Neutron Stars: A Cosmic Hadron Physics Laboratory

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Abstract: A progress report is given on neutron stars as a cosmic hadron physics laboratory. Particular attention is paid to the crustal neutron superfluid, and to the information concerning its properties which may be deduced from observations of pulsar glitches and postglitch behavior. Current observational evidence concerning the softness or stiffness of the high density neutron matter equation of state is reviewed briefly, and the (revolutionary) implications of a confirmation of the existence of a 0.5ms pulsar at the core of SN1987A are discussed.


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**Introduction:**

Neutron stars are an appropriate topic for a symposium which celebrates the remarkable oeuvre and persona of Lev Davidovich Landau, in part, because, according to Leon Rosenfeld, it was Landau who first suggested their existence. At a gathering at the home of Niels Bohr, on the evening of the day that the news of Chadwick's discovery of the neutron had reached Copenhagen, Landau, who had been interested in understanding how stars work, proposed that one might have stars made up entirely of neutrons. Presumably Landau, who had, independently of Chandrasekhar, recognized that the degeneracy pressure of electrons makes it possible for a white dwarf of mass less than ~1.4M_⊙ to remain stable against gravitational collapse, had in mind the possibility that a neutron star could accomplish the same feat through the degeneracy pressure of the neutrons. Some seven years were to pass before Robert Oppenheimer and George Volkoff carried out the initial analytic investigation of the stability of neutron stars, and, quite independently of Landau, showed that this did indeed occur.

Additional reasons to discuss neutron stars in a meeting dedicated to Landau come from the opportunities which they offer for the study of matter under extreme conditions. With average densities in excess of that of nuclear matter (ρ_0 = 2.8 × 10^{14} gm cm^{-3}), neutron stars are the highest density observable matter. Relatively speaking, they contain the lowest temperature superfluid. For example, the neutron liquid core of the pulsar, PSR0525-21, which is some 10^6y old, is ~10^{5}K, so that its temperature relative to its degeneracy temperature (100 MeV ~ 10^{12}K), is lower than that of superfluid helium at a few tenths of a microdegree. On the other hand, the neutron superfluid which coexists with a periodic lattice of neutron-rich nuclei in the inner crust of a neutron star (at densities ρ such that ρ_0/500 ≤ ρ ≤ ρ_0/2), is the ultimate high temperature superfluid, since its transition temperature may reach 8 × 10^{9} °K. Moreover, the neutron superfluids in the crust and core of neutron stars are also by far the most abundant superfluids we encounter in the universe.
Like the work of Landau, the study of neutron stars embraces almost every sub-field of physics: low temperature physics, condensed matter physics, nuclear and particle physics, gravitational physics, plasma physics, hydrodynamics. The physicist or astrophysicist working on neutron stars thus encounters Landau at essentially every step in his research.

On the other hand, his friends and colleagues tell us that Landau disliked astrophysics, or at least those branches of astrophysics in which investigators explore multiple scenarios, conclude that a large sub-set of these are consistent with observation, and present progress reports on these not very satisfying conclusions at regular intervals. Indeed, astrophysics differs from physics in that one cannot perform experiments to verify theory. Rather the theoretical astrophysicist must attempt to develop scenarios for a given body of observational data which are sufficiently detailed that future observations can verify or disprove a particular scenario, and provide thereby a direct link between theory and observation. In some areas (and these might have been of interest of Landau) astrophysics has matured to the point that there exists sufficient observational data to rule out large classes of theories and, conversely, to persuade us that a given physical picture is correct. In my talk today I shall consider the extent to which the study of the interiors of neutron stars has reached this stage of maturity, and thus give you a progress report on neutron stars as a cosmic hadron physics laboratory.

An Overview

In Figure 1 I reproduce a cross-section of a 1.33$M_\odot$ neutron star calculated using a Bethe-Johnson equation of state for neutron matter at densities $\rho \geq \rho_0$. The softness or stiffness of the equation of state in this region determines the mass-radius relation, the crustal extent and the stellar moment of inertia for a given mass, and the central density of the star. For this "moderately stiff" equation of state, one has a solid crust
which extends some 1.6 km below the surface of the star and a quantum liquid interior which contains electrons, superfluid neutrons in a $^3P_2$ pairing state, superconducting protons in the $^1S_0$ pairing state, and possibly, a pion condensate at densities greater than $2\rho_0$. Of particular interest to us will be that region of the inner crust lying between densities $\rho_0/10 \leq \rho \leq \rho_0/2$, which current calculations show contains a neutron superfluid in the $^1S_0$ pairing state which is pinned to the lattice of neutron-rich nuclei.

I summarize some current efforts to use neutron stars as a cosmic hadron matter laboratory in Tables 1 and 2. In each table the first column refers to the results of microscopic calculations which have the potential observational consequences spelled out in the second column, while the third column describes the current status of relevant observations. There is not time to discuss all aspects of these tables. In what follows I will focus primarily on the behavior of the crustal superfluid (for more details, and references, see Alpar et al., 1984a, and Pines and Alpar, 1985), which, as noted in Table 1, is believed to be responsible for

- glitches, observed sudden changes in pulsar rotation rates, $\Omega_c$ and $\dot{\Omega}_c$.
- the long time scale (days to years) relaxation observed in pulsars following a glitch
- the heating of old or cold neutron stars

I shall then discuss briefly the physics issues raised in Table 2.
Table 1. Neutron Stars as Cosmic Hadron Matter Laboratories:
The inner crust ($\rho_0/500 \leq \rho \leq \rho_0/2$)

<table>
<thead>
<tr>
<th>Theoretical Consequences of hadron interaction</th>
<th>Possible observational consequences of hadron interaction</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>• $^1S_0$ crustal neutron superfluid ($10^8K \leq T_c \leq 10^{10}K$), with $\xi \geq R_N$, the size of a nucleus.</td>
<td>• Pulsar spindown produces catastrophic vortex unpinning, leading to glitches (sudden pulsar spinups)</td>
<td>• 16 giant glitches ($10^{-9} \leq \Delta P/P \leq 4 \times 10^{-6}$) seen in 7 pulsars.</td>
</tr>
<tr>
<td>• Pinning of vortices in the crustal superfluid to the crust leads to lag of crustal superfluid in spinning down pulsars.</td>
<td>• Following a glitch, vortex creep leads to a long time scale ($2d \leq 2y$) recovery of $\Omega_c, \dot{\Omega}_c$, with $(</td>
<td>P</td>
</tr>
<tr>
<td>• Whether the pinning is strong or weak depends sensitively on the magnitude of the pairing gaps.</td>
<td>• Vortex creep also heats old or cold neutron stars; observation of pulsar surface temperatures provides a direct measure of pinning strengths.</td>
<td>• $T_{1929+10}$ rules out large energy gaps and appreciable strong pinning layers.</td>
</tr>
<tr>
<td></td>
<td>• Direct determination of the internal pulsar temperature is made possible by fits of post-glitch behavior using vortex creep theory.</td>
<td>• $T_{Crab}$ consistent with standard cooling.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• $T_{Vela}$ consistent with unconventional cooling plus vortex creep heating.</td>
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<tr>
<td></td>
<td></td>
<td>• $T_{0525-21}$ can be determined self-consistently from internal heating by vortex creep and fit to post-glitch behavior.</td>
</tr>
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Table 2. Neutron Stars as Cosmic Hadron Matter Laboratories:
The quantum liquid core ($\rho_0 \lesssim \rho \lesssim 3\rho_0$)

<table>
<thead>
<tr>
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<th>Possible observational consequences of hadron interaction</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>• E.O.S. of neutron matter determines</td>
<td>• Stellar wobble</td>
<td>• $35^d$ cycle of Her X-1</td>
</tr>
<tr>
<td>M-R relation, crustal extent, $M_{\text{max}}$, $P_{\text{min}}(M)$, $\rho_{\text{max}}(M)$</td>
<td>• Limit on initial period of neutron stars, and on minimum period of millisecond pulsars</td>
<td>• $P_{\text{ms}} \gtrsim 1.5$ ms?</td>
</tr>
<tr>
<td>• $^3P_2$ neutron superfluid</td>
<td>• Starquakes?</td>
<td>• $P_{1987A} = 0.5$ ms?</td>
</tr>
<tr>
<td>• $^1S_0$ proton superfluid</td>
<td>• Prompt supernovae?</td>
<td>• Crab pulsar glitches?</td>
</tr>
<tr>
<td>• Pion condensates?</td>
<td>• Strong coupling between two superfluids implies rapid crust-core coupling</td>
<td>• SN 1987A?</td>
</tr>
<tr>
<td>• Quark liquids?</td>
<td>• Rapid cooling of stars with gapless pion condensates, quark cores.</td>
<td>• No evidence for superfluid core lag in post-glitch behavior of pulsars</td>
</tr>
<tr>
<td>• Strange matter?</td>
<td>• Strange stars?</td>
<td>• Is the Vela pulsar too cold to be consistent with conventional theory?</td>
</tr>
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<td></td>
<td></td>
<td>• Ruled out as &quot;universal&quot; by glitch observations.</td>
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The physical origin of the superfluidity of neutron matter is a BCS pairing instability brought about by the attractive hadron interaction at distances $\lesssim 1\ \text{fm}$. The superfluid transition temperature is given by

$$T_c \sim E_F \exp \left( -\frac{1}{N(o)V_{\text{eff}}} \right)$$

where $E_F$ is the neutron Fermi energy, $N(o)$ the density of states per unit energy, and $V_{\text{eff}}$ the strength of the most attractive pairing interaction. For densities $\rho \lesssim \rho_o$, one does not sample appreciably the repulsive core ($r_c \lesssim 1\ \text{fm}$) of the hadron interaction, and the pairs which make up the condensate will be in a $^1S_0$ state. At higher densities the tensor force provides the major source of attraction, and the pairs condense in a $^3P_2$ state. Quite generally $T_c$ may be expected to be maximum at densities $\rho \sim \rho_o/10$. At lower densities, the density of states is too low to enable neutrons to take maximum advantage of the attractive hadron interaction, while at higher densities the neutrons begin to sample the repulsive core, so that $V_{\text{eff}}$ falls off more rapidly with increasing density than $N(o)$ increases. In Figure 2 I compare two calculations of the transition temperature; the upper curve represents early results obtained using separable potential or variational calculations; the two lower curves bracket some very recent results obtained in Urbana with a many-body theory which takes into account both short-range correlations produced by the repulsive core and long-range correlations produced by the induced particle-hole interactions (Ainsworth, Pines, and Wambach, 1989, and references therein). One test of our ability to use neutron stars as a hadron physics laboratory is the extent to which observations enable one to decide between these different theoretical results. This test is met in part: it is now possible to use observational results on the upper limit of the surface temperature of a relatively old pulsar to rule out the high transition temperatures characteristic of variational calculations.

For pulsars rotating at an angular velocity $\Omega$, it is energetically favorable for the neutron superfluid condensate to develop a vortex structure which enables it to mimic the effects of solid body rotation. One thus has lines of vortices parallel to the axis of rotation,
around which circulation is quantized in units of $h/2m$, of areal density such that the superfluid velocity, $v_s = \Omega r$. The vortex density $n$ is therefore $n(h/2m) = 2\Omega$. The size of the core of a vortex line is the superfluid coherence length

$$\xi = \frac{2E_F}{\pi \Delta k_F} \sim \frac{13}{\Delta} \text{fm}$$

where $k_F$ is the Fermi wave vector and $\Delta$ the pairing energy gap; $\xi$ is small compared to the average distance between vortices,

$$d \sim 3 \times 10^{-2} \Omega^{1/2} \text{ cm.}$$

In the stellar crust, vortices in the neutron superfluid coexist with a lattice of neutron-rich nuclei, of size $R_N$ (~7 fm), spaced a distance $b_\perp$ apart. Since the size of the vortex core, $\xi$, is comparable to the size of a nuclear cluster ($2R_N \sim 14$ fm), it is energetically favorable for a vortex line to pass through a nuclear cluster if the energy gap for superfluid neutrons inside the cluster is smaller than outside, or to avoid the cluster if the gap for the cluster neutron superfluid is greater than the gap for the exterior crustal neutron superfluid. In the former case, it costs less energy to create a vortex line if it passes through the nuclear cluster; in the latter case, it would cost more. We can speak of the pinning (or threading) of vortex lines in the stellar crust; since the density of the neutron superfluid inside clusters is greater than that outside, we expect to encounter threading (avoidance of clusters) for crustal superfluid densities less than the density at which $\Delta(p)$ is maximum (~$p_\sigma/10$) and pinning at higher densities. The pinning is intrinsic, in that the vortex core does not need a crystalline imperfection to pin, as is the case for a flux line in a typical type II superconductor.

A rotating superfluid can change its angular velocity, $\Omega$, only through the motion of vortex lines; pinning therefore acts to prevent the superfluid from following the changes in the crustal angular velocity, $\Omega_c$, produced by pulsar spin-down. As a result, a lag, $\omega = \Omega - \Omega_c$, develops between the neutron superfluid and the crust as a pulsar spins down. This lag produces in turn a force, the Magnus force, on the vortices at density $\rho$ and position $r$. 
\[ f_M = (\Omega_s - \Omega_0) \rho \kappa \tau = \omega \rho \kappa \tau, \tag{4} \]

where \( \kappa = \hbar/2m_N \) is the quantum of vorticity. As the pulsar spins down, the Magnus force increases until eventually, at some critical angular velocity lag, \( \omega_{cr} \), the vortices unpin and move outwards. Because the distribution of pinning sites is not uniform, after traversing distance \( \delta r \), the vortices will repin, and the entire process begins anew, either in that, or another, portion of the stellar crust. This, then, is the basic glitch scenario: the storage of rotational energy in the superfluid as a result of pinning, followed by its sudden release; with the process repeating itself as the pulsar spins down, and vortices move in to replace those which have unpinned.

The critical angular velocity lag is determined by balancing the Magnus force against the force per unit length of vortex line which must be overcome for vortices to move,

\[ f_{\max}^p (\rho) = \frac{E_p (\Delta, \rho)}{\xi b (\Delta, \rho)}, \tag{5} \]

where \( E_p \) is the pinning energy per nuclear cluster and \( b \) is the average spacing between pinning centers along the vortex line. Thus one finds

\[ \omega_{cr} = \frac{E_p}{\rho \kappa \xi b}, \tag{6} \]

where the pinning energy per cluster, \( E_p \), is given by

\[ E_p (\Delta, \rho) = \frac{3}{8} \gamma \frac{\Delta^2 (\rho)}{E_F (\rho)} \cdot \frac{4\pi R^3}{3} \tag{7} \]

and \( \gamma \) is a factor of order unity which takes into account the overlap of the vortex core and the cluster, and the differences in condensation energies within and without the nuclear cluster.

The pinning parameters are quite sensitive to the superfluid energy gap, \( \Delta (\rho) \), for the density region of interest, \( 1/10 \lesssim (\rho/\rho_0) \lesssim 1/2 \). The physical quantities of interest in this region are the following:

Nuclear spacing: \( 30 \text{ fm} \lesssim b_\perp \lesssim 50 \text{ fm} \)
Neutron Fermi wavevector: \( 0.82 \text{ fm}^{-1} \leq k_f \leq 1.2 \text{ fm}^{-1} \)

Nuclear cluster size: \( R_N \sim 7 \text{ fm} \)

Superfluid coherence length: \( \xi (\text{fm}) \sim 13 k_f (\text{fm}^{-1})/\Delta(\text{MeV}) \)

Pinning energy: \( E_p (\text{MeV}) \sim k_f (\text{fm}^{-1}) \Delta^2(\text{MeV}) \)

Energy to displace a nucleus a distance \( \xi \) from equilibrium site:
\[
E_L(\text{MeV}) \sim \frac{Z^2 e^2 \xi^2}{b_s^3} \sim b_k^2 (\text{fm}^{-1})/\Delta^2(\text{MeV}) (b_s/50)^3.
\]

Depending then on \( \Delta(r) \), pinning comes in one of three strengths.

- **Strong pinning**
  
  It is energetically favorable to displace a nuclear cluster so that it pins more effectively to the vortex line. The condition for this is \( E_p \geq E_L \). In this regime, \( \xi \leq R_N \), while the distance between pinning centers is \( b_s \sim b_z \), from which it follows that the criterion for being in the strong pinning regime is

\[
\Delta_s(\rho) \geq 1.5 k_f^{1/4}(\text{fm})(b_z/50)^{-3/4} \text{ MeV} \quad (8)
\]

and

\[
\omega_{sr}^{s}(\rho) \geq 6.5 k_f (\text{fm}^{-1})/\rho_{14} r_6 \Delta (\text{MeV})(b_z/50)^4 \quad (9)
\]

where \( \rho_{14} \) is the density in units of \( 10^{14} \text{ gm cm}^{-3} \), and \( r_6 \) the radial distance in units of \( 10^6 \text{ cm} \).

- **Weak pinning**
  
  Vortices pin to whatever nuclear clusters are encountered. The spacing between pinning centers is then

\[
b_w = b_z^3/\pi \xi^2 \cong 230 \frac{(b_z/50)^3}{k_f^2} \frac{\Delta^2(\text{MeV})}{\text{ fm}} \quad (10)
\]

and

\[
\omega_{sr}^{w} \equiv 0.4 \frac{\Delta (\text{MeV})}{r_6 k_f (\text{fm}^{-1})(b_z/50 \text{fm})^3} \text{ rad s}^{-1} \quad (11)
\]

is typically an order of magnitude smaller than \( \omega_{sr}^{s}(\rho) \).
Weak pinning exists for coherence lengths $\xi$ such that

$$\xi_W \leq b_z/2;$$

in other words, a vortex pins to only one nuclear cluster at a time.

- **Superweak pinning**

  When the coherence length becomes comparable to, or larger than, the average spacing between nuclear clusters,

  $$\xi^{SW} \geq b_z/2$$

  vortices pin weakly to two or more adjacent nuclear clusters. Under these circumstances, it is obviously easier for a vortex line to move; there is no simple way to estimate either the effective pinning energy, $(E_p)^{SW}_{\text{eff}}$, per cluster, or the corresponding critical lag, $\omega^{SW}_{cr}$; we may, however, expect that

  $$\frac{(E_p)^{SW}_{\text{eff}}}{E_p} \ll \frac{\omega^{SW}}{\omega_{cr}}$$

**Vortex Creep**

The postglitch behavior and heating of old neutron stars is associated with the thermally activated motion of vortex lines. When $\Omega_s = \Omega_c$, a vortex line sees a mixture of potential wells (the nuclear clusters) and barriers (produced by the Bernouilli force); as a result, its average drift velocity in the radial direction, $<v_r> = 0$. When, however, there is a lag between the crust and the superfluid, the Bernoulli barriers are preferentially reduced in one direction, so that vortices will creep at an average velocity

$$<v_r> = v_o \exp \left( \frac{E_p \omega_{cr} \omega}{kT \omega_{cr}} \right)$$

where $v_o$ is a characteristic approach velocity ($\sim 10^7 \text{ cms}^{-1}$). This vortex creep is analogous to the thermally assisted creep of flux lines in a hard (Type-II) superconductor studied by P.W. Anderson and his collaborators, and now of very great interest for the high temperature
superconductors. (Indeed, as Pethick and I have suggested elsewhere (Pethick and Pines, 1987), the short coherence lengths found in the latter materials make possible intrinsic pinning of the kind discussed here for neutron stars.) As the lag, \( \omega \), approaches \( \omega_{cr} \), such vortex creep enables the pinned superfluid to flow with greater ease, and so keep up with the spin-down induced decrease in \( \Omega_c \).

Quite generally, then, the crust is subject to both an external (magnetospheric) torque, \( N_{ext} \), which produces spin-down, and an internal torque produced by the lagging crustal superfluid. The crustal angular velocity obeys the equation of motion,

\[
I_c \dot{\Omega}_c = N_{ext} - I_p \dot{\Omega}
\]

(17)

where \( I_c \) is the total effective inertial moment of the crust plus the quantum liquid core (since the latter is found to couple to the crust on time scales (< 100s) which are short compared to those typically of interest for pulsar period irregularities), \( I_p \) is the moment of inertia of the pinned crustal superfluid, and \( \dot{\Omega} \) is determined by the radial drift of the vortex lines,

\[
\dot{\Omega} = -\frac{2\Omega}{r} \langle \nu_r \rangle .
\]

(18)

On combining the above equations, one obtains an equation for the lag, \( \omega = \Omega - \Omega_c \),

\[
\dot{\Omega} \equiv |\dot{\Omega}_c|[1 - \text{sinh} \frac{\omega}{\omega_{cr} kT}] / \eta
\]

(19)

where the vortex creep response parameter, given by

\[
\eta = \frac{|\dot{\Omega}_c| r \exp \frac{E_p}{kT}}{4 \Omega_c v_o}
\]

(20)
determines the character of the superfluid response.

Following a glitch, the resulting behavior of \( \dot{\Omega}_c \) (or \( \dot{\omega} \)) is determined by the glitch-induced creep of the vortex lines produced by the sudden jump in \( \Omega_c \). The nature of that superfluid response is in turn determined by \( \eta \). Where \( \eta \ll 1 \), one has \( \omega \ll \omega_{cr} \), and the superfluid responds linearly to the perturbation, \( \delta \omega(\omega) \), produced by a glitch; one finds

\[
\omega(t) = \omega_\infty - \delta \omega(\omega) e^{-t/\tau_f}
\]

(21)
where the linear response relaxation time,
\[ \tau_L = \frac{kT}{4E_p} \frac{\omega_{cr}}{\Omega_c v_o} \exp \frac{E_p}{kT}, \]
(22)
depends exponentially on the pinning energy and stellar temperature. On the other hand, where \( \eta \gg 1 \), one has \( \omega \sim \omega_{cr} \), and the crustal superfluid response depends non-linearly on the initial conditions. One finds
\[ \hat{\omega}_c \equiv \hat{\omega}_\infty \left\{ 1 - \frac{t_o}{t_c} \cdot \frac{1}{1 + [\exp(t_o/t_{nL}) - 1] \exp(-t/t_{nL})} \right\} \]
(23)
where \( t_o \) is an offset time which reflects the non-linear dependence on initial conditions,
\[ t_o = \frac{\delta \Omega(0)}{|\dot{\hat{\Omega}}_\infty|} , \]
(24)
and the non-linear response time,
\[ \tau_{nL} = \frac{kT}{E_p} \frac{\omega_{cr}}{|\dot{\hat{\Omega}}_\infty|} = \frac{kT}{pkrb_5|\dot{\hat{\Omega}}_\infty|} \]
(25)
depends linearly on the temperature and is independent of the pinning energy. In the weak pinning regime, one finds
\[ \tau_{nL}^W \equiv 55 \text{ days} \frac{kT(\text{kev})}{\Delta(\text{Mev})} \frac{1}{|\dot{\hat{\Omega}}_\infty/\Omega_{\text{Vela}}|} \frac{1}{r_6(b_2/50\text{fm})^3[k_5(\text{fm}^{-1})]^2} \]
(26)
on normalizing the pulsar spindown rate to that of the Vela pulsar, \( \dot{\Omega}_{\text{Vela}} \equiv 10^{-10} \text{ rad s}^{-2} \).

The transition from linear to non-linear response occurs at \( \tau_{nL} = \tau_L \), or equivalently, at a transition ratio of the pinning energy to the stellar temperature,
\[ \left( \frac{E_p}{kT} \right)_{tr} = 35 + \ln \left( t_s/10^6 \text{y} \right) \]
(27)
where \( t_s \equiv (W/2|\dot{\Omega}|_\infty \) is the pulsar spindown time.

To the extent that one knows the pulsar internal temperature and the superfluid energy gap and pinning energy as a function of density, one can then determine regions of linear and non-linear response in a given pulsar. Conversely, if the post-glitch behavior of a given pulsar can be shown to be linear or non-linear, one can use that information to deduce the character of \( \Delta(p) \) (Alpar, Cheng, and Pines, 1989).
Energy dissipation by vortex creep is significant when conditions are such that appreciable portions of the neutron star are in the non-linear creep regime, for which \( \omega \sim \omega_{cr} \). Under these circumstances, the energy dissipation rate in steady state is

\[
\dot{E}_{\text{diss}} \equiv I\rho \omega_{cr} |\Delta \omega| \tag{28}
\]

This dissipation is large enough to be observable in the surface luminosity of older pulsars which have radiated away much of their initial heat content.

**Comparison between theory and observation**

In considering the observational evidence for superfluidity in neutron stars, one can ask questions at several different levels. First, can one uniquely deduce the presence of pinned crustal neutron superfluid from observations of pulsar glitches (eight in the Vela pulsar, three in the Crab pulsar, one each in pulsars 0525+21, 0355+54, 1641-45, 1325-43, and 2224+65) and postglitch response. Here my answer would be an unambiguous, "Yes." The evidence that glitches originate in the interior of neutron stars is overwhelming, and pinning of the crustal neutron superfluid is the only viable candidate. The magnitude of the jumps in both \( \Omega_c \) and \( \dot{\Omega}_c \) is easily understood as resulting from the catastrophic unpinning of vorticity in the crustal neutron superfluid involving pinning regions with inertial moments, \( (I_p/l) \lesssim 10^{-2} \), in accord with our theoretical expectations. Although we lack as yet a detailed theory of how such catastrophic events are triggered, the presence of inhomogeneous pinning regions makes glitches inevitable, while the time between the frequent glitches observed for the Vela and Crab pulsars is in accord with that calculated for pulsar spin down to enable unpinned vortices in these pulsars to move into pinning regions in sufficient number to initiate a subsequent glitch.

Vortex creep theory likewise provides a natural explanation for the long time scale recovery of \( \Omega_c \) and \( \dot{\Omega}_c \) following a glitch; the observed time scales for that recovery, which range from a few days to a few months, are those expected for linear or non-linear response
in weak or superweak pinning regions. The four most recent Vela glitches and the latest Crab pulsar glitch have all been caught within a day of the glitch occurrence; fits to the postglitch data can be made with combinations of simple exponential decays, with two distinct time scales of a few days and a few months respectively, combined with the persistent shift in $|\dot{\Omega}_c|$ which is the hallmark of a substantial region of non-linear vortex creep.

Is it possible to proceed to a second level of comparison of theory and observation, and use vortex creep theory to draw from observation some quantitative conclusions concerning the magnitude of critical angular velocities and pinning energies? Again the answer is "yes". For example, the failure to detect the thermal luminosity of the radio pulsar PSR 1929+10 enabled Alpar et al. (1987) to place an upper limit on vortex creep heating of this pulsar, and to conclude that $<\omega_{cr}> \leq 0.7 \text{ rad s}^{-1}$; hence there can be no appreciable regions of strong pinning in this neutron star (and by inference, in any neutron star.) Alpar, Cheng and Pines (1989) have recently reviewed the application of vortex creep theory to postglitch behavior and arrive at the following conclusions:

- There must be present, in the Crab pulsar, substantial regions ($l_p/l \approx 10^{-3}$) in which the pinning energy, $E_p^w \approx 1 \text{ MeV}$; these are the weak pinning regions expected for energy gaps, $\Delta$, such that $1 \text{ MeV} \leq \Delta \leq 1.5 \text{ MeV}$.
- There must be present in PSR 0355-44, for which a linear postglitch response with a relaxation time of ~ 44d has been observed, appreciable regions of superweak pinning, with pinning energies, $E_p^{sw} \approx 50 \text{ kev}$.
- The observed relaxation time $\approx 100d$ for the Vela pulsar could reflect either the non-linear response of regions of superweak pinning or the linear response of regions of weak pinning.
- For PSR 0525-21, the long relaxation time, 150d, extracted from a fit to the postglitch data, reflects the non-linear response of a region of weak pinning and is consistent
with an internal temperature of \( \sim 1.3 \times 10^{-2} \) kev calculated assuming vortex creep heating of this old (\( \geq 1.5 \times 10^6 \) y) pulsar.

Given the difficulty in carrying out accurate calculations of energy gaps in neutron matter, and ambiguities in fitting postglitch response with linear or non-linear regimes, the agreement between vortex creep theory based on the APW gap calculations and observation is surprisingly good. As of this writing, a reasonable working hypothesis is that one has, for all neutron stars, a region of weak pinning, \( E_p^w \sim 1 \) MeV, extending from densities of order \( 3 \times 10^{13} \) gm cm\(^{-3}\) to \( 9 \times 10^{13} \) gm cm\(^{-3}\), followed (in density) by regions of superweak pinning, \( E_p^{sw} \leq 50 \) kev, extending from densities of order \( 9 \times 10^{13} \) gm cm\(^{-3}\) to \( \sim 2 \times 10^{14} \) gm cm\(^{-3}\).

The Quantum Liquid Core

Among the principal physics issues connected to the quantum liquid core of neutron stars are the following:

- The character of the equation of state of neutron matter at densities greater than \( \rho_0 \), the density of nuclear matter.
- The existence of neutron and proton superfluid states.
- The possible existence of a pion condensate (for \( \rho \geq 2 \rho_0 \)).
- A possible transition from hadron matter to a quark liquid at \( \rho \geq 3 \rho_0 \).
- The possible existence of strange stars, made up of strange quark matter.

The first issue, although it is not likely to have appealed to Landau (who would likely have regarded it as undecidable and hence uninteresting), is of considerable importance for understanding neutron stars, since the neutron matter equation of state at densities in excess of \( \rho_0 \) determines the mass-radius relation, the crustal extent, the maximum mass, the
minimum rotational period, and the maximum crustal density for a given stellar mass. As Pandharipande et al. (1976) showed some time ago, one can construct a variety of models for the neutron interaction which are consistent with terrestrial constraints, and which lead to equations of state (EOS) which span a range from "soft" EOS (derived from models for which the average neutron interaction energy is attractive at nuclear densities) to "stiff" EOS (derived from models for which the average neutron energy is repulsive at sub-nuclear densities). In contrast to the 1.33M\(\odot\) star depicted in Fig. 1, which has been calculated using a moderately stiff equation of state, a star of comparable mass calculated with a soft equation of state based on the phenomenological Reid model for neutron-neutron interaction will have a much smaller radius (~8km), a significantly smaller crustal extent (≤ 800 meters), and higher central density (~ 2 x 10^{15} g cm^{-3}). The minimum rotational periods for stars of ~ 1.4M\(\odot\) also differ substantially, ranging from ~ 1.3ms for a stiff EOS, to ~ 0.7ms for a Reid EOS.

Observational evidence in favor of a stiff EOS comes from the identification by Trumper et al. (1986) of the 35d cycle of the pulsating x-ray source. Her X-1 as originating in free precession of the rotating neutron star, analogous to the Chandler wobble of the earth, and from the minimum magnitude of the inertial moment of the pinned crustal superfluid required to explain postglitch behavior (see Pines, 1987, for a discussion.) Should the minimum rotational period of a millisec pulsar be ~ 1.5ms, as Friedman et al. (1988) and Lipunov and Postnov (1988) have suggested, again a quite stiff EOS is required. Under these circumstances one would be unlikely to find neutron stars with quark cores, since the calculated central density of a maximum mass neutron star is still rather less than 3\(\rho_0\), the most plausible density at which the hadron matter-quark liquid transition is likely to occur. It
is also worth noting that such stars would possess a sufficiently substantial crust that, when young, they would be subject to comparatively frequent starquakes induced by pulsar spindown; it is possible that such quakes could be responsible for the Crab pulsar glitches (Pines, 1987).

On the other hand, as discussed recently by Brown (1988), current calculations show that "prompt" supernovae in which the stellar envelope is blown off immediately after implosion of the core, are only possible if the EOS remains soft up to densities around $4\rho_0$; thus a stiff EOS favors "delayed" supernovae, in which neutrons act for some time following collapse to drive off the stellar envelope.

Should the 0.5ms periodicity reported for the expected neutron star remnant of SN1987a (Kristian et al. 1989) be confirmed and identified with the rotational period of the star, "almost all the main features of current theories of neutron star formation, structure, evolution, and, most fundamentally, current models of nuclear interactions and the equation of state of matter at the highest densities" will require drastic revision (Alpar et al., 1989). As many authors (Haensel and Zdunik, 1989; Friedman et al., 1989; Ipser and Lindblom, 1989; Sata and Suzaki, 1989; Shapiro et al., 1989) have now demonstrated, the existence of a uniform rotating star with a rotational period less than 0.5ms places unusually severe constraints on the equation of state, requiring that it be remarkably soft and lie within a pathologically narrow range of neutron star models. Thus the question considered above, of "soft" vs "stiff" equations of state becomes moot; all of the standard models presently in use are ruled out, and in all likelihood the pulsar would collapse into a black hole upon spindown.
In conclusion, (and assuming that 0.5msec pulsars do not exist!) I should like to address briefly the other three issues mentioned above. Pion condensates in neutron stars at densities greater than $\sim 2\rho_0$ continue to be a viable theoretical possibility; the best observational handle we have on their existence is the anomalously rapid (compared to the presence of, say, superfluid neutron matter) cooling of a neutron star such condensates make possible if they are in part gapless. Do there exist pulsars which appear to be far colder than conventional cooling theory would predict? Thus far the most promising candidate for such a star is the Vela pulsar. Ögelman and Zimmerman (1988) have deduced an interior temperature of $\sim 11$ keV from their recent EXOSAT observations of its surface temperature; such a temperature is somewhat lower than the predictions ($\sim 16$ keV) of most standard cooling theories. A still lower temperature ($\sim 1.4$ keV) was obtained by Alpar et al. (1984) who interpreted the observed 60d postglitch relaxation time as reflecting weak pinning and non-linear response. As noted earlier, should the postglitch response be linear, or reflect a superweak pinning regimes, there is no need to invoke such a low temperature.

Witten's proposal that neutron stars might in fact be composed of strange quark matter was an intriguing one; however Alpar (1987) pointed out that since such stars would almost certainly not possess a large enough solid outer crust to produce the observed amplitudes of glitches and post-glitch relaxation, it can at least be concluded that not all pulsars are strange. Indeed, to the extent that glitch statistics are consistent with the hypothesis that all pulsars glitch occasionally, perhaps no pulsars are strange.

Finally, what evidence do we have for the superfluidity of the neutron and proton liquids in the quantum liquid core? Because the core superfluid is now believed to couple on a very short time scale ($\sim$ min.) to the stellar crust (Alpar and Sauls, 1989), one has no direct
evidence of superfluidity from glitch observation. Since, however, Hamilton et al. (1989) were observing the Vela pulsar at the time of its eighth glitch, it is possible that the shortest time scales found in these observations will provide direct evidence concerning the existence of a superfluid neutron core, as well as providing invaluable observational limits on the duration of the glitch itself. Indeed, for "glitch theorists" this observation may well play a role comparable to that of SN1987A for "supernova theorists."

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References


Figure 1. A cross-section of a 1.33M⊙ neutron star calculated using a moderately stiff equation of state proposed by Bethe and Johnson (R.A. Smith, private communication.)
Figure 2. Calculations of the superfluid neutron transition temperature. The upper curve represents results obtained using a separable potential or variational approach; the two lower curves, APW-1 and APW-2, represent a bound on recent many-body calculations (Ainsworth, Pines, and Wambach, 1989) which take into account both short range correlations produced by the repulsive core and long-range correlations produced by induced particle hole interactions.