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DISSIPATION OF ALFVEN WAVES IN SOLAR CORONAL ARCHES

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I. INTRODUCTION

It has long been surmised that the temperature of the solar corona and the consequent radiated power in thermal bremsstrahlung is maintained by the input of mechanical energy flux of order $1W/cm^2$ from photospheric turbulent motion [1]. What is still a matter of contention is the physical mechanism by which this energy is delivered to and absorbed by the corona. It is also generally accepted that the solar magnetic field acts as conduit for the transmission of this energy flux to the corona. Of the three modes by which a magnetic field supports wave motion in a plasma only the shear Alfvén wave can penetrate to the high corona and indeed along open field lines this activity is observed even in the solar wind. In the closed field line regions, however, the resolution of present observations is insufficient to detect any wave activity. The upper limit of the *rms* coronal turbulent fluid velocity is $\lesssim 25km/sec$ [1,2]. The main difficulty is to explain the required dissipation of Alfvén wave flux <u>in the corona</u> given the extremely low value of the plasma resistivity which, on the basis of conventional theory [3], would predict an absorption length orders of magnitude larger than the length of the coronal loop. This means that a wave-packet would bounce back and forth in the magnetic loop almost forever before it is absorbed. However, Hollweg [3] has computed that the wave-packet is not completely reflected at the base of the loop in the denser chromosphere, but that a fraction of the energy leaks away by virtue of mode transformation, etc. If the loop is treated as a wave cavity, because of this leakage its quality factor $Q_L \sim 50$ [3]. Thus, only if it can be demonstrated that in some manner, due to inhomogeneities, wave dissipation reduces Q to less than Q_L can it be concluded that the waves deposit their energy in the corona and contribute to its heat content.

Because of this inherent limitation in Alfvén wave dissipation, Parker has renounced [1] the hypothesis of coronal heating by Alfvén waves altogether and put forward an alternative scenario. Since the coronal pressure is much smaller than the magnetic pressure i.e., $\beta \equiv 8\pi p/B^2 \sim 10^{-2} << 1$, the equation governing the magnetic field is the so-called force-free equation;

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \, . \tag{1}$$

Equation (1) has to be solved in the context of Fig. 1, where the feet of the field lines, presumably anchored in the dense photospheric plasma, suffer a quasi-random two-dimensional motion.

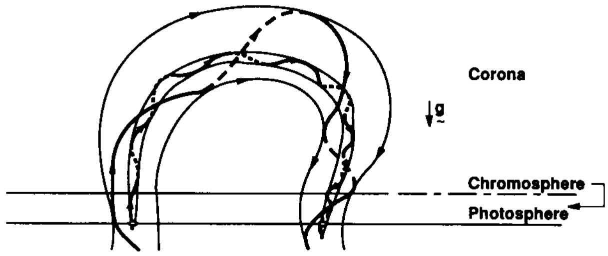

Fig. 1 Schematic of solar coronal magnetic loop.

This motion leads not only to the twisting of the field lines but to tangential discontinuities in the volume i.e., surfaces across which the direction of the field changes discontinuously and which are therefore the seat of singular current layers. In actuality, the width of these singular layers is determined by the resistivity, but the field gradients and local current densities are high, guaranteeing high local dissipation and ultimately to reconnected magnetic field lines. Parker claims that a balance is struck between winding the field by photospheric motion and the release of this energy by dissipation at the singular surfaces yielding an energy input of $\sim 1W/cm^2$ to the corona.

Although such a process looks eminently plausible doubts have been expressed by van Ballegooijen [4] and Vekstein [5] on the basis of restricted models. In any case, analytical or numerical demonstration of the evolution of such singular layers together with a quantitative determination of dissipation rates is necessary to validate this suggestion. There is, however, a very important corollary to the evolution of singular layers viz., that the magnetic field can no more be considered to be regular over the entire volume. In the vicinity of the singular layers, neighboring field lines will diverge exponentially over a scale length λ and diffuse over distances greater than λ with a diffusion coefficient $D_M = \langle(\Delta x_\perp)^2/\Delta s\rangle$ where Δs is the correlation distance along the field line and $\Delta x_\perp$ is the random step in the perpendicular

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direction. Thus the photospheric motion will give rise to irregularities in the magnetic structure of the coronal loop.

Studies of wave dissipation in inhomogeneous but regular magnetic fields by Tataronis and Grossman [6], Kappraff and Tataronis [7], Heyvaerts and Priest [8], Ionson [9], Davila [10], Hollweg [11] and Grossman and Smith [12], reveal that the dissipation length is proportional to $R_M^{\frac{1}{3}}$ whereas in homogeneous fields it is proportional to R_M; R_M, the magnetic Reynolds number, is $\sim 10^{10}$. If, however, the irregularities of the magnetic field discussed above are taken into account, it has been shown by Similon and Sudan [13] that the dissipationlength is proportional to $\ln R_M$. The Q of the loop computed for such an irregular field can be much less than the Hollweg limit Q_L so that Alfvén dissipation could indeed lead to coronal heating!

Our model for coronal heating by Alfvén waves therefore depends on those photospheric motions with excitation frequencies below $\omega_A = \langle v_A \rangle R^{-1}$ to cause a slow twisting of the flux tubes which builds up magnetic stress and singular layers and ultimately to reconnection and irregular field lines. The photospheric motions with excitation frequencies in excess of ω_A are able to excite Alfvén waves which dissipate rapidly in the presence of the stochastic field structure. Of course, this does not preclude direct dissipation of the stressed magnetic energy by joule dissipation in the singular layers. However, the rate of such dissipation, according to the Sweet-Parker model, will be proportional to $R_M^{\frac{1}{2}}$ and may turn out not to be sufficient.

In the next section we develop a set of rescaled MHD equations similar, but not identical to, the Strauss equations [14,15] for analyzing both the slow evolution of the magnetic field and the fast time scale of the Alfvén waves. The dissipation of Alfvén waves is treated in Section IV.

II. RESCALED MHD EQUATIONS FOR THE CORONA

The equilibrium force balance is given by

$$4\pi \nabla P_0 = (\nabla \times \mathbf{B}_0) \times \mathbf{B}_0 , \tag{2}$$

with $P_0 = p_0(\mathbf{x}) - g \int^z dz \rho_0(z)$, p_0 is the gas pressure, $\rho_0(z)$ is the stratified density and g is the acceleration due to gravity. Let $\beta = 8\pi P_0/B_0^2$ be a small quantity of order $\epsilon << 1$. Then, to lowest order in ϵ, Eqn.(2) reverts to the force-free equation (1). From (1), setting $\mathbf{B}_0 = B_0\hat{b}$ we obtain

$$\nabla \times \mathbf{B}_0 = \alpha_0 \mathbf{B}_0 \; ; \quad \mathbf{B}_0 \cdot \nabla \alpha_0 = 0 , \tag{3}$$

$$\nabla \times \hat{b} = \alpha_0 \hat{b} + (\hat{b} \times \nabla_\perp B_0)/B_0 , \tag{4a}$$

$$\hat{b} \cdot \nabla \times \hat{b} = \alpha_0 , \tag{4b}$$

$$\hat{b} \times \nabla \times \hat{b} = -\hat{b} \cdot \nabla \hat{b} \equiv \hat{n}\, R_c^{-1} = -(\nabla_\perp B_0)/B_0 , \tag{4c}$$

$$\nabla \cdot \hat{b} = -(\hat{b} \cdot \nabla B_0)/B_0 . \tag{4d}$$

R_c is the radius of curvature of the field line. Figure (1) shows a coronal loop of length L and minor radius a. We now order the scale of perturbations of this flux tube such that

$$\partial/\partial x_\perp \sim 0(1) , \tag{5a}$$

$$\partial/\partial t \sim \epsilon^{\frac{1}{2}} , \tag{5b}$$

and

$$\frac{\partial}{\partial s} \sim \frac{a}{L} \frac{\partial}{\partial x_\perp} \sim \epsilon^{\frac{1}{2}} , \tag{5c}$$

where s is the coordinate along the lines of force of $\mathbf{B}_0$ and $\mathbf{x}_\perp$ are the perpendicular coordinates. Thus, a/L is taken to be of order $\epsilon^{\frac{1}{2}}$. Furthermore, a/R_c is also of order $\epsilon^{\frac{1}{2}}$. The perturbations of the equilibrium quantities are scaled as follows:

$$\rho = \rho_0(z) + \delta\rho(s, \mathbf{x}_\perp, t) , \tag{6a}$$

$$p = \epsilon[p_0(\mathbf{x}) + \delta p(s, \mathbf{x}_\perp, t)] , \tag{6b}$$

$$\mathbf{v} = \epsilon^{\frac{1}{2}} \delta \mathbf{v}_\perp(s, \mathbf{x}_\perp, t) + \epsilon \delta v_\parallel(s, \mathbf{x}_\perp, t) + \dots , \tag{6c}$$

$$\mathbf{A} = \mathbf{A}_0 + \epsilon^{\frac{1}{2}} \delta \mathbf{A}_\parallel(s, \mathbf{x}_\perp, t) + \epsilon \delta \mathbf{A}_\perp(s, \mathbf{x}_\perp, t) + \dots . \tag{6d}$$

$\mathbf{v}$ is the fluid veloccity, $\mathbf{A}$ is the vector potential such that $\nabla \cdot \mathbf{A} = 0$ and $\mathbf{B} = \nabla \times \mathbf{A}$. Then Ohm's law for the plasma may be expressed as

$$\mathbf{E} = -\nabla\varphi - \frac{\partial}{\partial t}\mathbf{A} = -\mathbf{v} \times \mathbf{B} + \mathbf{j}/\sigma , \tag{7}$$

where $\mathbf{j} = -(4\pi)^{-1}\nabla^2 \mathbf{A}$ is the current density and σ is the plasma conductivity. Then, from Eqns. (6) and (7), to order $\epsilon^{\frac{1}{2}}$, we obtain

$$\delta \mathbf{v}_\perp = (\hat{b} \times \nabla_\perp \varphi)/B_0 , \tag{8}$$

and to order ϵ taking the $\hat{b}$ component of (7),

$$\frac{d}{dt}\delta A_\parallel \equiv \left[\frac{\partial}{\partial t} + (\delta \mathbf{v}_\perp \cdot \nabla_\perp)\right] \delta A_\parallel = -\frac{\partial \varphi}{\partial s} + \eta \nabla_\perp^2 \delta A_\parallel , \tag{9}$$

with $\eta = (4\pi\sigma)^{-1}$. Taking the $\hat{b}$ component of the curl of Eqn. (7) we get, to order $\epsilon^{\frac{3}{2}}$,

$$\frac{d}{dt}\delta B_\parallel = (\delta \mathbf{B}_\perp \cdot \nabla_\perp)\delta v_\parallel + \eta \nabla^2 \delta B_\parallel , \tag{10}$$

with

$$\delta \mathbf{B}_\perp = \nabla_\perp \delta A_\parallel \times \hat{b} . \tag{11}$$

To order $\epsilon^{\frac{1}{2}}$ the continuity equation becomes

$$\frac{\partial}{\partial t}\delta\rho = -\delta \mathbf{v}_\perp \cdot \nabla \rho_0 , \tag{12}$$

and to order ϵ the perpendicular momentum balance is given by:

$$\rho_0 \frac{d}{dt}\delta \mathbf{v}_\perp = -\nabla_\perp \delta p + \delta \mathbf{j}_\perp \times \mathbf{B}_0 + (\mathbf{j}_0 + \delta \mathbf{j}_\parallel) \times \delta \mathbf{B}_\perp + \delta\rho \mathbf{g} + \rho_0 \nu_\perp \nabla_\perp^2 \delta \mathbf{v}_\perp , \tag{13}$$

and to order $\epsilon^{\frac{3}{2}}$ the parallel momentum balance is given by

$$\rho_0 \frac{d}{dt}\delta v_\parallel = -\nabla_\parallel \delta p + \hat{b} \cdot \delta \mathbf{j}_\perp \times \delta \mathbf{B}_\perp + \delta\rho \hat{b} \cdot \mathbf{g} + \rho_0 \nu_\parallel \nabla_\perp^2 \delta v_\parallel . \tag{14}$$

Here $\mathbf{j}_0 = \alpha_0 B_0 \hat{b}$, $\nu_\perp$ and $\nu_\parallel$ are the perpendicular and parallel kinematic viscosities, $\nabla \cdot \delta \mathbf{v}_\perp$ is of order ϵ, and $\delta\rho\hat{b}\cdot\mathbf{g}/\rho_0 g$ is taken to be of order $\epsilon^{\frac{3}{2}}$. The pressure is obtained from a static equation obtained by taking the divergence of Eqn. (13) and recognizing that $\nabla \cdot \delta \mathbf{v}_\perp$ may be neglected because it is of higher order by $\epsilon^{\frac{1}{2}}$ than the remaining terms. Operating on Eqn. (13) with the operator $\nabla \cdot \hat{\delta}\times$ we obtain to order ϵ,

$$\begin{aligned}\nabla_\perp \cdot v_A^{-2} \frac{d}{dt} \nabla_\perp \varphi = & -\frac{\partial}{\partial s} \nabla_\perp^2 \delta A_\| \\ & - B_0^{-1}(\delta \mathbf{B}_\perp \cdot \nabla_\perp)\, \nabla_\perp^2 \delta A_\| \\ & + B_0^{-1}(\delta \mathbf{B}_\perp \cdot \nabla_\perp)\, \alpha_0 \\ & + \nabla_\perp \cdot \frac{\delta\rho}{B_0} \hat{b} \times \mathbf{g} \\ & + \nu_\perp v_A^{-2}\, \nabla_\perp^4 \varphi\, ,\end{aligned} \tag{15}$$

where we have eliminated $\delta \mathbf{j}_\perp$ through the relations $\nabla_\| \cdot \delta \mathbf{j}_\| = -\nabla_\perp \cdot \delta \mathbf{j}_\perp$ and $\delta \mathbf{j}_\| = -(4\pi)^{-1}\nabla_\perp^2 \delta A_\|$. Equations (9), (10), (12), (14) and (15) advance, $\delta A_\|$, $\delta B_\|$, $\delta\rho$, $\delta v_\|$ and $\nabla_\perp \varphi$, respectively; $\delta \mathbf{v}_\perp$ and $\delta \mathbf{B}_\perp$ are obtained from (8) and (11). This set of equations constitute a rescaled set of MHD equations suitable for the study of coronal magnetic fields. Equations (9), (12), and (15) form a self-consistent set to lowest order.

III. VERY SLOW MOTIONS OF CORONAL LOOP MAGNETIC FIELDS

When the excitation frequency of the perturbations to the coronal loop is below ω_A (as discussed in Sec. I) Alfvén waves will not be excited and the time evolution of the system is adiabatic. In this limit the $\partial/\partial t$ operator is even smaller than order $\epsilon^{\frac{1}{2}}$. We are therefore justified in neglecting the inertial term on the LHS of Eqn.(15). The term involving $\mathbf{g}$ may also be dropped if the magnetic field is approximately collinear with $\mathbf{g}$. Then the lowest order equations (15) and (9) furnish

$$\frac{\partial}{\partial s}\nabla_\perp^2 \delta A_\| + \left(\frac{\delta \mathbf{B}_\perp}{B_0} \cdot \nabla_\perp\right) \nabla_\perp^2 \delta A_\| = \frac{\nu_\perp}{v_A^2}\, \nabla_\perp^4 \varphi \tag{16}$$

$$\frac{\partial \varphi}{\partial s} + \left(\frac{\delta \mathbf{B}_\perp}{B_0} \cdot \nabla_\perp\right) \varphi = \frac{-\partial}{\partial t}\delta A_\| + \eta\, \nabla_\perp^2 \delta A_\|\, , \tag{17}$$

where $v_A = B_0/(4\pi\rho_0)^{\frac{1}{2}}$ is the Alfvén velocity. Equations (16) and (17) form a closed set and describe the very slow motions of the loop. Nevertheless, we require the time scale $\tau < \lambda_\perp^2/\eta$ where $\lambda_\perp$ is the perpendicular scale i.e., the magnetic Reynolds number $R_M > 1$. We notice that in the limit $\nu_\perp \to 0$, Eqn. (16) is homologous to the two-dimensional incompressible Euler equation for the vorticity if the coordinate s is identified with time and the Eulerian fluid velocity with $\nabla_\perp \delta A_\| \times \hat{b}$. Proceeding with this analogy we may identify the Lagrangian motion of the Eulerian fluid elements along the streamlines (magnetic lines of force). Thus,

$$\frac{d}{ds}\mathbf{x}_\perp = (\nabla_\perp \delta A_\| \times \hat{b})/B_0 \tag{18}$$

which leads to a Hamiltonian system of equations for the components $\mathbf{x}_\perp = (\chi_1, \chi_2)$

$$\frac{d\chi_1}{ds} = \frac{\partial \delta A_\|}{B_0 \partial \chi_2}\, ; \qquad \frac{d\chi_2}{ds} = -\frac{\partial \delta A_\|}{B_0 \partial \chi_1}\, , \tag{19}$$

with $\delta A_\|/B_0$ as the Hamiltonian. It is well known that even if the velocity field in unsteady two-dimensional Navier-Stokes flow is continuous and laminar, the trajectories of the fluid elements may be stochastic, a process known as "chaotic advection" [16] or "Lagrangian turbulence" [17].

In our case $\delta A_\|$ is a function of s in addition to χ_1 and χ_2 and it is therefore highly likely that the chaotic motions of the feet of field lines which is equivalent to prescribing the injected current $\nabla_\perp^2\, \delta A_\|$ at $s = 0$ will most likely lead to chaotic magnetic field lines.

Equation (16) may be integrated numerically along s if the injected current $\delta\, j_\|$ is specified at $s = 0$. The RHS of Eqn. (17) is therefore known and Eqn. (17) can be integrated now to furnish φ which specifies the velocity field.

For $\mathbf{B}_0 = B_0 \hat{z}$ with B_0 constant and an injected current at $s = 0$ given by

$$\begin{aligned}\delta j_\|/j_0 = & \sin(2\pi x/L)\sin(2\pi y/L) \\ & + \epsilon\, \cos(2\pi n x/L)\, \cos(2\pi m y/L),\end{aligned} \tag{19a}$$

we have plotted, in Fig. 2, what we define as the "irregularity" index

$$\lambda = \frac{1}{2} z^{-1} \ln\{\langle \mid \delta x_\perp(z) \mid^2 \rangle \,/\, \langle \mid \delta x_\perp(0) \mid^2 \rangle \}, \tag{19b}$$

which is a measure of how neighboring field lines $\delta \mathbf{x}_\perp$ apart diverge from each other according to $d\delta \mathbf{x}_\perp/dz = B_0^{-1}(\delta \mathbf{B}_\perp \cdot \nabla)\delta \mathbf{x}_\perp$. In the limit $z \to \infty$, λ is the Liapunov exponent. But in the case of finite length loops, $z_{max} = L$.

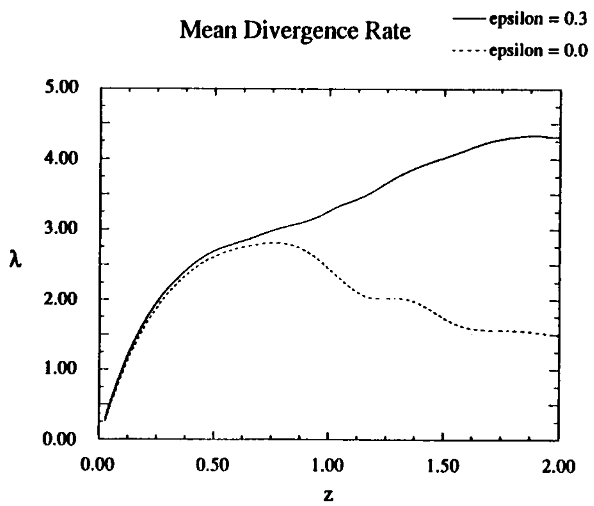

Fig. 2 Plot of λ defined in Eqn. (19b) as a function of z for $\delta j_\|$ given by Eqn. (19a) for $\epsilon = 0$ and $\epsilon = 0.3$.

IV. ALFVÉN WAVES IN ARBITRARY MAGNETIC FIELDS

Formulation of the Eikonal Approximation

In this case the fast component of photospheric motions excite Alfvén waves so that $\partial/\partial t \sim 0(\epsilon^{\frac{1}{2}})$. To lowest order we need only Eqns. (9), (12), and (15) for $\delta A_\|, \delta\rho$, and $\nabla_\perp \varphi$. As mentioned in Sect. I, the maximum bound of the perturbations in velocity in the coronal loop is of order 25 km/sec as compared to the typical Alfvén velocity of 2000 km/sec. Thus we are justified in linearizing these equations to obtain

$$\frac{\partial}{\partial t}\delta A_\| + \frac{\partial \varphi}{\partial s} = \eta \nabla_\perp^2 \delta A_\|\, , \tag{20}$$

$$\frac{\partial}{\partial t}\delta\rho = -\delta \mathbf{v}_\perp \cdot \nabla_\perp \rho_0\, , \tag{21}$$

$$\begin{aligned}\nabla_\perp \cdot v_A^{-2} \frac{\partial}{\partial t}\nabla_\perp \varphi = & -\frac{\partial}{\partial s}\nabla_\perp^2 \delta A_\| + \delta \mathbf{B}_\perp \cdot \nabla_\perp \alpha_0 \\ & + \frac{\hat{b}}{B_0} \times \mathbf{g} \cdot \nabla_\perp \delta\rho + \nu_\perp v_A^{-2} \nabla_\perp^4 \varphi\, .\end{aligned} \tag{22}$$

To simplify our discussion we take α_0 to be uniform and $\hat{b}\times \mathbf{g} \approx 0$. If the time dependence is taken as $\exp -i\omega t$ these equations may be combined to give:

$$\begin{aligned}\omega^2\nabla_\perp \cdot v_A^{-2}\nabla_\perp\varphi + \frac{\partial}{\partial s}\nabla_\perp^2 \left(1 - i\frac{\eta}{\omega}\nabla_\perp^2\right)\frac{\partial\varphi}{\partial s} \\ - i\frac{\nu_\perp}{\omega}\, v_A^{-2}\nabla_\perp^4\varphi = 0 ,\end{aligned} \tag{23}$$

To solve this equation we employ the ballooning approximation developed in fusion physics [18,19]. In what follows we outline the analysis of Eqn. (23) by Similon and Sudan [13]. The perturbed fields are represented as

$$\varphi(\mathbf{x},t) = \hat{\varphi}(s,\mathbf{x}_\perp)\, \exp[iS(\mathbf{x}) - i\omega t] , \tag{24}$$

where the amplitude $\hat{\varphi}$ is a slow function of s and $\mathbf{x}_\perp$ while the phase function S represents the fast variation. In this representation $\hat{b} \cdot \nabla S = \mathbf{k}_\parallel$ and $\mathbf{k}_\perp = \nabla_\perp S$ and Eqn. (23) may be written as

$$\frac{\partial}{\partial s}\, k_\perp^2(s)\left(1 - i\frac{\eta}{\omega}k_\perp^2\right)\frac{\partial}{\partial s}\hat{\varphi} + \frac{\omega^2}{v_A^2}k_\perp^2\left(1 + i\frac{\nu_\perp}{\omega}k_\perp^2\right)\hat{\varphi} = 0 , \tag{25}$$

$$\frac{d}{ds}\mathbf{k}_\perp = -(\nabla\hat{b})\cdot\mathbf{k}_\perp - (\mathbf{k}_\perp\cdot\kappa)\hat{b} , \tag{26}$$

with $\kappa = \hat{b}\cdot\nabla\hat{b}$. In Eqns. (25) and (26) we have set $k_\parallel \to 0$ as is required for long parallel wavelengths. If, on the other hand, one were to treat parallel wavelengths much shorter than the scale of variation of the equilibrium quantities with s, then $\partial/\partial s$ is replaced by $ik_\parallel$ in Eqn. (25) and $i\mathbf{k}_\perp$ may revert back to $\nabla_\perp$ if we wish to take account of perpendicular gradients in the equilibrium quantities. In this limit Eqn. (26) is modified to

$$\frac{d}{ds}\mathbf{k}_\perp = -(\nabla\hat{b})\cdot\mathbf{k}_\perp - (\mathbf{k}_\perp\cdot\kappa)\hat{b} - (\nabla v_A/v_A)k_\parallel . \tag{26'}$$

The driving frequency ω is constant and the boundary conditions are the given amplitude at $s = 0$ and outgoing wave condition for $s > L$.

Dissipation of Alfvén Waves in Complex Magnetic Field

From Eqn. (25) it is straightforward to establish the wave energy equation

$$\frac{\partial F}{\partial s} = |\, j_\parallel\, |^2/\sigma , \tag{27}$$

where we have dropped viscous dissipation; the wave energy flux $F = v_A(s)\varepsilon(s)$ and the wave energy density

$$\begin{aligned}\varepsilon(s) &= \frac{1}{2}\rho_0\delta\mathbf{v}_\perp^2 + (8\pi)^{-1}\delta\mathbf{B}_\perp^2 \\ &= (8\pi)^{-1}\left[k_\perp^2\,|\,\varphi\,|^2/v_A^2 + k_\perp^2\,|\,\delta A_\parallel\,|^2\right] \\ &\approx (4\pi)^{-1}k_\perp^2\,|\,\delta A_\parallel\,|^2 ,\end{aligned} \tag{28}$$

because $\varphi \approx v_A\delta A_\parallel$. From Eqn. (27) we obtain the relation between the wave flux at s and the wave flux at $s = 0$,

$$F(s) = F(0)\exp - \int_0^s ds'(\eta k_\perp^2/2v_A) . \tag{29}$$

We now define the dissipation length s_d through

$$\int_0^{s_d} ds'(\eta k_\perp^2/v_A) = 1 , \tag{30}$$

i.e., the distance at which the wave flux decreases by e^{-2}.

We first address a simple example in which the magnetic field $B_0(x)\hat{z}$ varies with x. Thus v_A varies with x and from Eqn. (26') we obtain $k_\perp = sk_\parallel v_A'/v_A = s\omega v_A'/v_A^2$ with $v_a' = dv_A/dx$. Substituting this expression for $k_\perp$ in Eqn. (30) we get

$$s_d = (3v_A^5/\eta\omega^2 v_A'^2)^{\frac{1}{3}} \propto R_M^{\frac{1}{3}} . \tag{31}$$

Similarly, the dissipation layer width w is given in order of magnitude by $k_\perp^{-1}$ at $s \sim s_d$, i.e.

$$w = (\eta v_A/3\omega v_A')^{\frac{1}{3}} \propto R_M^{-\frac{1}{3}} , \tag{32}$$

and decreases as $\eta^{\frac{1}{3}}$. These results are identical with those of Tataronis and Grossmann [6], Kappraff and Tataronis [7], and Heyvaerts and Priest [8] arrived at by considerations of Alfvén resonances and matched asymptotic expansions about the resonance $\omega = k_\parallel v_A(x)$. The wave-packet picture described here [13] gives precisely the same results but is capable of greater generalization as we show next.

If the magnetic field is irregular as discussed in Sect. II then neighboring field lines diverge exponentially, i.e., $\xi(s_0 + \delta s) = \xi(s_0)\exp\, \lambda\delta s$ where ξ is the distance between two neighboring lines. This exponentiation is limited to some correlation length after which it breaks down. On a scale greater than the correlation lengths the field lines have a diffusive behavior and ξ varies as $(2D_m s)^{\frac{1}{2}}$ where $D_M = \langle(\Delta x_\perp)^2/\Delta s\rangle$ is the field line diffusion coefficient. The field line exponentiation stretches the wave packet unidirectionally and steepens the gradients. This gives rise to a wave packet that spreads over an area $(2D_M s)$ constituted of thin filaments of width $k_0^{-1}\exp - \lambda s$ (see Fig. 3). Thus,

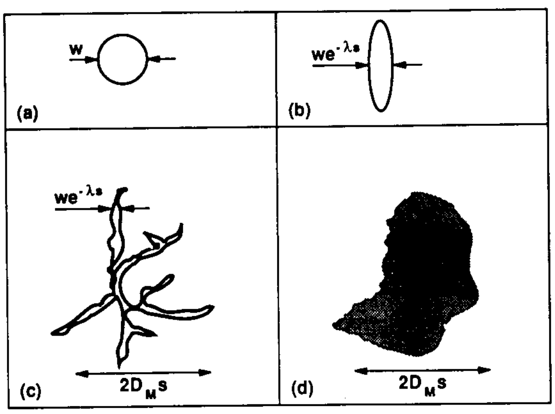

Fig. 3 Wave Packet evolution in s along the stochastic magnetic field. (a) is the initial state; (b) and (c) show the exponentiation phase, during which the gradients increase as $\exp(\lambda s)$; (c) and (d) show the field line diffusion, during which the packet spreads on a length $(2D_M s)^{\frac{1}{2}}$; (d) is the state beyond s_d, when "microscopic" diffusivity is effective, and when energy is dissipated by resistivity (from Ref.[13]).

$k_\perp^2(s) \approx k_0^2\exp 2\lambda s$. When this expression is substituted in Eqn. (30) we obtain

$$s_d = \frac{1}{2}\lambda^{-1}\ln(2\lambda v_A/\eta k_0^2) \propto \ln\, R_M , \tag{33}$$

k_0^{-1} is the perpendicular scale length of the perturbation at the photosphere $s = 0$. The width of the dissipation layer w is

$$w \approx k_\perp^{-1}(s_d) = (\eta/2\lambda v_A)^{\frac{1}{2}} \propto R_M^{-\frac{1}{2}} . \qquad (34)$$

Significant energy dissipation does not take place until s approaches s_d when the wave amplitudes crash to a low level. For $s \sim s_d$ the resistivity smooths out the large gradients, dissipates a large fraction of Alfvén wave energy and redistributes the rest over a large area $2D_M s_d$ determined by magnetic diffusion (see Fig. 3). For $s > s_d$, the wave packet in which high $k_\perp$ components have diffused away now evolves as a result of magnetic field line diffusion. The wave energy flux will vary, from this point on, as $F(s) = F(0)[1 + 2D_M s]^{-1}$.

For $\omega \approx 2v_A/R$, $R_M \sim 10^{10}$, $av'_A/v_A = 4$, $R \sim 10^4 km$ one finds, for the laminar magnetic field, $w_d/a \approx 1.6 \times 10^{-4}$ and $s_d/R \approx 780$. On the other hand, for an irregular magnetic field with $\lambda R \sim 1/2$, $k_0 a \approx 3\pi$ we get $s_d/R = 37$ and for $\lambda R \sim 2$, $s_d/R \approx 10$. The corresponding quality factors $Q = S_d/\lambda_\parallel = 18$ and 5, respectively, with $\lambda_\parallel \sim \pi R$ are easily less than Hollweg's limiting Q_L. Thus, an irregular magnetic field is very effective to dissipate Alfvén waves because it generates the small scale lengths needed for dissipation.

Dispersive Effects

So far we have assumed that the wave packets propagate closely along the field lines i.e., there is no dispersion in the perpendicular direction. This requires $\partial^2\omega/\partial k_\perp^2 << \eta$. A number of effects e.g. finite Larmor effects, finite pressure, gravity, and equilibrium currents could, in principle, contribute to the dispersion. These have been shown [13] to be insignificant in the solar coronal context. Because the Larmor radius of the ion is only $\lesssim 10^2 cm$ the FLR effect is negligible with $k_0^{-1} \sim 10^2 km$. The additional corrections to Eqn. (23), due to finite pressure and gravity, are homogeneous in $\mathbf{k}_\perp$. Furthermore, field line divergence aligns $\mathbf{k}_\perp$ to the direction of greatest contraction. Thus the spectrum contracts locally to one-dimension and hence $\partial^2\omega/\partial\mathbf{k}_\perp\partial\mathbf{k}_\perp$ tends to vanish leaving the wave packet nondispersive. Finally, in Eqn. (23) we observe that the equilibrium current does not affect the wave evolution if α_0 is uniform. For arbitrary α_0 the dispersion introduced by $\nabla_\perp\alpha_0$ decreases rapidly at large wavenumbers so that as the wave packet develops finer structure it conforms more closely to the field lines.

CONCLUSION

We have shown that the slow motion of the feet of coronal arches leads to irregular magnetic fields and that Alfvén waves propagating in the irregular magnetic structure are dissipated through filamentation of the wave packet that generates short scales necessary for efficient dissipation.

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