of the feedback law includes an adjustable gain times a state feedback and a saturation term. The saturation term increases linearly with the norm of the error between the system's response and the response of a reference model. This term can be shown to improve the stability of the system based on Lyapunov analysis. An integral term is also included to essentially eliminate steady state error. The theoretical analysis initially assumed that all states were available from measurements. Strain gages provide measurements of the link deflections, but strain rates are not available. Instead of the more computationally intensive reduced order observer first implemented, a numerical derivative was ultimately implemented with significant improvement computation time. The controller was developed based on an equivalent serial link model developed and verified in the laboratory, even though RALF contains a parallel actuating link for the second joint.

Mr. Yuan has been assisted by James Huggins, who was responsible for making RALF operational and verifying a mathematical model for his M.S. thesis and who is beginning his Ph.D. program.

A recently approved Ph.D. dissertation proposal [1] by Mr. Soo-Han Lee embarks on a study of the dynamic interactions between SAM and RALF. It should be mentioned that Mr. Soo-Han Lee has not been supported by NASA funds to this time, but since he expects to use the NASA supported test bed his work should be included in this report. NASA has justifiable concerns that an active payload on a flexible manipulator could cause that manipulator to become unstable. Mr. Lee is studying this type of interaction but with the perspective that the inertial forces generated could actually be used to more rapidly damp out the flexible manipulator's vibration. To initiate his studies, Mr. Lee is analyzing a single flexible link interacting with an outboard rigid link. A singular perturbation, two time scale approach was
used in the initial control studies. An alternative way to study the initial simple case is to treat the large/small arm combination as a single arm. In other cases it may be desirable for the small arm to be controlled independently. The configuration the small arm should have to most effectively influence the large arm's vibration is another question to be studied. When used to actively damp those vibrations a large influence is desired. If the small arm is independently controlled, the minimum influence may be preferred. Mr. Lee's work will be represented in one of two papers to be presented at the forthcoming NASA Conference on Telerobotics. [2]

Mr. Jae-Won Lee nears completion on the modeling studies on constrained dynamics of flexible arms, the major part of his Ph.D. thesis. Substantial effort has been extended on selection of the best assumed mode shapes to represent the experiments on RALF. Several aspects of RALF's design make this difficult: the offset of the joint axis from the link center line, the flexibility of the first link in board of the parallel link's attachment, and the general non-serial design. We expect that reasonable final accuracy results in a wider range of operating conditions that Yuan's simpler equivalent serial model. We also expect to understand ways to make any future designs easy to model. One of the exciting aspects of Mr. Lee's modeling studies is an understanding of substantial simplifications that can be made based on the form of the general serial flexible link equations. (The serial equations are used in formulating the parallel link equations also.) Mr. J.W. Lee's and Mr. Yuan's equations are developed directly from the system inertia matrix, avoiding the explicit application of Lagrange's equations and considerable processing difficulty. Both SMP and MACSYMA symbolic processing codes are used in the research. Mr. J.W. Lee's thesis should be completed in the second half of 1989. He is now considering alternate control algorithms that accommodate the constrained equations he has developed.

Mr. Doug Paul has implemented several joint control algorithms for SAM on the M68000 multiprocessor system he has designed. One 68000 is planned for each of SAM's 3 joints. They communicate with each other and with a supervisory IBM PC. The controllers implemented include P.I.D., state feedback with observer, and a zero phase feed forward controller. The 500 Hz sample rate seems more than adequate. Experiments have involved a single joint test setup with the actual motors and reducers from SAM. Mr. Paul's M.S. thesis is scheduled for completion in the first quarter of 1989. Again, he has not be supported by NASA funds, but is included because he uses some NASA equipment and because he contributes to the overall research goals.

Dong-Soo Kwon has been examining the combined use of two arms with the bracing strategy. In bracing, the large arm, RALF, stabilizes the work surface relative to the small arm, a technique that appears very important in the zero gravity of space. His current work on force control of flexible arms has brought out the desirability of introducing passive damping and additional compliance at the point of impact between the large arm and the bracing surface. The impact between the flexible arm and the surface is very scarcely studied because it is difficult to model. Preliminary studies with a lumped model have been performed, and now a more accurate model is under consideration. Mr. Kwon's work will also be represented in one of the papers at the NASA Conference on Space Telerobotics. [2]

Studies of model order reduction for flexible manipulator were carried out by Dr. Takahiko Tsujisawa, a visiting scholar from NEC Corporation, with RALF as
the example. These results have been collected in a paper.[3] NEC supported Dr. Tsujisawa and provided additional support for activities in the laboratory as well. The studies gave a much welcomed quantitative confirmation for the use of a small number of modes (1 or 2) in representing the dynamics of each of the links of RALF.

A second paper [4] to be presented at the NASA Conference on Space Telerobotics has been invited by the Organizing Committee. It will survey the status of flexible arm modeling and control.

The contribution of several other students should be acknowledged in this report, although they have not received NASA support to date. Mr. Jae Lew is beginning research in the area of robot controls and is in the process of finalizing his Ph.D. Thesis proposal involving the use of two staged arms (i.e. RALF and SAM) as a coordinated system. J.J. Wang has performed valuable analysis of the design space of staged arms to determine the circumstances under which a staged design is superior to a single arm. Some of his contributions appear in a paper.[2] Jonathan Cameron, recently returning to pursue his Ph.D. after several years at JPL, is exploring new concepts for telerobotics for space maintenance and construction.


APPENDIX I

ABSTRACTS OF PAPERS AND THESIS PROPOSALS
ACTIVE VIBRATION CONTROL OF A LARGE FLEXIBLE MANIPULATOR
BY INERTIAL FORCE AND JOINT TORQUE

by
Soo Han Lee
Ph.D. Thesis Proposal
George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology
June, 1988

ABSTRACT

The efficiency and positional accuracy of a lightweight flexible manipulator are limited by its flexural vibrations, which last after a gross motion is completed. The vibration delays subsequent operations.

In the proposed work, the vibration is suppressed by inertial force of a small arm in addition to the joint actuators and passive damping treatment. The proposed approach is:
1) Dynamic modeling of a combined system, a large flexible manipulator and a small arm,
2) Determination of optimal sensor location and controller algorithm, and 3) Verification of the fitness of model and the performance of controller.
USES FOR A FAST WRIST ON A LONG LIGHT-WEIGHT ARM

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ABSTRACT

A flexible manipulator (large arm) has a limited speed of response that may not permit time critical tasks to be successfully performed. One solution proposed is to have additional fast degrees of freedom (small arm) on the end of the flexible manipulator. This can be thought of as a fast wrist on the end of a flexible arm or as a fast arm on the end of a arm positioner. Three distinct means of utilizing this capability have been proposed. The “fast wrist” capability proposed by Cannon and others allows the large arm to wander but corrects the end point position using the small arm. The “bracing strategy” proposed by Book and others clamps the large arm to the work piece through a bracing point and uses the bracing point as a new base for the small arm. Both of these approaches concern the fine motion of the end effector near a fixed configuration. A third use called the “inertial damping strategy” is proposed here. Inertial forces are generated with the small arm for the purpose of damping large arm vibrations. This approach can be used with the other two approaches. It can also be used exclusively in applications where relatively high frequency disturbance forces (such as generated by metal removal) are generated at the end of the large arm and need to be isolated. This is in a manner similar to that proposed by Dickerson for machining with robots.

This paper will examine the necessary control strategies, sensor needs and suitable tasks for the three ways to use a small arm/large arm combination. Results of simulations and preliminary experiments for the bracing strategy and the inertial damping strategy will be presented. The experimental large arm is the Robotic Arm, Large and Flexible (RALF), which has two ten ft. links. The experimental small arm is the Small Articulated Manipulator (SAM), is approximately anthropometric. The bracing strategy is examined from the point of view of the reduction of uncertainty in the position of the end effector and from the possible need for force control of the flexible large arm. The inertial damping is examined from the viewpoint of effectiveness in increasing the large arm damping. The most appropriate small arm position for a docking maneuver leading to bracing of the large arm is studied.

Control of the two arm system for inertial damping is developed on the basis of a two time scale, singular perturbation approach. Control of the large arm during bracing is analogous to a hybrid force-position control. Initial impact between the large arm and the bracing surface is a particularly critical aspect of the overall operation that is under study.
Modeling, Design, and Control of Flexible Manipulator Arms: Status and Trends

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ABSTRACT

The desire for higher performance in manipulators has lead to dynamic behavior in which the flexibility is an essential aspect. This paper first examines the mathematical representations commonly used in modeling flexible arms and arms with flexible drives. Then design considerations directly arising from the flexible nature of the arm are discussed. Finally, control of joints for general and end point performance are discussed.

Time domain analysis of finite dimensional models are the most productive combination of model and analysis at present. The state variables are commonly chosen to be the amplitude of admissible functions or "assumed mode shapes" and their time derivatives. The most appropriate choice of shapes is not well understood. The shapes may be chosen from simple beam shapes, from finite element or experimental modal analysis, or from simple analytic functions such as polynomials. Nonlinear Coriolis and centrifugal effects can readily be incorporated into these models, or they can be linearized for standard control formulation or analysis about an operating point. Recent analysis has raised new concern about the importance of "beam foreshortening" effects at higher rotational velocities. This effect is commonly ignored but conclusively shown to be significant in some circumstances. Infinite dimensional models have also been proposed for arms with simple construction such as uniform cross sections. Time domain analysis is typically performed on a model reduced to a finite dimensional approximation. Frequency domain analysis of distributed parameter systems can be performed numerically with the limitation on upper frequency of interest replacing the finite dimensional approximation. True infinite dimensional analysis is rarely applied to practical situations. Finite element models come very close to representing the small motion behavior of arms with model orders of moderate size (<50) which can be subsequently reduced in a methodical way. Constraints due to non-serial chains or due to contact with the environment greatly complicate the models with algebraic constraint equations that must be solved in conjunction with the differential equations. Trends in flexible arm modeling include the use of symbolic equation generation for purposes of simulation.

Design often seeks to minimize the flexibility or to reduce its undesirable consequences. The design tradeoffs inherently include a compromise between fast, accurate fine motions (high bandwidth) which are enhanced by a rigid but massive arm, and fast large motions (high speed and acceleration) enhanced by a light but flexible arm. The choice of stiffer materials such as composites may shift the inevitable point of flexibility beyond the range of some applications at the expense of higher cost and difficult fabrication. The
damping of the arm depends on the material and can be enhanced through various treatments. The effect of improved damping on control is substantial, as complications due to spill over and wrap around with a digital controller are greatly reduced. "Smart materials" have also been shown to effectively increase the damping through local control loops in which piezo-ceramic or piezo-polymers react to local strain measurements. Using additional degrees of freedom can enable one to control vibrations by generating inertial forces. The actuators may be solely for the purpose of vibration control or they may be incidentally used for vibration control as well as the arm's basic function. Bracing a large arm and then using a small arm is a second way to use these degrees of freedom with a flexible arm. Finally a "fast wrist" may stabilize the end point while the arm continues to vibrate.

Joint controllers of the usual P.I.D. type succeed up to bandwidths of about 1/2 the first arm structural frequency (joints clamped). Beyond this fuzzy limit more advanced control algorithms are needed. Actually, the coulomb friction in speed reducers may make PID controllers ineffective at absorbing vibrational energy of low amplitude at any frequency. State feedback which includes flexible states can be quite effective at improving the performance although they may be sensitive to changes in plant dynamics in normal operation. Placement of sensors for measuring the flexible behavior commonly results in a "non-minimal phase" system, most apparent in the initial reverse reaction of the tip of a flexible beam to a step in torque at its base. Propagation of the flexural wave down the beam forms an upper limit on the tip control performance. Adaptive control using rigid model reference approaches is not greatly effective in overcoming this barrier. If flexibility is included in the control analysis the "model matching condition" of MRAS cannot be met in the simple manner used for rigid arms. This has been circumvented only for the near linear one link case. Bounded uncertainty analysis is another means for dealing with the variation in plant parameters. It has been successfully applied to the two link flexible arm.

Various methods have sought to decouple the flexible arm control problem into rigid and flexible components. Singular perturbation analysis is one methodical way to achieve this, but it requires a substantial separation between the fast (flexible) modes and the slow (rigid) modes. This builds in limits on the performance that caused one to consider flexibility in the first place. While lacking the strong theoretical base that singular perturbation analysis provides, other ad hoc approaches to specify nominal trajectories for the joints while controlling the flexible link behavior have been successfully applied to flexible arms of up to three links. Considerable progress has recently been made on specifying joint torques as a function of time to achieve the desired tip trajectories. This is the flexible version of the inverse kinematics and dynamics problems, which must be solved simultaneously. The large computational burden of some approaches restricts this to off line precalculation. Other researchers have taken the strategy of just smoothing joint trajectories to avoid exciting vibrations unnecessarily. The time optimal problem for simple paths of one link has been conclusively solved. The torque history is of a discontinuous (bang-bang) nature and not tolerant of minor model errors. For complex paths of the end point, no equivalent to the rigid arm solution is available since the flexible arm cannot be reduced to a single second order system. Approximations with a close analogy to the rigid approach have been proposed, but they are not true optimal solutions, nor do they exactly follow the prescribed path.
APPENDIX II

COPIES OF PAPER AND THESIS PROPOSAL
Active Vibration Control of a Large Flexible Manipulator
By Inertial Force and Joint Torque

A Thesis Proposal
Submitted to
the Faculty of the Division of Graduate Studies
By

Soo Han Lee

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy
Georgia W. Woodruff School of Mechanical Engineering

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Abstract

The efficiency and positional accuracy of a lightweight flexible manipulator are limited by its flexural vibrations, which last after a gross motion is completed. The vibration delays subsequent operations.

In the proposed work, the vibration is suppressed by inertial force of a small arm in addition to the joint actuators and passive damping treatment. The proposed approach is: 1) Dynamic modeling of a combined system, a large flexible manipulator and a small arm, 2) Determination of optimal sensor location and controller algorithm, and 3) Verification of the fitness of model and the performance of controller.
I. Introduction

Most industrial robots have limitations on efficient usage and wide application because of short arm lengths and heavy steel construction. An alternative approach to expand the capabilities is to design the robot with light weight flexible links which have safe strength.

A large, two degree of freedom flexible manipulator which has a small arm at one end has been constructed at the Flexible Automation Laboratory in Georgia Institute of Technology as shown in Fig. 1. The large flexible manipulator is for gross motions, and the small arm is for fine motions. The large manipulator consists of two ten foot long links made of aluminum tubing actuated hydraulically through a parallel link drive. The small arm is actuated by three brushless D.C. motors through harmonic drives at each joint.

Such a configuration has many advantages compared to conventional industrial robots, that is, larger workspace, faster motion time for large motion and higher payload to weight ratios. It would be useful for welding, riveting, assembly, and inspection of large vessels, structures and vehicles. The manipulator, however, has some technical problems that need to be solved. Such problems are due to the flexibility of the large manipulator, dynamic coupling between the large manipulator and the small arm, and the friction and elasticity of a drive at the joints of the small arm.

The amplitude of the flexural vibration of the manipulator increases with operating speed and payload. In order to apply a bracing strategy[5] for efficiency and positioning accuracy without collision between contacting parts, the vibration of the
manipulator needs to be suppressed as quickly as possible. Before working on fine motion jobs, the actuators of the small arm have little to do, and can generate inertial forces to reduce vibrations. In this research, the inertial force of the small arm is to be used for suppressing the vibrations of the large manipulator together with the hydraulic actuators of that arm and a passive damping treatment.

A variety of studies have been done to control the flexural vibration of a manipulator and a structure in several engineering areas. Most of them considered the control of one dimensional vibration of a beam-like structure[2],[21],[22],[26],[28],[31], a plate[1],[25], one link manipulator[7],[10],[14],[17], and two serial link manipulator [4],[12],[29]. Only a few of them have considered two dimensional vibration[9],[30], and dynamic coupling between a beam and an actuator[24]. However the proposed manipulator has some additional problems. First of all, the parallel link mechanism gives more complicated nonlinear dynamic equations, and the manipulator shows three directional vibrations induced by the inertial force of the small arm. Moreover the dynamics of the small arm are inherently coupled to the dynamics of the large manipulator. Such coupling must be modeled. The coupling is due to the fact that the reference frame of the small arm is dependent on the dynamics of the large manipulator. Also, the joint friction and flexibility of the small arm are not negligible. These features give complicated coupled nonlinear dynamic equations. In order to design a controller, the coupled dynamic equations should be determined and verified.

Besides the complexity of the dynamics, the manipulator has an infinite number of modes. It is practically impossible to design a controller based on an infinite dimensional dynamic model. The actuators, the hydraulic cylinder and the small arm, have limited bandwidths. Also, the limitations on cost and space restricts number of sensors to be used. Hence a controller should be designed with limited number
of sensors and a reduced order model which exhibits the best dynamic performance within these limitations. In this case, truncated higher modes can cause instability[2]. This phenomenon is called spillover. The spillover instability depends on the dynamics of a controlled system[16] and the location of sensors[17]. Since the proposed manipulator has highly coupled dynamic equations. A more sophisticated control algorithm is required to obtain stability. The spillover phenomenon will be studied by computer simulation, and examined by experiments. In relation to this problem, the effectiveness of the passive damping is to be studied. Also, the optimal location of sensors and the optimal posture of the small arm will be studied.

In positioning stages, the movement of the large manipulator is small compared to the gross motion. Then the dynamic equations can be linearized at a certain configuration. These linearized equations will be used for a controller design. The characteristics of the equations varies with the change in the payload and the configuration of the manipulator. Modeling errors are inevitable. In order to obtain a good performance, the controller designed should compensate the modeling errors and adapt the change of the dynamic characteristics. At the first stage of this research, a simple control algorithm will be used to examine the system performance. Later more sophisticated controller will be designed.
II. Related Previous Work

The dynamics of a flexible manipulator is viewed as coupled rigid and flexible motion. The flexible motion is governed by a partial differential equation. In order to obtain ordinary differential equations from the partial differential equation, modal analysis is commonly used.

The flexible manipulator can be modeled in terms of either constrained modes of vibration where the joint is held motionless, or the unconstrained modes of vibration where the entire body vibrates. Most researchers have used constrained modes in a dynamic modeling. In one link arm, the dynamic model using the constrained modes showed good agreement with the results of experiments[9],[14], [17]. A few researchers studied multi-link arms using the constrained modes [4],[27],[29]. In the case of multi-link arm, dynamic interactions between the links affect the boundary conditions at the joints. Hence the selection of mode shapes is a difficult problem. The accuracy of the constrained mode method has not been experimentally confirmed in modeling a multi-link arm.

A few researchers used unconstrained modes in a dynamic modeling. In this case, the dynamic model using unconstrained mode is more rigorous. However the determination of the eigenfunctions becomes more complicated as the number of links increases. Using the unconstrained mode, Cannon Jr. and Schmitz[7], and Fukuda and Arakawa[12] derived a dynamic model for a one link arm and for a two link arm respectively.

Another approach in modal analysis is to use the finite element method. Usoro et al[34] modeled a two link arm by the finite element method. Lee[19] is also mod-
eling the manipulator having parallel link mechanism by the finite element method. The finite element method is useful for the complicated structure whose boundary conditions are difficult to identify.

Several researchers have investigated joint flexibility and friction. Joint flexibility can cause large amplitude vibration and inaccurate positioning. The flexibility due to a harmonic drive shows nonlinear behavior similar to that of a hardening spring [35]. Sweet and Good [32] derived a nonlinear model for a robot drive system, which had strong anti-resonance/resonance properties. However, most researchers assume the joint flexibility to be a linear spring [11],[20],[23]. They confirmed the assumption by experiments.

Friction is always present to some extent, and causes poor motion accuracy. To find an exact model is difficult, hence several models of friction have been discussed in the literature [8],[13]. Canudas, et al [8] modeled static and viscous friction as nonlinear functions of angular velocity. Kubo, et al [18] used ideal Coulomb friction model in controlling a robot arm.

To control flexural vibration, most researchers have used joint actuators. The joint actuator also controls rigid body motion. An alternative is to use additional actuators which control flexible motions. A few researchers have studied this. Zalucky, and Hardt [36] designed two parallel beams with a hydraulic actuator mounted at one end. This arrangement was used to compensate deflection and to improve dynamic response. A similar configuration was applied to tracking control [10]. Singh and Schy [30] studied control of the vibration by external forces acting at one end. All of them neglected dynamic coupling between the actuator and manipulator. However, the movement of the manipulator can affect actuator dynamics. Ozguner and Yurkovich [24] have studied the vibration control of a beam coupled dynamically with
an actuator, but their work is still preliminary. Chiang[37] studied a fast wrist to achieve better end point control when a large link was vibrating. He decoupled the dynamic motion of the end point from the movement of the large link by locating the end point at the center of percussion of the wrist.

The flexural vibration has an infinite number of modes. But it is more difficult to design a controller based on an infinite dimensional model. Hence, all of the researchers have used a reduced order model for designing a controller[4],[29],[7]. In this case, control and observation spillover can occur. Balas[2] showed that the effect of spillover could destabilize a large flexible space structure system. He suggested a comb filter to eliminate the instability. However Trunckenbrodt[33] indicated that the control spillover was not necessarily bad, and the comb filter was not useful for preventing the observation spillover instability. Book, Dickerson et al[6] proposed a passive damper to overcome the spillover instability. They showed that an unstable system due to truncated higher modes could be stabilized by passive damper. Alberts et al[0] suggested a combined active/passive control scheme. They improved system's stability using a constrained viscoelastic layer method.
III. The Objective of the Research

Many researchers have studied the control of a flexible manipulator. However, most of them have used only joint actuators for the vibration control. The objective of this research is to develop a control scheme which suppresses the vibration of the large flexible manipulator in minimum time. This is approached by using the inertial force of a small arm as well as the joint actuators and passive damping. The issues surrounding the change of configuration of both the large and small arm will be addressed. When the large manipulator changes configuration, the modes of the manipulator change. When the small arm changes configuration, the ability to influence these modes will change. The result of the proposed research may contribute to the modeling and control of the flexible manipulator with high efficiency and positional accuracy. No previous study of this type is known.
IV. Proposed Approach

The small arm will be used as the actuator for suppressing the flexural vibration. Hence it is important to find the dynamic characteristics of the small arm. Its links are essentially rigid but its joints have flexibility and friction. Its friction and flexibility will be identified by experiment.

Although much work has been done on modeling the large flexible manipulator at Georgia Tech[15],[19], they have not included the dynamics of the small arm and the out of plane motions of the manipulator. The dynamics of the combined system will be obtained by applying Lagrange’s equations to the energy terms which are to be derived with assumed mode shapes. The mode shapes will be found by using the finite element method.

The dynamic equations of motion could be linearized at a certain configuration. The linearized equations can be written as

\[
\begin{bmatrix}
M_{rr} & M_{rf} \\
M_{fr} & M_{ff}
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_r \\
\ddot{x}_f
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
0 & D
\end{bmatrix}
\begin{bmatrix}
\dot{x}_r \\
\dot{x}_f
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
0 & K
\end{bmatrix}
\begin{bmatrix}
x_r \\
x_f
\end{bmatrix}
= \begin{bmatrix}
N_r \\
N_f
\end{bmatrix}
\]

(1)

where subscripts r and f denote rigid and flexible coordinates respectively, \( q \) is generalized coordinate vector, \( M \) is generalized mass matrix, \( D \) is damping matrix, \( K \) is elastic stiffness matrix, \( N \) is input matrix, and \( r \) is generalized force vector related to joint torque. The dynamics of the combined system will be verified by experiments.

The equation (1) can be expressed in state variable form as

\[
\begin{bmatrix}
\dot{X}_c \\
\dot{X}_t
\end{bmatrix}
= \begin{bmatrix}
A_{cc} & A_{ct} \\
A_{tc} & A_{tt}
\end{bmatrix}
\begin{bmatrix}
X_c \\
X_t
\end{bmatrix}
+ \begin{bmatrix}
B_c \\
B_t
\end{bmatrix}
U
\]

(2)
where subscript c and t denote controlled modes and truncated modes respectively. Theoretically, the system has an infinite number of modes, but considering passive and structural damping, one can assume that the sum of n and t modes accurately represents dynamic behavior. The control law of the reduced order system can be given by

\[ U = -FX_c, \]  

(3)

where \( F \) is the feedback gain matrix.

By using equations (2),(3) and a matrix \( C \) included in the output equations, the full order closed loop system can be written as

\[
\begin{bmatrix}
\dot{X}_c \\
\dot{X}_t
\end{bmatrix} = 
\begin{bmatrix}
A_{cc} - B_cFC_c & A_{ct} - B_cFC_t \\
A_{tc} - B_tFC_c & A_{tt} - B_tFC_t
\end{bmatrix}
\begin{bmatrix}
X_c \\
X_t
\end{bmatrix}
\]

(4)

or simply

\[ \dot{X} = \tilde{A}X \]

(5)

Even though the reduced order closed loop system, \( A_c - B_cFC_c \), is stable, the full order system can be unstable. The matrices \( A \) and \( B \) are related the posture of the small arm and the configuration of the large manipulator. The matrix \( C \) is a function of sensor location. Hence an optimally selected gain, sensor location and posture give the minimum real part of the eigenvalues of \( \tilde{A} \). These will be studied computationally.

At first stage of this research, the constant gain matrix \( F \) will be used in the experiment. The displacement and velocity of the vibration will be estimated from the measured signal of strain gages and acoustic gap sensors. A controller should process many input and output signals with high speed. In order to increase the processing
rate of the controller, multiple processors will be used. Later more sophisticated control algorithms, such as decentralized adaptive control and robust sliding mode control algorithm, will be considered for implementation.
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Figure 1: The Robotic Arm, Large and Flexible (RALF) carrying the Small Articulated Manipulator (SAM).