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OPTIMIZATION OF STRUCTURE AND CONTROL SYSTEM

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INTRODUCTION

The objective of this study is the simultaneous design of the structural and control system for space structures. This study is focused on considering the effect of the number and the location of the actuators on the minimum weight of the structure, and the total work done by the actuators for specified constraints and disturbance. The controls approach used is the linear quadratic regulator theory with constant feedback. At the beginning collocated actuators and sensors are provided in all the elements. The actuator doing the least work is removed one at a time, and the structure is optimized for the specified constraints on the closed-loop eigenvalues and the damping parameters. The procedure of eliminating an actuator is continued until an acceptable design satisfying the constraints is obtained. The study draws some conclusions on the trade between the total work done by the actuators, and the optimum weight and the number of actuators.

OBJECTIVES

- Minimum weight design
- Simultaneous structural and control disciplines
- Closed–loop damping and eigenvalue requirements
- Effect of the number and location of actuators
- Study of the work done by actuators

OPTIMIZATION PROBLEM

Minimize W, weight of the structure, such that the constraints on the closed-loop frequencies, $\bar{\omega}_i$, and the closed-loop damping, $\bar{\xi}_i$, are satisfied. This optimization problem was solved by using the NEWSUMT-A program, which is based on the extended interior penalty function method with Newton's method of unconstrained minimization.

Structure/Control Optimization Problem

Minimize weight

$$W = \sum \rho_i A_i l_i \qquad (1)$$

Such that

$$g_j(\tilde{\omega}_i) \leq 0 \qquad (2)$$

$$g_j(\xi_i) = 0 \qquad (3)$$

$$g_j(A_i) \geq 0 \qquad (4)$$

Where

$$g_j(\tilde{\omega}_i) = \tilde{\omega}_i - \bar{\tilde{\omega}}_i \qquad (5)$$

$$g_j(\xi_i) = \xi_i - \bar{\xi}_i \qquad (6)$$

$$g_j(A_i) = A_i - \bar{A}_i(\text{min}) \qquad (7)$$

CONTROL THEORY

In the state input Eq. 1 (below) {x} is the state variable vector and {f} is the input vector. The matrices [A] and [B] are the plant and input matrices. The plant matrix is a function of the structural frequencies. Eq. 2 defines the performance index where [Q] and [R] are the state and control weighting matrices. The result of minimizing the performance index and satisfying the input equation gives the state feedback control law given in Eq. 3. The Riccati matrix [P] in Eq. 4 is obtained by an interactive solution of the Algebraic Riccati equation.

Control System Design

State input equation

$$\{\dot{x}\} = [A]\{x\} + [B]\{f\} \tag{1}$$

Performance index

$$PI = \int_o^t \left(\{x\}^T[Q]\{x\} + \{f\}^T[R]\{f\}\right) dt \tag{2}$$

State feedback control law

$$\{f\} = -[\tilde{G}]\{x\} \tag{3}$$

Optimum gain matrix

$$[\tilde{G}] = [R]^{-1}[B]^T[P] \tag{4}$$

Algebraic Riccati equation

$$[A]^T[P] - [P][B][R]^{-1}[B]^T[P] + [P][A] + [Q] = 0 \tag{5}$$

The optimal closed-loop system is defined in Eq. 1 (below). The solution to this equation is given in Eq. 3 where x(o) is the initial value of the state vector at time t = 0. The complex eigenvalues of the closed-loop matrix $[\tilde{A}]$ are defined in Eq. 5 where $\bar{\sigma}_i$ and $\bar{\omega}_i$ are the real and imaginary parts. Eq. 6 defines the closed-loop damping parameter.

Control System Design

Optimal closed–loop system

$$\{\dot{x}\} = [\overline{A}]\{x\} \tag{1}$$

$$[\overline{A}] = [A] - [B][\overline{G}] \tag{2}$$

Solution

$$\{x\} = e^{[\overline{A}]t}\{x(0)\} \tag{3}$$

$$e^{[\overline{A}]t} = 1 + \frac{\overline{A}t}{1!} + \frac{(\overline{A}t)^2}{2!} + \ldots \tag{4}$$

Closed–loop eigenvalues

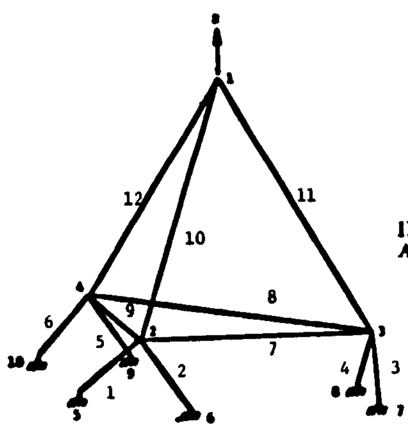
$$\lambda_i = \overline{\sigma}_i \pm j\overline{\omega}_i \tag{5}$$

Damping parameter

$$\xi_i = -\frac{\overline{\sigma}_i}{(\overline{\sigma}_i^2 + \overline{\omega}_i^2)} \tag{6}$$

PROBLEM DESCRIPTION

This figure shows the finite-element model of ACOSS-FOUR. The number along the elements indicates the collocated actuator and sensor numbers. The structure has twelve degrees of freedom, and four masses of two units each are attached at nodes 1 through 4. The constraints imposed on the optimum design are $\omega_1 = 1.341$, ω_2, $\geq$, 1.6 and $\bar{\xi}_1 = 0.2574$. To calculate the work done by the actuators and study the transient response, an initial displacement of unit magnitude is given at node 2 in the x direction at time t = 0. The diagonal elements in the left top half of matrix [Q] are equal to the square of the structural frequencies, and the weighting matrix [R] is an identity matrix.



INITIAL UNIT DISPLACEMENT AT NODE 2 AT T = 0.0

ACOSS FOUR (ACTUATOR LOCATIONS)

MINIMIZE THE WEIGHT WITH CONSTRAINTS ON

$$\tilde{\omega}_1 = 1.341$$

$$\tilde{\omega}_2 \geq 1.6$$

$$\bar{\xi}_1 = 0.2574$$

WEIGHTING MATRICES

$$Q = \left[\begin{array}{c|c} \omega^2 & 0 \\ \hline 0 & I \end{array}\right]$$

$$R = \left[\begin{array}{c} I \end{array}\right]$$

NUMERICAL RESULTS

This table gives the rank of the work done by each actuator for different designs. The first row gives the number of actuators as defined in the figure on the previous page. The first column on the left gives the number of actuators present in each design. The first design, 12*, is the initial design which is nonoptimum. All other designs are optimized. The design with 12 actuators was first obtained. It is seen that the ranking of the work done for the nonoptimum and optimum design with 12 actuators is not the same. In the optimum design the maximum work is done by actuator No. 7, and the least work by actuator No. 6. Hence, for the design with 11 actuators the sixth actuator was removed. This process was continued until a minimum weight design satisfying the constraints on the closed-loop eigenvalues and the damping parameters was obtained. The bottom row in the table shows the ranking of the work done by the actuators when five actuators are present. A design satisfying the constraints with less than five actuators could not be obtained.

RANK ORDER OF ACTUATOR INPUT

NUMBER OF ACTUATOR	1	2	3	4	5	6	7	8	9	10	11	12
12*	6	5	8	7	11	12	1	3	2	4	9	10
12	2	3	5	6	4	12	1	8	7	9	10	11
11	2	3	4	9	11		1	7	5	6	8	10
10	2	3	4	9			1	7	5	6	8	10
9	2	4	9	3			1	5	6	8	7	
8	2	4		3			1	5	6	8	7	
7	2	3		4			1	5	6		7	
6	1	3		6			2	4	5			
5	1	4					2	3	5			

*INITIAL DESIGN

NUMERICAL RESULTS (CONT)

This table summarizes the weight, the total work done by all the actuators and the magnitudes of the performance indices for each design. A minimum weight design with minimum total work done is obtained with 11 actuators. The weight and the total work done with 10 actuators are also nearly equal to the design with 11 actuators. So also for these designs, the magnitude of the performance index, PI, is the smallest.

PERFORMANCE INDEX, TOTAL WORK AND WEIGHT

NUMBER OF ACTUATORS	PI_1	PI_2	PI	TOTAL WORK	WEIGHT
12*	159.3	159.8	319.1	79.44	43.69
12	41.15	40.28	81.43	21.92	14.52
11	39.76	38.86	78.62	19.82	14.39
10	40.72	40.11	80.83	19.82	14.40
9	48.56	47.93	88.49	24.08	14.43
8	52.02	49.10	101.12	24.07	14.43
7	64.29	64.63	128.92	28.18	15.22
6	77.27	80.52	157.79	35.71	21.50
5	91.56	96.01	187.66	35.60	21.55

*INITIAL DESIGN

NUMERICAL RESULTS (CONT)

This table shows the percentage of work done by the first five actuators. The remaining actuators did less than 5% of the total work. For designs with more than six actuators, actuator No. 7 did the maximum work. For the remaining two cases with six and five actuators, actuator No. 1 did the maximum work.

PERCENTAGE OF WORK DONE BY THE ACTUATORS

NUMBER OF ACTUATOR	1	2	7	8	9
12*	5.5	6.5	30.2	12.2	24.0
12	17.0	11.0	34.0	5.1	3.3
11	21.0	15.0	35.0	5.0	5.8
10	21.0	16.0	31.0	4.0	6.3
9	17.0	8.0	37.0	6.2	5.3
8	18.0	9.0	40.0	6.5	6.3
7	25.0	11.0	39.0	7.3	5.8
6	36.0	11.0	26.0	10.0	8.2
5	37.0	11.0	29.0	14.0	8.0

*INITIAL DESIGN

NUMERICAL RESULTS (CONC)

This table shows the closed-loop damping parameters for all the designs for the 12 modes. The damping parameter associated with the first mode is equal to 0.25, indicating that this constraint is satisfied for all the designs. Comparing the damping parameters for all the designs, it is seen that as the number of actuators is reduced, the damping parameters associated with the unconstrained modes go on decreasing. For the designs with 11 and 10 actuators, the damping parameters associated with the first 5 modes are equal.

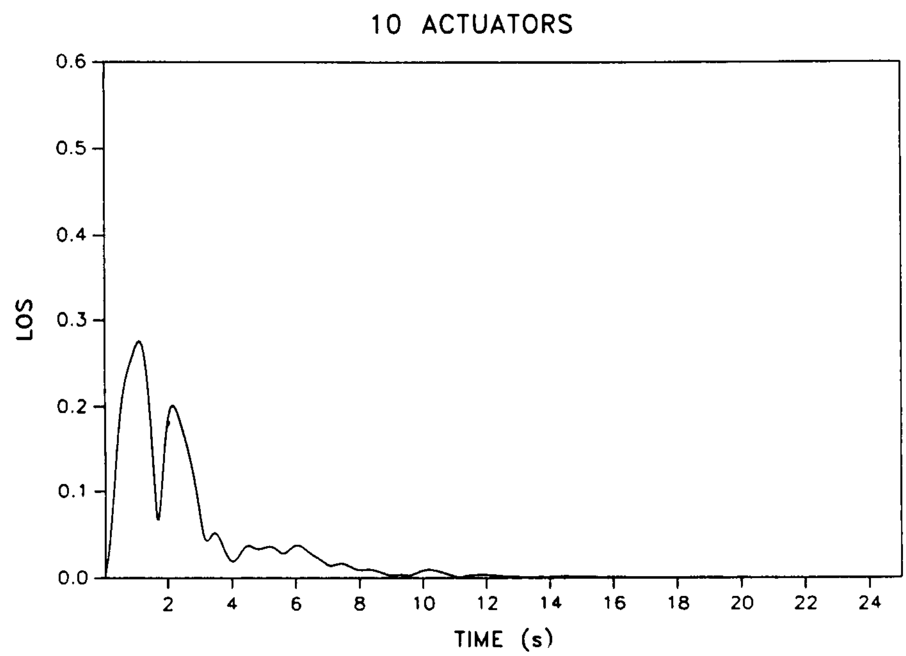
CLOSED-LOOP DAMPING PARAMETERS

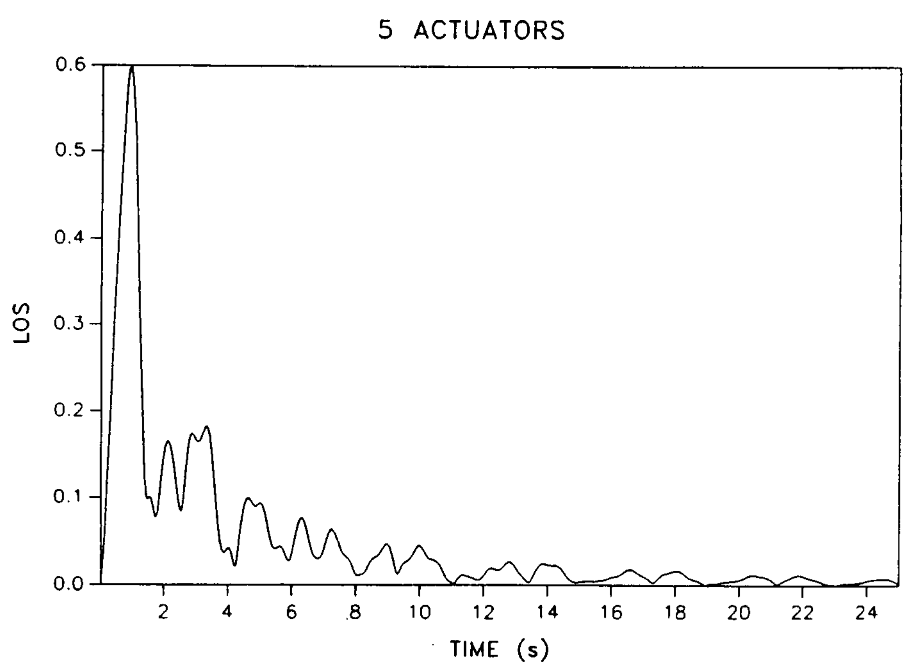
12*	12*	11*	10*	9*	8*	7*	6*	5*
0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
0.23	0.23	0.23	0.23	0.15	0.15	0.10	0.06	0.06
0.19	0.16	0.15	0.15	0.17	0.17	0.16	0.03	0.03
0.15	0.14	0.15	0.15	0.10	0.10	0.09	0.08	0.08
0.15	0.16	0.16	0.16	0.14	0.14	0.09	0.08	0.08
0.13	0.09	0.09	0.09	0.06	0.05	0.10	0.03	0.03
0.01	0.09	0.10	0.09	0.09	0.09	0.07	0.06	0.05
0.10	0.09	0.05	0.04	0.07	0.07	0.03	0.06	0.06
0.06	0.07	0.06	0.06	0.07	0.06	0.03	0.04	0.04
0.04	0.07	0.07	0.04	0.07	0.07	0.07	0.04	0.03
0.04	0.07	0.07	0.07	0.06	0.03	0.04	0.04	0.04
0.03	0.06	0.07	0.07	0.04	0.03	0.06	0.03	0.01

*NUMBER OF ACTUATORS

TRANSIENT RESPONSE

These two figures show the dynamic response of the designs with 10 actuators and 5 actuators. The transient response was simulated for a period of 25 seconds at a time interval t = 0.05 secs. The magnitude of the LOS (line-of-sight error) is given by the square root of the sum of the squares of the X and Y components of the displacement at node 1.





CONCLUSIONS

This work studied the optimum design of an ACOSS-FOUR structure starting with twelve actuators, one in each member. The performance index, the total work done by all the actuators, and the optimum weight satisfying the specified closed-loop requirements are compared. With a decrease in the number of actuators, the unconstrained damping values decreased substantially compared to twelve actuators. Due to this fact, the work done by the actuators increased to reduce the transient response or in effect to control the disturbance. The optimum weight realized increased to meet the specified closed-loop damping and eigenvalues. The closed-loop system performance index has also had similar effects. For 10 actuators, the total work, performance index and optimum weight were the best, but reducing the number of actuators beyond this number demanded increased work done by the controllers and an increase in the structural weight.

- Simultaneous structural and control optimization

  with closed–loop damping and eigenvalue requirements
- NEWSUMT–A — An optimizer for solving the problem
- Optimum number of actuators for best performance
- Fewer actuators provide less active damping
- Actuators closer to disturbance perform more work