WINDOWS ON THE AXION

Michael S. TURNER

NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
Batavia, IL 60510-0500

and

Departments of Physics and Astronomy & Astrophysics
Enrico Fermi Institute
The University of Chicago
Chicago, IL 60637-1433

Abstract

Peccei-Quinn symmetry with attendant axion is a most compelling, and perhaps the most minimal, extension of the standard model, as it provides a very elegant solution to the nagging strong CP-problem associated with the $\Theta$ vacuum structure of QCD. However, particle physics gives little guidance as to the axion mass; a priori, the plausible values span the range: $10^{-12}$ eV $\lesssim m_a \lesssim 10^6$ eV, some 18 orders-of-magnitude. Laboratory experiments have excluded masses greater than $10^4$ eV, leaving unprobed some 16 orders-of-magnitude. Axions have a host of interesting astrophysical and cosmological effects, including, modifying the evolution of stars of all types (our sun, red giants, white dwarfs, and neutron stars), contributing significantly to the mass density of the Universe today, and producing detectable line radiation through the decays of relic axions. Consideration of these effects has probed 14 orders-of-magnitude in axion mass, and has left open only two windows for further exploration: $10^{-8}$ eV $\lesssim m_a \lesssim 10^{-3}$ eV and $1$ eV $\lesssim m_a \lesssim 5$ eV (hadronic axions only). Both these windows are accessible to experiment, and a variety of very interesting experiments, all of which involve “heavenly axions,” are being planned or are underway.
I. Motivation for, and Properties of, the Axion

Quantum Chromo Dynamics (QCD) is a remarkable theory and is almost universally believed to be the theory of the strong interactions. Aside from the calculational difficulties associated with actually calculating the spectrum of states in the theory, QCD has but one serious blemish: the strong CP problem.\(^1\) Namely, the fact that non-perturbative effects violate CP, T, and P, and unless suppressed would lead to an electric dipole moment for the neutron which is in excess of experimental limits by some 10 orders-of-magnitude. Proposed in 1977, Peccei-Quinn symmetry with its associated axion is perhaps the most elegant solution to this nagging problem.\(^2\)

Non-Abelian gauge theories have a rich vacuum structure owing to the existence of non-trivial, vacuum gauge configurations. These degenerate vacuum configurations are characterized by distinct homotopy classes that cannot be continuously rotated into one another and are classified by the topological winding number \(n\) associated with them,

\[
n = \frac{ig^3}{24\pi^2} \int d^3x \text{Tr} \epsilon_{ijk} A^i(\vec{x}) A^j(\vec{x}) A^k(\vec{x})
\]

where \(g\) is the gauge coupling, \(A^i\) is the gauge field, and temporal gauge \((A^0 = 0)\) has been used. The correct vacuum state of the theory is a superposition of all the vacuum states \(|n\rangle\),

\[|\Theta\rangle = \Sigma_n \exp(-in\Theta)|n\rangle\]

where \textit{a priori} \(\Theta\) is an arbitrary parameter in the theory which must be measured. The state \(|\Theta\rangle\) is referred to as “the \(\Theta\)-vacuum.” By appropriate means the effects of the \(\Theta\)-vacuum can be recast into a single, additional non-perturbative term in the QCD Lagrangian,

\[
\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{PERT}} + \bar{\Theta} \frac{g^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{a\mu\nu}
\]

\[\bar{\Theta} = \Theta + \text{Arg} \det \mathcal{M}\]

where \(G^{a\mu\nu}\) is the field strength tensor, \(\tilde{G}^{a\mu\nu}\) is the dual of the field strength tensor \((G\tilde{G} \propto \tilde{E}_{\text{color}} \cdot \tilde{B}_{\text{color}})\), and \(\mathcal{M}\) is the quark mass matrix. Note that the effective \(\Theta\) term in the theory involves both the bare \(\Theta\) term and the phase of the quark mass matrix. Such a term in the QCD Lagrangian clearly violates CP, T, and P, and leads to a neutron electric dipole moment of order\(^3\)

\[d_n \simeq 10^{-15}\bar{\Theta} e - \text{cm}\]

The present experimental bound to the electric dipole moment of the neutron,\(^4\) \(d_n \lesssim 10^{-25}\) e-cm, constrains \(\bar{\Theta}\) to be less than (or of order of) \(10^{-10}\).
Before going on to discuss the axion, some general comments about the strong-CP problem are in order. The unwanted, non-perturbative term in the Lagrangian arises due to two separate and independent effects: the $\Theta$ structure of the pure QCD vacuum; and electroweak effects involving quark masses. In the limit that one or more of the quarks are massless the $G\tilde{G}$ term has no physically measurable effects: The $\Theta$ term can be rotated away by a chiral rotation, and there is no strong CP problem. In the absence of a massless quark species (for which the evidence in our world is strong), the effective $G\tilde{G}$ term is made of two unrelated contributions which a priori have no reason to cancel.

One might be tempted to ignore this mysterious topological contribution to the QCD Lagrangian, on grounds that one has no need for it, or the hope that its absence will be understood at some future date. This is not a particularly good thing to do either; the $\Theta$ structure of the QCD vacuum has at least one beneficial feature: the resolution of the $U(1)_A$ puzzle. In the absence of such a term one would expect 4 Goldstone bosons when the $U(2)_L \otimes U(2)_R$ global symmetry (of a massless, 2 flavor QCD world) is spontaneously broken by QCD effects. These Goldstone bosons are the $\pi$ and $\eta$ mesons, the $\eta$ meson being the Goldstone of the spontaneously broken $U(1)_A$ global symmetry. When non-zero $u$ and $d$ quark masses are taken into account one can show that the mass of the $\eta$ must satisfy $m_\eta \leq \sqrt{3}m_\pi$, which, needless to say, is contradicted by reality. The existence of the $\Theta$ vacuum structure of QCD corrects this erroneous prediction and solves the $U(1)_A$ problem.

So it seems likely that the $\Theta$ structure of the QCD vacuum is to be taken seriously. Moreover, it seems unlikely that any of the quark flavors is massless. How then, is one to solve the strong CP problem? The most elegant solution is the one proposed by Peccei and Quinn in 1977 (and my personal favorite, as I was a very impressionable young graduate student at Stanford when they made their exciting proposal!). Their idea is to make $\Theta$ a dynamical variable, which owing to its classical potential relaxes to zero. This end is accomplished by introducing an additional global, chiral symmetry, now known as PQ (or Peccei-Quinn) symmetry, which is spontaneously broken at a scale $f_{PQ}$. Weinberg and Wilczek realized that because $U(1)_{PQ}$ is spontaneously broken there should be a Goldstone boson, “the axion” (or as Weinberg referred to it for a while, “the higglet”). Because $U(1)_{PQ}$ suffers from a chiral anomaly, the axion is not massless, but acquires a small mass of order $\Lambda_{QCD}^2/f_{PQ}$, about which we shall be more precise shortly. Moreover, due to this anomaly a term in the QCD Lagrangian of the form

$$\mathcal{L}_{QCD} = ... + \frac{a}{f_{PQ}} \frac{g^2}{32\pi^2} G_\mu^a \tilde{G}_\mu^a$$

arises, where $a$ is the axion field and $const \, t$ is a model-dependent constant. Note the
similarity of this term to the previously discussed $\bar{\Theta}$ term. These two terms amount to a potential for the axion field, which is minimized by having

$$<a> = -\frac{\bar{\Theta} f_{PQ}}{\text{const}'}$$

for which the coefficient of the offending $G\bar{G}$ term vanishes! Expanding the axion field about its vacuum expectation value $<a>$ the axion part of the QCD Lagrangian is

$$\mathcal{L}_{\text{axion}} = -\partial^\mu a \partial_\mu a/2 + \frac{a}{f_{PQ}} \frac{\text{const}'}{f_{PQ}} \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a$$

(where I have not yet included the axion’s other interactions). Note that the $\bar{\Theta}$ parameter has been effectively replaced by the dynamical axion field, whose mass arises due to the non-perturbative $G\bar{G}$ term.

In an axion model then, the price for resolving the strong CP problem is the existence of an additional, spontaneously-broken global symmetry (which often arises in supersymmetric and superstring-inspired models in any case) and its associated pseudo-Goldstone boson. A priori the mass of the axion (or equivalently the PQ symmetry breaking scale) is arbitrary: all values solve the strong CP problem equally well. Taking $f_{PQ}$ to be somewhere between 100 GeV and $10^{19}$ GeV, the associated axion mass then lies between $\sim 1$ MeV and $10^{-12}$ eV—a span of some 18 or so orders-of-magnitude to search.

So much for the high brow theory and philosophy! In order to search for the axion one must know about its properties and how it couples to ordinary matter. As eluded to above, an axion model has one basic, free parameter: the axion mass, or equivalently the PQ symmetry breaking scale. They are related by

$$m_a = \frac{\sqrt{z} f_{PQ} m_\pi}{1 + z f_{PQ}/N} \approx \frac{0.62 \text{ eV} \times 10^7 \text{ GeV}}{f_{PQ}/N}$$

where $z = m_u/m_d \simeq 0.56$, $m_\pi = 135$ MeV and $f_\pi = 93$ MeV are the pion mass and decay constant, $N$ is the color anomaly of the PQ symmetry, and our normalization conventions for $f_{PQ}$ follow those of refs. 6. The axion field $a$ is related to $\bar{\Theta}$ by: $a = (f_{PQ}/N)\bar{\Theta}$. The effective Lagrangian for the interactions of axions with ordinary matter (nucleons, electrons, and photons) is

$$\mathcal{L}_{\text{int}} = \frac{i g_{\alpha NN}}{2m_N} \partial_\mu a (\bar{N} \gamma^\mu \gamma_5 N) + \frac{i g_{\alpha ee}}{2m_e} \partial_\mu a (\bar{e} \gamma^\mu \gamma_5 e) + g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

with the associated Feynman diagrams shown in Fig. 1. [In all instances where there is only one Goldstone boson in the problem, the pseudo-vector coupling may be written instead
as a pseudo-scalar coupling; e.g., $i g_a N N a (\bar{N} \gamma_5 N)$, by means of a suitable phase rotation of the fermion fields. For further discussion of this point, see refs. 7.

The axion couplings $g_{aii}$ are given by

$$g_{aee} = \frac{[X_e/N + (3\alpha^2/4\pi)(E\ln(f_{PQ}/m_e)/N - 1.95\ln(\Lambda_{QCD}/m_e)]}{(f_{PQ}/N)}$$

$$g_{a\gamma\gamma} = \frac{\alpha/2\pi}{f_{PQ}/N}(E/N - 1.95)$$

$$g_{ann} = \frac{[-F_{A0} - F_{A3})(X_d/2N - 0.18) + (-F_{A0} + F_{A3})(X_u/2N - 0.32)]m_N}{f_{PQ}/N}$$

$$g_{app} = \frac{[-F_{A0} - F_{A3})(X_u/2N - 0.32) + (-F_{A0} + F_{A3})(X_d/2N - 0.18)]m_N}{f_{PQ}/N}$$

where $E$ is the electromagnetic anomaly of the PQ symmetry, $1.95 = 2(4 + z)/3(1 + z)$, $0.32 = 1/2(1 + z)$, $0.18 = z/2(1 + z)$, $\alpha \simeq 1/137$ is the fine structure constant, $F_{A0} \simeq -0.75$ is the axial-vector, isoscalar part of the pion-nucleon coupling, and $F_{A3} \simeq -1.25$ is the axial-vector, isovector part of the pion-nucleon coupling. [Note that the axion-nucleon couplings have been computed in the context of the "naive quark model;" more sophisticated treatments lead to slightly different axion-nucleon couplings (see, e.g., Mayle, et al.\textsuperscript{35}).]

The quantities $X_i$ ($i = u, d, e$) are the PQ charges of the $u$ and $d$ quarks and the electron. Depending upon the PQ charge of the electron, the axion can be classified as one of two generic types: hadronic,\textsuperscript{8} the case where $X_e = 0$ (no tree level coupling to the electron); or DFSZ,\textsuperscript{9} the case where all the $X_i$ are of order unity. [In the original DFS model,\textsuperscript{9} $N = 6$, $X_e/N = \cos^2 \beta/3$, $X_u/N = 1 - \cos 2\beta$, and $X_d = 1 + \cos 2\beta$. Here $\beta$ parameterizes the ratio of the ‘up’ and ‘down’ PQ vacuum expectation values.]

First note that all the axion couplings ($g_{aii}$) are proportional to $1/(f_{PQ}/N)$, or equivalently $m_a$: the smaller the axion mass, or the larger the PQ SSB scale, the weaker the axion couples. The coupling of the axion to the photon arises through the electromagnetic anomaly of the PQ symmetry, and allows the axion to decay to two photons, with a lifetime,

$$\tau_a = 6.8 \times 10^{24} \text{sec} \frac{(m_a/eV)^{-5}}{[(E/N - 1.95)/0.72]^2}$$

Note too that the coupling of the axion to 2 photons depends upon the ratio of the electromagnetic to the color anomaly; when the axion is incorporated into the simplest of GUTs, $E/N = 8/3$ and $(E/N - 1.95) \simeq 0.72$. However, it is possible that $E/N$ could have a different value, even 2, in which case its 2 photon coupling $(E/N - 1.95 \simeq 0.05)$ would be
strongly suppressed. We should keep this fact in mind, as it will be of some importance when discussing the astrophysical effects of a hadronic axion.

Next note that the coupling of the axion to the electron has a tree level contribution which is proportional to \( X_e \) and vanishes for the hadronic axion, and a loop correction which is proportional to \( \alpha^2 \) and arises due to the anomalous 2 photon coupling of the axion (see Fig. 1). The axion-electron coupling is of great importance in determining the astrophysical effects of the axion, and explains why it is usually necessary to discuss the astrophysical constraints to DFSZ and hadronic axions separately.

Finally, note that the axion-nucleon coupling arises from two roughly equal contributions: the tree level coupling of the axion to up and down quarks, and a contribution which arises due to axion-pion mixing (both the axion and pion are Goldstone bosons with the same quantum numbers, and the physically-propagating states mix). This means that even a hadronic axion which does not couple to light quarks at tree level (as was the case with the original hadronic axion which only coupled to one very heavy, exotic quark) still has a coupling to nucleons which is comparable to that of a DFSZ axion. Because of this fact, the bound to the axion mass based upon SN 1987A, which involves the axion-nucleon coupling, is essentially the same for the hadronic and DFSZ axion.

While I have taken the view here that a priori the axion mass (or symmetry breaking scale) is an arbitrary parameter, to be determined by experiment, that viewpoint belies the history of the axion. The original axion proposed by Peccei and Quinn was based upon a PQ symmetry breaking scale equal to that of the weak scale \( f_{PQ} \sim 250 \text{ GeV} \), leading to an axion mass of about 200 keV. As I will now discuss, such an axion was quickly ruled out by unsuccessful experimental searches. Shortly thereafter the 'invisible axion' was invented, an axion with symmetry breaking scale \( \gg 250 \text{ GeV} \) and mass \( \ll 200 \text{ keV} \), whose interactions are necessarily extremely weak (recall \( g_{\text{all}} \propto m_a \propto f_{PQ}^{-1} \)). And of course, it goes without saying that once the weak scale had been ruled out, all educated bets as to the PQ SSB scale were off!

[Very recently it has been argued that wormhole effects, might cause the wave function of the Universe to be very highly peaked at \( \Theta = \pi \) (a CP-conserving value); if correct, this would solve the strong-CP problem and obviate the need for the axion. Even if this is true, Nature may still provide us with an axion, as PQ symmetry seems to be very generic to supersymmetric and superstring inspired models.]
II. Laboratory Searches

As mentioned above, the original Peccei-Quinn-Weinberg-Wilczek axion was characterized by a SSB scale of order the weak scale and a mass of order 200 keV. As such its interactions were roughly semi-weak, making it accessible to laboratory searches. So accessible in fact, that the original axion was very quickly ruled out. Without doing justice to the history and to the variety of important experiments, I will very briefly mention the most sensitive laboratory searches. First is the Kaon decay process

\[ K^+ \rightarrow \pi^+ + a \]

where the axion goes unseen. The present experimental upper limit to the branching ratio for \( K^+ \rightarrow \pi^+ + \text{nothing} \) is \( 3.8 \times 10^{-8} \). In an axion model this process arises either through axion/pion mixing (the decay \( K^+ \rightarrow \pi^+ + \pi^0 \) is observed), or an off diagonal coupling of the axion to s and d quarks.

Next are the decays of quarkonium (QQ) states

\[ J/\psi \rightarrow a + \gamma \quad \Upsilon \rightarrow a + \gamma \]

The upper limit for the branching ratio for these two processes are \( 1.4 \times 10^{-5} \) and \( 3 \times 10^{-4} \) respectively.

Based upon the three processes just mentioned, one can safely conclude that

\[ f_{PQ} \gtrsim 10^3 \text{ GeV} \quad \text{or} \quad m_a \lesssim 6 \text{ keV} \]

There have been other axion searches involving disallowed \( J^P = 0^-, 1^+, 2^-, \ldots \) nuclear transitions, reactor experiments, and beam dump experiments. In general, the limits that follow from these are less stringent or more difficult to interpret.

While laboratory-based experiments exclude axion masses in the range of about 10 keV to 1 MeV and most certainly rule out the original axion, they leave open an enormous window: \( 10^{-12} \text{ eV} - 10 \text{ keV} \), one which has only been explored by astrophysics and cosmology.

III. Axions and Stars

The life of a star is rather uncomplicated, the simple struggle to lose the enormous nuclear free energy associated with the primordial composition of the Universe (for every 10 atoms, roughly 9 H atoms, 1 He atom, and a trace of D, \(^3\)He, and \(^7\)Li). Given the intrinsically short time scale associated with nuclear interactions, it should be a great surprise that stars live as long as they do: a star like our sun burns hydrogen for \( 10^{10} \) yrs.
The reason for the hang up is simple; the rate at which a star can liberate its nuclear free energy is controlled not by nuclear reaction rates, but rather by the rate at which the nuclear energy liberated can be transported through the star and radiated into the vacuum of space. Under the conditions that exist in a typical star, say our sun, the mean free path of a photon is only about a cm!, and the time required for a photon liberated at the center of the sun to make its way (figuratively of course) to the surface is of order $10^7$ yrs. The enormous opacity of ordinary matter to photons of course traces to the strength of the electromagnetic interactions; recall that the Thomson cross section is $\sigma \approx 0.67 \times 10^{-24}$ cm². The very long time required for a star like our sun to burn its hydrogen fuel then owes to the large interaction cross section of the photon with ordinary matter.15

The existence of a light (i.e., compared to typical stellar temperatures, $T \sim \text{keV-MeV}$), weakly-interacting particle has the potential to greatly accelerate the evolutionary process of stars of all types by more efficiently transporting energy away, and thereby to shorten their lifetimes. To effectively carry off the free energy liberated in the nuclear reactions in a star, the hypothetical “super coolant” must interact weakly enough so that it streams right out without interacting, but strongly enough so that it is produced in sufficient numbers to carry away large amounts of energy. As one might guess, the optimal interaction strength is such that the super coolant particle has about 1 interaction as it streams out. Nature has provided us with at least 3 candidates, the 3 neutrino species, and contemporary theorists have postulated another, the axion.

Before turning to the axion, let us orient ourselves by discussing neutrino cooling in stars. Because of the nature of the weak interaction neutrino cross sections are highly temperature sensitive, proportional to $G^2 \nu T^2$. [It is interesting to note that neutrino emission, unlike axion emission, is necessarily a second order weak process, i.e., $\mathcal{L}_\nu \propto <\nu>^{-4}$, where $<\nu> \approx 250$ GeV is the SSB scale of the weak interactions.] In ordinary main sequence stars the neutrino luminosity $L_\nu$ is proportional to $T^6$, whereas we shall see shortly that the photon luminosity $L_\gamma \propto T^7_c$ ($T_c$ is the central temperature of the star). Only in stars hotter than about $10^8$ K does neutrino cooling begin to compete with photon cooling; for these stars (O, Si burning stars and beyond) neutrino emission is the dominant cooling mechanism, and as a result the time scale for these burning phases is greatly reduced as neutrinos can just stream out (O burning time scale is of order $10^5$ yrs; Si burning time scale is of order sec’s). In fact, long ago it was argued that the existence of carbon burning stars places a limit to $G_F$;16 had $G_F$ been a factor of 3 or so larger, the evolution time scale for C stars would have been greatly reduced due to neutrino emission, so much so that C burning stars would evolve through C burning so quickly that none would be observed.
As a preview of things to come, we should mention SN 1987A; the primary cooling mechanism for the hot, nascent neutron star was, as we all now appreciate, neutrino emission. Moreover, based upon the neutrino burst time scale, it has been argued that the number of light neutrino flavors (mass $\lesssim$ MeV) must be less than about 9, otherwise neutrino cooling of the neutron star would have proceeded more rapidly (by a factor of roughly 3) than observed.26

The effect of axion emission on stars is clear: the acceleration of their evolution and shortening of their lifetimes. In main sequence stars and red giants the primary axion emission processes are:17 the Compton-like process $\gamma + e^- \to a + e^-$; and axion bremsstrahlung $e^- + Z \to a + e^- + Z$, both of which are proportional to $g_{aee}^2 \propto m_a^2$. Of lesser importance unless $g_{aee}$ vanishes at tree level, as it does for a hadronic axion, is the Primakoff process $\gamma + Z(\text{or } e^-) \to a + Z(\text{or } e^-)$. In very low-mass stars ($M \lesssim 0.2M_\odot$) emission through the axio-electric effect (the analogue of the photo-electric effect) is also very important.18

To begin, consider the sun. At the center of the sun the temperature is about $1.6 \times 10^7$ K, and nuclear reactions ($p + p \to D + e^+ + \nu_e; D + D \to ^4He + \gamma$) liberate free energy at the rate of a few ergs g$^{-1}$ sec$^{-1}$. On the other hand, axion emission carries away energy at the rate of

$$\dot{\varepsilon}_a \sim 1 \text{ erg g}^{-1} \text{ sec}^{-1} (T_c/10^7 \text{K})^6[10^7 \text{ GeV}/(f_{PQ}/N)]^2$$

Roughly speaking then, if $(f_{PQ}/N)$ were less than about $10^7$ GeV, axions would carry energy away from the center of the sun faster than nuclear reactions could generate it. [Note the above rate is that for a DFSZ axion.] The thermal time constant of a star like the sun is only about $10^7$ yrs; this is the time required for the star to radiate away its thermal energy reserves, and is known as the Kelvin-Helmholtz time. Thus an axion luminosity greater than the rate at which nuclear energy is released can only be tolerated on a short time scale ($\lesssim 10^7$ yrs, or so); if $(f_{PQ}/N)$ were less than $10^7$ GeV then the sun would have to "adjust itself" to re-establish energy balance. As we shall see, in a hypothetical star in which one "turns on" axion emission, the star contracts to raise its temperature and nuclear energy liberation rate to balance axion losses. In the process it would also raise its photon luminosity, and as a result of both axion emission and enhanced photon emission its lifetime would be shortened. Thus, the all important observable for constraining axion emission from the sun is its age at a given $^4He$ mass fraction. For $(f_{PQ}/N) \lesssim 10^7$ GeV, a sun with our sun's $^4He$ abundance would be younger than our sun is known to be.19

Let us consider the sun in slightly more detail. As we have discussed the photon luminosity of the sun is determined by the opacity of solar material. Just by analyzing hy-
drostatic equilibrium and energy transport in stars like the sun (i.e., stars less massive than 
about 2 $M\odot$), Chandrasekhar has derived a remarkable formula (the so-called luminosity
formula) which relates the photon luminosity of a star to its central temperature\textsuperscript{15}

$$\mathcal{L}_{\gamma} \propto (G\mu_e)^7 M^5 \frac{T_c}{T_c^{1/2}}$$

where $\mu_e$ is the mean molecular weight per electron (1 for a pure H star; 2 for a pure $^4$He
star), $M$ is the mass of the star, and $T_c$ is the central temperature.

Energy balance requires that the energy liberated by nuclear reactions

$$Q_{\text{nuc}} = \int_{\text{star}} \dot{\varepsilon}_{\text{nuc}} dM$$

be equal to the photon (plus axion) luminosity. In general the nuclear energy liberation rate
(here per g of material per sec) is very temperature dependent and can be parameterized
as

$$\dot{\varepsilon}_{\text{nuc}} \propto \rho T^n \propto T^{n+3}$$

where for the sun $n \approx 3$ (in a star like the sun the entropy per baryon $T^3/\rho$ is constant,
so that $\rho \propto T^3$). To begin, consider a star in the absence of axion emission. Energy
equilibrium requires that $\mathcal{L}_{\gamma} = Q_{\text{nuc}} \equiv Q_0$. Now suppose that the star radiates axions,
with an axion luminosity $Q_a = \epsilon Q_0$. Energy balance now implies that

$$\mathcal{L}_{\gamma} + Q_a = Q_{\text{nuc}}$$

$$\delta \mathcal{L}_{\gamma} + Q_a = \delta Q_{\text{nuc}}$$

By fiat $Q_a = \epsilon Q_0$, and using Chandrasekhar's luminosity formula we see that $\delta \mathcal{L}_{\gamma} = 0.5(\delta T_c/T_c)Q_0$. Finally, since $\dot{\varepsilon}_{\text{nuc}} \propto T^{n+3}$, we have $\delta Q_{\text{nuc}} = (n+3)(\delta T_c/T_c)Q_0$. Using the
perturbed energy balance equation we find that

$$\delta T_c/T_c = \epsilon/6.5 \quad \delta \mathcal{L}_{\gamma}/Q_0 = \epsilon/13$$

$$\delta \mathcal{L}_{\text{TOT}}/Q_0 = 14\epsilon/13 \quad \delta R/R_0 = -\delta T_c/T_c = -\epsilon/6.5$$

where we have used the fact that the radius of the sun $R \propto T_c^{-1}$ to find $\delta R/R_0$.

As advertised, we see that a star perturbed by axion emission contracts to raise its
temperature and restore energy equilibrium, and in the process increases its photon lu-
minosity also. Suppose that $\epsilon = 1/2$; then the central temperature increases by about
8%, and the total luminosity by about 54%, thereby decreasing the evolution time scale
by more than a factor of 2, strongly suggesting that $\epsilon \gtrsim 1/2$, or so, is inconsistent with
our knowledge of the sun. [To make such an argument rigorous one must also consider
the very strong compositional dependence of stellar models, reflected in $L, \propto \mu_c^4$; this has
been done in ref. 19.]

An even more sensitive barometer for stellar axion emission is the rate of $^8$B neutrino
emission \cite{20} (the high energy neutrinos which have been detected by Davis' $^{37}$Cl experiment
and by the KII detector). The rate of emission of these high-energy neutrinos is propor-
tional to $T^2_c$ ($p \simeq 13$, depending upon which quantities in the stellar model are held fixed).
Using our previous formula for $\delta T_c$ we see that the $^8$B neutrino flux would increase by
almost a factor of 3 (for $\epsilon = 1/2$), exacerbating an already large discrepancy. We see that
the simple-minded limit provided by $\epsilon_a < \epsilon_{\text{nuc}}$ is more than justified!

Finally, it is interesting to note that Germanium double beta decay experiment of
Avignone, et al. \cite{21} provides a similar limit to $f_{\rho Q}/N$ based upon the non-observation of
solar axions in their detector. [Solar axions would be detected by their interactions with
electrons in the Ge detector: $e^- + a \rightarrow e^- + \gamma$.]

To summarize the axion mass limits that follow from the sun, they are: $m_a \lesssim 1 \text{ eV}$
(DFSZ); and $m_a \lesssim 20/[(E/N - 1.95)/0.72]$ (hadronic) (see Fig. 2). Moreover, these limits
do not apply to an axion of mass greater than 10 keV or so, since the production of such
axions would be severely suppressed owing to the fact that the temperature at the center
of the sun is only a few keV. While we have explicitly displayed the dependence of the
hadronic axion's model dependent coupling to two photons, we have not been so careful
with the model dependence of the DFSZ axion's coupling to electrons. DFSZ axion mass
limits are necessarily proportional to $\cos^2 \beta$, and in the limit that $\beta \rightarrow \pi/2$ ($X_e \rightarrow 0$),
they revert to those of the hadronic axion.

The discussion above should provide the reader with the flavor of stellar limits to the
axion mass. They all rely on the fact that axion emission modifies stellar evolution in such
a way as to significantly affect an observable, usually the lifetime of the star. Now let us
turn to the most stringent stellar evolution limits that exist at present. These limits are
provided by the evolution of red giant stars, stars whose central temperatures reach $10^6$
K and whose central densities are $\sim 10^3 - 10^4 \text{ g cm}^{-3}$. Because hadronic and DFSZ axions
couple very differently to electrons, their mass limits are very different.

The constraint to the mass of the hadronic axion is based upon the helium-burning
lifetimes of red giant stars.\cite{24} As we discussed above, when axion emission is taken into
account the central temperature of the star is necessarily increased to satisfy the extra
energy being carried away by axions, and this accelerates the evolution and shortens the
lifetime. The helium-burning phase of a red giant lasts of order $10^8$ yrs or so—too long for
most astronomers to observe. However, when one observes a cluster of stars (say M67, for example), the number of helium-burning red giants one sees is determined by the length of time red giants typically spend burning helium—the shorter the time, the fewer that will be seen. Raffelt and Dearborn\textsuperscript{22} argue that a hadronic axion of mass greater than about \(2 \, \text{eV}/[(E/N - 1.95)/0.72]\) would reduce the helium-burning time scale by more than an order of magnitude, in severe contradiction with observations of the number of helium-burning red giants seen in the cluster M67. Once again we see the factor which arises from the model dependence of the axion-photon-photon coupling.

The red giant limit for the DFSZ is based upon a slightly more subtle dynamical argument.\textsuperscript{23} Before helium ignition occurs in the core of a red giant, the \(^4\text{He}\) core is supported by electron degeneracy pressure. This is a very dangerous condition because any increase in temperature is not accompanied by a similar increase in pressure, and so once any \(^4\text{He}\) is ignited nuclear burning is a runaway process, until thermal pressure support becomes dominant. The brief period of thermal runaway is referred to as the helium flash (not to be confused with a hot flash). [In an ordinary star, the simple physical fact that the pressure is proportional to the temperature stabilizes nuclear burning, as any increase in temperature is accompanied by an increase in pressure which causes the star to expand and thereby cool—Nature’s stellar thermostat!] Before the helium flash, hydrogen continues to burn just outside the helium core. As the helium core grows in mass, its radius decreases (for degenerate matter \(R \propto M^{-1/3}\)), and the accompanying release of gravitational binding energy heats the core. Eventually, the helium core becomes hot enough for the triple-\(\alpha\) process to burn helium to carbon. The effect of axion cooling in the helium core decreases the temperature rise in the helium core associated with the contraction, and according to Dearborn, et al.\textsuperscript{23}, can prevent helium ignition from ever taking place for a DFSZ axion of mass greater than about \(10^{-2} \, \text{eV}\).

Let us consider their argument in slightly more detail. As the helium core increases in mass the size of the core contracts \((MR^3 = \text{const})\), thereby releasing gravitational energy:

\[
\dot{E}_{\text{grav}} \simeq \frac{d}{dt} \left( \frac{GM}{R} \right) \propto M^{1/3} \dot{M}
\]

In the absence of axion cooling, the dominant cooling mechanism for the approximately isothermal core is neutrino emission (because of the long mean free paths of electrons in degenerate matter, degenerate matter is almost always isothermal). For our purposes, let us assume that axion emission dominates, which is the case for an axion of mass sufficient to “screw up” the helium flash. The Compton-like process is the dominant axion emission
process in the helium core, and the energy radiated in axions is

$$Q_a \sim m_a^2 MT^6$$

Energy equilibrium, i.e., $\dot{E}_{\text{grav}} = Q_a$, determines the temperature of the core:

$$T_{\text{core}} \sim m_a^{-1/3} M^{-1/9} \dot{M}^{1/6}$$

Note, the larger the axion mass the lower the temperature of the core; this differs from the usual case where the existence of axions actually causes the star to raise its temperature to compensate for axion emission. Of course, in the present situation that is not possible since nuclear reactions are not yet occurring in the $^4\text{He}$ core. One can easily appreciate how a sufficiently massive axion can prevent the core from reaching the temperature required to ignite helium. However, the unstable nature of degenerate matter gives one pause; to ignite the core one only has to “light” the smallest region, which then triggers thermal runaway and ignites the entire core. Since the SN 1987A bound will be more stringent than this one, we do not have to lose sleep over this point.

To summarize the red giant constraints, the helium-burning lifetime argument precludes a hadronic axion of mass greater than about $2 \text{eV}/[(E/N - 1.95)/0.72]$, while the ignition of helium burning precludes a DFSZ axion more massive than about $10^{-2} \text{eV}$ (see Fig. 2). Because the temperatures in the cores of red giant stars are of order $10 \text{keV}$ or so, emission of axions more massive than about $100 \text{keV}$ is severely suppressed, and so these arguments do not apply to axions more massive than about $200 \text{keV}$ (which of course are precluded by laboratory searches).

Before going on to discuss the most stringent astrophysical bound, that based upon the cooling of yet another kind of star, the newly-born, hot neutron star associated with SN 1987A, we should mention two other astrophysical bounds. First, the cooling of relatively young (few $100 \text{yrs}$ old) neutron stars, including the Crab pulsar and RCW 103. It has been argued that an axion (of either type) more massive than about $10^{-2} \text{eV}$ or so would cool several of the well known young neutron stars so rapidly as to be inconsistent with Einstein measurements of their surface temperatures. Similar arguments based upon the cooling of white dwarf stars seem to preclude a DFSZ axion of mass greater than about $3 \times 10^{-2} \text{eV}$. However, for both of these bounds there are theoretical, as well as observational, uncertainties which cast some doubt upon them. In the case of the neutron star bound, it is not clear whether thermal emission has actually been detected from all, or even any, of these young neutron stars. On the theoretical side, it has been noted that superfluidity in cool neutron stars would greatly reduce axion emission and significantly degrade the
constraint. There are similar residual theoretical and observational uncertainties associated with the white dwarf bound. Since other constraints with fewer uncertainties exist that are as strong, or stronger, we will not dwell on the reliability of these limits further.

The various astrophysical constraints to the axion mass based upon stellar evolution are summarized in Table 1.

IV. Axions and SN 1987A

SN 1987A not only confirmed astrophysicists' more cherished beliefs about type II (core collapse) supernovae, but also provided a unique laboratory for the study of the properties of ordinary neutrinos, right-handed neutrinos, axions, and other exotic hypothetical particles. Here we will be interested in the bound to the axion mass provided by SN 1987A. As we shall see, it is the most stringent, and I believe, the most reliable astrophysical constraint to the axion mass.

Shortly after the gravitational collapse and hydrodynamic bounce of the 1.4 $M_\odot$ Fe core of the blue super giant Sanduleak -69 202 (thought to be a $\sim 15 M_\odot$ star), the central temperature of the nascent neutron star was 20-70 MeV and the central density was $\sim 8 \times 10^{14}$ g cm$^{-3}$. During the catastrophic collapse of the Fe core about $3 \times 10^{53}$ ergs of binding energy were liberated, and according to the standard picture, this energy is radiated in thermal neutrinos of all 3 types. The neutrino mean free path within the core is much smaller than the size of the core ($\sim 10$ km) and so even neutrinos are trapped in the core. Thus, neutrinos are radiated from a neutrino sphere ($R \sim 15$ km, $\rho \sim 10^{12}$ g cm$^{-3}$, $T \sim 4$ MeV).

Neutrino emission is characterized by two phases: the first is powered by residual accretion and hydrodynamic contraction of the outer core, and lasts 1-2 sec; the second phase is powered by the diffusion of heat trapped in the inner core region, and lasts $\sim 5-10$ sec, the time scale for neutrino diffusion from the core to the neutrino sphere. The energies associated with the two phases are comparable, and as a result one expects a neutrino burst of order 5-10 sec. The observations of KII$^{27}$ and IMB$^{28}$ are both qualitatively and quantitatively consistent with the standard picture. As it does with other types of stars, if the axion exists, it can play an important role in the cooling of this nascent neutron star. In the case of SN 1987A the observable effect of axion cooling would be the shortening of the neutrino burst, and fortunately we have 19 beautiful neutrino events spread out over about 10 sec to study the potential effects of axion cooling!

Under the conditions that existed in the post collapse core the dominant axion emission process is nucleon-nucleon, axion bremsstrahlung.$^{29}$ In the one pion exchange approxima-
tion (OPE) there are 4 direct and 4 exchange diagrams (see Fig. 1). The relevant axion coupling here is that to nucleons, which we should we recall is relatively insensitive to the type of axion and is of order $m_N/(f_a/N) \simeq 10^{-7}(m_a/eV)$. The full matrix element squared (64 terms) has been evaluated exactly in the OPE approximation. From $|\mathcal{M}|^2$ the axion emission rate (here, per volume per time) is given as

$$\dot{e}_a = \int d\Omega_1 d\Omega_2 d\Omega_3 d\Omega_4 (2\pi)^4 S |\mathcal{M}|^2 \delta^4(p_1 + p_2 - p_3 - p_4 - p_a) E_a f_1 f_2 (1 - f_3)(1 - f_4)$$

where $d\Omega_i = d^3p_i/(2\pi)^3 2E_i$, the labels $i = 1-4$ refer to the incoming (1,2) and outgoing (3,4) nucleons, $i = a$ denotes the axion, $S$ is the symmetry factor for identical particles in the initial and final states, $|\mathcal{M}|^2$ is summed over initial and final nucleon spins, and the nucleon phase space distribution functions are $f_i = [\exp(E_i/T - \mu_i/T) + 1]^{-1}$. The emission rate is relatively easy to evaluate in the fully degenerate or non degenerate regimes; however, the nucleons in the core are semi-degenerate, $\epsilon_{FERMI} \sim T$. In addition, since the post collapse core has roughly equal numbers of neutrons and protons, 3 bremsstrahlung processes are important: $\nu n \rightarrow \nu n + a$, $pp \rightarrow pp + a$, and $np \rightarrow np + a$. The axion emission rate, for all 3 processes and arbitrary nucleon degeneracy, has been evaluated numerically.\textsuperscript{30} [As it turns out, the non-degenerate axion emission rate provides a good approximation to the actual rate for the semi-degenerate conditions that exist in the hot neutron star.\textsuperscript{30}]

Axions less massive than about 0.02 eV, once radiated, freely stream out of the nascent neutron star, and thereby accelerate the cooling. Qualitatively then, one would expect axion emission to shorten the duration of the neutrino pulse—this is in fact what occurs (see Fig. 3). [Of course axion emission, which proceeds predominantly from the high temperature, high density inner core does not directly affect neutrino emission, which proceeds from the neutrino sphere (in the outer core).] We have incorporated axion cooling into realistic numerical models of the initial cooling of the nascent neutron star;\textsuperscript{31} the biggest theoretical uncertainty in these models is the equation of state (EOS) above nuclear density, densities which are achieved in the core during and after collapse. We have allowed for a wide range of EOS's, from a very stiff EOS to a very soft EOS. For our various axion-cooled, numerical models we have computed the resulting neutrino flux and the predicted response of the KII and IMB detectors: expected number of events; and burst duration, $\Delta t(90\%)$, the time required for the number of events to achieve 90% of its final value. The quantity $\Delta t(90\%)$ is the most sensitive indicator of axion emission. Axion emission tends to rapidly cool the inner core, depleting the energy which powers the second part of the burst. This effect is clearly seen in Fig. 3 where $\Delta t(90\%)$ is plotted as a function of $m_a$.\textsuperscript{30}
Axion emission has virtually no effect on $\Delta t(90\%)$ until a mass of $\sim 3 \times 10^{-4}$ eV, and by an axion mass of $10^{-2}$ eV the duration of the neutrino burst has dropped to less than a sec ($\sim$ time scale for the first phase of the burst). For comparison, for an axion mass of $10^{-2}$ eV, the expected number of neutrino events has only dropped from $\sim 10$ to $\sim 8$ for KII and from $\sim 6$ to $\sim 4$ for IMB. Likewise, for an axion mass of $10^{-2}$ eV, neutrinos still carry away more than 50% of the binding energy. The large effect on the burst duration traces to the fact that axion emission from the core efficiently radiates away the heat which powers the latter phase of the burst.

[One might wonder if a finite mass for the electron neutrino could lengthen an axion-shortened neutrino burst. A 20 eV or so mass might work just fine for the KII events; however, because the energies of the IMB events are much larger on average, a mass of 30-50 eV would be required to lengthen the IMB burst, a value precluded by the KII data and laboratory experiments.]

For axion masses greater than $\sim 0.02$ eV axions interact sufficiently strongly so that they do not simply stream out: rather, they become trapped in the core and are radiated from an axion sphere, with temperature $T_a$. In the trapping regime, the axion luminosity is $\propto T_a^4$. With increasing axion mass, the axion sphere moves outward and therefore has a lower temperature. Thus, for $m_a > 0.02$ eV the axion luminosity decreases with increasing axion mass; whereas in the freestreaming regime the axion luminosity increases with increasing axion mass, as $m_a^2$ (see Fig. 4). For sufficiently large axion mass the effect of axion cooling becomes acceptable. The complexity of axion transport has thus far prevented us from incorporating axion cooling into our numerical models (although such work is in progress). However, simple analytical models indicate that for an axion mass of $\sim 2$ eV or greater the axions are so strongly trapped that their presence is equivalent to less than a couple of additional neutrino species and is therefore consistent with the observations of KII and IMB.\(^{32}\)

Two uncertainties cast a shadow of doubt on this limit: the equation of state at supernuclear density and the calculation of the axion emission rate. While the former is indeed an important uncertainty, we have explored a variety of EOS's and our limit does not vary significantly. The latter is of greater worry. The axion emission rate has been calculated in the OPE approximation at supernuclear densities, neglecting any finite density effects. Broadly speaking then, there are two concerns: the validity of the OPE approximation itself; and possible effects due to finite density. The validity of the OPE approximation has been addressed by calculating the cross section for the analogue process, $pp \rightarrow pp + \pi^0$, using OPE, and comparing the result to the existing experimental data.\(^{33,34}\) The agreement
between the OPE cross section and the experimental data is quite impressive (better than a factor of 2) at the energies of interest. Since both the axion and pion are derivatively-coupled Goldstone bosons, one would expect this analogue process to provide a good test of OPE for axion production, a test which OPE passes with flying colors.

The finite density effects are more difficult to access. However, using the non-linear sigma model as guide for the behaviour of the interactions of pions and nucleons at high density, the authors of ref. 34 have concluded that any such effects are likely to be small (less than a factor of 3), and could possibly enhance axion emission, thereby strengthening the SN 1987A axion mass bound. The one finite density effect which might significantly modify the SN 1987A bound is the existence of an exotic state of matter at the core of the neutron star; e.g., a pion condensate, quark matter, or strange matter. Although such a possibility seems to be a long shot, we are currently studying how the axion limit would be affected.

In sum, the SN 1987A axion mass constraint is the most stringent astrophysical constraint, and applies equally to both types of axions. The inadequacies of OPE and the effects of finite density together might account for a factor of 3 or so uncertainty in the axion emission rate. Since that rate itself scales as the axion mass squared, that uncertainty probably amounts to less than a factor of \(\sqrt{3}\) uncertainty in the axion mass limit which follows. Moreover, because the physics of the cooling of the nascent neutron star is so simple and the observable (the neutrino burst) is so clean and direct, the SN 1987A constraint is probably the most reliable astrophysical bound.

[While I have only discussed the work I have been involved in with regard to axions and SN 1987A, similar work has been carried out by other authors,\textsuperscript{35} and at present all are in agreement as to the mass constraint which pertains in the free streaming limit, i.e., \(m_a \lesssim 10^{-3}\) eV. Thus far the other authors have not addressed the trapped regime.]

V. Axions and Cosmology

The topic of relic particles from the early Universe is a most interesting one. When the relic being considered is the axion, the topic is even more interesting! Relic axions arise due to three, different and distinct processes: thermal production;\textsuperscript{36} coherent production due to the initial misalignment of the axion field;\textsuperscript{37} and the decay of axionic strings.\textsuperscript{38} Each of these three processes can be the dominant production mechanism, depending upon the axion mass and whether or not the Universe ever underwent inflation (see Fig. 5). Let us consider the three processes in turn.

Owing to the state of near thermal equilibrium that existed in the early Universe
all kinds of interesting particles were present in great abundance at early times. Roughly speaking the criterion for a particle species to be in thermal equilibrium is that the reaction rate $\Gamma$ for processes which create and destroy that particle species occur rapidly compared to the expansion rate of the Universe, $H \sim T^2/m_{pl}$, i.e., $\Gamma \gtrsim H$. For axions the important creation and destruction processes are: photoproduction $\gamma + Q \rightarrow Q + a$ ($Q$ is a heavy quark), Primakoff production $\gamma + q \rightarrow q + a$ ($q$ is any charged particle), and nucleon-nucleon, axion bremsstrahlung $N + N \rightarrow N + N + a$ ($N$ is a nucleon) (see Fig. 1). Since each of these processes involves a single axion coupling, the rate for any of them scales as $m_a^2$.

Based upon a careful analysis of the rates for these processes we find that for an axion more massive than about $10^{-3}$ eV there is a period in the history of the early Universe, from a temperature greater than a few GeV (or even more for heavier axions) down to 100 MeV or so, where axions were in good thermal contact with the universal plasma, and should have been present in numbers comparable to photons (more precisely, $n_a\text{EQ} = n_\gamma/2$). Then, just as light neutrinos do, thermal axions decouple while they are still very relativistic, and their abundance per comoving volume "freezes in". Today, they should have an abundance relative to photons of order $n_a \simeq 2n_\gamma/11 \simeq 100\text{ cm}^{-3}$.

The thermal axion contribution to the present mass density of the Universe scales as $m_a$ and is given by

$$\Omega_{\text{thermal}}h^2 \simeq 10^{-2} (m_a/\text{eV})$$

where $100h\text{ km sec}^{-1} \text{ Mpc}^{-1}$ is the present value of the expansion rate (see Fig. 5). We see that thermal neutrinos can only close the Universe for an axion mass of order $100h^2$ eV, for which the axion lifetime is shorter than the age of the Universe. [It is intriguing to note that for $h < 1/2$ and $E/N \simeq 2$, thermal hadronic axions would close the Universe, would not have decayed by the present epoch, and would escape the previously discussed astrophysical bounds.]

While thermal axions probably cannot provide a significant fraction of the present density, it has been pointed out that the lifetimes of multi-eV axions are "well-matched" to the present age of the Universe, i.e., sufficiently long so that not all the relic axions have decayed, and sufficiently short so that a substantial fraction are decaying at present. Since the decay of an axion is a 2-body process, the decay-produced photons are mono-energetic (but slightly-broadened due to any velocity that the decaying axions may have, $\Delta \lambda \simeq (v/c)\lambda$). While multi-eV thermal axions will not contribute substantially to the
present mass density of the Universe, they will find their way into the many gravitational potential wells that exist, e.g., in galaxies (including our own) and in clusters of galaxies. Moreover, their decays will produce potentially detectable line radiation, at a wavelength \( \lambda_a = 2hc/m_a = 24800\text{Å}/(m_a/eV) \). The intensity of this radiation from the halo of our own galaxy should be

\[
I_{\text{halo}} = 2 \times 10^{-23} \text{erg cm}^{-2} \text{sec}^{-1} \text{arcsec}^{-2} \text{Å}^{-1} (m_a/eV)^{10} [(E/N - 1.95)/0.72]^2 J(\theta)
\]

where \( J(\theta) \) is the angular dependence of the signal which owes to the fact that we do not reside at the center of our galaxy. The width of such a line is expected to be of order the virial velocity in our galaxy, or \( \Delta\lambda \simeq 10^{-3}\lambda_a \). When one turns a telescope to the blank night sky one sees many lines—not all from axion decays, rather from airglow! Shown in Fig. 6 is a high-resolution spectrum of the night sky (air glow) and the expected axion line (for \( (E/N - 1.95) = 0.72 \)); from the existing data it is clear that an axion mass of greater than 4 eV or so is definitely precluded—so much for thermal axions providing closure density.

Even more favorable is the axion-produced line from relic thermal axions which reside in clusters

\[
I_{\text{cluster}} = 2 \times 10^{-20} \text{erg cm}^{-2} \text{sec}^{-1} \text{arcsec}^{-2} \text{Å}^{-1} (m_a/eV)^7 [(E/N - 1.95)/0.72]^2
\]

Here the line width is expected to be of order the virial velocity in a cluster, or \( \Delta\lambda \simeq 10^{-2}\lambda_a \). Moreover, by observing a cluster one has two other advantages: first, one can remove many of the airglow lines by subtracting “off cluster” measurements from “on cluster” measurements; second, the wavelength of the cluster axion line depends upon the red shift of the cluster, \( \lambda_a(\text{cluster}) = (1 + z_{\text{cluster}})\lambda_a \), and by looking at two or more clusters with different red shifts one can further discriminate against other night sky lines. Currently an observational effort is being mounted to search for a cluster axion line, and it is hoped that this effort will be sensitive to an axion as light as about 2 eV. [The rapidly decreasing strength of the axion line with decreasing axion mass, together with the increasing glow of the night sky at longer wavelengths, precludes searching for axion line radiation for masses smaller than about 2 eV.]

Axions are also produced by a very interesting and highly non-thermal process involving the relaxation of the \( \bar{\Theta} \) angle. Today the axion mass anchors \( \bar{\Theta} \) at the CP-conserving value \( \bar{\Theta} = 0 \). The axion mass, which arises due to instanton effects, is very temperature dependent,

\[
m_a(T) \simeq 0.1m_a(\Lambda_{\text{QCD}}/T)^{3.7}
\]
At very high temperatures the axion mass is essentially zero. Specifically, at $T \sim f_{PQ}$, when PQ symmetry is spontaneously broken, the axion mass is for all purposes negligible and the axion is a Goldstone boson. That means that no special value of $\bar{\Theta}$ is specified by dynamics and all values of $\bar{\Theta}$ are equally palatable! Therefore, the initial value of $\bar{\Theta}$ must be chosen by some stochastic process, and in general the initial value of $\bar{\Theta}$, call it $\bar{\Theta}_1$, is likely to be of order unity. Thus at early times the axion field is misaligned with the minimum of its potential ($\bar{\Theta} = 0$).

When the axion mass does "turn on" and become comparable to the expansion rate of the Universe, the axion field will start to roll toward $\bar{\Theta} = 0$, and of course will overshoot $\bar{\Theta} = 0$. Thereafter, it will oscillate like as heavenly harmonic oscillator. These cosmic oscillations of the axion field correspond to a zero momentum condensate of axions (with phase space density well in excess of $10^{30}$)! It is simple to estimate their present mass density. The initial energy density trapped in the misaligned axion field is, $\rho_a = m_a(T_1)^2 a_1^2 = m_a(T_1)^2 \theta_1^2 (f_{PQ}/N)^2$. The initial axion number density is just $\rho_a/m_a(T_1)$, or

$$n_a(T_1) \sim m_a(T_1) \theta_1^2 (f_{PQ}/N)^2$$

where $T_1$ is the temperature when the axion field begins to oscillate, $m_a(T_1) \sim 3H(T_1) \sim T_1^2/\mpl$. For an axion of mass $10^{-5}$ eV, $T_1$ is of order a GeV, and $T_1$ scales as $m_a^{-0.18}$. Assuming that there has been no entropy production since the axion field began to oscillate, the axion number density to entropy density ratio is conserved (even in the presence of the time-varying axion mass),

$$n_a/s \sim \frac{m_a(T_1) \theta_1^2 (f_{PQ}/N)^2}{T_1^3} \sim \frac{\theta_1^2 m_\pi f_\pi}{m_a^2 T_1 \mpl}$$

[The quantity $n_a/s$ corresponds to the number of axions per comoving volume ($\propto n_a R(t)^3$). This is because so long as the expansion is adiabatic, the entropy density $s$ scales as $R(t)^{-3}$. $R(t)$ is the scale factor of the Universe.]

The energy density today then is given by this constant ratio times the present entropy density ($s_0 = 7.04 n_\gamma \simeq 2970$ cm$^{-3}$) times the axion mass. Remembering that $T_1 \propto m_a^{-0.18}$, we see that $\Omega_a h^2 \propto m_a^{-1.18}$, where the unusual power of the mass traces to the way in which the axion mass turns on (see Fig. 5).

When this calculation is done very carefully (anharmonic effects taken into account, the motion of the axion field integrated precisely, etc.) the following expression results for the axion's contribution to the present energy density$^{43}$

$$\Omega_a h^2 = 0.85 \times 10^{\pm 0.4} \Lambda_{300}^{-0.7} (m_a/10^{-5}$ eV$)^{-1.18}$$

20
where $\Lambda_{200} \equiv \Lambda_{QCD}/200$ MeV, and the $10^{\pm 0.4}$ factor reflects the theoretical uncertainties. In deriving this formula, it has been assumed that there has been no significant entropy production since the epoch of axion production ($T \sim T_I$). If there has been significant entropy production, say the entropy per comoving volume ($S \equiv R(t)^3 s$) increased by a factor $\gamma$, then $\Omega_a$ is reduced by the same factor of $\gamma$.\textsuperscript{37}

Moreover, it has been assumed that the initial misalignment angle of the axion field is just the rms average of a flat distribution of initial values from 0 to $\pi$, that is, $\theta_1 = \pi/\sqrt{3}$. Assuming that the Universe never underwent inflation this is the reasonable thing to do: At the time the axion field began to oscillate the presently observable Universe was comprised of about $10^{30}$ or so causally-distinct volumes, each of which should have an independently chosen value for $\theta_1$; clearly, the value of $\theta_1$ relevant to computing the average mass density of axions in the Universe today is $\pi/\sqrt{3}$.

On the other hand, if the Universe underwent inflation, either after or during PQ symmetry breaking, then the entire observable Universe should be within a single inflationary region (or bubble, if you prefer) within which $\theta_1$ takes on the same value. That value is equally likely to be in any interval between 0 and $\pi$, i.e., $\theta_1$ is just as likely to be between 0.1 and 0.2, as it is to be between 1.5 and 1.6. [Of course, it is fair to say that the a priori probability of $\theta_1$ being in the interval $[1,1.5]$ is 100 times greater than it being in the interval $[0.01,0.015]$.] If one averages over all inflationary patches in the entire Universe one can say that the rms value of $\theta_1$ should be $\pi/\sqrt{3}$. However, that tells us nothing about the initial value of $\bar{\theta}$ in our neck of the woods. In order to determine $\theta_1$, we would have to measure the mass density of axions and the axion mass! Thus it is clear that in this case the energy density in axions is not precisely determined. Putting in the $\theta_1$ dependence in the energy density of relic, coherent axions we have

$$\Omega_a h^2 = 0.13 \times 10^{\pm 0.4} \Lambda_{200}^{-0.7} f(\theta_1) \theta_1^2 (m_a/10^{-5} \text{ eV})^{-1.18}$$

where the function $f(\theta)$ accounts for anharmonic effects: $f(\theta)$ is monotonically increasing and $f(\theta_1 = 0) = 1.0$.

Note that the theoretical uncertainties inherent in $\Omega_a$ are large: from particle physics a factor of $10^{\pm 0.4}$, and from cosmology a factor of $h^2$—all told, easily a factor of 10. For canonical values and $\theta_1 \simeq \pi/\sqrt{3}$, an axion mass of about $10^{-5}$ eV or so corresponds to closure density in axions.

The final mechanism for axion production is even more intriguing: axion production through the decay of axionic strings, and it was first discussed by Davis.\textsuperscript{44} In the case that the Universe never inflated, the initial value of $\bar{\theta}$ not only uniformly samples the interval of $[0,\pi]$, but also has non-trivial topology. That is, the initial mapping of $\theta_1$ to
our 3-dimensional space cannot in general be smoothly deformed to a uniform value of $\theta_1$ throughout space. The topological entities which exist are axionic strings. Let me be a little more specific about their formation and consequences.

In most axion models PQ symmetry breaking is effected by a complex scalar field, denoted by $\sigma$, which carries PQ charge. During PQ SSB $\sigma$ acquires a vacuum expectation value: $<|\sigma|> = f_{\text{PQ}}$. However, the argument of $\sigma$ is left undetermined—it is the axion degree of freedom. Consider the configuration around some axis where far from the axis $<\sigma> = f_{\text{PQ}} \exp(i\phi)$, and $\phi$ is the angle around the axis. This configuration cannot be smoothly changed into the configuration where the argument of $<\sigma>$ is constant and corresponds to an axionic string. In the core of the string (i.e., along the axis) $<|\sigma|>$ must necessarily vanish; and owing to the vacuum energy associated with $<\sigma> = 0$, together with the gradient energy associated with $<\sigma>$ ($|\sigma|$ changes from 0 in the core to $f_{\text{PQ}}$ far from the core) the string has an energy per unit length associated with it of order $\mu \sim f_{\text{PQ}}^2 \ln(f_{\text{PQ}}d)$, where $d$ is the characteristic distance between axionic strings.

[The energy per length of a single, isolated string is logarithmically divergent, a well-known feature of global strings. That the spontaneous breakdown of PQ symmetry should result in topologically stable string configurations is expected since $\Pi_1(U(1)) = Z_N$. A spontaneously broken $U(1)$ symmetry is the simplest model for a string; for a review of strings and their properties, see ref. 45.]

If the Universe did not inflate, then after PQ SSB, the Universe should be filled with a network of axionic strings. Much is known about the evolution of string networks. Very rapidly a scaling solution is reached where the energy density in string resides primarily in a few infinite portions of string per horizon volume and is given by

$$\rho_{\text{string}} \sim \frac{\mu}{t^2}$$

Here $t$ is the age of the Universe. It is referred to as the scaling solution because the ratio of energy density in string to the total density of the Universe is constant and equal to $G\mu$.

That the string energy density should evolve in such a way is somewhat surprising: On naive grounds, owing to the conformal stretching of the string network by the expansion of the Universe, one would expect the energy density of a string network to scale as $R(t)^{-2}$. In a radiation-dominated Universe this would imply that $\rho_{\text{string}} \propto t^{-1}$; if this did occur the energy density in string would rapidly grow relative to the radiation density, and string would soon come to dominate the energy density of the Universe. However, this does not occur because of dissipation: the cutting up of long pieces of strings into loops by string self-intersection and the dissipation of the oscillation energy of loops into gravitational
waves or other forms of radiation. In the present case, the dominant form of dissipation is the radiation of axions!

In each Hubble time \( (H^{-1} \sim t) \) essentially the full energy density of axionic string is converted into axions, so that the change in the number density of axions per entropy density is given by

\[
\Delta (n_a/s) \sim \frac{\mu t^2}{\omega T^3} \Delta (Ht)
\]

where during the radiation-dominated epoch the Hubble constant \( H \sim t^{-1} \sim T^2/m_{pl} \) and \( \omega \) is average energy per axion radiated. The total number of axions produced per comoving volume is obtained by integrating the above equation:

\[
\frac{n_a}{s} \sim \mu \int_{T_1}^{f_{eq}} \frac{dT}{\omega m_{pl}^2}
\]

where \( T_1 \) is the temperature at which the axion mass becomes comparable to the expansion rate. After this, the axion mass becomes significant, and the string network becomes a network of domain walls bounded by strings which quickly decays.

In order to calculate the axion production via this mechanism we must know \( \omega(t) \), the average energy of an axion produced by string dissipation at time \( t \). This is where the discussion heats up: Davis\(^4^4\) argues that the axions produced have the longest wavelengths that they could be expected to have, of order the horizon, \( \omega(t) \sim t^{-1} \); whereas Harari and Sikivie\(^4^7\) argue that there is a \( 1/k \) spectrum of axion energies, which leads to \( \omega \sim \ln(f_{PQ}t)t^{-1} \)—a difference of a factor of \( \ln(f_{PQ}t) \) or about 100! The number of axions produced by string decay (per comoving volume) is

\[
\frac{n_a}{s} \sim \left[ 1 \text{ or } \ln(f_{PQ}t_1) \right] \frac{f_{PQ}^2}{T_1 m_{pl}^2}
\]

The form of this expression should be familiar: up to a factor of \( [1 \text{ or } \ln(f_{PQ}t_1)]/\theta_1^2 \) it is identical to the expression for the number of axions produced by the initial misalignment of the \( \Theta \) angle! Depending upon the spectrum of axions produced by the decay of axionic strings, these axions either contribute a comparable number of axions, or 100 times as many axions, as the misalignment mechanism does.

More precisely, Davis\(^4^4\) claims that the string-produced population of axions contributes a mass density

\[
\Omega_a h^2 \simeq (m_a/10^3 \text{ eV})^{-1.18}
\]

Harari and Sikivie\(^4^7\) would claim that the contribution of string-produced axions is about a factor of 100 less. The difference is crucial: if Davis is correct axions provide closure
density for a mass of $\sim 10^{-3}$ eV, rather than $\sim 10^{-5}$ eV. If Harari and Sikivie are correct, then string-produce axions only slightly increase the axion density over that due to the misalignment mechanism.

In any case we see that the energy density of axions produced by coherent processes increases with decreasing axion mass. Based upon our knowledge of the present age of the Universe, that it is greater than 10 Gyr, $\Omega h^2$ must be less than about 1. In the non-inflationary case, this restricts the axion mass to be

$$m_a \gtrsim 10^{-6} \text{ eV}$$

if Harari and Sikivie are correct; and

$$m_a \gtrsim 10^{-3} \text{ eV}$$

if Davis is correct. Because of the peculiar scaling of $\Omega_a$ with the axion mass, the cosmic density provides a lower limit to the axion mass.

In the inflationary case there are no string-produced axions (as the value of $\theta_1$ is uniform throughout the observable Universe). Moreover, the bound to $m_a$ based upon the present mass density of axions depends upon $\theta_1$:

$$m_a \gtrsim 10^{-6} [\theta_1/(\pi/\sqrt{3})]^{1.7} \text{ eV}$$

These bounds are shown in Fig. 2.

We see that if Harari and Sikivie are correct, or if the Universe never underwent inflation, there is a substantial window between the SN 1987A bound and the axion mass density bound. On the other hand, if Davis is correct and the Universe never inflated, that window collapses to a single value for the axion mass: $m_a = 10^{-3}$ eV.

To summarize, the mass density of relic, thermal axions can probably never approach closure; however, the mass density can be significant enough to make an axion of mass 2-5 eV detectable through its radiative decays. The mass density of relic axions produced by misalignment, or by the decay of axionic strings, can be very significant. However, there are still substantial uncertainties in the calculation of the relic axion abundance: if the Universe inflated, the value of $\theta_1$; and if the Universe did not inflate, the spectrum of string-produced axions. In either case, the additional uncertainties in the values of $\Lambda_{QCD}$ and the Hubble constant further complicate matters, making a precise determination of the value of the axion mass which leads to $\Omega_a \simeq 1$ impossible at present.

One thing is certain, if axions do indeed contribute $\Omega_a = 1$, they behave as cold dark matter because of their intrinsically small velocities.\textsuperscript{48} And therefore, there is every reason
to believe that axions provide the dark matter known to exist in our halo, and therefore have a local density of \[ \rho_a = \rho_{\text{halo}} \approx 0.3 \text{ GeV cm}^{-3} \]
\[ n_a \approx 3 \times 10^{-13} \left( 10^{-5} \text{ eV} / m_a \right) \text{ cm}^{-3} \]

This estimate of the local axion density should be accurate to about a factor of 2. I should also mention that cold dark matter with inflation-produced adiabatic density perturbations continues to be a very attractive scenario of structure formation.49

[In axion models where \( N > 1 \), there are actually \( N \) distinct, degenerate vacua:50 
\[ \Theta = 2\pi n \ (n = 0, 1, \ldots, N - 1) \]. This means that in the absence of inflation different regions of space will wind up in different minima (i.e., different values of \( n \)), and will be separated by axionic domain walls of surface density \( \sigma \sim m_\pi f_\pi f_{\text{PQ}} \). Such domain walls are cosmologically very bad, leading to a wall-dominated Universe. There are number of ways of avoiding this catastrophe, including inflation.51 Consideration of axion domain walls doesn't really lead to any constraint to the axion mass: if they exist, axions are precluded, and so we will not address that issue here.]

### VI. Summary: Windows of Opportunity

A priori the axion window spans an 18 order-of-magnitude mass window: \( 10^{-12} \text{ eV} - 10^6 \text{ eV} \). Laboratory experiments have probed masses greater than about \( 10^4 \text{ eV} \) without success. This of course includes the originally favored value, \( m_a \sim 200 \text{ keV} \), corresponding to a PQ SSB scale equal to that of the electroweak scale. Through a variety of very clever and interesting astrophysical and cosmological arguments, all but about 3 of the remaining 16 orders-of-magnitude have been probed. The evolution of red giant stars precludes an axion mass in the intervals: \( 10^{-2} \text{ eV} - 10^5 \text{ eV} \) (DFSZ) and \( 2/[(E/N - 1.95)/0.72] \text{ eV} - 10^5 \text{ eV} \) (hadronic, or DFSZ axion with \( \beta \sim \pi/2 \)). The duration of the neutrino burst associated with SN 1987A precludes an axion mass in the interval: \( 10^{-3} \text{ eV} - 2 \text{ eV} \) (for either type axion). Radiative decays of relic, thermal axions preclude axion masses of 5-30 eV.

Very low mass axions are precluded by the relic density of axions produced by coherent processes in the early Universe. In the case that the Universe underwent inflation, masses smaller than about \( 10^{-6} \text{ eV} \left[ \theta_1 / (\pi / \sqrt{3}) \right] \)\(^2\) are precluded. Unfortunately, the value of the initial misalignment angle \( \theta_1 \) in our inflationary bubble is not known, and could with equal a priori probability lie anywhere in the interval \([0, \pi]\). If the Universe never inflated, then the rms value of \( \theta_1 \) is just \( \pi / \sqrt{3} \). However, in this case there are two mechanisms for axion production: the misalignment of the \( \Theta \) angle, and the decay of axionic strings. Production from the first mechanism is straightforward to compute (although there are
residual uncertainties), and an axion mass smaller than $\sim 10^{-6}$ eV is precluded. There is still spirited debate about the number of axions produced by the second mechanism: Davis claims 100 times that of the misalignment mechanism; while Harari and Sikivie claim that the number produced is comparable to that of the misalignment mechanism. If Davis is correct and the Universe never inflated, then an axion mass less than about $10^{-3}$ eV is precluded.

At present we are left with two windows of opportunity: (2 eV-5 eV) (for hadronic axions only) and ($10^{-6}$ eV - $10^{-3}$ eV), corresponding to PQ SSB scales of few $\times 10^7$ GeV and $10^{10} - 10^{13}$ GeV respectively. As noted previously, $10^{-6}$ eV is a “soft boundary;” if the Universe inflated, then the precise value depends upon $\theta_1^7$, and if the Universe never inflated, and Davis is correct the $10^{-6} - 10^{-3}$ eV window shrinks to the single value of $10^{-3}$ eV.

The 2-5 eV window for hadronic axions will be probed by at least two experiments: a telescopic search for photon line radiation from the decays of relic axions; and an experiment designed to detect axions emitted by our own sun, which will be sensitive to axions of mass 0.1 - 5 eV. The other window is even more intriguing because it encompasses the value of the axion mass for which axions provide closure density. Historically, that value has been taken to about $10^{-5}$ eV; as we have discussed, there are many caveats. If the Universe inflated, about all we can say is that the value which yields $\Omega_a = 1$ is less than about $10^{-5}$ eV and depends upon $\theta_1^7$. If the Universe didn’t inflate, then the situation is equally or even more complicated. Depending upon the spectrum of string-produced axions, the axion mass resulting in $\Omega_a = 1$ could be anywhere between $10^{-5}$ eV and $10^{-3}$ eV. Sikivie has proposed a very clever idea for detecting these cosmic axions, based on the conversion of cosmic axions to monoenergetic photons of energy $m_a$ in a strong magnetic field. Already experiments are underway to search for relic axions of mass $10^{-5}$ eV or so using his idea, and results have already been reported for axion masses of $0.4 - 1.5 \times 10^{-5}$ eV. However, the potentially interesting mass range up to $10^{-3}$ eV is not yet being probed, and stands as a real target of opportunity!

Does the axion actually exist? I don’t know. However, I am very certain that if it does, it will be found in the heavens and not on earth!

This review of the axion window is by no means complete; for more extensive reviews of the axion, see refs. 58. This work was supported in part by the DoE (at Chicago).
References


6. For a discussion of the properties of axions and their couplings to ordinary matter, see, e.g., D. Kaplan, Nucl. Phys. B260, 215 (1985); M. Srednicki, ibid, 689 (1985); P. Sikivie, in Cosmology and Particle Physics, eds. E. Alvarez, et al. (WSPC, Singapore, 1986); or J.-E. Kim, Phys. Rep. 150, 1 (1987). We follow the normalization conventions of Kaplan, Kim, and Sikivie, which differ from those of Srednicki by: \((f/N)_{\text{Srednicki}} = 2(f_{\text{PQ}}/N)\).


26. For a review of the standard picture of core collapse and cooling of the nascent neutron star; see, e.g., J.N. Bahcall, *Neutrino Astrophysics* (Cambridge Univ. Press, Cam-

33. K. Choi, et al., ref. 7.
257 (1988).
51. For a review of the solutions to the axion domain wall problem, see, e.g., P. Sikivie, in ref. 6.
56. P. Sikivie, N. Sullivan, and D. Tanner, experiment in progress.
57. In a recent AGS proposal A.C. Melissinos, et al, have proposed to search for a light, pseudoscalar (axion-like) boson in the mass range $10^{-6}$ eV - 1 eV, by looking for the rotation of polarized laser light which arises due to the boson's two photon coupling. As proposed, the sensitivity of the experiment in the $g_{a\gamma\gamma}$-mass plane does not include the axion, for which $g_{a\gamma\gamma} \simeq 1.4 \times 10^{-10} \left[\frac{(E/N - 1.95)}{0.72}\right](m_a/eV)$. For discussion of this experiment, see, e.g., F. Nezrick, in *Neutrino Mass and Neutrino Astrophysics (Telemark IV)*, eds. V. Barger, et al. (WSPC, Singapore, 1987), p. 398.
Table 1: Summary of axion masses *excluded* by astrophysical arguments based upon stellar evolution. Here \( \zeta = (E/N - 1.95)/0.72 \).

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>DFSZ</th>
<th>HADRONIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>1 eV - 10 keV</td>
<td>20 eV/( \zeta ) - 10 keV</td>
</tr>
<tr>
<td>Red Giants</td>
<td>( 10^{-2} ) eV - 200 keV</td>
<td>( 2 ) eV/( \zeta ) - 200 keV</td>
</tr>
<tr>
<td>SN 1987A</td>
<td>( 10^{-3} ) eV - 2 eV</td>
<td>( 10^{-3} ) eV - 2 eV</td>
</tr>
<tr>
<td>White Dwarfs</td>
<td>( 3 \times 10^{-2} ) eV - few keV</td>
<td>no limit</td>
</tr>
<tr>
<td>Neutron Stars</td>
<td>( \gtrsim 10^{-2} ) eV (?)</td>
<td>( \gtrsim 10^{-2} ) eV (?)</td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1. Axion couplings to ordinary matter (electrons, nucleons, and photons) and the dominant axion emission processes in stars. For the DFSZ axion the dominant emission processes in main sequence stars, red giant stars and white dwarfs are: the Compton like and bremsstrahlung processes, the rates for both of which are proportional to the axion-electron coupling squared. For the hadronic axion the dominant emission processes in these objects is the Primakoff process, owing to the fact that the tree level axion-electron coupling is highly suppressed. In neutron stars the dominant emission process for both types of axions is nucleon-nucleon, axion bremsstrahlung.

Fig. 2 Summary of the laboratory, astrophysical, and cosmological constraints to the axion mass, and the two remaining axion windows. Note the constraint based upon the cosmic density of string-produced axions is still very uncertain, as it depends upon the spectrum of string-produced axions (an issue which is still in dispute). If the Universe underwent inflation after, or during PQ symmetry breaking, then there are no string-produced axions and the cosmological limit based upon coherent production due to the initial misalignment of the axion field depends upon the initial misalignment angle (to the 1.7 power).

Fig. 3 The characteristic length of the predicted neutrino burst in the KII and IMB H$_2$O Cerenkov detectors as a function of axion mass for three different neutron star cooling models which include axion emission (from ref. 31). The quantity $\Delta t(90\%)$ (in sec) is the time required for the expected number of neutrino events to achieve 90% of its asymptotic value. Note that for an axion mass greater than $\sim 10^{-3}$ eV the duration of the neutrino bursts becomes significantly shorter than those observed ($\sim 6$ sec for IMB and $\sim 12$ sec for KII), thereby precluding such a value for the axion mass.

Fig. 4 The axion luminosity from the nascent neutron star associated with SN 1987A (based upon a simple analytic model$^{32}$) as a function of axion mass. For $m_a \lesssim 0.02$ eV axions simply freely stream out, and $Q_a \propto m_a^2$; for $m_a \gtrsim 0.02$ eV axions interact so strongly that they become trapped and are radiated from an axion sphere (like neutrinos). In this regime $Q_a \propto T_a^4 \propto m_a^{-16/11}$. For an axion mass in the interval $[10^{-3}$ eV, $2$ eV], the axion luminosity (more precisely, cooling rate) is unacceptably large, precluding such a mass.

Fig. 5 Summary of the relic axion contribution to $\Omega h^2$ from the 3 production processes: thermal,$^{36}$ misalignment,$^{37}$ and axionic string decay.$^{38}$ Note that if the Universe underwent inflation after, or during PQ symmetry breaking, there would be no string-
produced axions and the production due to misalignment would be proportional to the initial misalignment angle squared. Also note that the contribution of string-produced axions depends crucially upon the spectrum of axions from string decay, an issue which is still being debated, and which leads to an uncertainty of a factor of $\sim 100$ in axion production by this mechanism.

Fig. 6 High resolution spectrum of the night sky at Kitt Peak$^{42}$ and the axion line expected from axion decays in our galactic halo.$^{41}$ Note that an axion of mass 4 eV would have produced a very prominent line—more prominent than is seen.
Bremsstrahlung
AXION MASS CONSTRAINTS

$10^{-10}$ eV $10^{-5}$ eV 1 eV $10^5$ eV

--- LAB EXP'T---

--- RG (DFSZ) ---

--- RG (HADRONIC) ---

--- SN 87A ---

--- $\Omega_a > 1$ ---

--- $\Omega_a$ (Davis) > 1 ---

--- RELIC DECAYS ---

AXION WINDOWS

$10^{-5}$ eV 1 eV

--- (HADRONIC ONLY) ---

--- (DAVIS CORRECT & NO INFLATION) ---
- FIG 3 -