The Role of Optimization in the Next Generation of Computer-based Design Tools

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There is a close relationship between design optimization and the emerging new generation of computer-based tools for engineering design. With some notable exceptions, the development of these new tools has not taken full advantage of recent advances in numerical design optimization theory and practice. Recent work in the field of "design process architecture" has included an assessment of the impact of next-generation computer-based design tools on the design process. These results are summarized, and insights into the role of optimization in a design process based on these next-generation tools are presented.

Design optimization can be integrated into "intelligent" computer-based design tools as a constraint propagation mechanism. Design optimization techniques that offer significant potential for constraint propagation in next-generation computer-based design tools include the Schmit-Fleury technique for handling discrete constraints. Decomposition techniques provide a means for controlling the extent of constraint propagation in an "intelligent" computer-based design tool. The sensitivity of optimal solutions to problem parameters can be used to balance parallel constraint propagation tasks and to manage the iteration between levels of a multi-stage constraint propagation scheme.

Optimal sensitivity derivatives, combined with the use of the Schmit/Fleury technique to propagate design constraints, provide an algorithm for breaking a complex engineering problem into a sequence of design decisions. Why do this? The end product of design is not the "product definition". The result of a design effort is the understanding of the design issues gained by the design team. This understanding is represented by the signature of an engineer - certifying that in his or her professional opinion, the design is safe and will perform satisfactorily. This understanding is considerably enhanced through the execution of a step-by-step decision process. The development of a sequence of design decisions leading to an optimal (or near-optimal) solution can be thought of as one aspect of meta design - designing the design process.

An example problem has been worked out to illustrate the application of this technique. The example problem - layout of an aircraft main landing gear - is one that is simple enough to be solved by many other techniques. Although the mathematical relationships describing the objective function and constraints for the landing gear layout problem can be written explicitly and are quite straightforward, an approximation technique has been used in the solution of this problem that can just as easily be applied to integrate supportability or producibility assessments using theory of measurement techniques into the design decision-making process.

The design decision making process must be adapted to changes in requirements, goals and criteria, as well as to changing technologies. The primary way of accomplishing this adaptation is through design process planning. The "design of the design process" (meta design) is based on the structure of the problem. As used here, problem structure may include connectivity, monotonicity, and sensitivity of subproblem optimal solutions to problem parameters. The
problem structure information is developed by the design team (which has to include manufacturing and support personnel, as well as the aerodynamicists, operations analysts, structural analysts, vendors, and others currently closely involved with the design project group), in a "Generate Design Alternatives" step of the design process (Figure 1). The meta design step ("Plan Design Decisions") was envisioned in reference 1 as taking place after design alternatives had been generated (and captured in a "design-in-process" object/knowledge/data base), and before design decisions were made.

![Figure 1. "A/I Study" (ref. 1) design process.](image)

Some alternative approaches to accomplishing the meta design step have been investigated in the current study. The results of this effort have clarified the iterative nature of the relationship between meta design and design decision-making. This point of view is emphasized in Figure 2. Here the arrows (indicating sequence, not information flow) go both ways between meta design and design decision-making.

![Figure 2. Meta design and decision-making are intertwined.](image)

This study outlines one approach to accomplishing the development of such design tools. The approach is based on exploring a view of the "subtext" of design: the things designers do without thinking about them. The work so far has concentrated on decision support, i.e., once a design concept is defined, how does the designer plan and execute a design decision-making process? It should be emphasized that work on decision support for design should not take place in a vacuum relative to work on how design concepts are defined. Work supported by the NASA SBIR (Small Business Innovative Research) program and monitored through NASA Ames Research Center has addressed this need so far, and is also providing a vehicle for testing these ideas in prototype software (reference 2).

The basic idea is to formalize the design process planning and decision-making steps that designers go through. This approach provides an explicit mathematical model of the design decision-making process. Such a model is then accessible to analysis and criticism. Although concepts in this decision model are explained by reference to examples from "traditional" aircraft design, the nature of the model allows producibility and supportability considerations to be brought into the trade-off process at the same level as performance.

**Design Planning and Meta Design**

A procedure for meta design must take information from a description of alternative design concepts and formulate an interrelated set of design decisions based on this information. This seems to be a "trial and error" process now. The idea of a formal, structured approach to design process planning is not new. Several attempts have been made to come up with algorithms that use a model for the structure of the design-in-progress and some heuristic rules that (one hopes) characterize a desirable design process to structure design decisions.
Techniques for meta design that have been used in the past have included influence diagrams, interpretive structural modelling (ISM), the design structure system, monotonicity analysis, constraint propagation, and multilevel optimization using linear decomposition (MOLD). The techniques attack the problem at different stages. Some of the techniques, such as the design structure system, MOLD, constraint propagation, and monotonicity analysis, are primarily concerned with the individual design parameters and equations for design goals and constraints. Other techniques, such as influence diagrams and ISM, work at the level of design decisions.

An interesting contrast can be drawn between these various techniques based on their use of connectivity information contained in a network description of the design concept or decision process. MOLD, ISM, and the design structure system base their construction of a design decision-making plan primarily on this topological connectedness. On the other hand, monotonicity analysis is based on a mathematical analysis of the equations describing the design concept. Constraint propagation lies somewhere in the middle, using the topological connectedness as the basis for setting up a computational agenda (ref. 3) for solving the equations. Influence diagrams use the connectivity of relationships among design attributes, goals, and decisions to construct joint probability distributions for the effect of uncertainty in design attributes (or design evaluation results) on the attainment of design goals. A preliminary assessment of each of these techniques in terms of their application to meta design follows.

Influence diagrams (ref. 4) model all the elements of the design process, including design alternatives, goals, and design decisions. Relationships among these elements are modelled as probabilities. Before the influence diagram can be drawn, however, the basic decision structure must be known. Since finding this decision structure is one of the aims of meta design, the influence diagram technique does not appear to have immediate application to the meta design problem. There is, however, an interesting potential application of influence diagrams in assessing proposed decision structures for robustness against various known or suspected uncertainties. Such an assessment could be applied to eliminate design decision structures that are vulnerable to uncertainty from further consideration in the meta design process.

Interpretive Structural Modelling (refs. 5, 6) is based on the idea of exploiting priorities among decisions to construct a decision-making plan. ISM thus works at the level of design decisions. This level could be interpreted as choosing values for individual design attributes, or as determining the values for several tightly coupled attributes as part of an integral decision-making task. The method assumes a partial ordering of the decisions (i.e., the design decision-making process is thought of as a directed graph), and the structure of the decision-making plan is based almost entirely on this ordering. ISM addresses precisely the meta design problem. However, there are some limitations to the technique that must be addressed. First, since the decision structuring rules are strongly based on the idea of a partial ordering, we cannot model the process of "backing numbers out", i.e., inverting the design relationships, which is pervasive in aircraft design.

The design structure system (ref. 7) is based on the idea that if feedback from downstream choices is eliminated, a workable decision sequence can be identified. Structuring the adjacency matrix of the design problem to eliminate feedback loops results in a block-diagonal structure that is interpreted as defining the decision network. Problematically, the decision structure system algorithm may produce a single, highly coupled design decision when applied to aircraft preliminary design problems.

Monotonicity analysis (ref. 8) has been applied to identify active constraints and global monotonicities or degeneracies. Problems which are constraint-bound and can be solved as a system of simultaneous nonlinear equations can also be identified using this technique. The monotonicity analysis approach to identifying design decision structure introduces the
important idea that the analytic properties of design relationships (i.e. upper and lower bounds for independent variables and the signs of derivatives, which determine monotonicity properties) must enter into a successful method for structuring the design process, along with the topology (i.e. connectedness) of the network of design attributes and relationships among them.

Multilevel optimization by linear decomposition (refs. 9, 10, 11) introduces several important ideas. Decisions are modelled as mathematical optimization problems. A parameter passing algorithm is put forward to describe the iterative decision-making process. This approach has been further investigated (ref. 12), in which the non-hierarchical nature of complex design decision networks was illustrated by an in-depth example. Ref. 12 also explored an alternative way to group problem description elements into design decisions.

Even from this limited review of design process planning methodologies, it is clear that some model for the design decision-making process is needed. The model chosen as the basis for a meta design technique must be comprehensive enough to deal with discrete and continuous decision parameters, and with qualitative as well as quantitative design requirements, goals, and criteria. Means for distinguishing between desirable and undesirable design decision-making processes, compatible with the model for the design decision-making process, must be available.

The choice of mathematical optimization as a model for the design decision-making process has several advantages. Consideration of the design decision-making plan (the result of meta design) as an optimization procedure allows mathematically rigorous stability and rate of convergence criteria to be applied to distinguish between desirable and undesirable design decision-making processes. Design optimization is actually used in design practice, so that a meta design procedure based on this technique would not require an entirely new design toolkit to be transitioned to the design community. The techniques for analyzing parameter passing schemes in terms of optimal sensitivity derivatives can be readily applied to evaluate a prospective design decision-making plan. Finally, dual methods for design optimization are available that fit remarkably well with the meta design approach. Extensions to these methods are available that allow discrete parameters to be included in the decision-making process.

The most significant limitation of the design optimization model is, of course, the question of how to handle qualitative design considerations. Numerical ranking of alternative designs against qualitative criteria is one possible solution. "Quantifying the qualitative" raises significant issues in itself (ref. 13). However, a convincing argument can be made for including this approach as an integral part of the meta design procedure proposed in this study. The argument is made on the following points: First, the use of numerical rankings for qualitative criteria is well established in design practice (e.g., Cooper-Harper ratings, ref. 14. See also refs. 15, pg. 8-5; and 16). Second, representation and uniqueness problems almost certainly limit the applicability of measurement techniques to relative comparisons between design alternatives. Absolute predictions of product characteristics should probably not be based on these rankings alone. However, wind tunnel measurements of airplane drag are subject to this same restriction (ref. 17), and the wind tunnel is considered to be an invaluable tool for aircraft design.

The proposed technique for meta design will be described in detail by means of an aircraft layout example (see "Executing the design decision-making process", below). The technique is strongly based on the idea of structuring the design decision-making process to achieve stable convergence of the corresponding design optimization problem. The design process is structured dynamically as part of the solution to the design problem itself. To summarize the key elements of this approach:

- Model decisions as mathematical optimization problems.
- Use parameter passing to model the decision sequence.
- Handle uncertainty through error-bands on the location of constraints.
- Sequence decisions so that constraints imposed by previous decisions are not infeasible in later ones.
- Restrict allowable parameter passing topologies to those that are stable according to (ref. 12) (and other stability criteria).
- Formulate decisions dynamically as part of the solution process.
- Use dual methods to handle discrete parameters.
- Apply the theory of measurement to quantify qualitative attributes and relationships.

The technical approach for evaluating whether these recommendations are valid is that the resulting meta design approach should correspond to traditional design practices in interesting ways. Also, it must be clear how to bring qualitative assessments of downstream producibility and supportability issues into the trade-off process and to handle uncertainty in these assessments.

**A Formal Algorithm using the Meta Design Approach**

The problem is to select values for design decision attributes while meeting requirements, goals and criteria imposed on the design.

Considering this design problem as an optimization problem, we have

\[
\text{Problem } P : \\
\text{Minimize:} \\
\{f_a (x_1, \ldots, x_l), a \in \mathcal{A}\} \\
a \text{ set of multiple objectives,} \\
\text{Subject to:} \\
\{g_b (x_j^1, \ldots, x_j^k) \geq 0, b \in \mathcal{G}\} \\
\text{with } x_{i}^{\text{lower}} \leq x_i \leq x_{i}^{\text{upper}}
\]

(quality constraints could be included in a similar fashion). The main feature of this problem to be emphasized here is that the objective and constraint functions do not usually depend on all of the design decision variables \(x_i, i = 1, \ldots, n\).

The objectives fall into three broad categories (similar to those used by Taguchi, ref. 18), minimize, maximize, and goal. Formally, each type of objective can be handled as a minimization (for maximize objectives \(f\), we minimize \(-f\); for goal objectives, we minimize some measure of the departure from the goal value \(f_{goal}\), such as \((f - f_{goal})^2\)).

In the context of optimization theory, meta design involves partitioning the (large) optimization problem \(P\) into subproblems, and defining a convergent sequence for solving the subproblems. In general, there will be several design decision variables which appear in more than one subproblem. Values for these design decision variables will be determined by the solution of the first subproblem (in the solution sequence) in which these design decision variables appear. If a design decision variable \(x_j\) appears explicitly in a subproblem, but its value is determined outside of that subproblem, it is said to be a parameter in that problem.
Generally speaking, the imposition of any additional (active) constraint on an optimization problem will increase (i.e. worsen) the value that can be attained for minimization of the objectives of that problem. When a design decision variable \( x_i \) appears as a parameter in a problem \( P_j \), a penalty will be incurred in the value of the objective function (relative to the objective function value which could be achieved if \( x_i \) were allowed to vary within the problem \( P_j \)). Sobieski, et al., (ref. 11) have developed techniques for evaluating the rate of change of the objective function of \( P_j \) with respect to changes in the value of \( x_i \) (as a parameter), subject to the conditions that optimality continues to hold and the active constraint set does not change. This sensitivity of optimal solutions to problem parameters technique provides us with a tool to assess the penalties associated with selecting a given solution sequence for the subproblems. This approach is referred to as a parameter passing technique, since the values of design decision variables determined by solution of subproblems are passed as parameters to subsequent subproblems.

There are a number of different ways to implement a meta design technique based on parameter passing, depending on the details of the formulation of the subproblems. Sobieski, et al. have investigated several alternative approaches (refs. 9 and 10), including formulating a penalty function from the constraints of a subsequent problem, \( P_2 \), and using the optimal sensitivity derivative of this penalty function to define a linear constraint in the prior problem, \( P_1 \). In this approach, \( P_1 \) (the prior problem) is then re-solved with the new "optimal sensitivity" constraint in order to force selection of a value of the parameter that will lead to a feasible solution downstream when \( P_2 \) is finally re-solved.

Two somewhat different approaches are presented here. The first approach represents an alternative to existing goal-programming techniques for solving multiobjective optimization problems. The second approach is based on the idea of constraint propagation. As mentioned earlier, the primary difference between the constraint propagation and goal-programming approaches is in the partitioning and formulation of the subproblems.

Each objective function \( f_a \) is assigned to a single subproblem \( (P_a) \) in the goal-programming type approach. The constraints belonging to this subproblem are those which have one or more design decision variables in common with the set of design decision variables appearing explicitly in \( f_a \) (this set of constraints is denoted \( Q_a \)). The total set of design decision variables for the problem \( P_a \) includes all design decision variables appearing in any of these constraints or in the objective function \( f_a \), i.e.:

\[
\text{Problem } P_a : \\
\text{Minimize:} \\
f_a (x_{i_1}, \ldots, x_{i_r}) \\
\text{Subject to:} \\
\{g_b (x_{i_1}, \ldots, x_{i_r}, x_{j_1}, \ldots, x_{j_s}) \geq 0, \ b \in Q_a \} \\
\text{with } x_{i_{\text{lower}}} \leq x_i \leq x_{i_{\text{upper}}}, \ i \in \{i_1, \ldots, i_r, j_1, \ldots, j_s\} \\
(\text{of course, not all of the } x_i \text{'s have to appear explicitly in any given } g_b. \text{ By definition, } x_{i_1}, i \in \{i_1, \ldots, i_r\} \text{ all appear explicitly in } f_a, \text{ and none of the } x_i, i \in \{j_1, \ldots, j_s\} \text{ appear explicitly in } f_a. \text{ This second group of design decision variables are the ones that come into the problem through the constraints.})
Subproblems formulated in this way may still be relatively large in terms of design decision variables. Using the idea of constraint propagation, we can formulate a subproblem corresponding to each design decision variable $x_i$ as follows: Include each objective function and constraint in which $x_i$ appears explicitly as part of the subproblem. The other design decision variables appearing explicitly in these objective and constraint functions will have to be included as well. Thus we end up with:

Problem $P_i$:

Minimize:

$$\{ f_a (x_i, \ldots, x_{i_r}), a \in \mathcal{F}_i \}$$

a set of multiple objectives,

Subject to:

$$\{ g_b (x_{k_1}, \ldots, x_{i_r}, \ldots, x_{k_s}) \geq 0, b \in \mathcal{G}_i \}$$

with $x_{k_{lower}} \leq x_k \leq x_{k_{upper}}$ $k \in \{i_1, \ldots, i_r, \ldots, i, \ldots, k_1, \ldots, k_s\}$

Once the subproblems have been formulated, the basic idea of the meta design procedure is as follows (Figure 3): Each of the optimization problems $P_i$ are initially assigned to determine a value for one or more of the decision design attributes $x_i$. Which attribute will be assigned to which problem, and the sequence in which the problems are to be solved, will be determined by comparing the solution to an optimization problem $P_i$ in isolation, to the solution obtained when a decision design attribute $x_p$ has been passed as a parameter from another optimization problem $P_j$ (i.e. $x_p$ is set equal to some value $x_{p0}$ by solving $P_j$ and then $P_i$ is solved subject to the additional constraint $x_p = x_{p0}$). A penalty associated with this parameter passing sequence is determined using the optimal sensitivity derivative (ref. 11) for the objective function of problem $P_i$ with respect to the parameter $x_p$ and the difference between the optimal value for the design decision variable $x_p$ when $P_i$ is solved in isolation (call this value $x_{p*}$) and $x_{p0}$ (the value of $x_p$ determined by solution of $P_j$). For a given parameter passing scheme (including both: [1] the assignments of the determination of the decision design attributes to the optimization problems, and [2] the solution sequence for the optimization problems) a net penalty is calculated by summing the penalties associated with each parameter passing step. An optimal design process plan is one which minimizes this net penalty.

Executing the design decision-making process

The design problem of determining the location of the main landing gear (MLG) on an unlimited-class experimental racing aircraft (Figure 4) will be analyzed using the meta design approach. This design problem is extremely simple, yet elements of "quantifying the qualitative" are present. Producibility and operational characteristics are also significant elements of the problem.
Solve:
\[ P_i \]
\[ x_p = x_p^* \]
\[ x_p = x_p^0 \]

Solve:
\[ P_j \]
\[ x_p = x_p^0 \]

Solve:
\[ P_i \]
with \( x_p = x_p^0 \)

Compute:
\[ \frac{df}{dx_p} = \frac{\partial f}{\partial x_p} - D^T \frac{\partial G_{active}}{\partial x_p} \]

where:
- \( f \) - objective function of problem \( P_i \)
- \( D \) - vector of Lagrange multipliers of active constraints (same as optimal values of dual variables)
- \( G_{active} \) - vector of active constraint functions

\[ \text{Penalty} = \left( \frac{df}{dx_p} \right) (x_p^* - x_p^0) \]

Figure 3. Overview of meta design Procedure.
Figure 4. Unlimited-class experimental racing aircraft.

Landing Gear - Arr. 3

Figure 5. Landing Gear Track Angle.

Landing Gear - Arr. 4

this angle must be small

Figure 6. Landing Gear "Tail Down" Angle.
In design terms, early integration of producibility and supportability into the design process involves making design decisions that make the performance, cost and schedule of the aircraft as insensitive as possible to the details of how it will be manufactured, operated, and maintained. This point of view is based on the Taguchi (ref. 18) definition of quality as robustness to "noise" or uncertainty. Uncertainty refers to factors over which the designer has little or no control.

There are several failure modes that can occur when an aircraft is on the ground or when the aircraft is landing. When the aircraft is taxiing or parked on the airfield, a gust can tip the aircraft over about a line from the main landing gear wheel location to the nose wheel location. In order to avoid this, the track angle (Figure 5) is specified. Reference 19 gives a value of 55 degrees as the maximum allowable for this angle. Another failure mode can occur on landing. High performance aircraft often approach the runway and touch down at relatively high angles of attack. If the center of gravity is behind the vertical plane of the main landing gear wheels, the aircraft will encounter a moment which tends to sit the aircraft on its tail. This occurs as the weight of the aircraft is transferred from the wings to the landing gear. Restricting the angle between the center of gravity and the plane of the main landing gear wheels, as shown in Figure 6, will cause the aircraft to hit its tail on the runway (and presumably bounce back) before it can come to rest in a stable position on its tail.

One final consideration in landing gear location is considered in this example problem. The main landing gear must be retractable and must fit into a small space. Reliability, maintainability and cost will be adversely affected if complex retraction kinematics are required to accomplish this. Thus, minimizing retraction complexity is a goal for this example problem. In order to keep the analysis simple, retraction complexity is considered to be proportional to the distance from the main landing gear wheel to the closest fuselage frame forward of the wheel.

The example problem is then: to locate the main landing gear (in x,y,and z aircraft reference coordinates) in such a way as to minimize the retraction complexity, while satisfying constraints on "tail down angle" and "track angle". Side constraints on each of the design variables MLGx, MLGy, and MLGz, are derived as follows. The x coordinate of the aircraft c.g. (center of mass) location is at 9.5 ft. Thus a lower bound for MLGx is set at 10 ft. The upper bound is set at 15 ft. In a more detailed example, the upper bound would be related to the horizontal tail volume coefficient required to rotate the aircraft on takeoff. The lower bound for MLGy is set by the fuselage envelope at 1 ft. The upper bound for MLGy is related to the wing span and is set at 15 ft. The lower bound for MLGz is set by the shock strut length, which is related to the aircraft weight, design sink rate on approach, and landing gear design load factor, and is set at 4 ft. The upper bound for MLGz is related to maintenance access and is set at 7 ft. The statement of the example problem as an optimization problem is then:

Minimize: retractionComplexity (MLGx,MLGy,MLGz)

Subject to:  
trackAngle (MLGx,MLGy,MLGz) ≤ 55
tailDownAngle (MLGx,MLGy,MLGz) ≥ 15

10 ≤ MLGx ≤ 15
1 ≤ MLGy ≤ 15
4 ≤ MLGz ≤ 7
Figure 7. Alternative Landing Gear Arrangements.

Constructing Approximations

Simple mathematical expressions for retraction complexity, track angle and tail down angle as a function of MLGx, MLGy, and MLGz can be derived from the aircraft geometry. In the meta design procedure proposed here, these simple relationships are not used directly, but are approximated by functions that are linear either in the design variables, $x_i$, or in their inverses, $1/x_i$. As discussed in reference 20, this form of the approximations is separable and thus permits an explicit solution for the primal design variables in terms of the dual variables. The approximations are constructed by curve-fits to evaluation of the objectives and
constraints on a set of alternative landing gear arrangements, as shown in Figure 7.

The process of constructing these approximations provides insights that are similar to those obtained through monotonicity analysis, although the approximations are only valid near the region of design space defined by the alternative configuration arrangements selected.

It should be emphasized that the use of these approximations allows performance and cost requirements to be balanced against a broad range of producibility and supportability requirements. Using the approximation technique, any aspect of the design that can be evaluated at some level can be brought into the trade-off process.

At least for this example problem, approximations can be found which provide a reasonably accurate qualitative picture of the design space (Figures 8 to 10). For meta design, this is probably accurate enough. These qualitative results are only valid locally, however, and can be quite inaccurate when extrapolated very much beyond the region of design space near the alternative configurations which were evaluated to construct the approximations (Figure 11).

In order for the dual objective function to be well-defined, the primal optimization problem must be convex (ref. 21). For the form of the approximations used here, this means that the coefficients of the design variables (or their inverses) must be positive if the constraint is "greater than or equal to". In the present study, this criterion was used to determine whether a constraint or goal design attribute was directly or inversely proportional to a decision design attribute. This approach was an unqualified success, as the correct (from the point of view of monotonicity) form of the approximation was deduced on the basis of this criterion in every case. (It is difficult if not impossible to ascertain this kind of information from the design evaluations alone for most arrangements of design points. An exception is, of course, a finite difference grid.)

![Figure 8. Retraction Complexity is (roughly) independent of MLGx.](image-url)
Formulation and Solution of Optimization Problems.

The initial structure of design decision-making tasks is based on the idea that goals (such as minimize retraction Complexity) and constraints (such as tailDownAngle) are propagated through choices of the design parameters MLGx, MLGy and MLGz (Figure 12). Thus the initial problem structure is as shown in Figure 13. There is a correspondence between optimization subproblem P1 and design variable MLGx in that problem P1 propagates goals and constraints that are linked in the "attribute-relationship diagram" of Figure 12 by

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Figure 9. Track Angle is an increasing function of MLGx.

Figure 10. Tail Down Angle as a function of MLGz.
the design variable MLGx. The meta design problem is to determine which of the optimization subproblems P1, P2 and P3 should determine which of the design variables MLGx, MLGy, and MLGz, and in what sequence. Surprisingly, for this example, a design decision plan in which P1 determines MLGx, P2 determines MLGy, and P3 determines MLGz is not optimal.

Figure 11. The approximations are not usually accurate globally.

Figure 12. Attribute-relationship diagram for example problem.

The first step in the meta design procedure is to solve the individual optimization/constraint propagation problems P1, P2, and P3 as self-contained problems. This can be done very easily using the approximate forms for the goal and constraint design attributes. The method of ref. 20 is used here, with some changes. The first change is that the problems have been solved explicitly, rather than numerically, since this solution is straightforward. The second difference involves a simple extension of Schmit and Fleury's technique to handle a slightly broader range of forms for the constraint and goal approximations.

The initial solutions are

\begin{align*}
\text{P1:} \\
\text{d}_{1} &= 0, \text{d}_{2} = 0
\end{align*}
MLGx = 15, MLGy = 15, MLGz = 4.

P2:
d_1 = 0.1497
MLGx = 10, MLGy = 4.189, MLGz = 4

P3:
d_1 = 0.1614, d_2 = 0.02626
MLGx = 10.39, MLGy = 4.351, MLGz = 4

Problem 1
Satisfy:
- track angle \leq 55
- tail down angle \geq 15

Problem 2
Minimize:
- retraction complexity
Subject to:
- track angle \leq 55

Problem 3
Minimize:
- retraction complexity
Subject to:
- track angle \leq 55
- tail down angle \geq 15

Figure 13. Optimization problem structure.
Evaluation of Parameter Passing Schemes

Once initial solutions to the constraint propagation/optimization problems P1, P2, and P3 have been obtained, we can begin to assess parameter passing schemes. The approach to accomplishing this is outlined in Figure 3. The purpose of computing the optimal sensitivity derivatives is to determine the penalties associated with each parameter passing scheme that is of interest (see, for example, Figure 14, where the penalties associated with passing MLGx as a parameter are labels on the directed arcs along which MLGx would be propagated).

Parameter passing schemes can be evaluated using the penalties summarized in Figure 14, Figure 15 and Figure 16. This has been done in Figure 17 for the most obvious parameter passing scheme (i.e. determining MLGx from problem P1, MLGy from P2, and MLGz from P3).

![Figure 14. Penalties associated with passing MLGx as a parameter.](image)

The penalty associated with passing MLGx as a parameter from P1 to P2 (Figure 17) is found to be -4.862 using Figure 14. The penalty for setting MLGy by solving problem P2 and passing the result to problem P1 is found (from Figure 15) to be 0. The net penalty for this parameter passing scheme is determined by summing individual penalties over all the arcs of the Figure 17 parameter passing diagram. If the net penalty is negative, the larger the absolute value of the net penalty, the more the solution of the global (i.e., including the whole network of problems) constraint propagation problem will depart from the locally (i.e., considering the individual problems in isolation) optimal solutions for the individual problems. The penalty calculated for the "naive" parameter passing scheme of Figure 17 is actually an upper bound for the loss of optimality. There may be some sequences of solution of the problems P1, P2, and P3 in which optimal solutions of prior problems may lead to infeasibility of subsequent ones. If this happens, the net penalty goes to $-\infty$.

An improved parameter passing scheme for constraint propagation (Figure 18) can be found by thoughtful inspection of Figures 14, 15, and 16. In this scheme, P3 is solved first to find MLGx. The value 10.39 for MLGx is then passed as a parameter to problems P2 and P1. Solution of P2 then yields the value 4.351 for MLGy. Since this value is the same as the optimal value of MLGy for problem P3, there is no penalty associated with passing MLGy as a parameter from P2 to P3 (even though the optimal sensitivity derivative is not zero in this
The solution to problem P1 is still feasible with $MLG_x = 10.39$ and $MLG_y = 4.351$, so there are no penalties associated with passing these parameters to problem P1.

Figure 15. Penalties associated with passing $MLG_y$ as a parameter.

Figure 16. Penalties associated with passing $MLG_z$ as a parameter.
The working hypothesis on the application of parameter passing and constraint propagation ideas to meta design has been that: 1) an optimization problem could be associated with each design variable, 2) optimal sensitivity derivatives could be used in a straightforward way to identify a stable and convergent sequence for solving these problems. This example clearly indicates that the meta design problem is more subtle than was originally thought. At the same time, the example indicates that the meta design idea can be integrated with parameter selection techniques, such as Taguchi methods, through the use of approximations to goal and constraint design attributes. This approach has significant potential as a technique for performing trade-offs between performance, cost, schedule, producibility and supportability. In view of this, further investigation along the lines of this overall approach is likely to be productive. Thus it is important to highlight some of the questions raised by this example.

The formulation of the problems P1, P2, and P3 was based on the constraint propagation idea. P1 propagates constraints that are linked by MLGx, P2 those linked by MLGy, and P3 the constraints linked by MLGz. Yet, in the optimal parameter passing solution, P1 determines MLGx, and P3 determines MLGy, even though MLGx does not appear explicitly in the objective function for P3. This may be due to the fact that MLGz is in direct proportion to every constraint and goal design attribute where it appears, and thus is driven to its lower bound of
4. Thus, as a result of monotonicity, manipulating MLGz does not give the designer much leverage. Yet, from a constraint propagation point of view, MLGz is pivotal. P3 contains all of the design attributes (goals, constraints, and decision parameters) in the original problem. The fact that the constraint propagation problems derived from one of the decision variables are used to determine other decision variables also suggests that the technique of problem formulation requires further critical examination. Perhaps the point of view that the meta design process can indicate which information is required to determine which design decision attribute is more appropriate. Connectivity could then be used to show that it is not necessary to consider all possible permutations and combinations of constraints, goals, and design variables.

The size of the example problem was necessarily quite small. Several interesting potential applications of the meta design approach are slightly outside its scope. For example, the dual method of Schmit and Fleury has been extended (ref. 20) to handle discrete variables. Demonstration of this capability would require only a marginal increase in problem size, to include, say, number of fuselage frames as a design variable. The discrete variable capability also makes it possible to bring qualitatively different design choices (by quantifying them using some representation). This could be investigated by including the issue of whether the landing gear should be attached to wing or fuselage as a decision variable (with discrete, specifically binary values).

Another aspect of the meta design procedure is that the meta design process clearly produces a solution to the design problem. How this would work on a larger, more complex problem is certainly of interest. Closely related to this issue is the approach that has been used in this study: the meta design technique has been developed and refined in this study through application to real (if extremely simple) design problems. Further investigations into meta design should follow the same basic approach: research priorities should be set by problems encountered in trying to apply the technique to design problems that are as realistic as possible.

The approximations to the constraint and goal design attributes were constructed from evaluations of candidate designs without using the analytical relationships. The fact that this aspect of the process was successfully accomplished suggests that the meta design process can now be applied to areas of producibility and supportability where analytical relationships are not available and judgments based on simulation and engineering or operational experience must be used to evaluate the design alternatives.

The decision-based model of the design process used in this study and elsewhere (c.f. ref. 16) promises to shed new light on the relationship between requirements definition and the design process. In fact, requirements are set by decisions. These requirements then appear as parameters in subsequent design decisions. Thus the relationship between requirements setting and design decisions can be studied using the meta design approach. In this context, the example problem implies some remarkable conclusions which merit further study. Considering requirements definition and design as a single, integrated problem to be decomposed using a meta design approach, it is clear that the problem does not have the simple, hierarchical structure (i.e. each requirement cannot be traced back to a single prior decision - see, for example, the complex decision network of ref. 12). The example problem indicates that, using meta design, we may be able to find a sequence of interleaving requirements setting and design decisions for which no iterative reallocation is necessary, assuming all of our information is accurate.

Integrating requirements-setting and design decision-making problems will make it necessary to deal with uncertainty in the values of the goal, constraint or decision design attributes. From the meta design point of view, this uncertainty may change the expected
optimal decision-making sequence, perhaps even to an infeasible one. The meta design approach could be used to investigate the structural stability of the decision-making sequence as attributes subject to uncertainty are varied. This capability suggests a unique tool for balancing technical and schedule risk: the cost of reducing uncertainty in the attributes can be traded off against the schedule impact of a marginally workable decision sequence.

The meta design example also suggests that the problem of multiobjective programming, i.e. finding a Pareto-optimal ("balanced") decision solution when there are several conflicting goals, can be attacked by associating a constraint propagation problem with each goal and determining a decision sequence that balances the parameter passing penalties associated with each optimization subproblem.
References


