Eddies, Streams, and Convergence Zones in Turbulent Flows

By J.C.R. Hunt¹, A.A. Wray², and P. Moin³

1. Introduction

Recent studies of turbulent shear flows have shown that many of their important kinematical and dynamical properties can be more clearly understood by describing the flows in terms of individual ‘events’ or streamline patterns. Examples of such events might be high values of filtered vorticity (Hussain 1986) or high velocity, perhaps in combination with straining (Adrian & Moin 1988), or high Reynolds stress (Blackwelder & Kaplan 1976).

These events or flow regions are studied because they are associated with relatively large contributions to certain average properties of the flow, for example kinetic energy, Reynolds stress, or to particular processes in the flow, such as mixing and chemical reactions, which may be concentrated at locations where streamlines converge (Leonard & Hill 1988) for fast chemical reactions (which we shall refer to as convergence or C regions), or in recirculating eddying regions for slow chemical reactions (Broadwell & Breidenthal 1982); figure 1a). These vortical regions (which we refer to as eddy or E regions) are also of importance in certain flame and combustion problems (Peters 1988) where turbulent flows transport bubbles or particles. Recent experimental and computational research (e.g. Hunt et al. 1988; Maxey 1987; Chung & Troutt 1988; Fung & Perkins 1989) has shown that bubbles (or other low-density fluid such as reactants) tend to concentrate in low-pressure regions, while denser particles, especially if buoyancy forces are important, tend to concentrate in the streaming or S regions between eddies (figure 1b). But those particles that are entrained into eddying regions can remain for long periods; in fact, the control of eddying regions forms the basis of a recently-patented metallurgical purification process.

Many authors have recognised that models for reactions and particles and bubbles require some assumption about the kinematics of the flow, for example, as a distribution of vortices of given life time (Brown & Hutchinson 1979; Picart et al 1986) or of vortices separated by stagnation points (Broadwell & Breidenthal 1982). Another approach following Kraichnan (1970) is to model the kinematics of the turbulent flow field by random Fourier space and time modes with the appropriate amplitudes and relations between space and time behavior (Fung et al. 1989). This ‘kinematic simulation’ is a ‘cheap’ enough way of representing different

¹ University of Cambridge
² NASA Ames Research Center
³ NASA Ames Research Center and Stanford University
kinds of turbulent flow field that it is then more feasible to compute reaction and combustion processes or particle and bubble motions.

However at present there is no generally agreed way of assessing the accuracy or appropriateness of a representation of turbulent flow fields. How much of the space should be occupied by eddy, convergence, or streaming zones? How strong should they be? And so on. Answering such questions is one of the main objectives of this research project.

Usually the diffusion and mixing problems for non-reacting and non-dynamical particles are discussed qualitatively in terms of how individual fluid particles move, and are analyzed quantitatively in terms of the statistics of the velocity and displacement of fluid particles (e.g. Monin & Yaglom (1971), vol. 1). The key parameters are the rms velocity fluctuations $u_0$ and the Lagrangian integral time scale $T_L$. The value of $u_0$ is the same (or similar) at a fixed measuring point and for a moving particle (Lumley 1961, a point that has been verified by the kinematic simulations). But $T_L$ cannot be defined in terms of simple Eulerian statistics because it depends on how a particle moves through the flow field. Therefore it must depend, given the length and velocity scales ($L, u_0$), on the nature of the flow pattern and therefore on certain aspects of $E$, $C$, and $S$ zones. It is also interesting to compare $T_L$ with the time $T_E$ over which the velocity at a point changes for an observer moving with the mean flow.

As an example, in figure 2 (a) the flow largely consists of streaming zones which of course change with time. Then the particle trajectories are rather straight, but the length scale over which the velocity is decorrelated is smaller, so in this case $T_L \gg L/u_0$ and $T_L > T_E$.

On the other hand, if the flow consists largely of eddy zones ($E$) the velocity of a particle would keep changing on a time scale $L/u_0$. So $T_L \sim L/u_0$ (figure 2 (b)). The comparison between $T_L$ and $T_E$ depends on how slowly the eddies move. If $\langle u \rangle_E$ is the bulk velocity of the eddies, then $T_E \sim L/\langle u \rangle_E$ and $T_L > T_E$ if $\langle u \rangle_E > u_0$.

This information about $T_L$ is useful for computing mean concentrations, but it does not tell us about the overall shape of a dye cloud (such as that depicted by Corrsin, quoted by Monin & Yaglom (1971)). This typical shape has been described as tendrils and 'whirls' by Berry (1978). It can be explained by rapid motion along the streaming ($S$) regions forming the 'tendrils' and then slow motion into the eddy ($E$) regions forming the 'whirls' of the dye clouds.

Also, other important statistics of clouds such as mean-square displacement $|x|^2$ or width of the dye cloud $L_e^2$ and the mean shape of a contaminant cloud are not defined uniquely by $u_0$ and $T_L$; rather they depend sensitively on the kinematics of the flow field.

The identification of certain significant regions in a flow can also provide an important method for analyzing the dynamics of the flow. Recent computations for two-dimensional flows have established that significant vortical eddy regions interact with each other as if they are equivalent point vortices with the same circulation (McWilliams 1984). This then makes the computation of development of the whole flow much simpler, because the analysis of a set of point vortices is much simpler.
computationally and conceptually than that for the whole flow field.

In other words, the whole dynamical evolution of the flow has in this case been reduced to a low-order system. It is not yet by any means clear whether a comparable simplification would be possible for a global analysis of a three-dimensional flow. But if the significant vortical regions and their detailed flow structure can be evaluated, certain local analyses may become possible, for example, the interaction of the eddy with its surrounding flow or with other eddies (Moin, Leonard, Kim 1986; Hunt 1987), or in high Reynolds numbers flows the interaction between the eddying and small-scale turbulence within the eddy (e.g. Kida & Hunt 1988) (or the interactions between 'coherent' and 'incoherent' motion (Hussain 1986)).

In a three-dimensional turbulent flow there are large coherent eddies or vortices, but because vorticity diffuses out of these regions or because vorticity is torn off the eddies when they interact, there has to be much smaller-scale chaotic vorticity in the flow between the large vortices. This small-scale chaotic vorticity can be significantly amplified and dissipation increased in convergence zones outside the eddy regions. The nature of this amplification changes depending on whether the convergence is flattening or elongating a material sphere. So for the dynamical analysis it is also important to define these convergence zones and to quantify the magnitude of the straining, defined by \( E_{ij} E_{ji} \) where \( E_{ij} \) is the symmetric stress tensor \( \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i) \), and its nature, by \( III = E_{ij} E_{jk} E_{ki} : III > 0 \) for elongating and \( III < 0 \) for flattening.

This approach to the dynamical analysis, conditionally sampling the flow and then considering the dynamical equations governing these regions, differs from analyzing the whole flow into eigenmodes (Fourier modes for homogeneous flows or Karhuen-Loeve modes for inhomogeneous flows) and then computing the interactions between the modes. The latter classical approach (described at length by Batchelor (1953) and under investigation at CTR by Rogallo & Domaradzki (1988)) is straightforward to understand for a few interacting wave modes which pervade a whole flow. But it is impossible conceptually when the significant dynamics is localized. Imagine trying to analyze the interaction of point vortices using Fourier methods!

Many other investigators have been and are currently engaged in identifying strong eddying or vortical regions using various criteria for these regions, such as low pressure; strong rotational motions defined by the local deformation tensor \( \partial u_i/\partial x_j \), where \( u_i(x,t) \) is the velocity field (Herring 1988; Perry & Chong 1987); or regions where the vorticity of the filtered velocity field is large (Hussain 1986).

The aim of this project is to use the numerical simulations at CTR to develop suitable criteria for defining these eddying or vortical zones. But in this study we are also interested in defining the convergence (C) and streaming (S) zones, in order to define the whole flow field, for all the reasons given here.

2. Defining S, E, C zones in velocity fields

2.1 Qualitative and mathematical criteria

(i) Eddy zones (E)
These zones are approximately defined as strong swirling zones with vorticity. 
Irrotational swirling motion outside the zones is excluded. Also excluded are nearly 
straight shear layers on the edges of streaming zones. 

In general in a turbulent flow the swirling rotational zones are also being stretched 
by larger-scale irrotational straining, which helps maintain their vorticity (e.g. in 
mixing layers the cores of spanwise ‘rollers’ are strained by the longitudinal ‘braid’ 
vortices).

We apply two criteria to define E zones, (a) and (b) below.
(a) the irrotational straining is small compared with the vorticity, i.e. the second 
invariant of the deformation tensor is less than a negative threshold value \( -II_E \): 

\[
II < -II_E
\]  

(2.1a) 

where 

\[
II = \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} = E_{ij}^2 - \frac{1}{2} \omega_i^2 
\]  

(2.1b) 

and \( E_{ij} \) is the symmetric strain tensor \( \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i) \) and \( \omega_i \) is the vorticity 

\[
e_{ijk}\partial u_k/\partial x_j
\]

(b) If the pressure tends to a minimum somewhere in the zone, there is a pressure 
gradient across the streamline, i.e. \( \frac{1}{R} \partial p/\partial r \approx +\omega_i^2/R \) where \( R \) is the radius of 
curvature. A criterion that is independent of the pressure field outside the eddy 
zone would be 

\[
p_{\text{edge}} - p_{\text{int}} > p_E, \quad \text{or} \quad p_{\text{int}} < p_{\text{edge}} - p_E
\]  

(2.2a) 

where \( p_{\text{int}} \) is in the interior of the E region, \( p_{\text{edge}} \) is on the edge, and \( p_E \) is a threshold 
value. As a working approximation, we first try 

\[
p_{\text{int}} < -p_E
\]  

(2.2b) 

For most vortices, the criterion (a) is equivalent to the criterion suggested by 
Perry & Chong (1987) that eddies are where the eigenvalues of the deformation 
tensor \( \partial u_i/\partial x_j \) are complex. It would not be exactly the same as (2.1a) in a case 
where a vortex was being strained (figure 3) parallel to itself. But if the straining 
is weak compared with the rotation, Perry’s criterion is equivalent. 

The reason why the pressure criterion (b) needs to be added to the vorticity 
criterion of (a) is that it ensures that, if the flow is rotational, then the streamlines 
are curved. In a shear flow with straight streamlines \( II = 0 \). But because \( II \) and 
\( p \) are non-linear quantities different kinds of flow fields, when added together, can 
satisfy the criteria. Consider a diffuse double vortex sheet or jet of scale \( \ell \): 

\[
u_3 = U_0 e^{-(x_1^2 + x_2^2)/\ell^2}
\]  

(2.3a) 

embedded in a large, weak-motion vortex.
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\[ u_1 = \Omega x_3, \quad u_3 = -\Omega (x_1 - R) \]  
(2.3b)

For the flow (2.3a), \( II = 0 \), \( \nabla p = 0 \), but the addition of (2.3a) and (2.3b) leads to

\[ II \approx -\frac{4x_1 U_0 \Omega}{\ell^2} e^{-(x_1^2 + x_3^2)/\ell^2} - 2\Omega^2 \]  
(2.3c)

while

\[ \frac{\partial p}{\partial x_1} = \Omega U_0 e^{-(x_1^2 + x_3^2)/\ell^2} - \Omega^2 (R - x_1) \]  
(2.3d)

Note how, if \( U_0/\ell \gg \Omega \), the addition of a straight shear layer increases the value of \(|II|\) and \(|\nabla p|\).

Also, note how \( II \) rapidly changes sign across the jet. Depending on the background value of pressure, (2.2b) might or might not be satisfied. However, (2.2a) would not be satisfied, if \( |\Omega U_0\ell| < p_E \). Clearly this is not an eddy in the sense we know.

So it suggests that an eddy zone on a scale \( \ell_e \) should be defined by averaging \( II \) over \( \ell_e \); in our computations \( II \) is in fact computed from the unfiltered velocity field. Small eddies could be defined by considering frames of reference moving mainly with the large eddies; then fluctuations can appear as local 'cat's eyes'. We are not attempting to describe such rather artificial constructs.

(ii) Convergence zones (C)

These zones are approximately defined as regions where there is irrotational straining motion and where there is strong convergence and divergence of streamlines. There will be a stagnation point in such a region (defined in a suitable frame of reference).

The criteria for the C zones must be such as to avoid the irrotational swirling flow around vortices. Therefore we adopt two conditions: (a) the irrotational straining is large compared with the vorticity, so for some threshold value \( II_C \),

\[ II > II_C \]  
(2.4)

and (b) the pressure rises in the interior of the C zone so

\[ p_{\text{int}} - p_{\text{edge}} > p_C \]  
(2.5a)

Or, more simply but less generally,

\[ p_{\text{int}} > p_C \]  
(2.5b)

The pressure criterion avoids the possibility of irrotational swirl and also the possibility of a jet or rotating flow. (In general, the large-scale C zones are most appropriately defined by averaging the criterion (2.4) over a scale \( \ell_c \) (say, \( 1/4L \)).

(iii) Streaming zones
In these zones the flow is relatively fast, not very curved, and not diverging or converging strongly. These zones are the main ‘highways’ for fluid or marked particles to be transported across the flow. The suggested criteria are

\[
\begin{align*}
(a) & \quad u_i^2 > u_0^2 \quad \text{for speed, and} \\
(b) & \quad |II| < II_S \quad \text{for weak curvature and convergence}
\end{align*}
\]

(2.6)

(2.7)

It is important to use the unfiltered velocity field because the vorticity and the value of $|II|$ may be high on the sides of these zones and filtering can ‘smear’ these values of $II$ over the $S$ zone, causing the criterion (2.7) to be not satisfied.

The criteria given above are such that not every point in the flow field is in an $S$, $E$, or $C$. But the conditions on $II$ are made such that no point can belong to more than one type of zone, by choosing $II_S = \min(II_E, II_C)$.

2.2 Critical computations to be performed

(i) Diagnostic tests

The algorithms for $S$, $E$, $C$ zones should be used to define these zones for a number of flows where direct simulations have been performed. A number of diagnostic tests should first be run to evaluate the significance and usefulness of the algorithms:

(a) Do the computed $S$, $E$, $C$ zones correspond to the expected streamline patterns for these zones?

(b) What is the improvement in using the double ($II$, pressure) criteria over other criteria?

(c) How sensitive are the definitions of the zones to the magnitudes of $II_E$, $II_C$, $p_E$, $p_C$, $u_0$?

(d) Are the same criteria applicable to different types of turbulent flows?

(ii) Comparison with other investigations

(a) It would be desirable to compare our criteria with those suggested by Perry & Chong (1987).

(b) How do the ‘eddy’ zones defined by our criteria compare with the space occupied by ‘coherent structures’ as defined by other investigators, e.g. by Adrian & Moin (1988), for the channel flow (using conditional sampling to define the structure), and by Hussain, Jeong & Kim (1987) for homogeneous flow (using ‘eduction’ techniques), and Moser & Moin (1988) for the channel flow (using orthogonal eigen-mode expansion).

(iii) To define kinematics

(a) The geometrical and topographic properties of the zones need to be defined, for example the relative volumes occupied by the $S$, $E$, $C$ zones $V^{(S,E,C)}$, their typical extent $L^{(S,E,C)}$, and their spacing $D^{(S,E,C)}$.

(b) The movement of the eddy and convergence zones, or mobility (to help understand the Lagrangian time scale $T_L$), defined as the average velocity across such
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a zone \( \mu^{(E,C)} = |\langle v \rangle| \), averaged across an E or C zone. The normalized value of \( \mu^{(E,C)} = \mu^{(E,C)}/|II| \) would be of interest.

(iv) To test and develop fluid mechanical concepts

(a) Processes of reaction and vorticity dynamics depend on the nature of irrotational straining in the C and E zones. So it is important to compute the third invariant of the rate of strain tensor \( E_{ij} \):

\[
III = E_{ij}E_{jk}E_{ki}
\]

(2.9)

Peak and average values of \( III^{(C,E)} \) within C and E zones need to be known. A correlation between \( III^{(E)} \) and \( II^{(E)} \) in the eddy zones would indicate how localized the formation of vorticity really is.

(b) As in previous ‘coherent-structure’ analysis, it might be interesting to compute the relative contribution to various global statistics by the S,E,C zones, e.g. \( \omega^2 \) by E zones, \( \bar{u}_t^2 \) by S zones, \( \bar{E}_{ij}^2 \) by C zones. The typical helicity of the E zones should be computed by \( |\langle u \cdot \omega \rangle| \) averaged over the E zone.

(c) For Lagrangian statistics, it would be interesting to know how long an average particle spent in the different zones.

3. Results and discussion

(i) Simulations for testing the concept of flow zones.

The first simulation used was homogeneous stationary turbulence driven by a stationary random force field. There are mean field and fluctuating components \( \langle u(x), u(x,t) \rangle \) of the velocity field. The emphasis of our study was to examine the fluctuating component \( u(x,t) \), which was of course strongly affected by the mean component (see Hunt, Wray & Buell 1987). The characteristic Reynolds number for \( u \) is about 20. No strong coherent structures were observed in the previous study.

The second simulation was the turbulent channel flow computed by Moin & Moser (1987), with a characteristic Reynolds number of the turbulence of about 100. In this case, strong streamwise vortices are formed, but a number of weaker ‘structures’ or eddies also exist in the flow which need identifying.

(ii) Checking and ‘tuning’ the algorithm

The algorithm for the E,C,S zones defined by (2.1a) and (2.2b), (2.4) and (2.5b), (2.6), and (2.7) were used to compute the areas occupied by these zones in the two flows. The magnitudes taken for the criteria were: (a) \( II_C = \langle \bar{II}^2 \rangle^{1/2} \), i.e. the rms value of \( II \) over the whole flow, with \( II_E = 2II_C \), (b) \( p_C = \langle \bar{p}^2 \rangle^{1/2} \), the r.m.s value of \( p \) over the flow, with \( p_E = 0.2p_C \), and (c) \( u_0 = (|u|^2)^{1/2} \), the rms value of the speed of the fluctuating component.

These values were computed, and the criteria were tested by comparing the indicated zones with the patterns formed by computed velocity vectors plotted onto the zones, for many realizations of the flow. The vectors were in one plane, e.g. \( (u_1,u_2) \), but the criteria were defined for the full three-dimensional flow field. As
shown in figure 4, it is clear that where \((u_1, u_2)\) vectors circled round a region, the zone identification using the algorithms above indicated on eddy, E, region; where there was a local stagnation point, the zone computations indicated a convergence, C, region. Where vectors indicated high speed regions with low curvature, the zone computations indicated streaming, S, regions.

In the initial stages, a spatially filtered velocity field was used to calculate the quantities in the different zone algorithms. However, we found that in the indicated E,C,S zone areas, the flow structure did not correspond to swirling, converging, or streaming motion. The explanation was probably that the spatial filtering smeared the vorticity over the zones, so that their definition was effectively changed. Consequently spatial filtering was abandoned. Some examples of bad disagreement between the indicated zones and the actual patterns are shown in figure 5.

The same algorithm and the same parameters developed for homogeneous turbulence were then applied to channel flow. The agreement between expected flow structure in the different flow zones and the actual flow structure was even more satisfactory than for homogeneous turbulence.

The general features found for the flow zones were:

(i) Eddy zones

These zones were isolated and distributed uniformly over the flow. Across any section of these zones the ‘swirl’ velocity is in one direction, and the vorticity, on the scale of the zone, is in one direction. The swirl velocity of the vorticity, on the scale of the zone, is maximum around the circumference of the zones. The typical diameter of these zones using our criteria is about \(L/4\) where the spacing between them is about \(L\). Not all eddy zones were circular in cross section, a few were quite elongated in the plane being examined, but in that case the vorticity was approximately parallel to the elongation. By examining the eddy zones intersecting a plane at different times, it was clear that some eddy zones move around (usually the weaker vortices) while others move very little (usually the stronger zones). This random movement of vortical regions under the action of larger scales had been studied by Hunt, Wray \& Buell (1987).

(ii) Convergence zones

The C zones were also isolated, generally round or square shaped regions. Across a section of any zone there appeared to be a single large-scale straining motion, usually with a stagnation point in the zone somewhere (i.e. the principal axes of the strain did not change direction significantly). Their length scale and distribution was similar to that of E zones.

(iii) Streaming zones

The S zones are elongated and sometimes filamentary regions leading into C zones. The curvature is small. They do not generally lead into E zones. Their longest extend is about \(L\) to \(1.5L\), while their width is typically \(0.5L\). The maximum velocity in these regions (which must be greater than \(u_0\)), is generally less the
maximum velocity on the edge of the E zones.

(iv) Straining processes in the flow zones.

The third invariant $III$ of the deformation tensor $\partial u_i/\partial x_j$, defined by (2.9), was computed over the flow. Regions where $III$ was positive and negative are indicated by different contours, on a background of the indicated flow zones in figure 6.

It was found that, in the convergence C zones, more than $3/4$ of the area has $III < 0$, while in the eddy zones more than about $3/4$ of the area has $III > 0$. Also, the maximum positive and negative values of $III$ were located in these zones.

4. Discussion and conclusions

(i) The main conclusion is that homogeneous and sheared turbulent flow fields are made up of characteristic flow zones — eddy, convergence, and streaming zones. A set of objective criteria have been found to describe regions in which the streamlines circulate, converge or diverge, and form streams of high velocity flow. Previous investigations to identify characteristic regions or coherent structures seem to have been less successful in selecting criteria that define the zones in a way that accords with the qualitative kinematical definition of the regions.

(ii) The most interesting question arising from this study is: What is the characteristic three-dimensional eddy structure that is consistent with these flow zones, and with the values of the third invariant $III$? From the fact that the eddy zones are approximately vortex tubes, it is likely that the main structure is one of vortex lines concentrating in smaller regions into vortex tubes. These vortices induce motion in each other and can produce strong stream zones between them, and C zones where they meet other S zones.

In turbulent flow, especially at higher Reynolds number, smaller eddy structures can appear, especially at the edges of the E zones and in the C zones. The vorticity of these eddies is strained by the flow around the E zones and by the converging/diverging streams in the C zones. This leads (see figure 7) to elongating straining ($III > 0$) around the edges of the E zones and to flattening straining ($III < 0$) within the C zones.

(iii) As Leonard & Hill (1987,8) have pointed out, flame or reaction fronts are likely to occur in the C zones and that the fronts must tend to lie parallel to the plane normal to the principal direction of compressive strain of $\partial u_i/\partial x_j$. (Since $III < 0$, the other two directions are elongational.)

(iv) It would of course be interesting to compute different statistics for the C zones to provide a good basis for an objective description of each of the key flow structures.

REFERENCES


LUMLEY, J.L. 1962, Mechanique de la Turbulence, Paris ed. CNRS.
**Figure 1A.** Showing how reacting species (A,B) tend to react in C zones for fast or E zones for slow reactions.

**Figure 1B.** Showing how particles tend to concentrate in the streaming S zones.
Typical trajectories: (a) for particles mainly in S zones, $T_E << T_L$; (b) for particles mainly in E zones so that $T_E \sim T_L$.

Choice of criteria: in (a) there is straining and vorticity, and in (b) pure vortical motion; if the straining is strong enough, case (a) is not counted as an eddy!
Figure 4. S,E,C zones with the full velocity field used in the criteria. Cyan: Eddies ($p < -p_E; II < -II_E$); Yellow: Streams ($u^2 > u_{rms}^2; |II| < II_S$); Red: Convergence zones ($p > p_C; II > II_C$); White: Unclassified zones.
FIGURE 5. S,E,C zones with the filtered velocity field used in the criteria. Cyan: Eddies ($p < -p_E$; $|II| < -II_E$); Yellow: Streams ($u^2 > u^2_{rms}$; $|II| < II_S$); Red: Convergence zones ($p > p_C$; $II > II_C$); White: Unclassified zones.
FIGURE 6.  S,E,C zones shown with contours of III. Solid contours are positive and dashed negative. Cyan: Eddies ($p < -p_E$; $II < -II_E$); Yellow: Streams ($u^2 > u_{rms}^2$; $|II| < II_S$); Red: Convergence zones ($p > p_C$; $II > II_C$); White: Unclassified zones.
Figure 7. An interpretation of the characteristic flow zones in homogeneous turbulence in terms of vortex tubes at an angle to each other. Note the narrow streaming region between them. In the C region small vortices normal to the large-scale flow are stretched (and 'piled up'), and in the S region these are compressed (and 'spaced out'). These effects lead to $III > 0$ near E and $III < 0$ in C.