

Performance of Galileo's Concatenated Codes With Nonideal Interleaving

K.-M. Cheung and S. J. Dolinar, Jr.
Communications Systems Research Section

The Galileo spacecraft employs concatenated coding schemes with Reed-Solomon interleaving depth 2. This article compares the bit error rate (BER) performance of Galileo's concatenated codes, assuming different interleaving depths (including infinite interleaving depth). It is observed that Galileo's depth 2 interleaving, when used with the experimental (15, 1/4) code, requires about 0.4 dB to 0.5 dB additional signal-to-noise ratio to achieve the same BER performance as the concatenated code with ideal interleaving. When used with the standard (7, 1/2) code, depth 2 interleaving requires about 0.2 dB more signal-to-noise ratio than ideal interleaving.

I. Background

The Galileo spacecraft employs a communication system which uses either a (7, 1/2) convolutional code or a (15, 1/4) convolutional code as the inner code, and a (255, 223) Reed-Solomon code as the outer code. By using soft, maximum-likelihood decoding on the received symbols, the convolutional codes perform well at low signal-to-noise ratios. However, maximum-likelihood decoding of convolutional codes creates bursty errors. An interleaver is placed between the convolutional code and the Reed-Solomon code to randomize the bursty errors before they are fed to the Reed-Solomon decoder.

Concerns were initially expressed last summer^{1,2} and recently repeated³ [1] about the adequacy of Galileo's interleaving depth for the constraint length 15 code, even when it was

¹S. Dolinar, "Alternative Code Considerations for Galileo," JPL Interoffice Memorandum 331-87.2-308 (Appendix), (internal document), Jet Propulsion Laboratory, Pasadena, California, July 1, 1987.

²S. Lushbough, "Galileo Convolutional Coder Meeting with Project, 31 July 1987," JPL Interoffice Memorandum 313/5-181-SL:jlc, (internal document), Jet Propulsion Laboratory, Pasadena, California, August 3, 1987.

³L. Swanson, "Interleaving Depths for Reed-Solomon Decoders," JPL Interoffice Memorandum 331-88.2-042, (internal document), Jet Propulsion Laboratory, Pasadena, California, July 13, 1988.

first mistakenly assumed that Galileo's interleaver was the same as Voyager's. Depth 4 interleaving was selected for Voyager to sufficiently randomize the error bursts created by the (7, 1/2) convolutional decoder. Performance degradation for the (7, 1/2) code with depth 4 interleaving is insignificant (less than 0.1 dB) relative to ideal interleaving at bit error rates between 10^{-5} and 10^{-6} . However, the error bursts from the (15, 1/4) decoder are about twice as long (on the average) as the bursts from the (7, 1/2) decoder, and thus the longer-constraint-length code would seem to require about double the interleaving depth. Instead, Galileo's actual interleaving depth is only half of Voyager's, and this can potentially cause significant concatenated system performance degradation for both of Galileo's codes relative to theoretical predictions based on ideal interleaving.

Previous studies of the effects of interleaving depth on concatenated system performance included some test data for the (7, 1/2) code⁴ but no in-depth analyses that would allow extrapolation to the case of the (15, 1/4) code. Direct simulation tests of concatenated system performance using the (15, 1/4) code were completely unfeasible because of the huge amount of data that would have to be collected to verify bit error rates (BERs) in the 10^{-5} to 10^{-6} range, and because of the slowness of the software Viterbi decoder simulation (about 30 hours of CPU time on a Sun-3/260 computer per 100,000 decoded bits for the (15, 1/4) code).

Recently the completion of C. R. Lahmeyer's 1-kbit/sec (currently constrained to run at about 0.1 kbit/sec—still a hundredfold increase in speed relative to last summer's software simulation) hardware Viterbi decoder⁵ has allowed us to do some long decoding runs not previously feasible for the (15, 1/4) code. With the advent of this hardware decoder, the following research tools have been developed:

- (1) Long decoding runs (several megabits) for the (15, 1/4) convolutional code were performed on the hardware decoder, and the error bursts are stored in data files conforming to the data compression format described in recent memos.^{6,7} These data files can be used to

replicate the long, time-consuming runs exactly and are useful to anyone who wants to analyze the burst statistics of the Viterbi decoder.

- (2) Similar long decoding runs were performed for the (7, 1/2) convolutional code using the software simulation, and the error bursts from those runs are also saved in the compressed format.
- (3) Simulation software was developed which reads the compressed burst data obtained from the long decoding runs and simulates the operation of the entire concatenated coding system with different interleaving depths.

II. Performance Results

Simulated BERs of concatenated coding systems with various interleaving depths are given in Figs. 1 and 2. Figure 1 shows the performance of the concatenated system using the (15, 1/4) convolutional code as the inner code, and Fig. 2 shows the performance of the concatenated system using the (7, 1/2) convolutional code. The BERs of the concatenated coding systems with finite interleaving depths are compared to the BERs with ideal interleaving (infinite interleaving depth). In both figures, the concatenated code BER is shown as a function of two bit-energy-to-spectral-noise-density ratios, E_b/N_0 :

- (1) for the convolutional code alone and
- (2) for the overall concatenated system.

The difference between the two E_b/N_0 scales is the overhead of 0.58 dB accounting for the redundancy of the outer (255, 223) Reed-Solomon code.

The data points plotted in Figs. 1 and 2 were obtained from a series of long decoding runs varying in length from 5 million to 40 million decoded bits. Smooth curves were fitted through the data points corresponding to BERs greater than 10^{-5} . The 1σ statistical uncertainty in the data points at BERs lower than 10^{-5} is more than 100% for the cases of finite interleaving depth. The corresponding uncertainty in the data points at BERs higher than 10^{-5} ranges from about 5% to about 100% of the simulated BER. The 1σ uncertainty in the data points for ideal interleaving is between 10% and 30% of the BER.

An example illustrates the difficulty of obtaining accurate simulated concatenated system performance at low BERs. The rightmost data point in Fig. 1 (interleaving depth 2, convolutional code $E_b/N_0 = 0.7$ dB) required about 120 hours (5 days) of running time on the hardware decoder to decode 40 Mbits. The same decoding run would have consumed

⁴F. H. J. Taylor, "Project Galileo Orbiter/Deep Space Network Communications Design Document," Project Document 625-257, Jet Propulsion Laboratory, Pasadena, California, pp. 5-12, January 15, 1981.

⁵C. Lahmeyer, "1 Kilobit per Second Viterbi Decoder," JPL Interoffice Memorandum 331-88.3-042, (internal document), Jet Propulsion Laboratory, Pasadena, California, August 19, 1988.

⁶C. Lahmeyer and K. Cheung, "Long Decoding Runs for the (15, 1/4) Convolutional Code," JPL Interoffice Memorandum 331-88.3-040, (internal document), Jet Propulsion Laboratory, Pasadena, California, July 27, 1988.

⁷K. Cheung, "More Long Decoding Runs of the Convolutional Codes (including the (15, 1/4) convolutional code and the (7, 1/2) convolutional code)," JPL Interoffice Memorandum 331-88.2-048, (internal document), Jet Propulsion Laboratory, Pasadena, California, August 1, 1988.

1.4 years of CPU time on the software decoder. Furthermore, the entire 40-Mbit run produced only three observed code-word errors, and the 1σ statistical uncertainty in the simulated BER is around 100%.

III. Performance Comparison

Tables 1 and 2 show the minimum convolutional code E_b/N_0 to achieve Galileo's concatenated and unconcatenated system performance requirements assuming ideal (infinite-depth) interleaving and depth 2 interleaving, respectively. Table 3 shows the performance degradation caused by depth 2 interleaving relative to ideal interleaving for Galileo's two alternative convolutional codes. Galileo's experimental (15, 1/4) code requires about 0.4 dB to 0.5 dB additional signal-to-noise ratio to overcome the insufficiencies of depth 2 interleaving and achieve concatenated code BERs between 10^{-5} and 10^{-6} . Galileo's standard (7, 1/2) code is hurt less by depth 2 interleaving, but still suffers about 0.2 dB degradation. The relative performance advantage of the (15, 1/4) code over the (7, 1/2) code is reduced by about 0.2 dB to 0.3 dB from the amount predicted in earlier studies (e.g., [1]) based on ideal interleaving. With depth 2 interleaving, concatenated system performance will be improved by only about 1.2 dB when the (15, 1/4) code is substituted for the (7, 1/2) code. The corresponding improvement for an unconcatenated system or for a concatenated system with ideal interleaving is between 1.4 dB and 1.5 dB.

IV. Conclusions and Recommendations

Galileo is unfortunately stuck with depth 2 interleaving, and so the immediate consequence of our simulations is simply to quantify the amount of expected degradation for Galileo's concatenated codes. However, future missions should select interleaving schemes that produce minimal degradation. The required interleaving depth increases roughly in proportion to

the constraint length of the inner convolutional code. For example, interleaving depth 8 appears sufficient to keep the degradation under 0.1 dB for the (15, 1/4) code, as does interleaving depth 4 for the (7, 1/2) code.

As alternatives to simply increasing the interleaving depth of conventional block interleaving schemes, new techniques to combat bursty errors, such as convolutional interleaving, helical interleaving, and burst forecasting, should also be investigated. These techniques are superior to conventional block interleaving schemes. Also, a new Reed-Solomon decoder which can correct both errors and erasures is being developed in the Communications Systems Research Section at JPL. It is expected that the performance of concatenated systems will be substantially improved by the use of error-forecasting techniques together with erasure-correcting Reed-Solomon decoders. We propose to investigate the possibility of developing better interleaving schemes for future deep space missions and to analyze the performance of concatenated coding systems using these new interleaving schemes.

Even though the hardware Viterbi decoder has allowed us to simulate many million decoded bits at a time, the data is still insufficient to accurately simulate concatenated code performance at BERs less than about 10^{-5} . The amount of data required for an accurate estimate increases in proportion to the interleaving depth, and so it is even more difficult to simulate directly the performance of deeply interleaved schemes than it was for Galileo's interleaving depth 2. Hence, notwithstanding the recent advance in decoding speed, it is still important to develop theoretical models for the decoded output of the Viterbi decoder, from which concatenated code performance can be accurately estimated without directly simulating the entire concatenated system. The geometric burst model of [2] should be reexamined for applicability to long-constraint-length codes, and new models need to be developed for estimating Reed-Solomon code performance based on the theoretical model for the Viterbi decoder output.

References

- [1] S. Dolinar, "A New Code for Galileo," *TDA Progress Report 42-93*, January-March 1988, Jet Propulsion Laboratory, Pasadena, California, pp. 83-96, May 15, 1988.
- [2] R. L. Miller, L. J. Deutsch, and S. A. Butman, *On the Error Statistics of Viterbi Decoding and the Performance of Concatenated Codes*, JPL Publication 81-9, Jet Propulsion Laboratory, Pasadena, California, September 1, 1981.

Table 1. Minimum E_b/N_0 to achieve concatenated and unconcatenated system performance requirements under ideal interleaving assumption

	(7, 1/2) code	(15, 1/4) code	Difference
Unconcatenated BER = 5×10^{-3}	2.02 dB	0.52 dB	1.50 dB
Concatenated BER = 10^{-6}	1.79 dB	0.33 dB	1.46 dB
Concatenated BER = 10^{-5}	1.70 dB	0.27 dB	1.43 dB

Table 2. Minimum E_b/N_0 to achieve concatenated and unconcatenated system performance requirements for interleaving depth 2

	(7, 1/2) code	(15, 1/4) code	Difference
Unconcatenated BER = 5×10^{-3}	2.02 dB	0.52 dB	1.50 dB
Concatenated BER = 10^{-6}	2.01 dB	0.83 dB	1.18 dB
Concatenated BER = 10^{-5}	1.90 dB	0.69 dB	1.21 dB

Table 3. Concatenated system performance degradation for interleaving depth 2 versus ideal interleaving

	(7, 1/2) code	(15, 1/4) code	Difference
Concatenated BER = 10^{-6}	0.22 dB	0.50 dB	0.28 dB
Concatenated BER = 10^{-5}	0.20 dB	0.42 dB	0.22 dB

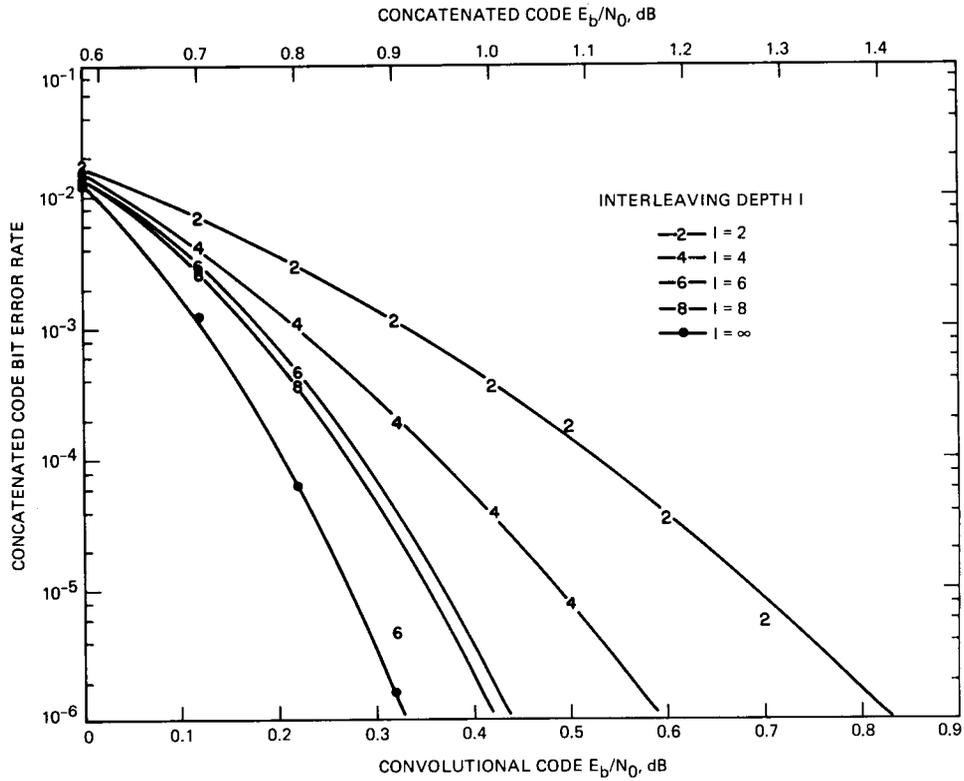


Fig. 1. Concatenated code performance for Galileo's experimental (15, 1/4) inner code with nonideal interleaving.

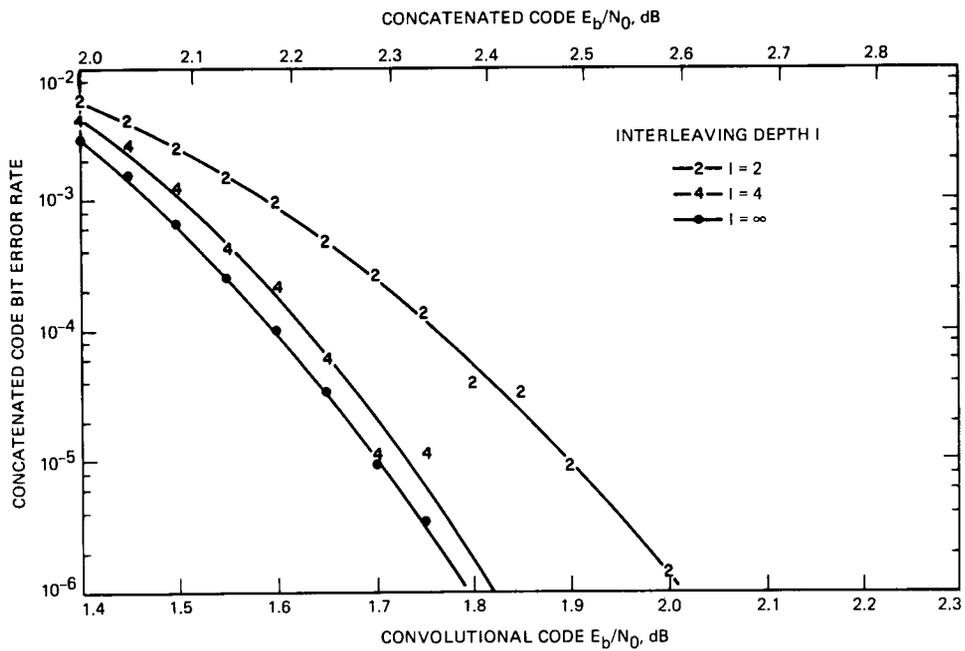


Fig. 2. Concatenated code performance for Galileo's standard (7, 1/2) inner code with nonideal interleaving.