ANALYSIS OF LEADING EDGE SEPARATION
USING A LOW ORDER PANEL METHOD

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ABSTRACT

An examination of the potential flow computer code VSAERO to model leading edge separation over a delta wing. Recent improvements to the code suggest that it may be capable of predicting pressure coefficients on the body. Investigation showed that although that code does predict the vortex roll-up, the pressure coefficients have significant error. The program is currently unsatisfactory, but with some additional development it may become a useful tool for this application.
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# NOMENCLATURE

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<thead>
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<tr>
<td>b</td>
<td>Local span</td>
</tr>
<tr>
<td>c</td>
<td>Chord length</td>
</tr>
<tr>
<td>dS</td>
<td>Differential surface element</td>
</tr>
<tr>
<td>n</td>
<td>Unit normal vector</td>
</tr>
<tr>
<td>P</td>
<td>Arbitrary point in space</td>
</tr>
<tr>
<td>r</td>
<td>Vector between P and surface element dS</td>
</tr>
<tr>
<td>S</td>
<td>Surface of the arbitrary body</td>
</tr>
<tr>
<td>S∞</td>
<td>Imaginary surface at infinity</td>
</tr>
<tr>
<td>W</td>
<td>Wake surface</td>
</tr>
<tr>
<td>V</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>x</td>
<td>Chordwise coordinate</td>
</tr>
<tr>
<td>y</td>
<td>Spanwise coordinate</td>
</tr>
<tr>
<td>z</td>
<td>Coordinate perpendicular to x-y plane</td>
</tr>
<tr>
<td>φ</td>
<td>Velocity potential</td>
</tr>
<tr>
<td>μ</td>
<td>Doublet singularity strength per unit area</td>
</tr>
<tr>
<td>v</td>
<td>Source singularity strength per unit area</td>
</tr>
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**Subscripts**

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<thead>
<tr>
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<tr>
<td>i</td>
<td>Interior region</td>
</tr>
<tr>
<td>L</td>
<td>Lower wake surface</td>
</tr>
<tr>
<td>P</td>
<td>Refers to point P</td>
</tr>
<tr>
<td>U</td>
<td>Upper wake surface</td>
</tr>
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INTRODUCTION

Panel methods have been primarily used for the determination of a flow field about a two or three dimensional body. The body is represented as being made up of a set of panels with a distribution of singularities across the set. If the singularities are constant within a panel a low order code is the result. If the singularities are variable within a panel a high order code is the result. It has been shown that low order panel codes can achieve good results if the panel density is sufficiently high at a substantial savings in computing time. The singularities can be sources, sinks, doublets, vortices, and any combination. However, if the body is to generate lift the panels must be either doublets or vortices.

The strength of panel methods is the capacity to model complex, arbitrary configurations. There has been some interest in using the low order panel code VSAERO to model a highly swept delta wing at high angles of attack. If this can be achieved it will be very useful in the design and analysis of high performance aircraft and extra-atmospheric vehicles such as the National Aerospace Plane.

When the leading edge of a delta wing is sharp, the flow separates from the wing, forms a spiral vortex sheet and then reattaches to the wing. The vortex sheet induces increased velocities on the surface of the wing. As a result the lift coefficient is higher than for attached flow for the same flight conditions. This incremental lift is called the vortex or non-linear lift and is highly desirable.

The difficulty to the problem is that panel codes are based on the inviscid flow equations, and model the effects of viscosity by the means of a wake composed of source panels. Potential flow codes historically have
not produced good results when separation has a significant effect.

However, preliminary studies have shown some promise.
THEORY

VSAERO uses the low order potential flow equations developed by Lamb (ref. 1). As we know from potential flow theory, any flow solution must satisfy the Laplace's Equation. Lamb assumes that a flow would have two solutions \( \phi \) and \( \phi' \) such that:

\[
\nabla^2 \phi = \nabla^2 \phi' = 0.
\]  

(1)

The Divergence theorem states that

\[
\int_V \nabla^2 \phi \, dV = -\int_S \nabla \phi \, dS
\]

(2)

applying the two equations, and multiplying by \( \phi' \) results in following equation:

\[
\int_V \phi' \nabla^2 \phi \, dV = \int_S \phi' \nabla \phi \, dS = 0.
\]

(3)

\( \phi \) and \( \phi' \) can be reversed to yield

\[
\int_S \phi' \nabla \phi \, dS = \int_S \phi \nabla \phi' \, dS = 0.
\]

(4)

Lamb chooses \( \phi' \) to be \( 1/r \) which satisfies the Laplacian and is a relatively simple function and the equation becomes
However, a singularity occurs at $r = 0$. By describing a small sphere with an incremental area of $d\varepsilon$ about point $P$ the singularity is excluded and the equation becomes:

$$\int S \frac{1}{r} \nabla \phi \, dS = \int S \phi \nabla \left( \frac{1}{r} \right) \, dS. \tag{5}$$

If we evaluate $\phi$ at point $P$ the equation becomes:

$$\int \phi \nabla \left( \frac{1}{r} \right) \, d\varepsilon + \int \phi \nabla \left( \frac{1}{r} \right) \, dS = \int \frac{1}{r} \nabla \phi \, d\varepsilon + \int \frac{1}{r} \nabla \phi \, dS. \tag{6}$$

This is $\phi$ at any point $P$ of the fluid in terms of $\phi$ and $\nabla \phi$ at the boundary. The first integral is the disturbance potential of a distribution of double sources or doublets with axes normal to the surface having a density $\phi$ per unit area, while the second integral is the disturbance potential of a distribution of simple sources having a density of $\nabla \phi$ per unit area. If $P$ is external to the surface then:

$$\phi_p = \frac{1}{4\pi} \int \phi \nabla \left( \frac{1}{r} \right) \, dS + \frac{1}{4\pi} \int \frac{1}{r} \nabla \phi \, dS. \tag{7}$$

$$\phi_p = 0 = \frac{1}{4\pi} \int \phi_i \nabla \left( \frac{1}{r} \right) \, dS + \frac{1}{4\pi} \int \frac{1}{r} \nabla \phi_i \, dS. \tag{8}$$
Subtracting equation 8 from equation 7 results in an equation for the velocity potential of the fluid both internal and external to the surface. As shown in Figure 1 the internal flow is the flow of interest, and the external flow is fictitious. The equation becomes:

\[
\phi_p = \frac{1}{4\pi} \int_{S+} \left( \phi - \phi_0 \right) n \cdot \nabla \left( \frac{1}{r} \right) dS + \frac{1}{4\pi} \int_{S-} \frac{1}{r} n \cdot (\nabla \phi - \nabla \phi_0) dS.
\]

(V9)

VSAERO assumes that \( S_\infty \) can be considered a large sphere centered on \( P \) such that:

\[
\frac{1}{r} = \nabla \left( \frac{1}{r} \right) = 0.
\]

Since the disturbance potential at infinity is zero, the velocity potential at infinity must that due to the onset flow. For \( W \) the upper and lower surfaces are infinitesimally close, and that the corresponding upper and lower elements can be combined \( \phi_i = 0 \). In addition entrainment is ignored

\[
n \cdot (\nabla \phi_u - \nabla \phi_v) = 0.
\]

These assumptions result in:

\[
\phi_p|_{W+} = \frac{1}{4\pi} \int_{W} (\phi_u - \phi_v) n \cdot \nabla \left( \frac{1}{r} \right) dW + \phi_{\infty P} \tag{10}
\]

The surface \( S \) does not have any simplifying assumptions, however, if the point \( P \) lies on \( S \) the integral becomes singular. \( P \) is excluded from the
surface $S$ by describing a local hemispherical deformation, and $\phi$ due to the local hemispherical deformation is:

$$\phi_{p|S} = - \frac{1}{4\pi} \int \int (\phi - \phi') n \cdot \nabla \left( \frac{1}{r} \right) R^2 \sin \gamma d\gamma d\theta$$

$$- \frac{1}{4\pi} \int \int \frac{1}{r} n \cdot (\nabla \phi - \nabla \phi') R^2 \sin \gamma d\gamma d\theta.$$

(11)

The integrals are evaluated between zero and $\pi$, and the limit as $R$ approaches zero results in:

$$\phi_{p|S} = 2 \left( \phi - \phi' \right)_{p}$$

(12)

If equations 9, 10, and 12 are combined appropriately we arrive at the equation for $\phi$ at any point $P$ based on the values of $\phi$ on the surface, the equation is:

$$\phi_{p} = \frac{1}{4\pi} \int_{S} (\phi - \phi) n \cdot \nabla \left( \frac{1}{r} \right) dS + \frac{1}{4\pi} \int_{S} \frac{1}{r} n \cdot (\nabla \phi - \nabla \phi) dS$$

$$+ \frac{1}{4\pi} \int_{W} (\phi_{u} - \phi) n \cdot \nabla \left( \frac{1}{r} \right) dW + \phi_{\infty} \frac{1}{2} (\phi - \phi_{i}).$$

(13)

The boundary condition used to solve for $\phi_{p}$ is the internal Dirichlet boundary condition. The total potential $\phi$ is taken as being composed of the onset potential $\phi_{\infty}$, and a disturbance potential $\phi_{d} = \phi - \phi_{\infty}$. In order to minimize the size of the surface singularities the potential of the fictitious
flow is set equal to that of the onset flow. Using these boundary conditions and looking at the fictitious flow equation 13 becomes:

\[
0 = \frac{1}{4\pi} \int_S \phi_d n \cdot \nabla \left( \frac{1}{r} \right) dS + \frac{1}{4\pi} \int_S \frac{1}{r} n \cdot \left( \nabla \phi - \nabla \phi_\infty \right) dS
\]

\[
= \frac{1}{4\pi} \int_W \left( \phi_u - \phi_\infty \right) n \cdot \nabla \left( \frac{1}{r} \right) dW + \frac{1}{2} \phi_P
\]

(14)

The doublet strength is defined as:

\[
4\pi \mu = \phi_d = \phi - \phi_\infty
\]

The source strength is defined as:

\[
4\pi \sigma = -n \cdot \left( \nabla \phi - \nabla \phi_\infty \right) = V_N - n \cdot V_\infty,
\]

and can be solved directly for \( \sigma \). For a more general case \( V_N \) may have two components representing a boundary layer displacement and inflow/outflow. Knowing the source strengths, VSAERO then solves equation 14 for the unknown doublet strengths.

\[
\phi_P = \frac{1}{4\pi} \int_S \mu n \cdot \nabla \left( \frac{1}{r} \right) dS + \frac{1}{4\pi} \int_S \frac{\sigma}{r} dS
\]

\[
= \frac{1}{4\pi} \int_W \mu_W n \cdot \nabla \left( \frac{1}{r} \right) dW + \phi_\infty + K_\mu_P
\]

(15)

If P is off the surface, then K is zero. If P is on an inside smooth surface, then K is \(-2\pi\). If P is on an outside smooth surface, then K is \(2\pi\). If P is on a
crease, then $K$ is equal to the solid angle. If the potential field has been computed for a grid, the velocity field can be calculated using a local differentiation scheme since:

$$V_p = -\nabla \phi_p,$$

or calculated directly which is the current VSAERO method with the following equation:

$$V_p = -\frac{1}{4\pi} \int_S \mu \nabla \left[ n \cdot \nabla \left( \frac{1}{r} \right) \right] dS - \frac{1}{4\pi} \int_S \sigma \nabla \left( \frac{1}{r} \right) dS$$

$$- \frac{1}{4\pi} \int_{W} \mu_w \nabla \left[ n \cdot \nabla \left( \frac{1}{r} \right) \right] dW + V_\infty.$$

(16)
DISCUSSION

Initially the 3000 panel version of VSAERO was used in the attempt to model the leading edge separation problem. The literature suggested that the code would converge on a solution. However, it appears that the user would have to enter a wake shape very close to the actually shape. There is a preprocessor commercially available called VORSEP which is suppose to generate a wake close to actual that allows the code to converge to the solution. This preprocessor was not immediately available, and it was decided to examine a modified version of VSAERO that was thought to be capable modeling the problem.

The modified version is time-dependent, and the wake is not specified. At time \( t = 0 \) the body does not have a wake. At time \( t = t_1 \) it has a wake one panel in length. The generation of the wake is shown in figure 23. The wake is generated by following the path of a particle originating on the separation line where the velocity of the particle at point \( P \) is equal to the velocity of the fluid at point \( P \). In this manner it was expected that wake generated would follow the path of the vortex sheet, and produce reasonable pressure distributions.

Be aware that the program has many variables which can be adjusted to modify the calculations. After an exhaustive parametric study it was determined that the program was unable to produce results to the desired accuracy. The wake shape generated was as far as the eye could tell correct. The pressure coefficients were incorrect in magnitude, but the shape of the curve of \( C_p \) versus span was correct. It was thought that the method of calculating the particle trajectory was too simple. Several different numerical schemes were initiated including a Runge-Kutte and
Adam's Predictor-Corrector methods. For each modification the same parametric study was performed. The results being a substantial increased in computing time without a significant improvement to the solution.

The velocity of the particle has three components: the onset velocity, the velocity due to body, and the velocity due to wake. Of these three only the velocity calculations due to the wake was new coding. The lines making up a wake panel are considered to be vortex filaments. Since the velocity due to an inviscid, vortex filament approaches infinity as the distance from that filament, \( R \), approaches zero, there is a cut off distance called \( R_{CUT} \). When \( R \) is less than \( R_{CUT} \) the induced velocity is set to zero. This was modified to such that when \( R \) was less than \( R_{CUT} \) the induced velocity became a linear function of \( R \). In either version \( R_{CUT} \) is a very important parameter. This modification had little effect on the results.

At this it was decided to test the capability of the program by manually forcing the wake into the proper shape. This produced improved results the best results thus far. The results, shown in figures 2 through 8, are not satisfactory. The program cannot produce better results without substantial modification.
CONCLUSIONS

VSAERO as it currently stands is inadequate for the modeling of leading edge separation. In addition to the poor correspondence with the Hummel wing, there is another problem. VSAERO assumes that the wake connects the body to a large spherical surface at infinity. Due to the necessity of taking very small time increments in modeling of leading edge separation, the length of the wake is on the order of the maximum body dimension. However, this does not appear to be as important as the near field problem.

The near field calculation coding is known to have problems, and in the investigation it was proven so. In order to obtain a reasonable flowfield about the body the near field calculations had to be bypass. Since the wake is in the near field, this has the potential for considerable error. If the near field problem can be corrected, VSAERO could be a useful tool in the analysis of leading edge separation in complex configurations.
REFERENCES


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