Computational Studies of an Impulsively Started Viscous Flow

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ABSTRACT Progress in validating incompressible Navier-Stokes codes is described using a predictor/corrector scheme. The flow field under study is the impulsive start of a circular cylinder and the unsteady evolution of the separation bubble. In the current code, a uniform asymptotic expansion is used as an initial condition in order to correctly capture the initial growth of the vortex sheet. Velocity fields at selected instants of time are decomposed into vectorial representations of Navier-Stokes equations which are then used to analyze dominant contributions in the boundary-layer region.

1. Introduction

Many attempts have been made to understand the complex physical processes in the interior of a viscous fluid by examining flows about simple geometries such as cylinders. With respect to the prediction of viscous drag, cylinder flows are closely related to airfoils (except for a broader wake, higher drag, and more severe adverse pressure gradients). This can be an advantage in fundamental studies because the large lateral length scales allow more details to be measured (or computed). In addition, the transient viscous flow about a cylinder will be of great help in understanding the physical processes that govern the development of separation and drag on practical configurations and their ultimate minimization and control.

One approach to the Navier-Stokes equations is to examine the short-time behavior due to impulsive starts. Such studies are reported in Wang [11] and [2], Collins and Dennis [3], Dennis and Staniforth [4], Bar-Lev and Yang [5], and Ta Pho Loc [6]. These early-time solutions, based as they are on asymptotic approximations, tend to diverge at moderate times. The flow at longer times must be dominated by large-scale vortex structures and its early time behavior does not seem to be important, at least in the global sense. The fully developed flow seems to be quite independent of the initial conditions, considering the broad general agreement among the previous investigations.

The connection between the short- and long-time solutions is the onset of a wake instability that converts the initially symmetric flow into an asymmetric vortex shedding mode. The exact mechanism that transforms a symmetric boundary-value problem into an unsteady wake-dominated flow is still unclear, although the trigger mechanism is believed to be unavoidable, but small, free-stream disturbances.

The objective of this paper is to combine an analytic short-time solution with a predictor-corrector algorithm to march the unsteady Navier-Stokes equations forward in time. Unlike previous solutions, a uniformly valid initial flow satisfying the Navier-Stokes equations is used to start the calculation. The algorithm is very simple, using an implicit Euler forward integration scheme with an upwind biased, third-order approximation for the convection term. The details of the solution, along with some experimental data, are presented in ref. [7].

The plan of the paper is to first compute the symmetric flow field of an impulsively started circular cylinder up to a nondimensional time (T) of about 7, based on the free-stream velocity and radius. Computations are presented for three combinations of grid spacing and outer boundary location. A detailed analysis of the force and acceleration vectors contributing to the Navier-Stokes equations is presented in graphical form. In this way, the relative importance of the unsteady, convective, pressure-gradient, and viscous terms is made apparent. The onset of instability, and the prediction and analysis of the dominant instability mode will be the subject of a subsequent paper.
The most important result of the vector analysis shows that the flow near the surface of the body varies slowly in time, but the inviscid flow field away from the body has a large dynamic component. This dynamic component is due to the unsteady boundary-layer growth and could have a significant bearing on the subsequent wake instability.

2. Computational Results

In this section computational results are given and compared with other investigations. Pressure and vorticity on the surface are found to be consistent with previous calculations, and the inclusion of realistic initial conditions is shown to eliminate a "ringing" effect.

Static pressure distributions for the three grids are shown in Figs. 1 and 2. The pressure distribution in Fig. 1 at $T = 0.1$ is compared with an analytical representation given by Wang in ref. [2]. The static pressure is a very sensitive parameter and is determined by three competing effects—the curvature of the wall normal velocity that has a cosine distribution that strongly affects both the forward and rear stagnation points; the convective acceleration that is symmetric fore and aft; and the unsteady term that has both symmetric and unsymmetric components. The three computed solutions are similar, except for a small discrepancy at the two stagnation points. The analytical solution shows the same trend, although the pressure falls to a deeper trough. The difference, however, between the minimum pressure and the rear stagnation pressure compares quite well. (Another analytical solution for the short-time impulsive pressure is given in ref. [5], but this solution was not reconcilable with Wang's formula.) Figure 2 shows excellent agreement among the computed solutions and the solutions given by Dennis and Staniforth [4] at time $T = 1.0$.

Turning to the time evolution of the flow, Fig. 3 shows the variation of forward stagnation pressure. Figure 3 shows results for three computational grids. The sensitivity of the outer boundary is apparent for $T > 3.0$ where there is a slight, but consistent discrepancy. Other investigations show the stagnation pressure to fall to about 1.10 at infinitely long times, but this is an unrealistic result because a wake instability sets in at about $T = 14$. Also shown in Fig. 3 is the trend from Wang's asymptotic small-time solution where the stagnation pressure drops very quickly when $T = 0(1)$. Figure 4 shows a reference calculation using the inviscid pressure distribution superimposed on the present computations. At the initial instant of time, the value of $C_p$ is 1.0 and jumps quickly to a very high value. After about two cycles, the oscillation damps out and approaches the previous results with a slight residual oscillation.

Since the kinematics of the flow field seem to be adequately represented by the computational procedure, it is worthwhile to look a little deeper into the equations themselves. At each point in space, the Navier-Stokes equations can be represented by a vector that is in equilibrium among its four components: convection, diffusion, time derivative, and pressure gradient. The relative importance of these effects is presented in Fig. 5 at $T = 0.14$. The left figure shows solution vectors at 12 points on a circle at $r = 1.1$ within the highly viscous flow regime very soon after the initiation of motion. These vectors indicate a strong balance between diffusion and unsteady effects in the direction parallel to the body and between convection and pressure gradient in the normal direction. This verifies the essential uncoupling of the viscous, rotational flow field (the former) and the inviscid, irrotational flow (the latter). The right figure shows the instantaneous boundary layer flow profiles at $T = 0.14$ which are before the onset of flow separation.

3. Concluding Remarks

Physically plausible initial conditions were used to demonstrate that the evolving unsteady flow quickly forgets its early history. Individual field quantities are very useful, but they will not replace an analysis of the full dynamics of the flow field. A first step toward such a dynamical analysis was presented by examining the closure of the vector components of the full Navier-Stokes equation during the initial instant of motion.
References


Fig. 1. Static pressure distribution on the surface of an impulsively started circular cylinder at $T = 0.10$, $Re = 100$: open circles, mesh A; open squares, mesh B; open triangles, mesh C; closed circles, analytical solutions, Wang [1], [2].

Fig. 2. Static pressure distribution on the surface of an impulsively started circular cylinder. $Re = 100$: open circles, meshes A, B, and C; closed circles, numerical computation of Dennis and Staniforth [4]. $T = 1.0$. 
Fig. 3. Time history of the stagnation pressure on an impulsively started circular cylinder. \( Re = 100 \): solid, mesh A; dash, mesh B; dots, mesh C; heavy line, analytical solution, Wang \([1],[2]\).

Fig. 4. Time history of the stagnation pressure on an impulsively started circular cylinder. \( Re = 100 \): solid, mesh A; dash, mesh B; dots, mesh C; heavy line, solution without uniformly valid pressure as initial condition.

\[ \frac{\partial \vec{V}}{\partial t} \quad \vec{V} \cdot \vec{V} \]

\[ \nabla p \quad -\nu^2 \frac{\vec{V}}{r} \]

Fig. 5. Flow field vector decomposition near the body at \( r = 1.1, Re = 100 \). Left: vector equations; right: instantaneous velocities. \( T = 0.14 \), scale factor = 1.0.
Progress in validating incompressible Navier-Stokes codes is described using a predictor/corrector scheme. The flow field under study is the impulsive start of a circular cylinder and the unsteady evolution of the separation bubble. In the current code, a uniform asymptotic expansion is used as an initial condition in order to correctly capture the initial growth of the vortex sheet. Velocity fields at selected instants of time are decomposed into vectorial representations of Navier-Stokes equations which are then used to analyze dominant contributions in the boundary-layer region.