INTRODUCTION

MULTI-BODY DYNAMICS PROGRAMS REQUIRE CHARACTERIZATION OF EACH BODY

- **RIGID BODY**: GEOMETRY AND MASS PROPERTIES

- **FLEXIBLE BODY**
  - EXACT TYPE OF INPUT DEPENDS ON PROGRAM
  - ALL INVOLVE MODAL CHARACTERISTICS IN SOME FORM
  - ALWAYS NEED FOR MODAL TRUNCATION
  - SYSTEMATIZE TRUNCATION PROCEDURE

**GALILEO SPACECRAFT**

- **ACTUATORS**: SBA, SAS, THRUSTERS
- **SENSORS**: GYROS, CLOCK AND CONE ENCODERS, SUN SENSOR, STAR SCANNER
- **CLOCK (SBA) CONTROL LOOP IS ACTIVE DURING ALL ATTITUDE CONTROL MANEUVERS**
  - CLOCK CONTROLLER BANDWIDTH ≈ 0.5 Hz
  - GYRO ROLLOFF FREQUENCY ≈ 15 Hz
- **NEED "ADEQUATE" MODEL OF PLANT FOR DESIGN AND SIMULATION**
TRUNCATION CRITERIA

- CONTROL SYSTEM SPECIFICATIONS CAN SET TRUNCATION CRITERIA AT SYSTEM LEVEL ONLY

- SYSTEM MODE WITH FREQUENCY ABOVE 15Hz CAN BE DROPPED

- ELIMINATE MODES THAT DO NOT INTERACT "STRONGLY" WITH THE CONTROL SYSTEM

SYSTEM LEVEL TRUNCATION

METHOD

\[ M\ddot{x} + Kx = F \quad (1) \]

\[ x = \phi z \quad (2) \]

\[ X(s) = \left( s\phi^2 + \omega_i^2 \right)^{-1} \phi^T F(s) \quad (3) \]

FOR RESPONSE AT i LOCATION DUE TO STEP INPUT AT j LOCATION,

\[ X_i(s) = D_{ij} F_j(s) = \sum_{k=1}^{m} \left\{ \phi_{ik} \phi_{jk} A_j \left[ s\left( s^2 + \omega_k^2 \right) \right] \right\} \quad (4) \]
SYSTEM LEVEL TRUNCATION (CONT'D)

CONTRIBUTION OF kth MODE TO RESPONSE:

\[ X^k_i(s) = \phi_{ik} \phi_{jk} A/[s(s^2 + \omega_k^2)] \]  

(5)

OR

\[ X x^k_i(t) = (\phi_{ij} \phi_{jk} A/\omega_k^2) [1 - \cos(\omega_k t)] \]  

(6)

SINUSOIDAL RESPONSE WITH PEAK-TO-PEAK AMPLITUDE TO

\[ X^k_i = 2 \phi_{ik} \phi_{jk} A/\omega_k^2 \]  

(7)

A MEASURE OF IMPORTANCE OF MODE K

APPLICATION TO GALILEO

![Diagram of Galileo system with nodes labeled A (1, 2, 3), B (4, 5, 6), C (7, 8).]
APPLICATION TO GALILEO (CONT’D)

- AVAILABLE DATA
  - EIGENVALUES, EIGENVECTORS FOR UP TO 60 MODES
- PLOT MODAL INFLUENCE COEFFICIENTS
- DISCARD MODES WITH "LOW" COEFFICIENTS
- USE BODE PLOT TO CHECK RESULTS

MODAL INFLUENCE COEFFICIENTS

$SA$ TO SCAN CLOCK ANGLE=0 CONE ANGLE=30

COEFFICIENT SCALE: 1=20 MICRORAD

MODE NUMBER

520
MODAL INFLUENCE COEFFICIENTS

SBA TO SCAN CLOCK ANG=0 CONE ANG=30

BODE PLOT OF PLANT ALPHA = 0 BETA = 30
BODE PLOT OF PLANT CLOCK = 0 CONE = 30

MAGNITUDE (DB)

FREQUENCY (HZ)

BODE PLOT OF PLANT ALPHA = 0 BETA = 30

MAGNITUDE (DB)

FREQUENCY (HZ)
TRUNCATION AT COMPONENT LEVEL

- AVAILABLE
  - COMPONENT "FREE-FREE" MODES
  - SYSTEM MODES TO BE RETAINED

- PROBLEM
  - DETERMINATION OF "IMPORTANT" COMPONENT FREE-FREE MODES FROM KNOWLEDGE OF SYSTEM MODES

- SOLUTION
  - RETAIN THOSE COMPONENT MODES THAT "CONTRIBUTE SUBSTANTIALLY" TO IMPORTANT SYSTEM MODES

COMPONENT LEVEL TRUNCATION (CONT'D)

\[
\begin{align*}
M_A \ddot{x}_A + K_A x_A &= F_A \\
x_A &= \phi_A q_A \\
Iq_A + \omega^2 q_A &= \phi_A^T F_A
\end{align*}
\]

BODY A
D.O.F. = \( n_A \) (8)

\[
\begin{align*}
M_B \ddot{x}_B + K_B x_B &= F_B \\
x_B &= \phi_B q_B \\
Iq_B + \omega^2 q_B &= \phi_B^T F_B
\end{align*}
\]

BODY B
D.O.F. = \( n_B \) (9)

\[
\begin{align*}
Mx + Kx &= F \\
x &= \phi q \\
Iq + \omega^2 q &= \phi_F^T
\end{align*}
\]

COMBINED SYSTEM
D.O.F. = \( n \leq (n_A + n_B) \) (10)
COMPONENT LEVEL TRUNCATION (CONT'D)

- SYSTEM AUGMENTED $\phi$ MATRIX - $\tilde{\phi}$

- SYSTEM MATRIX WITH SOME ROWS REPEATED

\[
\begin{pmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_{n_A} \\
  \vdots \\
  x_{n_A+n_B}
\end{pmatrix}
= \begin{bmatrix}
  \phi_{11} & \phi_{12} & \cdots & \phi_{1m} \\
  \phi_{21} & \phi_{22} & \cdots & \phi_{2m} \\
  \phi_{n_A1} & \phi_{n_A2} & \cdots & \phi_{n_Am} \\
  \phi_{n_A+1,1} & \cdots & \phi_{n_A+1,m} \\
  \phi_{n_11} & \phi_{n_12} & \cdots & \phi_{n_1m} \\
  \phi_{n_21} & \cdots & \phi_{n_2m}
\end{bmatrix}
\begin{pmatrix}
  q_1 \\
  q_2 \\
  q_{n_A} \\
  q_{n_A+1} \\
  q_{n_1} \\
  q_{n_2}
\end{pmatrix}
\]

- PARTITION $\tilde{\phi}$ INTO $\tilde{\phi}_A$ AND $\tilde{\phi}_B$

COMPONENT LEVEL TRUNCATION (CONT'D)

- DELETE COLUMNS OF $\tilde{\phi}$ THAT CORRESPOND TO SYSTEM MODES THAT WERE DROPPED

- REDUCED $\phi$ MATRICES: $\hat{\phi}_A$ AND $\hat{\phi}_B$

- USE $\hat{\phi}_A$ AND $\hat{\phi}_B$ AS TRANSFORMATION MATRICES FOR BODIES A AND B RESPECTIVELY

  \[
  \hat{\phi}_A^T M_A \hat{\phi}_A \hat{\chi}_A + \hat{\phi}_A^T K_A \hat{\phi}_A = \hat{\phi}_A^T F_A
  \]

  OR

  \[
  \hat{M}_A \hat{\chi}_A + \hat{K}_A \hat{\phi}_A = \hat{\phi}_A^T F_A
  \]

  \[
  \hat{\chi} = \hat{\phi}_A^T F_A
  \]

  \[
  \hat{M}_B \hat{\phi}_B + \hat{K}_B \hat{\phi}_B = \hat{\phi}_B^T F_B
  \]
COMPONENT LEVEL TRUNCATION (CONT'D)

- \( \hat{\mathbf{M}}_A, \hat{\mathbf{K}}_A, \hat{\mathbf{M}}_B, \hat{\mathbf{K}}_B \) NOT NECESSARILY DIAGONAL
- DIAGONALIZE VIA ANOTHER MODAL ANALYSIS

\[ \mathbf{q}_A = \mathbf{\psi}_A \mathbf{\hat{q}}_A \quad (15) \]
\[ \mathbf{q}_B = \mathbf{\psi}_B \mathbf{\hat{q}}_B \quad (16) \]

\[ \mathbf{i} \ddot{\mathbf{q}}_A + \mathbf{\bar{\omega}}_A^2 \mathbf{q}_A = \mathbf{\psi}_A^T \mathbf{\hat{\phi}}_A^T \mathbf{F} \quad (17) \]
\[ \mathbf{i} \ddot{\mathbf{q}}_B + \mathbf{\bar{\omega}}_B^2 \mathbf{q}_B = \mathbf{\psi}_B^T \mathbf{\hat{\phi}}_B^T \mathbf{F} \quad (18) \]

COMPONENT LEVEL TRUNCATION (CONT'D)

- \( \bar{\omega}_A, \bar{\omega}_B \) ARE DIAGONAL; THEY ARE ALSO SUB-MATRICES OF \( \omega_A, \omega_B \) RESPECTIVELY, AND CONTAIN FREQUENCIES OF COMPONENT MODES TO BE RETAINED

- SIMILARLY \( \mathbf{\Phi}_A = \mathbf{\hat{\phi}}_A \mathbf{\psi}_A \) AND \( \mathbf{\Phi}_B = \mathbf{\hat{\phi}}_B \mathbf{\psi}_B \) ARE SUBMATRICES OF \( \mathbf{\Phi}_A \) AND \( \mathbf{\Phi}_B \), AND CONTAIN THE EIGENVECTORS OF COMPONENT MODES TO BE RETAINED
SUMMARY AND CONCLUSION

- DETERMINE SYSTEM MODES TO BE RETAINED USING
  - AVAILABLE CRITERIA
  - MODAL INFLUENCE COEFFICIENTS
  - BODE

- DESCEND TO COMPONENT LEVEL VIA A TWO-PHASE DIAGONALIZATION PROCESS STARTING WITH SUBMATRICES OF TRUNCATED AUGMENTED SYSTEM MODAL MATRIX

FUTURE WORK

- STREAMLINE SIMULATION CODES — ESPECIALLY DYNAMICS FORMULATION METHOD

- DEVELOP VERY EFFICIENT AND EASILY IMPLEMENTABLE MODEL REDUCTION STRATEGY