FINAL REPORT

FOR

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High Accuracy Thermal Conductivity Measurements Near the Lambda Transition of Helium with Very High Temperature Resolution

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INTRODUCTION

Over the past few years we have made extensive thermal conductivity measurements near the lambda point of helium. The original goal of measuring the thermal conductivity with a resolution of $t = T/T_\lambda - 1$ of $3 \times 10^{-8}$ was reached, but with less somewhat accuracy than was hoped. Subtle effects in the apparatus near the transition were observed which reduced our ability to interpret the results. Nevertheless, for resolutions of $t \geq 10^{-7}$ reliable data was obtained, extending previous measurements by more than an order of magnitude. Deviations from theoretical predictions were observed for $t \leq 3 \times 10^{-6}$ leading us to question the validity of the present renormalization group analysis of transport properties, at least for the case of helium. This anomaly led us to look more closely at boundary effects in the measurement. Although this work, now covered under separate funding, is not yet completed the indications are that such effects are unimportant. Instead, the changing properties of helium in other parts of the apparatus are the major source of distortion of the results.

During the experiments we also observed a totally unexpected effect in very dilute $^3$He - $^4$He mixtures which led us to a new explanation of anomalous results seen by others. The concentration dependence of the thermal conductivity near $T_\lambda$ in the superfluid phase had been found to deviate strongly from the predictions. Our results gave an independent verification of this behavior and caused
us to re-analyze Khalatnikov's theory of the hydrodynamics of the mixtures. We found an alternative solution which is in better agreement with the experiment.

The measurements have so far led to four publications, copies of which are attached. One additional paper has been submitted, (also attached) and at least one more is expected in the next few months.

BACKGROUND

When a system is close to a cooperative transition, anomalies occur in a wide variety of its dynamic properties, as well as in the more commonly discussed static properties. The dynamical properties of a system are quantities such as transport coefficients, relaxation rates, and multi-time correlation functions, all of which depend on the equations of motion. The static properties, on the other hand, are quantities such as the thermodynamic coefficients and single time correlation coefficients which are determined by the time independent equilibrium distribution of particles. With the development of the renormalization group (RG) approach to static phenomena\(^1\) at cooperative transitions, it also became possible to advance our understanding of dynamic phenomena\(^2\). Not only was it possible to elucidate the areas of applicability of previous theories, but predictions were also made for the values of exponents characterizing the divergence of a number of dynamic properties, and of various dimensionless functions and coefficient ratios. This area of research is still being developed actively and the precise
values of some of the quantitative predictions are not yet well established. For this reason experimental measurements of dynamic phenomena near cooperative transitions are currently extremely useful as guides to the theoretical development of the field.

The testing of the dynamic RG predictons\textsuperscript{2, 3} involves parameter estimation from experimentally determined properties of systems over wide ranges of the independent variable. To perform the most rigorous tests of the theory it is necessary to collect high quality data over the maximum range available, and it is here that the $\lambda$-transition of helium becomes uniquely important. Only for the $\lambda$-transition is it possible to approach the transition temperature to a few parts in $10^8$ in the parameter $t$, two or three orders of magnitude closer than in any competing system. Another requirement for rigorous tests of the theory is high quality data, implying the measurement of easily determined quantities that are expected to be free from significant perturbing effects. For dynamic tests at the $\lambda$-point the natural choice of variables is the thermal conductivity, $K$, as a function of temperature above $T_\lambda$ and the bulk attenuation coefficient of second sound below.

Kerrisk and Keller\textsuperscript{4} made the first thermal conductivity measurements near the $\lambda$-point which showed anomalous behavior. They found a strong divergence in $K$ consistent with the exponent value of $1/3$ predicted by Ferrell et al.\textsuperscript{5} However, these measurements were not of high enough resolution to be sure the asymptotic region had been reached. In 1968 Ahlers\textsuperscript{6} reported thermal conductivity measurements extending to $t \sim 5x10^{-8}$, although
with large uncertainties below $10^{-6}$. Almost simultaneously Archibald et al.\textsuperscript{7} reported anomalous size dependent effects on $K$ in small cells over the range $1.5 \times 10^{-6} < t < 1$. More recently Ahlers\textsuperscript{8} has reported more accurate measurements over the range $2 \times 10^{-6} < t < 5 \times 10^{-4}$ in a cell large enough to be expected to be free of distortion. These results have been subjected to extensive theoretical analysis and have lead to a certain amount of controversy in their interpretation. Ahlers, Hohenberg and Kornblit\textsuperscript{9} (AHK) have performed the most recent analysis of this data in which they show that it is consistent with the asymmetric planar magnet model\textsuperscript{10} of the superfluid transition, as was previously expected. Two regimes are identified: a weak coupling regime $t_c \ll t \ll 1$, where $t_c$ is a cross-over temperature, and an asymptotic region, $t \ll t_c$. The value of $t_c$ was estimated to be $\sim 10^{-3}$, indicating a somewhat smaller asymptotic region than with static properties. AHK conclude that meaningful tests of the theory could be made in the weak coupling regime, but close to the transition both the theoretical and experimental situations are at present too unsettled to draw significant conclusions. Nevertheless, agreement with experiment could be obtained over essentially the whole range by adjustment of parameters in the asymmetric model. Unfortunately, this apparent agreement is clouded by the fact that similar agreement can be obtained with a second set of data\textsuperscript{9} from a somewhat smaller cell, even though the two data sets have a noticeably different temperature dependence. One would expect this apparent size-dependent effect to lead to some difficulty, since the model does not allow for it in any direct way. So far the model has not been fit to
the data of Archibald et al., which differs substantially from that of Ahlers. Possibly at this level some lack of agreement could be detected.

The situation outlined above indicates the status of the testing of the dynamic RG predictions for the thermal conductivity when the present project was commenced. The main difficulties appeared to be:

1) The experimental situation was still not clear in that significant size dependent effects were seen even with relatively large cells;

2) The asymptotic region was not well explored; and

3) The theoretical model had too many adjustable parameters to allow definitive testing.

The experimental program we undertook was designed to attack the first two issues by making use of our high resolution thermometry and ultra-stable thermal control capabilities. In the theoretical arena we note that dynamic RG calculations are currently at the cutting edge of theoretical developments in cooperative phenomena. Hopefully our results will stimulate new initiatives in this area.

Other experimental tests of the dynamic RG theory have been performed, including second sound damping below $T_\lambda$ and light scattering measurements throughout the transition region, and measurements on other systems. However, none of these tests have been able to approach the accuracy or resolution of the thermal conductivity measurements on helium.
APPARATUS

The initial portion of the grant period was spent building some new equipment to make the thermal conductivity measurements and adapting other existing equipment to the new requirements. This work is summarized briefly below.

**Thermal conductivity cell:** A thermal conductivity cell with the capability of making measurements in a four-terminal configuration was constructed early in the grant period. The design of this device is shown in Figure 1 and closely follows that in our original proposal. However, we found significant difficulty in constructing fin assembly for the thermometer contacts that met our requirements for minimal perturbation of the temperature gradient in the helium. Since all previous measurements used a two-terminal approach, we also fabricated components to operate in this mode. A schematic view of this assembly is shown in Figure 2. It has greater flexibility than the four-terminal design in that the cell height can easily be changed, allowing end corrections to be made by comparison of data from two low temperature runs. Also it can be operated with much smaller gaps, minimizing gravitational effects. We believe this flexibility largely offsets any loss caused by deviations from the true four-terminal approach. In the first experiment the vertical height of the sample space was about 2 mm and the cross-sectional area was about 1 cm². The top of the sample space was mechanically and thermally connected to a second chamber and a valve assembly and the whole unit was mounted inside a three stage thermal control system. Most of our measurements were made servo-controlling the
THERMAL CONDUCTIVITY CELL

Figure 1
THERMAL CONDUCTIVITY APPARATUS

Figure 2
temperature of the top of the sample space, applying varying amounts of heat to the bottom, and observing the resulting temperature differences across the sample.

**High resolution thermometers:** A second high resolution thermometer (HRT), closely matched to an existing device, was constructed. This allowed us to make the thermal conductivity measurements with the maximum sensitivity and lowest power inputs currently possible. Under a separately funded contract we evaluated copper ammonium bromide as a candidate sensing element for the thermometers and obtained a significant improvement over the salt previously used. We have also gained significant experience in the area of thermometer calibration. By using cerium magnesium nitrate as the sensitive element in one thermometer, we were able to linearize the temperature scale of second HRT to about 0.01% over 20 millideg. or so. While the absolute scale is unknown to this accuracy, the linearization ensures that systematic effects due to temperature scale *curvature* can be eliminated to the accuracy of the data. This is very important when attempting to extract parameters of theoretical interest by curve fitting to measurements. The total range of temperature to be investigated in the main experiment was about 8 millideg., from the lambda point to the density maximum of the helium. Over this range we were able to put useful limits on any departures from linearity in the thermal conductivity data.
**Dual digital flux counter:** A two-channel digital flux counting system was constructed to allow simultaneous measurements of two thermometer outputs and easy logging of the results by our data acquisition system. This system was based on our pre-existing single channel counter but incorporates opto-couplers to reduce pick-up noise and sample and hold circuits to allow very good timing of the data collection sequence.

**Thermal control system:** The thermal control system for the experiment was developed under separate funding and was shown to be fully capable of performing the thermal conductivity experiment. The system exhibited very good reliability and performed very effectively. We made small changes to reduce the susceptibility of the system to stray heat inputs from r.f. pick-up and to accommodate the thermal conductivity cell.

**RESULTS:**

In the second year of the program we obtained an extensive set of data on the thermal conductivity of helium very close to the lambda point using a 2 mm high sample. For the first time the gravitationally affected region was well resolved in small samples, giving us the highest resolution data that is feasible on earth. Preliminary analysis was performed on the data and a divergence of $K$ similar to that found further from the transition was observed up to the gravity limit. This work is summarized in the following subsection.
During the course of the measurements a new effect was observed in the superfluid phase which was traced to the presence of $^3$He in the sample. These results are described briefly below and in an attached preprint. The effect has been used to measure the effective thermal conductivity of a $^3$He-$^4$He mixture in the superfluid state at very low concentrations of $^3$He.

In the final part of the funding period additional thermal conductivity measurements were made in smaller cells. Gaps of 1, .07 and .04 mm were investigated. For the data very close to $T_\lambda$ some dependence on sample size was observed. Our analysis indicates that these effects may be due to other phenomena in the apparatus associated with the rapidly changing properties of the helium near the transition. More detailed work in this area is now being continued under separate funding.

**Thermal conductivity measurements - I:** In this section we describe some of the early experimental results obtained with a 2 mm high helium sample. Initially we measured the temperature dependence of the thermal conductivity of normal well grade* $^4$He in the region close to the $\lambda$-point. This experiment was conducted at moderately high resolution, to $t \sim 10^7$, and was intended as a pilot investigation aimed primarily at developing our technique. Some of the results are shown in Figures 3-5. Figure 3 shows the temperature differences across the sample for two power levels, less than 5 nano-watts, as a function of temperature through the $\lambda$-point. Each data point was obtained by moving the set point of the upper

* about 0.2 - 0.3 ppm $^3$He concentration, verified by Bureau of Mines analysis.
THERMAL CONDUCTIVITY MEASUREMENTS NEAR THE LAMBDA POINT

**Figure 3**
TEMPERATURE DIFFERENCE, 1 DIVISION = 0.12 MICRO-DEG.
High resolution thermal conductivity measurements near the lambda point

Figure 5
plate to a new temperature, waiting for equilibrium, and then measuring the temperature difference. The small vertical bars at each temperature represent all the data collected for a given temperature difference measurement with no averaging. The curves clearly show the expected form of a divergent thermal conductivity as the transition is approached. Figure 4 shows similar data but at higher resolution. The baseline for the difference measurements was established by comparison of the thermometer calibrations below the transition under conditions of zero applied power.

For Figure 5, data was collected in a different mode where the upper plate was allowed to cool slowly, giving a continuous record of the temperature difference. The data in the figure clearly shows the distortion of the thermal conductivity curve due to the presence of the two-phase region very close to the transition. With the exceptionally small power levels used here it is easy to arrange for the width of the two phase region to be controlled primarily by the gravitational height of the sample and not by the heat flux, as was the case in all previous reported work in this field. This shows that we are able to obtain high accuracy thermal conductivity data with the highest resolution that is possible on earth.

From the temperature difference data shown in the previous figures we can compute the thermal conductivity of the helium allowing for the parallel resistance of the wall of the container. The resulting data is shown in Figure 6, where we compare it with previous work. Our data is slightly higher than that of Tam and Ahlers, possibly due to the uncertainty in the measurement of the sample gap. Beyond the region of overlap we can see that the
THERMAL CONDUCTIVITY OF HELIUM
JUST ABOVE THE LAMBDA POINT

Figure 6
divergent form continues to a resolution of about $10^{-7}$ deg. Below this value gravitational effects are important and analysis is still in progress. The data shown in Figure 6 represents only a small amount of that collected.

The initial experiment described above went so well that it was continued until the maximum resolution, $t-10^{-8}$, was achieved. However, due to some effects associated with the residual $^3\text{He}$ in the sample that are described in the next section, it was considered prudent to obtain data with a higher purity sample. $^4\text{He}$ with an impurity level of no more than 1 part in $10^9$ was obtained and the data collection sequence was repeated. Except for some small effects in the two-phase region very close to the transition, no significant difference between the two samples was seen. From a theoretical point of view, no differences are expected at the resolution currently attainable, but extremely close to the transition some differences might be observed. The final results of this experiment are shown in Figure 7. Details of the analysis are given in an attached reprint.

**Measurements in the superfluid phase:** When the sample container becomes completely full of superfluid we expect the temperature differences across it to go to an extremely low value controlled primarily by the parasitic heat leak into the bottom of the cell and the Kapitza boundary resistance. Since the boundary resistance is in series with the helium it is a correction that must be applied to our thermal conductivity data. In all previous experiments it has been taken to be a constant, independent of temperature in the transition region. We have tested this
Temperature dependence of the thermal conductivity parameter $R_k$. 

- •: OUR RESULTS;  •: DATA OF TAM AND AHLERS 
- --: RG THEORY;  --- - : RANGE OF UNCERTAINTIES

Figure 7
assumption by monitoring the temperature drop across our sample at a very high power levels in the superfluid region very close to the transition. Within 1 microdegree of the transition we find an enhanced boundary resistance. Some of the data showing this is presented in Figure 8. In the upper portion we see the temperature offsets across the sample as a function of temperature for three power levels. Anomalous behavior is clearly visible at powers of the order 20 nanowatts or higher. The lower part of the figure shows similar data at one power level using a sample in which the $^3$He concentration is much less than in normal well-grade helium. It is clear from this data that the anomaly is not due to the $^3$He in the sample. The Kapitza anomaly we see is large enough to cause some distortion of our data below a resolution of about $2 \times 10^{-7}$. In a later part of this work we observed anomalous behavior further below the lambda-transition. At first this was thought to be due to thermometer problems, but additional observations convinced us that the effect was real. Figure 9 shows data similar to Figure 3 but starting well below the transition and extending to slightly above it. A marked nonlinearity in the baseline occurs about 10 microdeg below the transition indicating the presence of an anomalous temperature difference. This effect was found to occur only if the system was prepared by an appropriate thermal cycle, and decayed away with a time constant of about 4 days. At other times the baseline was flat as indicated in Figure 3. Our interpretation is that some of the residual $^3$He in the sample is flushed into the region of the valve during the thermal cycle and gradually leaks back into the main experimental chamber. Concentration gradient relaxation
KAPITZA RESISTANCE MEASUREMENTS VERY NEAR THE LAMBDA POINT

Figure 8
Observations of an anomalous temperature difference set up in the thermal conductivity cell well below the lambda transition in zero applied power (a small parasitic power is present along with the $^3$He current described in the text).

Figure 9
times\textsuperscript{10} are in reasonable agreement with this idea if allowance is made for the geometry of our system. The returning \textsuperscript{3}He mass flux, even though extremely small, sets up the inverse of the heat flush effect in an applied temperature gradient. Thus the small concentration gradient due to the incoming \textsuperscript{3}He forces the system to establish a small temperature differential with the bottom plate relatively warm. This effect would be expected at any temperature where there is sufficient normal fluid to be dragged by the \textsuperscript{3}He mass flux. It is not the effect which gives rise to dilution refrigeration, although there is some loose relationship between the two. Also it is not due to a parasitic heat leak since it is dependent on the thermal cycle and on temperature as shown in the figure. Very close to the lambda point the effect gets turned off, presumably by some critical velocity effect associated with the rapidly deceasing superfluid density.

An analysis of the effect in terms of Khalatnikov's hydrodynamics\textsuperscript{11} shows that it is possible to use it to derive values for the effective thermal conductivity of the \textsuperscript{3}He-\textsuperscript{4}He mixture under conditions of no heat flow. A reprint describing the results so obtained is attached.

\textbf{Thermal conductivity measurements - II:} After the results from the 2mm high sample had been fully analyzed, we turned our attention to the sample size dependence problem reported by others\textsuperscript{7,9}. To get more information on this problem we decided to progressively reduce the height of our sample and compare sets of apparent thermal conductivity data. Initially we made
measurements on a 1mm high sample. When the data was suitably normalized far from $T_{\lambda}$ we were surprised to find that there was virtually no change in the results right up to the maximum resolution of the 2mm data set. These results were the subject of a paper presented at the LT-18 Conference. A copy of this publication is attached and the results are shown in Figure 10 for reference.

Since the sample size effect was less than expected, we then decided to examine very small gaps. Data was collected using spacings of 0.07 and 0.04mm. Far from $T_{\lambda}$ the data agreed well with previous results, but for $t < 10^{-6}$ significant departures were seen. Also the two new data sets did not agree with each other in this temperature range. Since the gaps were so small these effects were unlikely to be due to gravitational corrections. In additional low temperature runs various experimental parameters were changed to test for measurement errors. Some unexpected effects were seen which led us to doubt the validity of some of the high resolution data from the narrow gap runs. It appears that changes in the thermal conductivity of the helium in other parts of the apparatus leads to perturbations of the thermal conductivity results. Analysis is still in progress to determine the range of reliable data from these runs. Also under separate funding we are rebuilding the apparatus to reduce this type of problem. It appears that the difficulties only appear in the small cells because of the small signals involved. For the 1 and 2mm cells the effects represent only minor corrections do not affect the validity of our earlier conclusions.
Temperature dependence of the thermal conductivity parameter $R_K$:

- $\diamondsuit$: 1mm CELL;
- $\bullet$: 2mm CELL;
- $\circ$: TAM AND AHLERS

**Figure 10**
REFERENCES: