A DECOMPOSITION-BASED DESIGN OPTIMIZATION

METHOD WITH APPLICATIONS

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TOPICS

There are many real-world engineering design problems which cannot be effectively handled using conventional design optimization methods. Special techniques and/or modifications of the conventional methods are necessary to handle such complex problems. One solution involves two-level decomposition, whereby a problem is divided into smaller subproblems, each with its own design objective and constraints (refs. 1,2).

Here, we will describe a two-level design optimization methodology and give a progress report of its application to Printed Wiring Board (PWB) assembly examples.

1. Two-Level Design Optimization
   - Formulation
   - Procedure
2. Example: PWB Assembly
3. Summary
FORMULATION

We consider a problem which may be decomposed into two levels, each having several local variables. Here, i,j are the indices corresponding to the number of subproblems and number of constraints in each subproblem, respectively. Furthermore, $x_i$ is the vector of "local" design variables in the lower-level subproblem $i$, and $y_i$ is the vector of "global" design variables in the top-level problem.

Minimize $f(y;x) = f_0(y) + \sum_{i=1}^{I} f_i(y;x_i)$

Subject: $g_z(y)\leq 0$ \hspace{1cm} $z=1,\ldots,L$

$g_{i,j}(y;x_i)\leq 0$ \hspace{1cm} $j=1,\ldots,J$
PROCEDURE

The procedure is to

(1) Select the starting value for the global variables y,
(2) find $x_i$ (y is fixed), $i=1,\ldots,I$, in subproblem $i$,
(3) find a new $y$ in the top-level problem such that $f(y,x)$ is decreased,
(4) return to step (2) until the minimum for $f(y;x)$ is obtained.

Subproblem $i$:

Minimize $f_i(y;x_i)$

Subject to: $g_{i,j}(y;x_i) \leq 0 \quad j=1,\ldots,J$

Top-level problem:

Minimize $f(y;x) = f_o(y) + \sum_{i=1}^{I} f_i(y;x_i)$

Subject to: $g_\ell(y) \leq 0 \quad \ell=1,\ldots,L$
EXAMPLE: PWB ASSEMBLY

Design optimization of a PWB assembly is considered. The objective is to determine the required component redundancy and fluid flow-rate for each PWB such that the reliability of the assembly is maximized. This is a mixed-integer nonlinear programming problem.
EXAMPLE: TWO-LEVEL MODEL OF A PWB ASSEMBLY

Here, a two-level design optimization model for an assembly of PWBs is presented. Allocation of fluid flow-rates (continuous variables) is performed at the top-level problem, while, allocation of component redundancy (integer variables) for each PWB is performed at the bottom-level subproblems.
EXAMPLE: REDUNDANCY ALLOCATION FOR A PWB (Ref. 3)

It is assumed that each PWB consists of a series of $N$ stages, where each stage $n$, is a parallel combination of $M_n$ redundant components. All components in a stage are active. Thus, for a stage to fail, all components in that stage must fail. Furthermore, it is assumed that for a PWB, all components at a given stage are identical and equally reliable.
EXAMPLE: TWO-LEVEL OPTIMIZATION FORMULATION

In the two-level formulation of the PWB assembly, subproblem i corresponds to PWB i in which the reliability \( R_i \) is maximized. In the top-level problem, fluid flow-rates \( Q_i, i=1, \ldots, I \) are allocated to maximize the assembly reliability \( R \). It is assumed that the assembly is a series system of I PWBs.

Subproblem i:

Maximize \( R_i = \prod_{n=1}^{N} (1-q_n^n) \)

Subject to: \( \sum_{n=1}^{M} A_n M_n e_n^n - A_{avi} < 0 \)
\[
M_n > 1 \quad n=1, \ldots, N
\]

Top-Level Problem:

Maximize \( R = \prod_{i=1}^{I} R_i \)

Subject to: \( \sum_{i=1}^{I} Q_i - Q_t < 0 \)
\[
Q_i > 0 \quad i=1, \ldots, I
\]

where:
- \( q_n \) = nth stage component unreliability of ith PWB
- \( M_n \) = nth stage component redundancy of ith PWB
- \( A_{avi} \) = available area of ith PWB
- \( A_n \) = area of a component at nth stage of ith PWB
- \( Q_t \) = total fluid flow-rate of assembly
EXAMPLES

Three PWB assembly examples were solved. The first example was an assembly of 2 PWBs, each PWB having 5 stages. The second example was an assembly of 4 PWBs, each PWB having 15 stages. The third example was an assembly of 4 PWBs, each PWB having 30 stages. The overall design objective in each example was to maximize assembly reliability.

<table>
<thead>
<tr>
<th>Example</th>
<th>No. of stages/PWB</th>
<th>No. of PWBs</th>
<th>Variables</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>2</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>4</td>
<td>64</td>
<td>69</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>4</td>
<td>124</td>
<td>129</td>
</tr>
</tbody>
</table>
EXAMPLE: RESULTS

Initial and final solutions for an assembly of two PWBs are given:

**INITIAL:**
5 STAGES PER PWB

- **ASSEMBLY RELIABILITY** = 0.908061
- **Q_1** = 2.0 lbs/min

- **PWB1 Reliability** = 0.953124
  - **Q_1** = 0.5 lbs./min
  - **M_n** = (1,1,1,1)

- **PWB2 Reliability** = 0.95272
  - **Q_2** = 0.5 lbs./min
  - **M_n** = (1,1,1,1)

**FINAL:**
5 STAGES PER PWB

- **ASSEMBLY RELIABILITY** = 0.99778
- **Q_1** = 2.0 lbs/min

- **PWB1 Reliability** = 0.997828
  - **Q_1** = 1.34 lbs./min
  - **M_n** = (3,2,1,1,2)

- **PWB2 Reliability** = 0.999953
  - **Q_2** = 0.66 lbs./min
  - **M_n** = (3,4,2,2,2)
EXAMPLE: RESULTS

Initial and final solutions for two assemblies of four PWBs are given.

**INITIAL:**
15 STAGES PER PWB

- **ASSEMBLY**
  - Reliability: 0.877281
  - \( Q_1 = 4.0 \text{ lbs./min} \)

- **PWB1**
  - Reliability: 0.921300
  - \( Q_1 = 0.5 \text{ lbs./min} \)

- **PWB2**
  - Reliability: 0.862237
  - \( Q_2 = 0.5 \text{ lbs./min} \)

- **PWB3**
  - Reliability: 0.910671
  - \( Q_3 = 0.5 \text{ lbs./min} \)

- **PWB4**
  - Reliability: 0.807443
  - \( Q_4 = 0.5 \text{ lbs./min} \)

**FINAL:**
15 STAGES PER PWB

- **ASSEMBLY**
  - Reliability: 0.939016
  - \( Q_1 = 4.0 \text{ lbs./min} \)

- **PWB1**
  - Reliability: 0.966725
  - \( Q_1 = 1.52 \text{ lbs./min} \)

- **PWB2**
  - Reliability: 0.989408
  - \( Q_2 = 0.7 \text{ lbs./min} \)

- **PWB3**
  - Reliability: 0.995174
  - \( Q_3 = 1.09 \text{ lbs./min} \)

- **PWB4**
  - Reliability: 0.988571
  - \( Q_4 = 0.7 \text{ lbs./min} \)

**INITIAL:**
30 STAGES PER PWB

- **ASSEMBLY**
  - Reliability: 0.630707
  - \( Q_1 = 5.0 \text{ lbs./min} \)

- **PWB1**
  - Reliability: 0.778563
  - \( Q_1 = 1.0 \text{ lbs./min} \)

- **PWB2**
  - Reliability: 0.914040
  - \( Q_2 = 1.0 \text{ lbs./min} \)

- **PWB3**
  - Reliability: 0.823231
  - \( Q_3 = 1.0 \text{ lbs./min} \)

- **PWB4**
  - Reliability: 0.905888
  - \( Q_4 = 1.0 \text{ lbs./min} \)

**FINAL:**
30 STAGES PER PWB

- **ASSEMBLY**
  - Reliability: 0.929625
  - \( Q_1 = 5.0 \text{ lbs./min} \)

- **PWB1**
  - Reliability: 0.968763
  - \( Q_1 = 0.87 \text{ lbs./min} \)

- **PWB2**
  - Reliability: 0.971216
  - \( Q_2 = 1.71 \text{ lbs./min} \)

- **PWB3**
  - Reliability: 0.996246
  - \( Q_3 = 0.73 \text{ lbs./min} \)

- **PWB4**
  - Reliability: 0.979138
  - \( Q_4 = 1.69 \text{ lbs./min} \)
SUMMARY

The design of PWB assemblies is a complex task which is generally conducted as a "sequential process." Individual PWBs are usually designed first, followed by the composition of the PWBs into an assembly. As a result, optimizing design considerations such as assembly reliability cannot be accomplished. This study showed that a two-level decomposition method can be employed to optimize for reliability at both the PWB- and the assembly-level in a coupled manner. The two-level decomposition method also resolved the mixed-integer nonlinear programming nature of the problem rather easily.

- The sequential design process makes system optimization impossible
- A mixed-integer nonlinear optimization problem modelled and solved using a two-level optimization technique
- More research is needed to improve the performance of the two-level optimization method
REFERENCES

