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DESIGN OPTIMIZATION OF AXISYMMETRIC BODIES IN NONUNIFORM TRANSONIC FLOW

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FORMULATION OF ANALYSIS PROBLEM

For an axisymmetric body immersed in a propfan slipstream, or a jet, the effect of freestream nonuniformity must be accounted for to calculate the wave drag in a transonic flow. Since the flow field is rotational, in the present approach a rotation function (F) and a velocity function (ϕ) are introduced in Euler's equations to result in a governing equation which is similar, in mathematical structure, to a full-potential equation as shown in figure 1 (refs. 1 and 2). The equation is solved with the algorithm of reference 3. Following reference 3, the equation is cast in curvilinear coordinate systems with a body-normal coordinate system covering the front portion and a sheared cylindrical system used in the aft. The rotation function is calculated through Crocco's relation. In figure 1, only the equation in body-normal coordinates is shown. Details can be found in reference 2.

- **Equation in Body Normal Coordinates:**

$$\left(1-\frac{u^2}{a^2}\right)\frac{1}{H}\left(\frac{1}{H}\phi_\xi\right)_\xi-\frac{2uv}{a^2H}\phi_{\xi\eta}+\left(1-\frac{v^2}{a^2}\right)\phi_{\eta\eta}+\left(\frac{2uv}{a^2}\frac{\kappa}{H}+\frac{\sin\theta}{r}\right)\frac{1}{H}\phi_\xi+\left[\left(1-\frac{u^2}{a^2}\right)\frac{\kappa}{H}+\frac{\cos\theta}{r}\right]\phi_\eta$$

$$+\left[\frac{uv}{a^2}\sin\theta+\left(1-\frac{u^2}{a^2}\right)\cos\theta\right]\frac{1}{H}F_\xi-\left[\frac{uv}{a^2}\cos\theta+\left(1-\frac{v^2}{a^2}\right)\sin\theta\right]F_\eta=0$$

- **Velocity Components:**

$$u=\frac{1}{H}\phi_\xi+(1+F)\cos\theta \qquad v=\phi_\eta-(1+F)\sin\theta$$

- **Crocco's Relation for the Rotation Function:**

$$\frac{\sin\theta}{H}F_\xi+F_\eta\cos\theta=\frac{\gamma}{u\left(1+\frac{\gamma-1}{2}M^2\right)}\left(\frac{M^2}{2}T_{o\eta}+\frac{T_\eta}{\gamma P_o}P_{o\eta}\right)$$

T_o = Stagnation temperature

P_o = Stagnation pressure

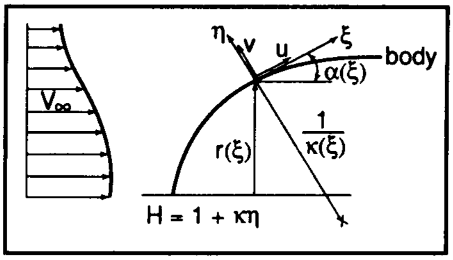

Figure 1

DESIGN FORMULATION

The analysis program is coupled with the CONMIN optimizer (ref. 4) to design an axisymmetric body for minimal wave drag. The constraints include a specified maximum thickness and a tail thickness. To reduce the number of design variables, the shape is represented by a modified Fourier series with its coefficients being the design variables (fig. 2).

- **Objective Function to be Minimized:**

  $OBJ = -0.1/(0.001 + C_{d_w})$

- **Constraints:**

  $G(1) = 10(r_{max}/r_u - 1) < 0$

  $G(2) = 10(1 - r_{max}/r_\ell) < 0$

  $G(3) = r_{te}/r_u - 1 < 0$

  $G(4) = 1 - r_{te}/r_\ell < 0$

  where r_u, r_ℓ, t_u, and t_ℓ are the specified upper and lower bounds of maximum thickness and the tail thickness, respectively

- **Body Shape Representation:**

  1. Rounded Nose and Tail

$$r = \frac{\ell}{2}\left\{\frac{A_{N+1}}{2}(\theta + \sin\theta) - \frac{A_{N+2}}{2}(\theta - \sin\theta) + \frac{A_1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right) + \sum_{n=2}^{N} A_n\left[\frac{\sin(n-1)\theta}{n-1} - \frac{\sin(n+1)\theta}{n+1}\right]\right\}$$

  2. Rounded Nose only

$$r = \frac{\ell}{2}\left\{A_{N+1}\sin\theta\cos\frac{\theta}{2} + \sum_{n=2}^{N} A_n\cos(n-1)\theta\right\}, \quad \theta = \cos^{-1}\left(\frac{2x}{\ell} - 1\right), \quad \ell = \text{body length}$$

Figure 2

DESIGN ALGORITHM

The design process is started by identifying the design variables from the input shape through a Fourier analysis. By perturbing the design variables (i.e., the Fourier coefficients) one at a time, gradients of the objective function and constraint equations can be calculated. To reduce the computing time, these gradient calculations are made with a small change in the design variables. Typically, the change (to be called the step size) is taken to be 0.1% ~ 0.5% of each design variable, but not less than 0.00035 ~ 0.0005. These gradients are all calculated with the same starting ϕ-values. These ϕ-values are updated if the design is feasible (fig. 3).

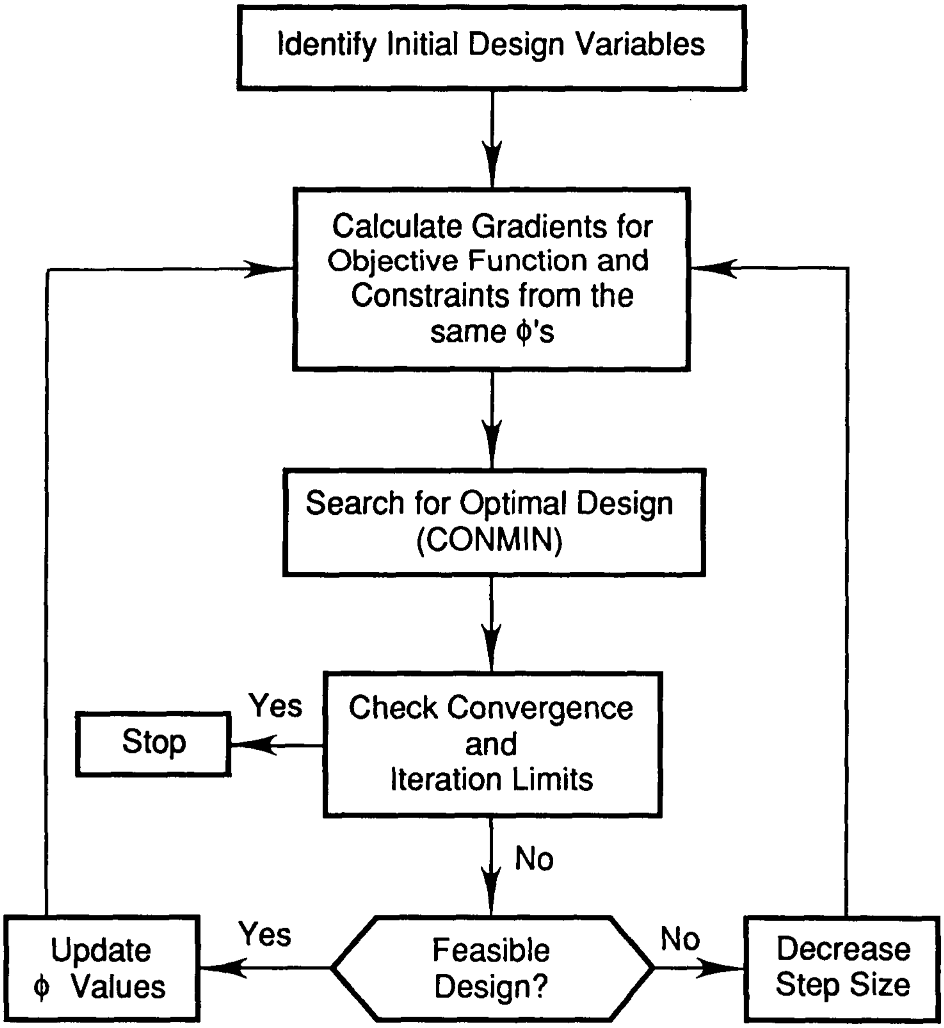

Figure 3

SUMMARY OF SOME OPERATIONAL EXPERIENCE

1. To reduce the computing time, an improved design was chosen by comparing all calculated results at the same numerical accuracy. That is, in analysis, convergence to a very small residual was not demanded. Instead, evaluation of all gradients was based on convergence to the same maximum residual. All starting values of the velocity function (ϕ) for gradient calculation were the same. The velocity function was updated only when the design has improved.

2. Typically, 2 to 3 iterations were performed in each run to allow manual adjustment in step size for gradient evaluation. If the design was not improved, the step size should be reduced. For this purpose, the solution was always saved in a file for possible re-use.

3. Since a smooth input shape was highly desirable, it was found advantageous to Fourier-analyze the input shape separately and then use the resulting Fourier coefficients, or modified values if desired, to generate a starting shape with more defining coordinate points.

4. All design exercises have been achieved with 81 x 81 grid points. Attempt with 41 x 41 grid points has not been successful.

DESIGN OF AN AXISYMMETRIC BODY WITH A FINENESS RATIO OF 8.33 IN A NONUNIFORM FLOW

The initial shape was assumed to be given by the NACA-0012 contour. Six design variables (A_n) were used. The tail thickness was constrained to be between 0 and 1%. The Mach number in the external flow was 0.98, and that over the body was 0.995. The results in figure 4 indicated that reducing the nose radius and increasing the thickness in the aft portion would reduce the wave drag (c_{d_w}) by 29%.

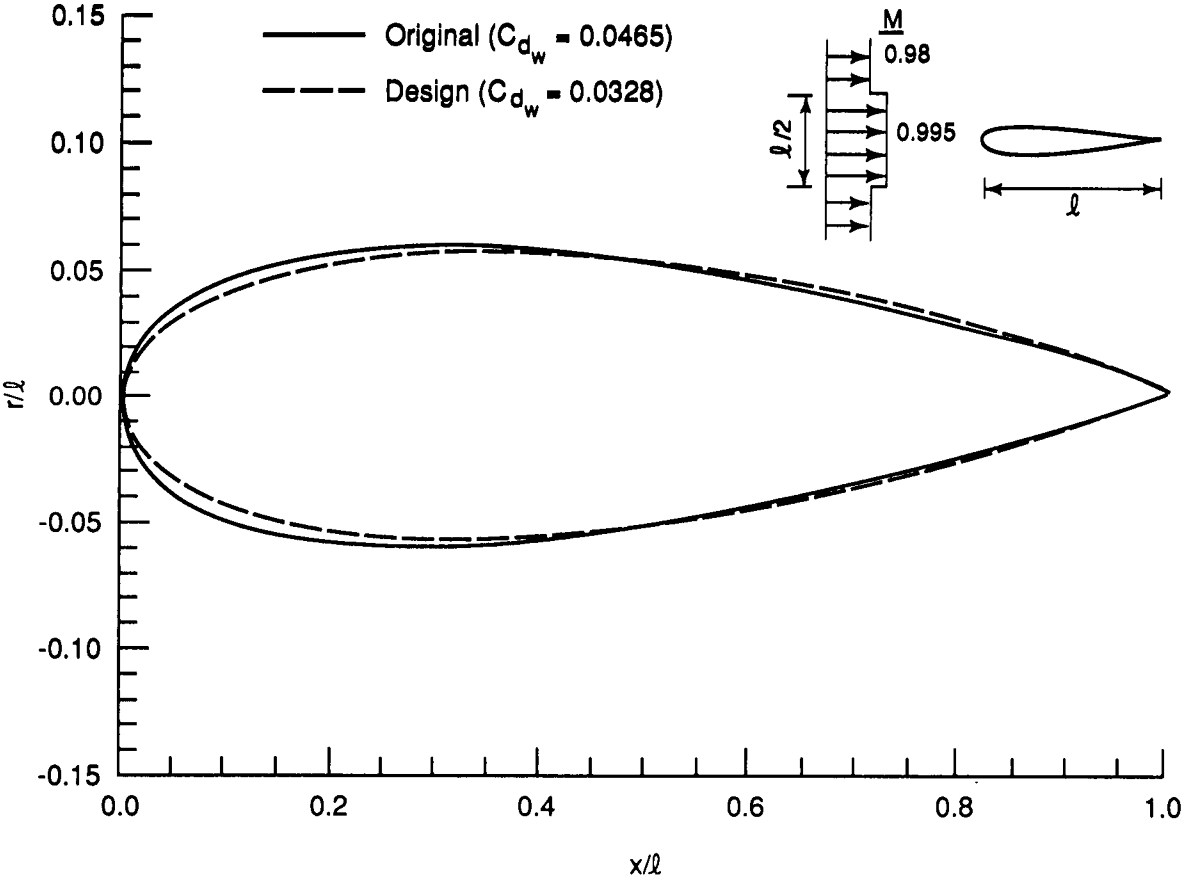

Figure 4

DESIGN OF AN AXISYMMETRIC BODY WITH A FINENESS RATIO OF 5.0 IN A TRANSONIC UNIFORM FLOW

The initial shape was generated from the NACA-0020 contour. The freestream Mach number was 0.925. The step size used in gradient evaluation was 0.1% of the design variables with a minimum change of 0.0005. The results in figure 5 showed that to reduce the wave drag, the pressure peak in the nose region must be reduced. As a result, the shock strength could also be decreased. A reduction in wave drag by 41% was achieved.

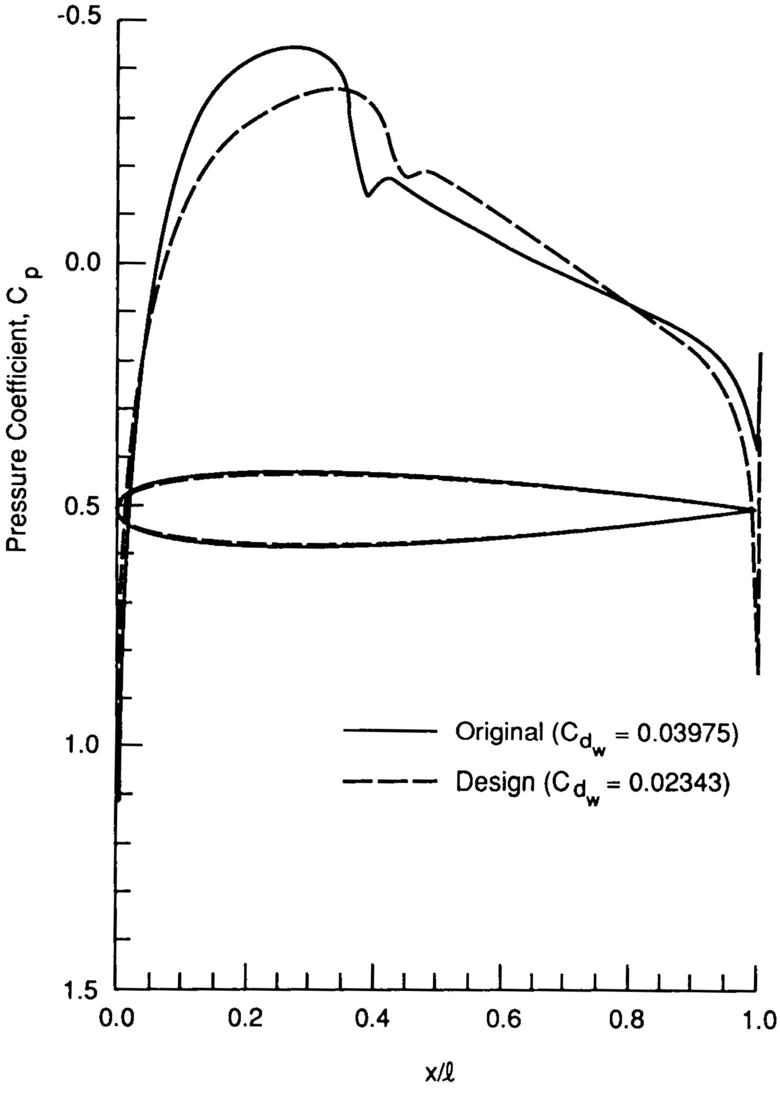

Figure 5

DESIGN OF AN AXISYMMETRIC BODY WITH A FINENESS RATIO OF 5.0 IN A TRANSONIC UNIFORM FLOW

The final body shape given in figure 6 indicated, again, that to reduce the wave drag, the nose radius should be reduced and the maximum thickness location moved aft. Further improvement could be made only if a larger tail thickness was allowed.

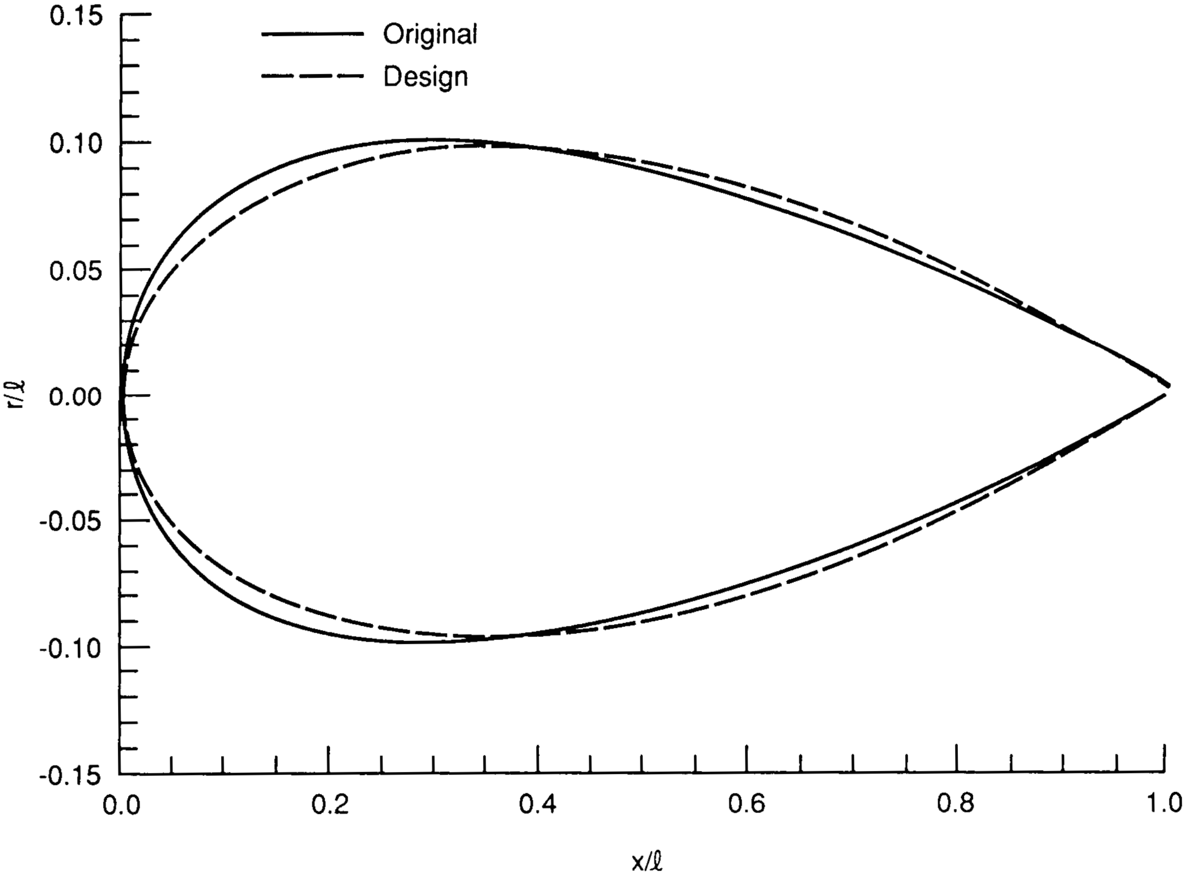

Figure 6

DESIGN OF AN AXISYMMETRIC BODY WITH A FINENESS RATIO OF 5.0 IN A TRANSONIC NONUNIFORM FLOW

The initial shape was again generated with the NACA-0020 contour. The freestream Mach number varied from 0.90 away from the body and a 0.95 near the body, with an average of 0.925. The step sizes in gradient evaluation used ranged from 0.1% at the beginning to 1%. The final value used was 0.5%. It was found that if the step size was greater than 0.5%, little improvement could be made. The results in Figure 7 showed that the change in shape successfully reduced the shock strength. The wave drag was reduced by 65%. A larger drag reduction was possible in this case perhaps because the external Mach number was lower than that in figure 5.

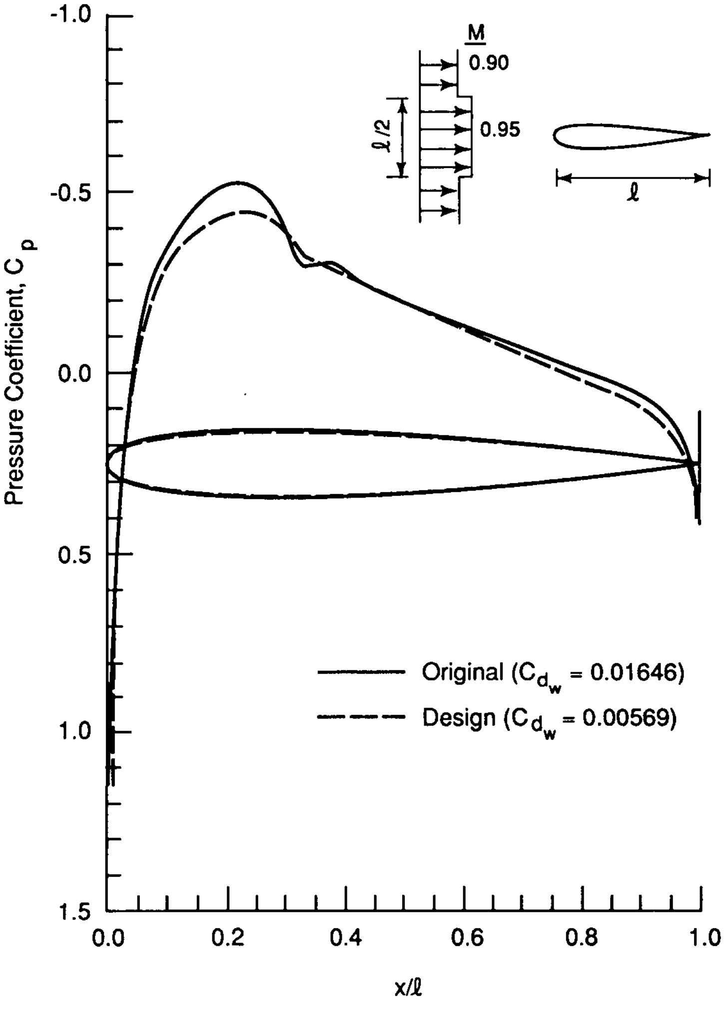

Figure 7

Again, a favorable shape was one with a reduced nose radius and a thicker aft portion as shown in figure 8. Further change was difficult because of the constraints of maximum thickness and tail radius.

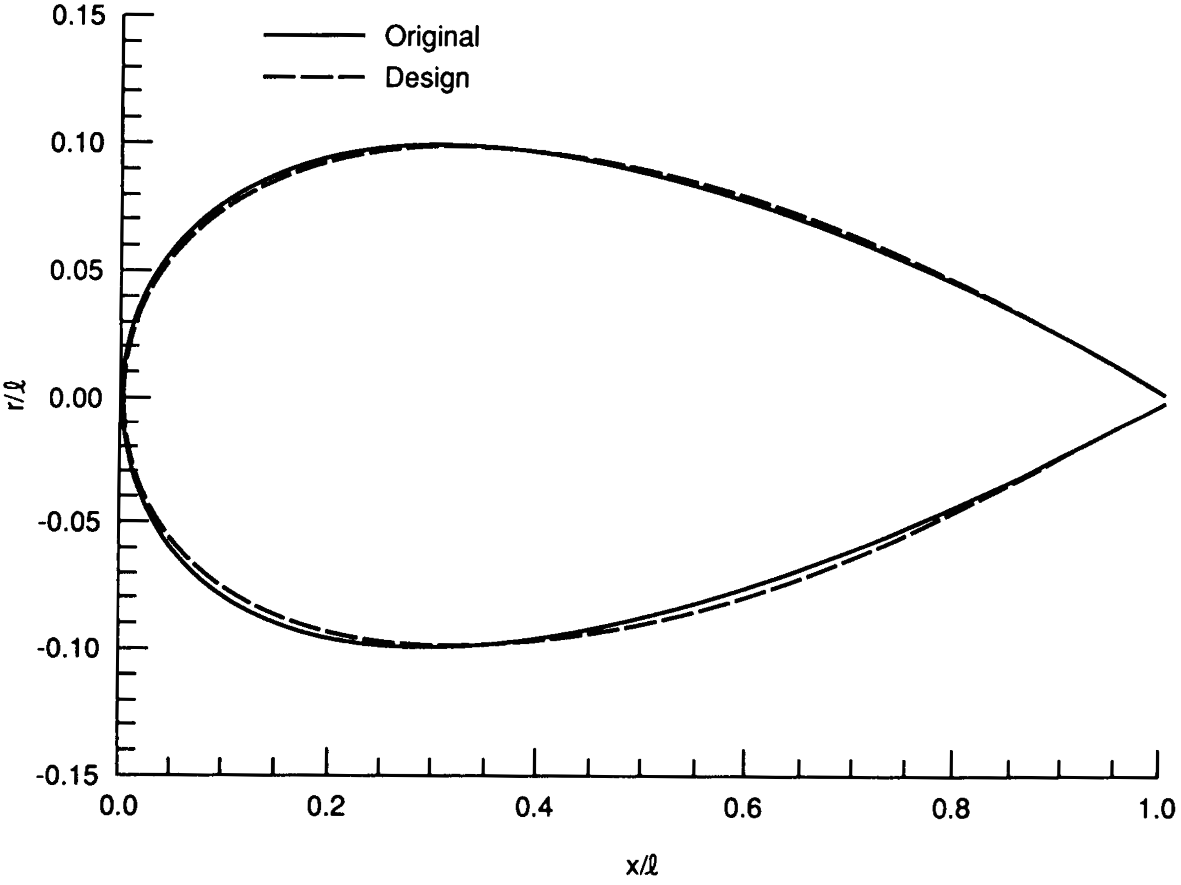

Figure 8

CONCLUSIONS

An inviscid transonic code capable of designing an axisymmetric body in a uniform or nonuniform flow was developed. The design was achieved by direct optimization by coupling an analysis code with an optimizer.

Design examples were provided for axisymmetric bodies with fineness ratios of 8.33 and 5 at different Mach numbers. It was shown that by reducing the nose radius and increasing the afterbody thickness of initial shapes obtained from symmetric NACA four-digit airfoil contours, wave drag could be reduced by 29% for a body of fineness ratio 8.33 in a nonuniform transonic flow of $M = 0.98$ to 0.995. The reduction was 41% for a body of fineness ratio 5 in a uniform transonic flow of $M = 0.925$ and 65% for the same body but in a nonuniform transonic flow of $M = 0.90$ to 0.95.

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