FREQ: A Computational Package for Multivariable System Loop-Shaping Procedures

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Summary

Many approaches in the field of linear, multivariable, time-invariant systems analysis and controller synthesis employ loop-shaping procedures wherein design parameters are chosen to shape frequency-response singular value plots of selected transfer matrices. This report documents a software package, FREQ, for computing within one unified framework many of the most used multivariable transfer matrices for both continuous and discrete systems. The matrices are evaluated at user-selected frequency-response values, and singular values and vectors are computed. FREQ also tabulates maximum and minimum singular values against frequency. Example computations are presented to demonstrate the use of the FREQ code.

Introduction

Many approaches in the field of linear, time-invariant systems analysis and controller synthesis currently employ loop-shaping procedures wherein design parameters are chosen to shape singular-value-magnitude plots of selected transfer matrices. The linear-quadratic-Gaussian (LQG) formulation, ranging from the LQG/loop-transfer-recovery (LTR) approach (Doyle and Stein 1981; “Multivariable Analysis...” 1981; Ridgely and Banda 1986; Stein and Athans 1987) to direct methods utilizing constrained optimization (Mukhopadhyay 1987; Hefner and Mingori 1987), has evolved into a sophisticated methodology for manipulating singular value (Bode-type) plots to specify such properties as stability margin and performance in multiloop feedback. Robust controller techniques (Dorato 1987) implicitly involve singular value plots to design multivariable compensators in the frequency domain. For linear time-invariant systems in general, the feedback properties of plant disturbance attenuation, stability margins, sensor noise response, and sensitivity to plant and sensor variations are quantitatively related to Bode-magnitude plots of the singular values of the return difference and associated inverse return difference matrices (Safonov, Laub, and Hartmann 1981).

A standard (Doyle and Stein 1981) plant/compensator configuration employed in many of the loop-shaping methodologies is shown in figure 1. The block diagram of figure 1(a) shows a plant with transfer matrix \( G_0 \) interconnected in unity-gain feedback fashion with a compensator \( K_0 \) defining a plant input \( u \). The system can be forced by exogenous commands \( r \), measurement noise \( n \), and disturbances \( d \). In figure 1(a), all disturbances have been reflected to the measured output \( y \). In the compensator design and system analysis process, a number of important transfer matrices arise. Depending on whether the loop in figure 1(a) is broken at the plant input or plant output, the loop-gain transfer matrix \( G_L \) is either \( G_L = K_0 G_0 \) or \( G_L = G_0 K_0 \), respectively. Figures 1(b) and 1(c) also give

\[
y = G_0 K_0 (I + G_0 K_0)^{-1} (r - n) + (I + G_0 K_0)^{-1} d
\]

and

\[
r - y = (I + G_0 K_0)^{-1} (r - d) + G_0 K_0 (I + G_0 K_0)^{-1} n
\]

where \( I \) denotes the identity matrix. Alternatively, for a reference input \( u_r \), if \( r \) is replaced by \( G_0 u_r \),

\[
u = K_0 G_0 (I + K_0 G_0)^{-1} (u_r - \hat{d} - \hat{n})
\]

and

\[
u_r - u = (I + K_0 G_0)^{-1} u_r - K_0 G_0 (I + K_0 G_0)^{-1} (\hat{d} + \hat{n})
\]

where \( \hat{d} \) and \( \hat{n} \) are defined by the relations

\[
G_0 \hat{d} = d
\]

and

\[
G_0 \hat{n} = n
\]
It has also been shown (Doyle and Stein 1981) that the stability robustness to unstructured uncertainties at the plant output can be measured in terms of inequalities involving singular values of

\[ I + (G_0 K_0)^{-1} \]

Tolerances for unstructured uncertainties at the plant input involve singular values of

\[ I + (K_0 G_0)^{-1} \]

For structured uncertainties at the plant input or output, structured singular value methodology (Doyle, Wall, and Stein 1982) should be employed. Loop-shaping techniques based on singular value tolerances work well for unstructured uncertainties, but they are likely to give overly conservative designs in the structured uncertainty case. However, singular value methodology is computationally easier to implement and can be used to give upper bounds on structured singular values.

Thus, for a given \( G_L \), the following arise:

- The closed-loop response (complementary sensitivity) matrix
  \[ G_L (I + G_L)^{-1} \]
- The return difference matrix
  \[ (I + G_L) \]
- The inverse return difference matrix
  \[ (I + G_L^{-1}) \]
- The sensitivity matrix
  \[ (I + G_L)^{-1} \]

Various loop-shaping, singular-value-based analysis and design methodologies employing these transfer matrices can be found in the previously cited references. The purpose of this report is to address computational issues common to all loop-shaping, singular-value-based methodologies—i.e., the numerical evaluation of the transfer matrices, the computation of their singular value bounds, and the tabulation of data for loop-shaping plots.

This report documents a software package, FREQ, for computing within one unified framework a variety of frequency-response transfer matrices including the aforementioned plant, compensator, loop gain(s), closed-loop response, return difference, inverse return difference, and sensitivity matrices. For the case in which the plant \( G_0 \) and/or compensator \( K_0 \) are constructed through given linear, time-invariant state-space realizations \( (A_g, B_g, C_g, D_g) \) and/or \( (A_k, B_k, C_k, D_k) \), respectively, FREQ internally calculates, for selected values of the frequency-response variable \( s = j\omega \), the plant and/or compensator transfer matrix as

\[ G_0(s) = C_g(sI - A_g)^{-1}B_g + D_g \]
\[ K_0(s) = C_k(sI - A_k)^{-1}B_k + D_k \]

where \( s \) represents the Laplace transform parameter, \( \omega \) represents the frequency, and \( j^2 = -1 \). For other forms of plant or compensator, the user has the freedom to externally compute and directly input the transfer matrix. Generally, at the option of the user, FREQ provides the capability of computing, for selected values of \( s = j\omega \), some or all of the transfer matrices, their singular values and vectors, as well as singular-value-magnitude tabulations of maximum and minimum singular values against frequency \( \omega \). FREQ also includes the capability of performing the foregoing family of computations for discrete system transfer matrices in which \( s \) is replaced by \( e^{j\omega} \), where \( 0 \leq \omega \leq \pi \). Specific details of the FREQ package are presented in the sections to follow.

The next section, “Functional Description of Software” (see p. 3), explains in linear algebraic terms the calculations that can be performed by FREQ. The following three sections may be viewed as a FREQ users manual. “Use of Software: Initialization and Output” (see p. 4) describes the function of some internal parameters used by FREQ and explains how to (re)initialize them. “Use of Software: Computational Routines” (see p. 7) explains how to call those FREQ subroutines that comprise the user interface to the working part of the package. This section is addressed to the analysis of transfer matrices arising from continuous systems; but, with a few additional considerations, this section applies also to discrete systems. “Use of Software: Discrete System Analysis” (see p. 17) gives the additional information necessary to use FREQ to analyze transfer
matrices arising from discrete systems. "Software Notes" (see p. 17) goes into more detail about how FREQ performs its computations. Information presented in this section helps the user to make the correct choices of program options and to understand any error messages that FREQ might generate. It also explains how to access the code, in which language FREQ is coded, and which third-party code is included. The section entitled "Example" (see p. 21) demonstrates the use of the code.

Five appendixes give information primarily of interest to programmers using the FREQ software package. "Appendix A—Guide to Singular Value Tabulation" (see p. 22) gives an annotated listing of SDTAB, one of the tabulation drivers, which can be used as a template by the user who has special needs in a tabulation driver. "Appendix B—Selected Subroutine Preambles" (see p. 26) gives the program preambles of the FREQ user interface subroutines. In the interest of economy of distribution, these have been left out of the versions of the source code distributed by the NASA-sponsored Computer Software Management and Information Center (COSMIC). "Appendix C—Symbolic Names of Global Entities" (see p. 52) lists the symbolic names of global entities such as subroutines, common blocks, etc., used by FREQ. The user needs to avoid using these as global names in his own code. In addition, these global entities are correlated with the software organization charts in figures 2 and 3. "Appendix D—Example Program Listing" (see p. 74) contains the listing of the program used to produce the example. "Appendix E—Output From Example Computation" (see p. 74) contains the output from executing the example program of appendix D.

Use of the FREQ package is demonstrated in the section "Example" (see p. 21) with data taken from a study in which a fine-pointing attitude control system was designed for a large, flexible hoop/column space antenna (Joshi, Armstrong, and Sundararajan 1986; Sundararajan, Joshi, and Armstrong 1987). The control system was designed to possess stability robustness with respect to high-frequency, unmodeled antenna dynamics, and a prototype version of FREQ was employed to obtain the robustness-barrier design plots. Similar studies and uses of FREQ for other large space antenna configurations may be found in Joshi and Armstrong (1987); Armstrong, Joshi, and Stewart (1988); and Joshi, Rowell, and Armstrong (1988). Each such application of FREQ indicated new user needs and preferences. Since the FREQ package has evolved through a sequence of control analysis and design applications, it is felt that it provides a body of tested software incorporating most of the desired properties and options needed by the practicing engineer.

**Functional Description of Software**

The primary function of the FREQ package is to provide a frequency-domain analysis tool for the generic plant/compensator configuration shown in figure 1 where both the plant and compensator were described by linear, time-invariant state-space realizations. In addition, FREQ contains other options that have been added at the request of early users.

FREQ operates at two levels. Subroutines SIGCAL and SIGDYN perform the analysis at a single value of the frequency parameter. Subroutines SIGTAB and SDTAB, which accept multiple values of the frequency parameter, perform the analysis at each of these values by making repeated calls to SIGCAL or SIGDYN, respectively.

For each single value of the frequency parameter, FREQ must perform a number of tasks. The first task of FREQ is to generate a transfer matrix $G_L(s)$ at a value of the Laplace variable $s = j\omega$ (where $j^2 = -1$) determined for a user-specified frequency $\omega$ or to generate a $z$-transform matrix $G_L(z)$ at a value of the variable $z = e^{j\omega}$ determined for a user-specified normalized frequency $\tilde{\omega}$. A typical "normalized frequency" is angular frequency multiplied by sampling period. For simplicity of presentation, we will refer only to the continuous case ($s = j\omega$) in the following discussion. The reader interested in the discrete case ($z = e^{j\omega}$) can substitute $z$ for $s$ in his own mind and take note of the later section of this paper, "Use of Software: Discrete System Analysis" (see p. 17).

The user has several options at this stage: $G_L(s)$ may be $G_0(s), K_0(s), K_0(s)^{\pm 1} G_0(s)^{\pm 1}$, or $G_0(s)^{\pm 1} K_0(s)^{\pm 1}$. The user may pass to FREQ system matrices that constitute a linear, time-invariant state-space realization of $G_0(s)$ (and/or $K_0(s)$), and FREQ will calculate $G_0(s)$ (and/or $K_0(s)$) internally. Alternatively, the user may calculate the matrix $G_0(s)$ (and/or $K_0(s)$, if it is used) externally to FREQ and pass it in. An option is offered to compute the system matrix eigenvalues.
Next, FREQ determines, at the option of the user, a matrix \( G(s) \) that may be either the \( G_L \) just calculated or one of \( G_L(I + G_L)^{-1}, (I + G_L)^{-1}, (I + G_L)^{-1}, \) or \( (I + G_L)^{-1} \). The option exists to compute the singular values of \( G \), and if singular values are computed, a suboption is provided to compute singular vectors.

The subroutines SIGTAB and SDTAB which accept multiple frequency values in an array are offered since one of the intended uses of FREQ is to provide the information needed to plot the maximum and/or minimum singular values of \( G(j\omega) \) or \( G(e^{j\omega}) \) as a function of \( \omega \) or \( \bar{\omega} \), respectively. Provision is made to perform the preceding calculations for each value of \( \omega \) or \( \bar{\omega} \) in an array (which might be user supplied or internally calculated from user specifications) and to return the corresponding arrays with the maximum and minimum singular values of the chosen \( G \) matrix. Not all “single \( \omega \)” options are available here. Primarily, \( G_0 \) (and \( K_0 \), if used) must be internally calculated from linear state-space realizations. (The user for whom this restriction is a burden is invited to use FREQ subroutines SIGTAB or SDTAB as templates for constructing a singular value tabulation driver. To aid in this construction, an annotated copy of SDTAB is given in appendix A.)

FREQ uses the efficient, multivariable frequency-response calculation subroutines developed by Laub (Laub 1981; Lehtomaki 1981; Laub 1986) for computations of the form \( C(sI - A)^{-1}B \), where \( (A, B, C) \) are constant real matrices of compatible dimension. The representation of the transfer matrix \( C(sI - A)^{-1}B \) is modified through a similarity transformation applied to \( A \) to produce the form \( C'(sI - A')^{-1}B' \), where \( A' \) is upper Hessenberg (i.e., a matrix whose elements are zero below the first subdiagonal). More information on this process is included below in the subsection “Plant/Compensator Parameters” (see p. 10) and also in “Software Notes” (see p. 17). This transformation is independent of \( s \) and need be done only once even if the transfer matrix is to be evaluated for many values of \( s \). The efficiency arises in that the calculation of \( (sI - A')^{-1}B' \) with \( A' \) upper Hessenberg is much faster than that of \( (sI - A)^{-1}B \) with a general \( A \) matrix (Laub 1981). Laub’s software includes a mechanism that will track whether the \( A, B, \) and \( C \) matrices have undergone this transformation.

Use of Software: Initialization and Output

This section and the two sections that follow provide a FREQ users manual and describe the overall structure of the code. This section describes how to initialize FREQ and set certain parameters and what effects these parameters have on output and error processing. The next section describes the functions of and the calling sequences for the five subroutines intended as the user interface into FREQ. The section after that gives the additional information that the user needs to use FREQ for the analysis of discrete systems.

Initialization and Parameter Management

FREQ manages its output and error processing through a number of parameters that are stored in common blocks. Before initializing FREQ, the user must open the input file (if used), the output file, and the error message file (if different from output). Then, the user must do at least one of the several actions that will initialize these parameters before calling any of the FREQ computational routines.

Figure 2 shows the structure of initialization and output routines in FREQ. The lines with arrowheads denote data flow, whereas the lines without arrowheads show software-calling hierarchy. The dotted box and lines indicate that any other software available that knows the FREQ initialization protocols may be used.

The parameters set by initialization routines are given as follows:

\[
\begin{align*}
\text{NIN} & \quad \text{the external unit identifier of the input file; the default value is 5.} \\
\text{NOUT} & \quad \text{the external unit identifier of the output file; the default value is 6.} \\
\text{NERR} & \quad \text{the external unit identifier of the file for error messages may be the same as the output file; the default value is 6.} \\
\text{NLP} & \quad \text{the number of lines on a page of output; the default value of 59 is suitable for 11-in-long paper printed at 6 lines per inch.}
\end{align*}
\]
NCL

the line length (in characters) of an output line, forced by FREQ to be either 80 or 132; the default value of 132 is suitable for the typical long-carriage line printer.

IEROPT

an integer error-option flag; the default value of 0 tells FREQ to stop when a fatal error is detected. An error is considered fatal if FREQ is unable to make requested calculations because of this error (except for certain eigenvalue calculations as explained later).

TITLE

a CHARACTER*80 variable that contains information printed at the top of each output page; default is all blanks.

These parameters can be set (or reset) by calls to any of three routines, FREQNV, FRDTIT, or FREQNT. In addition, FREQNV can also be used to identify the current values of the parameters, and subroutine ORACES returns the current value of IEROPT while setting a new value. These routines are called in the following manner.

FREQNT is called with no arguments:

CALL FREQNT

This sets all parameters to their default values.

FRDTIT is called with no arguments:

CALL FRDTIT

This sets all parameters except TITLE to their default values. It then reads the next line of unit 5 (since NIN has just been set to default) and uses it for TITLE. FRDTIT is the analogue of ORACLS (Armstrong 1978a, 1978b, 1980) routine RDTITL.

FREQNV is called with arguments:

CALL FREQNV(IOPT,NIN,NOUT,NERR,NLP,NCL,IEROPT,TITLE)

IOPT is a user-set option flag that specifies what is to be accomplished by this call to FREQNV.

IOPT = 1: FREQNV uses the values set by the user in its remaining arguments to initialize the corresponding FREQ parameters. If the user-input value of NCL is less than 132, FREQNV resets NCL to 80. Otherwise, NCL is set to 132.

IOPT = 0: FREQNV treats all parameters except TITLE exactly as it does in the case that IOPT = 1. TITLE is read from unit NIN which the user has just specified. The new value is returned to the user in calling sequence argument TITLE.

IOPT = -1: No FREQ parameters are reset; the current values are returned to the user in the arguments. This option should not be exercised until the parameters have been set by some other action.

Since all arguments of FREQNV except IOPT can be changed by FREQNV, safe programming practice suggests calling FREQNV with variables in these positions instead of actual values. The user who wishes mostly default values for the parameters with a few exceptions can use the following technique:

... CHARACTER*80 TITLE
CALL FREQNT
CALL FREQNV(-1,NIN,NOUT,NERR,NLP,NCL,IEROPT,TITLE)

C LINES OF CODE RESETTING THE FEW EXCEPTIONS, E.G.,
NCL = 80
NERR = 7
TITLE = 'TEST RUN'
CALL FREQNV(+1,NIN,NOUT,NERR,NLP,NCL,IEROPT,TITLE)

The call to FREQNT sets default values, the first call to FREQNV puts these in the user's variables, the next few lines reset the ones that the user wants changed, and the second call to FREQNV informs FREQ of the changes.

**ORACES** is called with arguments:

```fortran
CALL ORACES(NEROPT, LEROPT)
```

This subroutine resets the parameter IEROPT to a user-specified value while remembering the value that was in IEROPT before ORACES was called.

- **NEROPT**
  - On input, the new value to which IEROPT will be set, not changed by subroutine ORACES.
- **LEROPT**
  - Not used on input; on output, ORACES stores the previous value of IEROPT in this variable. This can be used later to reset IEROPT.

FREQ uses ORACES internally to temporarily disable error termination and then reset IEROPT to the user's value. The user who wishes all parameters to have default values except IEROPT could use FREQNT and ORACES to accomplish this instead of using FREQNV.

**Output**

Output generated by FREQ is formatted into pages and written on the file with unit number NOUT which the user has set when initializing FREQ. Before initializing FREQ, the user must ensure that this file is available to the program by using the OPEN statement in FORTRAN or perhaps by using some host-machine-dependent file-handling protocol.

The output is formatted using column 1 for carriage control information. A "1" in column 1 means "print this line at the top of a new page," and a blank in column 1 means "advance the paper a single space from the last line and print this line;" no other carriage control characters are used. At the top of each page a heading is printed that includes the information stored in the parameter TITLE. The maximum number of lines written to a page is in parameter NLP (exception: if a matrix has so many columns that one row will not fit on a page, each row is written to the output file without a page break). Depending on the value of parameter NCL, lines will be formatted into a 132- or 80-character length. There is a small inconsistency in this use of the numbers "132" and "80." A 132-character line contains a carriage control character plus up to 132 additional characters. This is the capacity of a typical long-carriage line printer. An 80-character line contains a maximum of 79 characters including carriage control in an attempt to avoid triggering the extra carriage return (resulting in a blank line) which occurs when full 80-character lines are viewed on some 80-character-wide cathode-ray-tube (CRT) screens or printed on some 80-character printers.

User output may be interspersed with FREQ output on unit NOUT without confusing the page formatter if the user will inform FREQ of how many lines are to be written. This is done by issuing the FORTRAN statement:

```fortran
CALL LNCNT(N)
```

just before issuing command(s) which cause the writing of N lines to unit NOUT. FREQ will move to a new page if necessary so that the block of N lines will be together. The user should both prefer making several calls with small values of N to making one call with a large value and avoid using $N > NLP - 2$ (two lines are used for the header). The user can also force subsequent writing to NOUT to start on a new page by using the FORTRAN statement:

```fortran
CALL LNCNT(-1)
```

FREQ will also move to a new page for writing done following a call to any of the initialization routines (except FREQNV with IOPT = -1).

Finally, all FREQ-generated output, except for error messages, can be turned on or off by user option when the FREQ computational interface routines described in the next section are called.
Error Processing

When FREQ detects an error, it writes a message to the file whose unit number is set in parameter NOUT and, if NERR ≠ NOUT, also to unit NERR. This allows the user to have a full output on one file and an error log on another (which might be an interactive terminal CRT screen). Whenever FREQ detects an error that is considered to be fatal (e.g., FREQ has failed to make some requested computation; almost all errors fall in this category) and if the parameter IEROPT is set to 0 (this is the default), FREQ stops (i.e., it stops using FORTRAN statement “STOP ’FREQ ERROR STOP’”). If the user has set IEROPT ≠ 0 or if the error is nonfatal, control returns to the user’s program. Most FREQ computational subroutines set one or more error parameters that the user’s program can test to see if FREQ detected any errors.

The user wishing to configure FREQ to minimize FREQ-generated output should OPEN a file with unit number NOUT, initialize FREQ with NERR = NOUT, and call FREQ computational interface routines with the output option parameter turned off (ISOPTA(3) = 0).

Use of Software: Computational Routines

Overview

There are five subroutines that constitute the user’s interface to the computational routines of FREQ (as opposed to initialization, output, etc.). They are SIGCAL, SIGDYN, SIGTAB, SDTAB, and TABGEN.

SIGCAL performs a single-frequency analysis of the simpler system that results from removing the K₀ block from figure 1(a). The “SIG” in the name comes from “sigma,” the English spelling of the Greek letter commonly used to represent singular values of a matrix. “CAL” is for “calculation.”

SIGDYN analyzes a system of the form of figure 1(a) at a single frequency. “SIG” is motivated as before, and “DYN” comes from the motivation for figure 1(a), which is a plant with dynamic compensator.

SIGTAB repeatedly calls SIGCAL to reanalyze the same system with a variety of frequency values. The principal outputs of SIGTAB are corresponding tabulations (hence the “TAB”) of maximum and minimum singular values of the selected matrix as a function of frequency.

SDTAB functions analogously to SIGTAB except that it calls SIGDYN. The “SD” in SDTAB comes from SIGDYN; the “TAB” is motivated as before.

These are the driver routines for the computations that are FREQ’s reasons for existence. In the following, references to “calling FREQ” should be understood to mean “calling one of the subroutines SIGDYN, SIGCAL, SIGTAB, or SDTAB.”

TABGEN is a utility program that generates a list (table) of frequency values. The user supplies the first and last values, the number of points in the list, and a parameter that tells whether the values are to be equally spaced arithmetically (constant differences) or geometrically (constant ratios). The latter option is offered since graphs based on these tabulations commonly use a logarithmic scale for the frequency axis; and points in geometric progression become equally spaced on a log plot.

The user may either request SIGTAB or SDTAB to generate its own table of frequencies (which is done by a call to TABGEN) or the user may supply the table. One possibility would be for the user to call TABGEN to generate a spread of frequency values over a range of interest and then merge into that table additional points that refine the grid in the neighborhood of “interesting” frequencies (e.g., natural frequencies and transmission zeros). We illustrate this in the example presented in the section “Example” (see p. 21).

Figure 3 shows the relationships among the five interface routines with the user’s software on the one hand and the remainder of the FREQ package on the other. Dotted lines indicate that the knowledgeable user can directly access those portions of the software, which have been taken from public sources. Instructions are available elsewhere (Laub 1981 and 1986; Dongarra et al. 1979; Smith et al. 1976) and will not be repeated here.

We now give the calling sequences and describe the arguments that are passed to the interface routines. TABGEN is a special case and will be described in full first. There is much overlap among the arguments passed to the other four routines, and so after giving the specific calling sequences we will present a unified discussion of these parameters.
**TABGEN**

Subroutine **TABGEN** generates a table of numbers spaced with the user's choice of equal increments or equal consecutive ratios. It is called by a statement of the form

```call tabgen(no,omgtab,jop,kerr)```

The arguments in the calling sequence have the following meanings:

- **NO**
  - an integer giving the number of elements in OMGTAB

- **OMGTAB**
  - real array of at least NO locations. On entry, OMGTAB(1) and OMGTAB(NO) must be set to the first and last entries of the desired table. (If JOP = 2, these values must have the same sign.) On normal return, the remainder of the OMGTAB array is filled according to the option requested by parameter JOP.

- **JOP**
  - an integer variable that the user sets to 0, 1, or 2 depending on the purpose of TABGEN. (On normal return, JOP = 0.)
    - **JOP = 0:** virtually a "do nothing" option which is provided so that the call to TABGEN can be made in a loop; and if JOP is initialized outside the loop, only the first call will do the actual work of table generation. TABGEN does check that NO is positive, but it does not alter OMGTAB.
    - **JOP = 1:** TABGEN fills in OMGTAB so that consecutive differences are equal.
    - **JOP = 2:** TABGEN fills in OMGTAB so that consecutive ratios are equal.

- **KERR**
  - an integer variable used to signal errors encountered by TABGEN.
  - On normal return, KERR = 0. If an error is detected, KERR = -1 and OMGTAB and JOP are unchanged. Possible causes: NO non-positive; JOP not equal to 0, 1, or 2; or JOP = 2 but OMGTAB(1) and OMGTAB(NO) do not have the same sign.

**Packed Arrays**

All real and complex arrays passed between the user and FREQ need to be in packed format. By this we mean that the first column is stored in sequential array elements that are followed immediately (with no elements skipped) by the second column and so forth for each succeeding column. The software package ORACLS (Armstrong 1978a, 1978b, 1980) stores matrices in this fashion.

The principal danger to be avoided is passing a matrix that has been stored in an over-row-dimensioned array as in the following example:

```dimen A(20,20)
...d0 20 J = 1, 4
d0 20 K = 1, 3
<calculation resulting in the (K, J) matrix value>
A(K,J) = <the calculated value>
20 continue```

The 3 × 4 matrix stored in array A is not stored in packed format. For example, column 1 is stored sequentially, but then 17 array elements are skipped before the second column begins.

One way to correct this is to replace the DIMENSION statement by

```dimension A(3,n)```

where n ≥ 4. The critical thing is to have the first declared dimension equal the actual row count. Another correct technique is
DIMENSION A(n)

where \( n \geq 12 \); then to set A, execute the code

\[
I = 0 \\
\text{DO 20 J = 1, 4} \\
\text{DO 20 K = 1, 3} \\
I = I + 1 \\
\text{<calculation resulting in the (K, J) matrix value>} \\
A(I) = <the calculated value>
\]

20 CONTINUE

It is critical that the inner loop correspond to the first index. Otherwise, the transpose of the matrix will be stored in A.

**Calling Sequences**

With the parameters as defined below, these routines are called as follows:

**CALL SIGCAL**

\[
\text{CALL SIGCAL(NG,LG,MG,AG,BG,CG,DG,DGNUL,IGOPTA,FIRSTG,SAVEG,} \\
1 \text{ EVRG, EVIG, IGERRA, RCONDG,} \\
2 \text{ OMEGA, RCONDS, ISOPTA, G, SIGMA, U, V, INFO,} \\
3 \text{ RDUM, CDUM, IDUM, JERR)}
\]

**CALL SIGDYN**

\[
\text{CALL SIGDYN(NG,LG,MG,AG,BG,CG,DG,DGNUL,IGOPTA,FIRSTG,SAVEG,} \\
1 \text{ EVRG, EVIG, GO, IGERRA, RCONDG,} \\
2 \text{ NK, LK, MK, AK, BK, CK, DK, DKNUL, IKOPTA, FIRSTK, SAVEK,} \\
3 \text{ EVRK, EVIK, KO, IKERRA, RCONDK,} \\
4 \text{ OMEGA, RCONDS, ISOPTA, G, SIGMA, U, V, INFO,} \\
5 \text{ RDUM, CDUM, IDUM, JERR)}
\]

**CALL SIGTAB**

\[
\text{CALL SIGTAB(NG,LG,MG,AG,BG,CG,DG,DGNUL,IGOPTA,FIRSTG,SAVEG,} \\
1 \text{ EVRG, EVIG, IGERRA, RCONDG,} \\
2 \text{ OMEGA, RCONDS, ISOPTA, G, SIGMA, INFO,} \\
3 \text{ RDUM, CDUM, IDUM, JERR,} \\
4 \text{ JOP, NO, OMTAB, SVMAX, SVMIN, KERR)}
\]

**CALL SDTAB**

\[
\text{CALL SDTAB(NG,LG,MG,AG,BG,CG,DG,DGNUL,IGOPTA,FIRSTG,SAVEG,} \\
1 \text{ EVRG, EVIG, GO, IGERRA, RCONDG,} \\
2 \text{ NK, LK, MK, AK, BK, CK, DK, DKNUL, IKOPTA, FIRSTK, SAVEK,} \\
3 \text{ EVRK, EVIK, KO, IKERRA, RCONDK,} \\
4 \text{ OMEGA, RCONDS, ISOPTA, G, SIGMA, INFO,} \\
5 \text{ RDUM, CDUM, IDUM, JERR,} \\
6 \text{ JOP, NO, OMTAB, SVMAX, SVMIN, KERR)}
\]

Under some of the options available, some of the parameters are not used. In this case, a dummy parameter may be used in the calling sequence. It is recommended that the standards of the American National Standards Institute (ANSI) be maintained in that such dummy parameters agree in type with what is expected. Thus, if in SIGCAL or SIGDYN the user elects ISOPTA(2) = 2, the U and V arrays are not used, but the position in the calling sequence must be occupied, and this position filler should be a complex array. CDUM could be used in both positions; or, for more safety, an array, say CD, could be declared as "COMPLEX CD(1)" and used.

In addition to the following discussion, program prelude documentation of these subroutines is given in appendix B. Here each subroutine is treated separately, and the parameters are explained in the order used in the subroutine call. This might be a useful tool for the programmer who is writing a code that uses FREQ.

**Common Parameters: General**

For the purposes of this discussion, we revert to the notation introduced in the section "Functional Description of Software" (see p. 3). The parameters naturally fall into three categories: (1) those involved with the calculation of \( G_0(j\omega) \) (and a parallel set for \( K_0(j\omega) \) if it is used), (2) those
involved with the calculation of \( G(j\omega) \) and its singular values, and (3) (for SIGTAB and SDTAB only) those involved with controlling the repeated calculation for the several values of frequency. The parameters in each category can be further classified as those in which data are passed to FREQ or results are returned from FREQ (some parameters do both), parameters which specify options, and parameters which report error conditions.

The user is reminded that FREQ default handling of a fatal error is to write an error message and stop. The user who wishes to manage error processing differently must initialize FREQ with parameter IEROPT set to nonzero. Thereafter, if an error occurs, the error message will still be written, but control will return to the user’s program which may then interrogate the error parameters.

Although there are a multitude of parameters provided for identifying possible error sightings by FREQ, the question of whether an error has occurred can be answered in a single test. If JERR = 0 (after SIGCAL or SIGDYN) or KERR = 0 (after SIGTAB or SDTAB), then no fatal errors were detected. Otherwise, further analysis of the error parameters is indicated.

**Plant/Compensator Parameters**

The option-setting parameters for the \( G(j\omega) \) calculation are IGOPTA and FIRSTG. IGOPTA is an integer array of at least two elements for SIGTAB and SDTAB or three elements for SIGCAL and SIGDYN. The third element is used to indicate if the user is calculating \( G(j\omega) \) externally; this option is not available to SIGTAB or SDTAB. The user sets IGOPTA(3) = 1 to indicate the external calculation of \( G(j\omega) \). Note that if IGOPTA(3) = 1, most of the plant parameters are ignored, including the first two entries in IGOPTA. The value of \( G(j\omega) \) is communicated to FREQ through parameter G in SIGCAL and G0 in SIGDYN. In this case, of the remaining \( G_0 \) parameters, only LG and MG are used. The LG and MG are integer variables containing the number of rows and columns in \( G_0(j\omega) \). The G and G0 are complex matrices in which the user-calculated \( G(j\omega) \) is stored in packed format.

The remainder of the discussion of \( G(j\omega) \) parameters assumes that \( G(j\omega) \) is calculated internally by FREQ based on a linear, time-invariant state-space realization. This is always assumed by SIGTAB and SDTAB and is indicated to SIGCAL and SIGDYN by IGOPTA(3) = 0. (Any value for IGOPTA(3) except 0 or 1 generates a fatal error condition.) Then, \( G_0(j\omega) \) has the form

\[
C(j\omega I - A)^{-1}B + D
\]

Nonsingular matrices \( T \) exist such that \( A' = T^{-1}AT \) is upper Hessenberg, i.e., zero below the first subdiagonal. (See, e.g., Golub and Van Loan (1983), p. 222, where \( T \) is chosen to be orthogonal.) Then, with

\[
C' = CT \quad B' = T^{-1}B
\]

the result is

\[
C(j\omega I - A)^{-1}B + D = C'(j\omega I - A')^{-1}B' + D
\]

Performing the calculation in the latter form is much faster than the former since the matrix to be inverted is also upper Hessenberg (Laub 1981).

**FIRSTG** is a logical variable. The user sets FIRSTG = .TRUE. to indicate that the upper Hessenberg decomposition has not yet been done. FREQ will then do the decomposition, store the \( A', B', \) and \( C' \) matrices in the SAVEG array, and reset FIRSTG to "FALSE." Then, if FREQ is called with FIRSTG = .FALSE., FREQ does not repeat the upper Hessenberg decomposition but assumes that the reduced matrices are available in array SAVEG. Thus, once any FREQ routine has been called with given \( A, B, \) and \( C \) matrices in a linear state-space realization with FIRSTG = .TRUE., the user can make future calls to the same or to a different FREQ routine with any \( D \) matrix, any frequency value(s), and any of the five choices of \( G(j\omega) \) matrices using the FIRSTG (which is now .FALSE.) and the SAVEG array returned when FREQ was called with FIRSTG = .TRUE..

**THERE IS ONE EXCEPTION TO THIS USE OF FIRSTG:** If SIGCAL or SIGTAB is called with DGNUL = .TRUE., signifying that the \( D \) matrix is all zeros, and ISOPTA(1) = 2,
signifying that

\[ G(j\omega) = G_0(j\omega)(I + G_0(j\omega))^{-1} \]

then an alternate formula for \( G(j\omega) \) is

\[ G(j\omega) = C(j\omega I - A + BC)^{-1}B \]

In this case FREQ bypasses the normal \( G_0(j\omega) \) calculation and calculates \( G(j\omega) \) directly by submitting the matrix \( A - BC \) instead of \( A \) to the module that normally calculates \( C(j\omega I - A)^{-1}B \). This means that instead of reducing \( A \) to upper Hessenberg form, it is \( A - BC \) that must be reduced; so a fresh decomposition (FIRSTG = .TRUE.) must be performed, and the user must realize that the resulting contents of SAVEG give an equivalent upper Hessenberg realization for the triple \( (A - BC, B, C) \) instead of for \( (A, B, C) \).

**IGOPTA** selects options in the calculation of \( G_0(j\omega) \).

**IGOPTA(1):** This has an effect only when FIRSTG = .TRUE.. In that case, the possible values and their effects are as follows:

- \( \geq 0 \): The \( A \) matrix (or \( A - BC \) in the exceptional case) will be balanced and its eigenvalues determined using EISPACK routines BALANC and HQR (Smith et al. 1976) and returned to the user.
- \( < 0 \): Do not do balancing and eigenvalue calculation.

The recommended choice is to set IGOPTA(1) \( \geq 0 \). We discuss this further in the next section.

**IGOPTA(2):** Now let \( (A', B', C') \) be the equivalent upper Hessenberg realization for the triple \( (A, B, C) \) (or for \( (A - BC, B, C) \) in the exceptional case) and set \( H = j\omega I - A' \). The value of IGOPTA(2) controls the sophistication of singular matrix detection applied to \( H \) in calculating \( C'H^{-1}B' \). Possible values and their effects are as follows:

- \( < 0 \): The reciprocal of the condition number of \( H \) is estimated and returned to the user in RCONDG. Then \( H \) is declared singular to working precision if the FORTRAN expression “1.0 + RCONDG .EQ. 1.0” evaluates as “.TRUE.” and this is declared to be a fatal FREQ error.
- \( \geq 0 \): The only singularity checking is to see if any diagonal elements become 0.0 when \( H \) is factored by Gaussian elimination with partial pivoting.

**IGOPTA(2) < 0** is recommended.

**IGOPTA(3):** This is used only by SIGCAL and SIGDYN. Its use is discussed in the first paragraph of this subsection.

**NG, LG, MG, AG, BG, CG, DG, and DGNUL** describe the linear state-space realization of \( G_0(s) \). The \( A, B, \) and \( C \) matrices are passed to FREQ in packed format in real arrays AG, BG, and CG. The integer in NG is the number of system states. This is also the row and column dimensions of AG, the row dimension of BG, and the column dimension of CG. The integer in LG is the number of system inputs. This is the column dimension of BG. MG contains the number of system outputs that is the row dimension of CG. DGNUL is a logical variable. A value of .TRUE. indicates that the system \( D \) matrix is null (all zeros). If DGNUL is .FALSE., the \( D \) matrix (MG \( \times \) LG) is passed to FREQ in packed format in real array DG.

**SAVEG** is a real array of at least \( NG \times (MG + NG + LG) \) elements. FREQ uses SAVEG to save the upper Hessenberg realization. The SAVEG array that is passed to FREQ with
FIRSTG = .FALSE. must be one that has been initialized by a prior call with FIRSTG = .TRUE. See also the exceptional situation discussed above in connection with FIRSTG.

**EVRG** and **EVIG** are real arrays. If FIRSTG = .TRUE. and ISOPTA(1) ≥ 0, the real and imaginary parts of the eigenvalues of A (or A - BC in the exceptional case) are returned to the user in EVRG and EVIG, respectively; so these must have at least NG elements each.

**G0** is a complex array of at least LG × MG elements used by SIGDYN and SDTAB to hold the G0(jω) matrix which may be either internally calculated or, in the case of SIGDYN, precalculated by the user and passed to FREQ in G0.

**IGERRA** is an integer array of at least two elements that is used to report back to the calling program any errors detected in the G0(jω) calculation.

**IGERRA(1):** This signals any errors in the eigenvalue calculation, so it is affected only if FREQ is called with FIRSTG = .TRUE. and ISOPTA(1) ≥ 0. The user is advised to set IGERRA(1) to 0 before first calling FREQ. Then, a return value of 0 signals either that the eigenvalue calculation detected no errors or that no eigenvalue calculation was called for. If the eigenvalue calculation is done, possible return values of IGERRA(1) are as follows:

= 0: Normal return.

> 0: Not all the eigenvalues have been computed. In this case, the values in EVRG and EVIG in index range 1, ..., IGERRA(1) are not to be trusted as eigenvalues of A (or A - BC); the remaining NG - IGERRA(1) were computed without errors being detected.

IGERRA(1) > 0 is considered a nonfatal error since FREQ makes no further use of these values.

**IGERRA(2):** This signals any error in calculating C'(jωI - A')⁻¹B'. Possible return values of IGERRA(2) are as follows:

= 0: Normal return.

= -1: FREQ was entered with ISOPTA(2) negative, indicating that the reciprocal condition number RCONDG of jωI - A' was to be estimated, and the FORTRAN expression “1.0 + RCONDG .EQ. 1.0” evaluates as “.TRUE.” so that the matrix is singular to working precision.

> 0: FREQ was entered with ISOPTA(2) nonnegative, indicating that the calculation was to be done by using Gaussian elimination (with partial pivoting and without condition estimation) to factor jωI - A' into the product of a permutation of a bidiagonal unit-diagonal lower triangular matrix and an upper triangular matrix. At location IGERRA(2), the diagonal of the upper triangular matrix is null, and thus jωI - A' is singular.

A nonzero return is considered fatal. The user wishing to interrogate this parameter must initialize FREQ with IEROPT ≠ 0; otherwise FREQ will generate an error stop before control is returned to the user’s program.

**RCONDG** is a scalar variable in which FREQ stores the estimated reciprocal condition number of the jωI - A' matrix used in calculating G0(jω) when the user has elected this option by setting ISOPTA(2) < 0.

**OMEGA** is a real variable in which the frequency ω is stored at which G0(jω) (and K0(jω), if used) is evaluated when calling SIGCAL or SIGDYN. This variable is also passed to SIGTAB and
SDTAB where it is set internally from values in the array OMGTAB. On return, OMEGA contains
the last frequency value at which calculation was attempted; this has potential diagnostic value in
the event of an error exit from FREQ.

The parameters specifying $K_0(j\omega)$ in SIGDYN and SDTAB parallel those for $G_0(j\omega)$. For a
description of these parameters, the reader should reread the “$G_0(j\omega)$ calculation” paragraphs just
preceding in which “$G_0$” is replaced by “$K_0$” and any “$G$” in an argument name is replaced by “$K$”
(so that NG becomes NK, IGOPTA becomes IKOPTA, etc.)

The only limitations here are to ignore the OMEGA discussion (there is no “OMEKA” parameter)
and to ignore anything referring specifically to SIGCAL and SIGTAB. In particular, eliminate
any reference to storing a user-calculated transfer matrix in $G$ and eliminate any reference to the
exceptional $(A - BC)$ calculation.

### Loop-Gain Matrix (and Analysis Matrix) Parameters

The next group of parameters is concerned with the calculations that are made after $G_0(j\omega)$
(and $K_0(j\omega)$, if used) has become available to FREQ. For SIGDYN and SDTAB, this involves first
determining the loop-gain matrix $G_L(j\omega)$. (SIGCAL and SIGTAB use $G_L = G_0$.) Afterwards,
the matrix $G(j\omega)$ to be subjected to singular value analysis is calculated from $G_L(j\omega)$, and finally
the singular value analysis is performed. These parameters consist of several arrays used to store
calculated values for internal use and/or to return them to the user (and, in one special case, input
information into FREQ), an integer array for signaling options to FREQ, and several parameters
for error reporting. The principal output parameters for SIGCAL and SIGDYN are included here.
These same parameters are included in the calling sequences for SIGTAB and SDTAB, but they are
reused for each frequency at which the calculation is being repeated so that the user sees only the
values computed for the last value of frequency used. (The principal output parameters of SIGTAB
and SDTAB are discussed below.)

**ISOPTA** is the option-determining parameter. It is an integer array of at least three elements
for SIGCAL and SIGTAB or four elements for SIGDYN and SDTAB. ISOPTA(1) determines which
matrix will be returned to the user in the $G$ array. This is also the matrix subject to singular value
decomposition, if elected.

ISOPTA(1): This has the additional function of determining whether the value in
OMEGA or the values in OMGTAB are to be used as frequencies for
continuous-system analysis or as normalized frequencies for discrete
system analysis. The continuous case is discussed here; the discrete case
is covered in the next section:

- $= 1$: $G = G_L$, the loop-gain transfer matrix.
- $= 2$: $G = G_L(I + G_L)^{-1}$, the feedback transfer matrix.
- $= 3$: $G = (I + G_L)^{-1}$, the sensitivity matrix.
- $= 4$: $G = I + G_L^{-1}$, the inverse return difference matrix.
- $= 5$: $G = I + G_L$, the return difference matrix.

For all options except ISOPTA(1) = 1, $G_L$ must be square.

ISOPTA(2): This determines which of three optional actions SIGCAL and SIGDYN
will take with the $G$ matrix:

- $= 1$: SIGCAL and SIGDYN return control to the user with no further
calculation.
- $= 2$: The singular values of $G$ are calculated and stored in the SIGMA
array.
- $= 3$: In addition to calculating singular values as in option 2, the left
and right singular vectors of $G$ are calculated and stored
in the $U$ and $V$ arrays.
Since the purpose of SIGTAB and SDTAB is to tabulate singular values of the $G(j\omega)$ matrix as a function of $\omega$, these routines overwrite the user's setting of ISOPTA(2) with option 2. (Option 1 would be pointless, and option 3 would involve wasted work.)

ISOPTA(3): This controls FREQ-generated output:

≠ 0: A considerable amount of output is written to logical unit NOUT (see the previous section on initialization, etc.) indicating the input data and the options being exercised, and also giving the results of many of the computations.

= 0: Output from FREQ is limited to error messages (if any). SIGTAB and SDTAB reset ISOPTA(3) to 0 after the first entry in the table of frequencies has been processed to limit what could otherwise be an overwhelming amount of output, much of it redundant.

ISOPTA(4): This determines how $G_L$ will be formed from $G_0$ and $K_0$, so it is unnecessary for SIGCAL and SIGTAB since $G_L$ is always $G_0$ in these. In SIGDYN and SDTAB, the user selects which among $G_0^\pm 1K_0^\pm 1$ and $K_0^\pm 1G_0^\pm 1$ will be used for $G_L$ by the following scheme:

Let $i_1$ = 0 or 1 depending on whether $G_0$ or $G_0^{-1}$, respectively, is to be used.

Let $i_2$ = 0 or 1 depending on whether $K_0$ or $K_0^{-1}$, respectively, is to be used.

Let $i_0$ = 0 or 1 depending on whether the order of multiplication is to be $G_0^\pm 1K_0^\pm 1$ or $K_0^\pm 1G_0^\pm 1$, respectively.

Finally, set ISOPTA(4) = $i_0 + 2i_1 + 4i_2$.

This implies that ISOPTA(4) is in the range of 0 through 7; other values create a fatal error in FREQ.

Let us denote the numbers of rows and columns of $G_L$ by $m$ and $\ell$, respectively. Then, in SIGCAL and SIGTAB, $m = MG$ and $\ell = LG$. In SIGDYN and SDTAB, $m$ and $\ell$ depend on $i_0$. If $i_0 = 0$, then $m = MG$ and $\ell = LK$; whereas if $i_0 = 1$, $m = MK$ and $\ell = LG$. Each choice of ISOPTA(4) imposes one or more obvious restrictions on the dimensions $MG$ and $LG$ of $G_0$ and $MK$ and $LK$ of $K_0$. (Dimensions must be compatible for forming the products; a matrix must be square to be inverted.)

$G$ is a complex array of at least $m \times \ell$ elements ($m$ and $\ell$ are defined in the previous paragraph). On normal return from FREQ, this contains, in packed format, the matrix $G(j\omega)$ for $\omega = OMEGA$ (on return from SIGCAL or SIGDYN) or $\omega = OMGTAB(NO)$ (on return from SIGTAB or SDTAB). The $G$ array is used as an input array in one circumstance; if SIGCAL is called with ISOPTA(3) = 1, the user-furnished value of $G_0(j\omega)$ is passed to SIGCAL in array $G$ in packed format.

$SIGMA$ is a real array of at least $\min(m, \ell)$ elements in which, if ISOPTA(2) ≥ 2, the nontrivial singular values of $G(j\omega)$ (for the last $\omega$ value used, if SIGTAB or SDTAB were called) are stored.

$U$ and $V$ are complex arrays of at least $m \times \min(m, \ell)$ and $\ell \times \min(m, \ell)$ elements, respectively. If ISOPTA(2) = 3, the left and right singular vectors corresponding to the singular values in $SIGMA$ are stored in the $U$ and $V$ arrays, respectively, in packed format. Thus, the columns of $U$ and of $V$ each form an orthonormal set of vectors and $G(j\omega) = U \times \text{diag}(SIGMA) \times V$(transposed).

$RCONDS$ : Some of the options for forming $G_L(j\omega)$ and $G(j\omega)$ involve matrix inversion. In actual practice, each matrix inversion is coupled with a matrix multiplication either naturally or by introduction of an identity matrix and is reformulated as a solution of linear equations, with the matrix to be inverted becoming the coefficient matrix. This is accomplished using software from
LINPACK (Dongarra et al. 1979) which also estimates the reciprocal of the condition number of the coefficient matrix. The reciprocal condition number (numerically between 0.0 and 1.0) is returned to the user (for the last $\omega$ value used, if SIGTAB or SDTAB were called) in RCONDS. If more than one inversion is required, the minimal reciprocal condition number found is returned in RCONDS. If no inversion is called for, RCONDS is set to 2.0. If RCONDS is so small that the FORTRAN expression \(1.0 + \text{RCONDS} \cdot \text{EQ.} 1.0\) evaluates as \“.TRUE."., the coefficient matrix is singular to within machine precision and FREQ declares a fatal error.

INFO is an integer variable used to report singular value decomposition errors. If ISOPTA(2) = 2 or 3 (singular value decomposition is called for), INFO is set to the \“INFO\” parameter returned by the LINPACK (Dongarra et al. 1979) singular value decomposition routine; otherwise it is unchanged. The normal return value is 0. If INFO > 0, then the first $\min(m, \ell) - \text{INFO}$ entries of SIGMA are singular values and the rest are undefined.

JERR is the \“error summary\” parameter for SIGCAL and SIGDYN and also passes error information for SIGTAB and SDTAB. On normal return, JERR = 0. Of the possible nonzero values, 2, 6, and 26 signal nonfatal errors; the rest are fatal. If both nonfatal and fatal errors are detected, the fatal error is reported in JERR (although the nonfatal error(s) still show up in the other error parameters). The possible values of JERR are as follows:

1. Matrix dimensions are nonpositive or inconsistent with calculations chosen by ISOPTA(1) or ISOPTA(4); or, ISOPTA(4) is out of range.
2. An error was detected in calculating the eigenvalues of the A (or A - BC) matrix in the $G_0$ calculation. (See IGERRA(1) above.)
3. The matrix to be inverted in the $G_0$ calculation is exactly singular; zero was detected on the diagonal of the upper triangular factor. (This corresponds to IGERRA(2) > 0.)
4. The matrix to be inverted in the $G_0$ calculation is singular to working precision. (See the discussion of RCONDG and the case that IGERRA(2) = -1.)
5. IGOPTA(3) was neither 0 nor 1.
6. An error was detected in calculating the eigenvalues of the A matrix in the $K_0$ calculation. (See IKERRA(1) above.)
7. An error was detected in calculating $K_0$ that corresponds to the error signaled by JERR = 3 for $G_0$.
8. An error was detected in calculating $K_0$ that corresponds to the error signaled by JERR = 4 for $G_0$.
9. IKOPTA(3) was neither 0 nor 1.
10. The matrix to be inverted in calculating $G_L$ from $G_0$ and $K_0$ was singular to working precision. (See RCONDS above.)
11. The matrix to be inverted in calculating $G$ from $G_L$ was singular to working precision. (See RCONDS above.)
12. An error occurred in singular value decomposition. (See INFO above.)
26. Both nonfatal errors 2 and 6 above occurred.

IDUM, RDUM, and CDUM are three workspace arrays that must be provided: IDUM is of type integer, RDUM is of type real, and CDUM is of type complex. The minimum sizes of these arrays depend on a combination of factors involving the problem-sizing parameters NG, LG, and MG (and NK, LK, and MK if $K_0$ is used) and which options are selected. For purposes of the following calculations, set NG (or NK) to 0 if the $G_0(j\omega)$ (or $K_0(j\omega)$) matrix is being user calculated. Recall that $m$ and $\ell$ are the dimensions of the $G_L(j\omega)$ matrix as defined following the discussion of ISOPTA(4) above.

IDUM must have at least $\min(m, NG)$ (SIGCAL, SIGTAB) or $\max(m, NG, NK)$ (SIGDYN, SDTAB) elements. RDUM must have at least NG (SIGCAL, SIGTAB) or $\max(NG, NK)$ (SIGDYN, SDTAB) elements.

The calculation for the minimum size of CDUM is quite complicated. However, a simple calculation gives an overestimate. Let $\bar{z}$ be the largest of NG, LG, and MG. Then CDUM needs at least

$$\bar{z}(\bar{z} + MG + LG)$$

15
elements. If a compensator is used, repeat the calculation using K sizing parameters. If a larger number results, increase the size of CDUM to this new value. If this gives CDUM only three elements, increase to four. In the event that this is a SIGDYN or SDTAB application with ISOPTA(4) = 3 or 4, do the calculation for 26 in the following paragraph and increase the size of CDUM, if necessary, to accommodate at least i6 elements. In typical applications, this is not a very wasteful overestimate. For example, if the size of CDUM is determined by the first calculation using the plant-sizing parameters and if the plant has at least as many states (NG) as it has inputs (LG) or outputs (MG), then the number of unused elements in CDUM is less than the number of elements in the smaller of BG and CG.

The exact calculation for CDUM is given next. For use in SIGCAL and SIGTAB, the following calculation applies: Let

\[ i_1 = NG \times (NG + LG + 1) \]

If ISOPTA(1) = 2 and DGNUL = FALSE, or if ISOPTA(1) = 3 or 4, let \[ i_2 = MG \times (2 \times MG + 1) \]; otherwise let \[ i_2 = 0 \]. If ISOPTA(2) = 2 or 3 and MG < LG, let \[ i_3 = (MG + 1) \times (LG + 2) - 1 \]; otherwise let \[ i_3 = 0 \]. If ISOPTA(2) = 2 or 3 and MG ≥ LG, let \[ i_4 = LG \times (MG + 2) + MG \]; otherwise let \[ i_4 = 0 \]. Then CDUM needs \( \max(i_1, i_2, i_3, i_4) \) elements. For use in SIGDYN or SDTAB, the size of CDUM is calculated as follows: Let \[ i_1 = NG \times (NG + LG + 1) \] and \[ i_2 = NK \times (NK + LG + 1) \]. If ISOPTA(1) = 2, 3, or 4, let \[ i_3 = m(2m + 1) \]; otherwise \[ i_3 = 0 \]. If ISOPTA(2) = 2 or 3 and \( m < \ell \), let \[ i_4 = (m + 1)(\ell + 2) - 1 \]; otherwise \[ i_4 = 0 \]. If ISOPTA(3) = 2 or 3 and \( m \geq \ell \), let \[ i_5 = \ell(m + 2) + m \]; otherwise \[ i_5 = 0 \]. Determine \( i_6 \) based on ISOPTA(4) from the following table:

<table>
<thead>
<tr>
<th>ISOPTA(4)</th>
<th>( i_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 or 1</td>
<td>0</td>
</tr>
<tr>
<td>2 or 5</td>
<td>( m(m + 1) )</td>
</tr>
<tr>
<td>3 or 4</td>
<td>( \ell(2\ell + m + 1) )</td>
</tr>
<tr>
<td>6 or 7</td>
<td>( m(2m + 1) )</td>
</tr>
</tbody>
</table>

Then CDUM needs \( \max(i_1, i_2, i_3, i_4, i_5, i_6) \) elements.

**Tabulation Driver Parameters**

Finally, there are the parameters used by SIGTAB and SDTAB to control the generation of tables of maximum and minimum singular values of the user's choice of \( G \) matrix. There is an option parameter, parameters defining the table of frequencies at which calculations will be performed, arrays to hold the resulting maximum and minimum singular values, and an error parameter.

**JOP** is an option parameter used to control internal generation of the table of frequencies OMGTAB at which calculations will be performed:

- **JOP = 0:** FREQ takes the current contents of the OMGTAB array as the table of frequency values.
- **JOP = 1:** FREQ fills in the interior NO-2 elements of OMGTAB based on what it finds in OMGTAB(1) and OMGTAB(NO) so that the entries are equally spaced.
- **JOP = 2:** FREQ fills in the interior NO-2 elements of OMGTAB based on what it finds in OMGTAB(1) and OMGTAB(NO) so that consecutive ratios of entries are equal. (These values will be equally spaced when plotted on a logarithmic scale.) For this option, OMGTAB(1) and OMGTAB(NO) must have the same sign.

FREQ returns JOP = 0 so that the same values can be used over without FREQ repeating the work of frequency-table generation.

**NO** (number of omegas) is an integer variable in which the user sets the number of entries in the arrays OMGTAB, SVMAX, and SVMIN.

**OMGTAB** (omega table) is a real array of at least NO elements which contains the frequencies at which calculations are to be performed. The user must either fully initialize OMGTAB and call
FREQ with JOP = 0 or initialize OMGTAB(1) and OMGTAB(NO) and call FREQ with JOP = 1 or 2.

SVMAX and SVMIN (singular value maximum, etc.) are real arrays of at least NO elements each. On normal exit, SVMAX(I) and SVMIN(I) contain the maximum and minimum singular values of the user-selected \( G(j\omega) \) matrix for \( \omega = OMGTAB(I), I = 1, \ldots, NO \).

KERR is the error-summary parameter for SIGTAB and SDTAB. The normal return is 0. If FREQ returns KERR = -1, it has detected an error in the input values of NO (e.g., negative), JOP (none of 0, 1, or 2), or OMGTAB (end points of different signs when JOP = 2). If FREQ returns KERR > 0, it means that a fatal error has been detected while attempting the requested calculation with OMEGA = OMGTAB(KERR). In this case, JERR, the more detailed error parameters, and/or the output or error file should be consulted to determine the exact location of the error. Even if KERR = 0, the user should consult JERR before trusting any returned eigenvalues (EVRG and EVIG; and, if used, EVRK and EVIK).

Use of Software: Discrete System Analysis

The singular-value-based loop-shaping methodology considered until now for transfer matrices arising from continuous, linear time-invariant systems also applies in the discrete system case. In the discrete case, transfer matrices are obtained through z-transform theory; and in the control design and system analysis process, the same family (loop gain, return difference, closed-loop response, inverse return difference, and sensitivity) of transfer matrices is important. The only structural difference in the discrete transfer matrices is the appearance of the z-transform parameter \( z \) in place of the Laplace transform parameter \( s \).

Discrete frequency-response calculations, such as those considered by FREQ, replace \( z \) by \( e^{j\omega} \) where \( \omega \) is the normalized frequency, also called the normalized angular frequency (Kwakernaak and Sivan 1972). The choice of the set of values for \( \omega \) is based on particular discrete system characteristics such as sampling interval and Nyquist frequency (Oppenheim, Willsky, and Young 1983), and, as in the continuous case, the choice will be left to the user.

Although FREQ will accept any value for \( \omega \), all available information about the singular values of any of the transfer matrices in the FREQ family can be obtained by limiting values of \( \omega \) to \([0, \pi]\). In the first place, \( e^{j\omega} \) is a periodic function of \( \omega \) of period \( 2\pi \), and so \( \omega \) may be limited to \([-\pi, \pi]\). In the second place, \( e^{-j\omega} \) is the complex conjugate of \( e^{j\omega} \), and so any discrete transfer matrix of the FREQ family calculated from \(-\omega\) is the complex conjugate of the matrix calculated from \( \omega \), and these have the same singular values. This means that information calculated from choosing \( \omega \) in \([-\pi, 0]\) (or, by periodicity, in \([\pi, 2\pi]\)) duplicates information calculated from \( \omega \) in \([0, \pi]\).

In order to access the FREQ discrete system analysis capability, the user simply follows all the instructions for the continuous case with two exceptions:

1. The frequency values input to FREQ in variable OMEGA or in array OMGTAB should be reasonable as normalized frequencies.
2. The user must add “10” to the value given ISOPTA(1), the transfer matrix choice parameter.

For purposes of applying the instructions of the previous section to the discrete calculation, the user should ignore the “10” added to ISOPTA(1) in step 2 above. Stated another way, if one wants to analyze, say, the feedback transfer matrix of a discrete system, then one must set ISOPTA(1) = 12, but follow any special instructions given in the previous section for ISOPTA(1) = 2.

Software Notes

This section goes into more detail about the code and the algorithms.

It describes in which language the code is written, where it is available, and what parts of it have been obtained from other sources. It then discusses the calculations done by FREQ and tells what is being calculated, how it is being calculated, and what can go wrong. This will hopefully help a user diagnose the source of trouble in the event that FREQ detects an error.

Notes on the Code

FREQ exists in single- and double-precision versions. The single-precision version is written in ANSI standard FORTRAN 77. The double-precision version fails to be ANSI standard only in its
use of data-type COMPLEX*16 and associated intrinsic functions (e.g., DCMPLX). It is believed that any dialect of FORTRAN 77 that can handle the complex double-precision LINPACK (that is, the LINPACK routines whose names start with "Z"; Dongarra et al. 1979) can also compile double-precision FREQ. FREQ is intended to be portable code. The single-precision version has been exercised on Control Data Corporation (CDC) CYBER computers under the NOS operating system, on Digital Equipment Corporation (DEC) VAX computers under the VMS system, and on Sun Microsystems computers under the UNIX operating system; and the double-precision version has been exercised on VAX and Sun. Based on these tests, the use of double precision on 32-bit machines is recommended.

The authors had the following problem with the version of LINPACK (Dongarra et al. 1979) used in this work. The associated Basic Linear Algebra Subroutines (BLAS) contain function subprograms whose FUNCTION statements have the form

\begin{verbatim}
FUNCTION ZXXXX*16( ... )
\end{verbatim}

Some compilers (e.g., f77 under UNIX on Sun minicomputers) do not recognize this syntax. If this is rewritten as

\begin{verbatim}
FUNCTION ZXXXX( ... )
\end{verbatim}

it compiles both on the Sun and on VAX using the VMS FORTRAN compiler.

Source code and documentation (a copy of this TM) are available from COSMIC (NASA's software dissemination center):

Computer Software Management and Information Center (COSMIC)
The University of Georgia
382 East Broad Street
Athens, GA 30602

Ordering information may be found in the current COSMIC catalog; at the time that this is being written, the current catalog is COSMIC (1989). Both the single- and double-precision versions are available in brief and complete form. The brief forms fail to be complete in that the LINPACK and EISPACK modules have been omitted. These versions provide more economical copies of the code that can be used by users who have alternate access to the standard software packages LINPACK and EISPACK. In the further interest of economy, comment lines have been removed from all versions of the COSMIC source code. Some of these have been included in this document (appendix B).

COSMIC identifies the four versions of FREQ as follows:

- **LAR-14119** single precision; LINPACK and EISPACK modules omitted
- **LAR-14120** single precision; complete
- **LAR-14121** double precision; LINPACK and EISPACK modules omitted
- **LAR-14122** double precision; complete

In constructing the software package FREQ, code has been incorporated from other sources. In addition to the aforementioned LINPACK (Dongarra et al. 1979) and EISPACK (Smith et al. 1976) modules, FREQ uses the efficient, multivariable frequency-response software of Laub (1981, 1986) that was first published in Lehtomaki (1981). Laub's software has been published (Laub 1986) under copyright, 1986, by the Association for Computing Machinery (ACM), Inc., and has been incorporated into FREQ for distribution by COSMIC by permission of the ACM. The authors are grateful to the ACM for the permission and to Laub for furnishing his double-precision version.

As with any subroutine package, the user should avoid naming any of his global entities (program name, subroutine names, common block names, etc.) with symbolic names used by FREQ for global entities. A list of these is given in appendix C. If the subroutine package ORACLS (Armstrong 1978a, 1978b, 1980) or any of its derivatives is also being used, there is a possibility of conflict since some of the symbolic names used for global entities in FREQ are also used in ORACLS. The user wishing to combine FREQ and ORACLS or a derivative in a single application has the best chance of success if the following procedures are observed: Link (load) the software in such a manner that if a subroutine or common block name occurs both in FREQ and ORACLS, the version from
FREQ is used. Execute any initialization protocols for ORACLS first, and then perform the FREQ initialization. The user is warned that some developmental versions of ORACLS are incompatible with FREQ in that common blocks are used that have the same name but different lengths.

Notes on the Algorithm

The subroutines SIGTAB and SDTAB perform data and parameter value management and error detection. If the user requests generation of an array of frequency values, this is done by a call to TABGEN. The calculations of transfer function matrices, the loop-gain matrix, etc., and singular values are accomplished by repeatedly calling SIGCAL or SIGDYN, respectively, for each frequency in OMGTAB.

On entry to SIGCAL or SIGDYN, the first task performed is to check if the integer parameters that give the system size are in proper range and are consistent for the calculations selected. It is verified that all matrix dimensions are positive, that the sizes are compatible if any matrix multiplication is called for, and that any matrix to be inverted or added to an identity matrix is square. The error signal JERR = 1 signals the failure of one of these tests.

Once again, the discussion will focus on the continuous case with transfer matrices being evaluated at \( j\omega \). These remarks also apply to the discrete case by replacing \( j\omega \) with \( e^{j\omega} \).

Unless the user signals that the \( G_0(j\omega) \) (or \( K_0(j\omega) \) in SIGDYN) matrix has been precalculated, the next job is to perform this calculation. Only the \( G_0(j\omega) \) calculation is discussed; the \( K_0(j\omega) \) calculation is analogous (in fact, it is performed by a call to the same subroutine). There are several alternatives in this calculation, and they are controlled principally by the values of FIRSTG, IGOPTA(1), IGOPTA(2), and DGNUL with some possible influence (in SIGCAL only) by ISOPTA(1).

If FIRSTG = .TRUE., FREQ calculates an equivalent realization for the linear system with realization \( (A, B, C) \) (or \( (A - BC, B, C) \) in the exceptional case) where the matrices \( A, B, \) and \( C \) are in arrays AG, BG, and CG, respectively. Recall that the term “exceptional case” refers to the case that SIGCAL has been called with DGNUL = .TRUE. and ISOPTA(1) = 2. The equivalent realization is characterized by having an upper Hessenberg (i.e., null below the first subdiagonal) system matrix. The inclusion of some optional steps in the Hessenberg decomposition is controlled by IGOPTA(1).

First, the matrices \( A \) (or \( A - BC \) in the exceptional case), \( B \), and \( C \) are copied from the user's AG, BG, and CG arrays (which are never changed by FREQ) and packed into the SAVEG array. If IGOPTA(1) \( \geq 0 \), the copy of the system matrix in the SAVEG array is balanced using EISPACK subroutine BALANC (Smith et al. 1976; this is a row-and-column equilibration procedure that also isolates eigenvalues by row-and-column permutations where possible), and the appropriate transformations are applied to the \( B \) and \( C \) matrices so as to maintain equivalence of representation. The eigenvalues of the system matrix are then computed using EISPACK subroutine HQR. FREQ returns these eigenvalues to the user but makes no further use of them. Laub recommends that the balancing and eigenvalue computation be done. In a private communication, he said “Eigenvalue computation ... is recommended mainly to get an idea of where the poles are (i.e., where the frequency response can be expected to shoot off to infinity). Balancing is done to enhance the accuracy of computed eigenvalues and has to do with the backward error analysis of the QR algorithm.” One would expect that the balancing would also be numerically beneficial to the transformation into upper Hessenberg form.

Whether the system matrix has been balanced or not, the next computation is to use orthogonal similarity transformations to reduce the system matrix to upper Hessenberg form, accumulating the transformations into the \( B \) and \( C \) matrices so that the linear system represented by the matrices stored in array SAVEG remains equivalent to the original system. The variable FIRSTG is set to “.FALSE.” so that the user can recall SIGCAL or SIGDYN with the same FIRSTG and SAVEG without doing anything else to them.

The \( G_0(j\omega) \) calculation continues with the calculation of \( C(j\omega I - A)^{-1}B \). (This point is where the calculation starts if FREQ were entered with FIRSTG set .FALSE.; in the exceptional case, \( C(j\omega I - A + BC)^{-1}B \) is calculated; further remarks are limited to the normal case.) Now the matrices \( A, B, \) and \( C \) are taken from array SAVEG, and so \( A \) is upper Hessenberg and therefore \( j\omega I - A \) is also. The term \( (j\omega I - A)^{-1}B \) is calculated by solving the matrix equation \( (j\omega I - A)Z = B \)
for Z. Laub's software does this by using subroutines patterned after the LINPACK general equation solvers, but taking advantage of the upper Hessenberg structure of the coefficient matrix. As with LINPACK, the user has the choice of whether or not to estimate the reciprocal condition number of the coefficient matrix. The former, which is signaled to FREQ by setting IGOPTA(2) < 0, is the more robust means of detecting whether or not the coefficient matrix is so ill-conditioned that the results are unreliable, and it is the generally recommended method. If the open-loop eigenvalues are all sufficiently separated from the point(s) jω at which G0(jω) is to be calculated, the faster IGOPTA(2) ≥ 0 may be elected. In either case, an attempt is made to detect ill-conditioning or singularity in the coefficient matrix. Results of this attempt are reported back to the user in parameters RCONDG and IGERRA(2) as discussed in the previous section.

Next, if DGNUL = .FALSE., the calculation of G0(jω) is completed by adding the matrix in the DG array to the matrix calculated above.

In SIGDYN, after G0(jω) and K0(jω) have been calculated, the next task is to form the loop-gain matrix G_L(jω). This step does not apply to SIGCAL since it takes G_L(jω) = G0(jω). Usually, forming the loop-gain matrix will involve only a matrix multiplication. However, some options do involve matrix inverses, which can give numerical problems. In this case, G_L is calculated by formulating the calculation as the solution of a linear system and using LINPACK (Dongarra et al. 1979) subroutines for solving systems of equations with general complex-valued coefficient matrices. For example, if

\[ G_L(j\omega) = G_0(j\omega) K_0(j\omega)^{-1} \]

the linear system \( K_0' G_L' = G_0 \) is solved for \( G_L' \) (the transpose of \( G_L \)), and thus \( G_L \) is determined. If

\[ G_L(j\omega) = G_0(j\omega)^{-1} K_0(j\omega)^{-1} \]

(\( K_0 G_0 \)) \( G_L = I \) is solved for \( G_L \), where I is an identity matrix. If a linear system is solved, then the reciprocal condition number of the coefficient matrix is estimated, returned to the user in parameter RCONDS (unless a worse value is generated later), and a fatal error is declared if RCONDS is so small that the FORTRAN expression "1.0+RCONDS.EQ.1.0" evaluates as " .TRUE.".

It was decided not to include the analogous option in SIGCAL that would allow \( G_L(j\omega) = G_0(j\omega)^{-1} \). Almost all the results that would be added by offering this option can be obtained by taking note of the following observations: The closed-loop response matrix for \( G_0^{-1} \) is the sensitivity matrix for \( G_0 \), and conversely. The return difference matrix for \( G_0^{-1} \) is the inverse return difference matrix for \( G_0 \), and conversely. The singular values of \( G_0^{-1} \) are the reciprocals of the singular values of \( G_0 \).

FREQ next calculates the matrix selected by the user in ISOPTA(1). If ISOPTA(1) = 1, there is nothing to do and control passes to the next step. In SIGCAL, FREQ is mindful at this point of the exceptional case, and if ISOPTA(2) = 2 and DGNUL = .TRUE., SIGCAL assumes that the linear system transfer function calculation has been done with the linear system matrix replaced by \( A - BC \) so that no further calculation is needed to obtain the feedback transfer matrix. Otherwise, identity matrices are added and/or linear systems are solved to obtain an inverse or the product of the inverse of a matrix with another matrix as required. The same module for calling LINPACK linear equation solvers is used here as was used in the \( G_L \) calculation in SIGDYN, and so the same remarks about error detection apply.

Finally, if the user has elected this option, singular values and, if chosen, singular vectors of \( G_L(j\omega) \) are calculated. This is done by the single-precision or double-precision subroutine for singular value decomposition of a complex matrix from LINPACK (Dongarra et al. 1979). If any errors are made at this point, they are signaled in parameter INFO which FREQ copies from LINPACK. The user encountering errors at this point and wishing to understand them should refer to the discussion of INFO by Dongarra et al. (1979, see p. 11.5). The arrays S and E referred to there are located, respectively, in the first min(m + 1, \( \ell \)) and the next min(m, \( \ell \)) locations of work array CDUM. See the discussion of ISOPTA(4) in the section “Use of Software: Computational Routines” (see p. 7) for the definitions of m and \( \ell \).
Example

Use of the FREQ code is demonstrated with numerical data taken from NASA TP-2560 (Joshi, Armstrong, and Sundararajan 1986) wherein a linear-quadratic-Gaussian (LQG)/loop-transfer-recovery (LTR) controller synthesis procedure is applied to the problem of designing a fine-pointing attitude control system for a large flexible antenna. The study is based on a 26th-order finite-element model of the 122-m hoop/column antenna, which consists of three rigid-body rotational modes and the first 10 elastic modes. A 12th-order compensator design model is formed by taking the rigid body and the first three elastic modes. The remaining truncated elastic mode dynamics are interpreted as an additive perturbation to the design system and are used as an approximate representation of the unmodeled dynamics for use in stability robustness tests. The LQG/LTR procedure produced estimator and controller gain matrices defining a 12th-order LQG-type compensator.

We use the 26th-order model of the hoop/column antenna as given in the appendix of NASA TP-2560 and refer to its transfer matrix as $G$. Truncation of this system to its first 12 states gives the design model (plant) $G_p$. Truncation of this same 26th-order model to its last 14 states gives the additive uncertainty model $\Delta G$. We use the estimator (filter gain) and control gain matrices given in table II of TP-2560 together with the linear system representation of $G_p$ in the last formula on page 5 of TP-2560 to construct a linear system model of the controller; its transfer matrix is denoted by $G_c$.

The table of frequency values used by SIGTAB and SDTAB is generated by merging the table used in generating the plot data for TP-2560 (containing selected open-loop eigenvalue and invariant zero frequencies) with a grid of values giving 100 intervals per decade spaced in a geometric fashion. This illustrates the technique suggested in the earlier discussion of TABGEN. We then tabulate the maximum and minimum singular values of $G$, $G_p$, $\Delta G$, and $G_c$. Frequency domain analysis is then applied to the plant-compensator combinations $G_pG_c$ and $GG_c$. Plots of the resulting tabulations are presented in figures 4–14 with the maximum and minimum singular values labeled $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$, respectively.

Figure 4 presents the open-loop response of the full 26th-order model of the hoop/column antenna ($G$). Figure 5 shows the same information for the design model ($G_p$). This corresponds to figure 18 in TP-2560. Figure 6 shows the frequency response for the additive error model ($\Delta G$), corresponding to figure 19 in TP-2560. Figure 7 shows the same information for the compensator ($G_c$). Figures 8 and 9 refer to the design model with compensator ($G_pG_c$). Figure 8 shows the open-loop response (corresponding to fig. 23 in TP-2560), and figure 9 shows the closed-loop response.

Finally, figures 10–14 apply the full spectrum of FREQ analysis to the transfer matrix $GG_c$, representing the compensator applied to the full 26th-order model of the hoop/column antenna. Figure 10 shows the loop-gain transfer matrix (open-loop response), figure 11 shows the feedback transfer matrix (closed-loop response), figure 12 shows the sensitivity matrix, figure 13 shows the inverse return difference matrix, and figure 14 shows the return difference matrix.

A listing of the code used to produce this example is given in appendix D. This was run on a CDC CYBER 180-series computer under the NOS operating system. Each single-precision floating-point number occupies 60 binary bits with 48 bits being used for the mantissa. The subroutines READ, TRANP, UNITY, NULL, and READ1 used by this example are identical in function (except in case an error is detected) with the subroutines of the same name in subroutine package ORACLS (Armstrong 1978a, 1978b, 1980). They have been modified to be compatible with FREQ input/output (I/O) and error processing.

The resulting output is given in appendix E. The input data are not listed separately since they are echoed as the first five matrices printed in the output listing.
Appendix A

Guide to Singular Value Tabulation

This appendix gives an annotated listing of the subroutine SDTAB. This can serve as a template for the user who wishes to drive FREQ to do frequency analysis on a linear system at several values of frequency, but cannot fit the task within the restrictions placed on the use of SIGTAB and SDTAB.

SUBROUTINE SDTAB(NG, LG, MG, AG, BG, CG, DG, DGNUL, IGOPTA, FIRSTG, SAVEG,
1 EVRG, EVIG, GO, IGERRA, RCONDG,
2 NK, LK, MK, AK, BK, CK, DK, DKNUL, IKOPTA, FIRSTK, SAVEK,
3 EVRK, EVIK, KO, IKERRA, RCONDK,
4 OMEGA, RCONDS, ISOPTA, G, SIGMA, INFO,
5 RDUM, CDUM, IDUM, JERR,
6 JOP, NO, OMGTAB, SVMAX, SVMIN, KERR)

CDUM(*), G(*), GO(MG,*), KO(MK,*), JERR, JOP, KERR, MG, MK, NG, NK, NO, LG, LK, IDUM(*), IGERRA(2),
+ IGOPTA(2), IKERRA(2), IKOPTA(2), INFO, ISOPTA(4)

INTEGER
+ U(1), V(1)
+ I, IGO(3), IKG, IKO(3), L, LEROPT, LERR, M, NEROPT, NSIG

The purpose of SDTAB is to repeatedly call SIGDYN in its “calculate both G0 and K0 internally” mode while varying the value of the frequency parameter OMEGA so as to tabulate the maximum and minimum singular values of the G matrix which SIGDYN chooses to calculate based on the value of the parameter “ISOPTA(1)”. These comments assume that the reader is familiar with the description of how to use SIGDYN. (See the section “Use of Software: Computational Routines” (see p. 7) and/or the program prelude documentation of SIGDYN found in appendix B.)

A running commentary is interspersed throughout the code to aid the user who wishes to write a routine to tabulate singular values outside the restrictions placed on SIGTAB and SDTAB, such as for a system with one or both of the G0 and K0 matrices calculated externally.

Variables for internal use are declared:

CHARACTER*77 MSG
+ COMPLEX

INTEGER
+ I, IGO(3), IKG, IKO(3), L, LEROPT, LERR, M, NEROPT, NSIG

The IGO and IKO arrays are used to force the “calculate both G0 and K0 internally” option in SIGDYN without altering the user’s IGOPTA or IKOPTA. The user writing a tabulation driver with external calculations of either G0 or K0 must make appropriate changes here:

IGO(1) = IGOPTA(1)
IGO(2) = IGOPTA(2)
IGO(3) = 0
IKO(1) = IKOPTA(1)
IKO(2) = IKOPTA(2)
IKO(3) = 0
KERR = -1

Generate an error return if NO is not positive.
IF (NO .LE. 0) RETURN

Bypass table generation if there are only one or two OMEGA's:
IF (NO .LE. 2) GO TO 100

Bypass table generation if the OMGTAB array is already filled:
IF (JOP .EQ. 0) GO TO 100

Save the user's value of error option parameter "IEROPT" in "LEROPT" and set IEROPT to 1 so that fatal errors will not cause a program stop. The user's wishes, as expressed in the setting of IEROPT, will ultimately be honored; but, by temporarily overriding that setting, if there is an error, the user will discover not only that it occurred in TABGEN but also that it was at a time when TABGEN was called from SDTAB. This overriding and then resetting of IEROPT is done again around the call to SIGDYN so that any errors detected there will be reported to the user as coming from a call to SIGDYN made by SDTAB:

NEROPT = 1
CALL ORACES(NEROPT,LEROPT)

Generate the table of frequencies:
CALL TABGEN(NO,OMGTAB,JOP,KERR)

Now reset IEROPT to the user's value:
CALL ORANCES(LEROPT,NEROPT)

If no error, go on:
IF (KERR .EQ. 0) GO TO 100

If there was an error, write a "FATAL ERROR" message:
CALL ORCERP(3,'ERROR IN TABGEN CALLED FROM SDTAB')

Depending on the user's setting of IEROPT, this will either cause a "FREQ ERROR STOP" program stop or will return control at the following line after writing out the error message:
RETURN

Normal processing continues at this point:
100 CONTINUE
JOP = 0

Turn off generation of the U and V matrices of the singular value decomposition (SVD):
ISOPTA(2) = 2

Determine the end of the main diagonal (NSIG) of the SVD diagonal matrix (code through "30 CONTINUE" not needed for a SIGCAL driver):

IKG = MOD(ISOPTA(4),2)

If IKG .EQ. 1, then M = MK and L = LG; otherwise, M = MG and L = LK:
M=MK
L=LG
IF (IKG .NE. 0) GO TO 30
M=MG
L=LK
30 CONTINUE

If the user is writing a SIGCAL driver, replace the following line by "NSIG = MIN(MG,LG)"
NSIG = MIN(M,L)

23
Begin the loop to calculate singular values as a function of OMEGA. Initialize JERR:

\[
\begin{align*}
\text{JERR} &= 0 \\
\text{DO 20} & \text{ I=1,NO}
\end{align*}
\]

In case of error on this OMEGA, KERR will be set:

\[
\text{KERR} = 1
\]

Pick up the current value of OMEGA:

\[
\text{OMEGA} = \text{OMGTAB(I)}
\]

This is the place to do any external calculation of G0 or K0 matrices (G for SIGCAL). Note also the correspondence between the parameters in the following call and those of this subroutine. The user writing a SIGCAL (or SIGDYN) driver code would probably want to use IGOPTA (and IKOPTA) in place of IGO (and IKO) below.

Save the user’s value of error option parameter “IEROPT” in “LEROPT” and set IEROPT to 1 so that fatal errors will not cause a program stop. (See the remarks surrounding the similar code near the call to TABGEN above.) Thus:

\[
\begin{align*}
\text{NEROPT} = 1 \\
\text{CALL ORACES(NEROPT,LEROPT)}
\end{align*}
\]

Determine the system matrix, etc., and singular values:

\[
\begin{align*}
\text{CALL SIGDYN(NG,LG, MG, AG, BG, CG, DG, DGNUL, IGO, FIRSTG, SAVEG,} \\
1 \text{ EVRG, EVIG, GO, IGERRA, RCONDG,} \\
2 \text{ NK, LK, MK, AK, BK, CK, DK, DKNUL, IKO, FIRSTK, SAVEK,} \\
3 \text{ EVRK, EVIK, KO, IKERRA, RCONDK,} \\
4 \text{ OMEGA, RCONDS, ISOPTA, G, SIGMA, U, V, INFO,} \\
5 \text{ RDUM, CDUM, IDUM, LERR)}
\end{align*}
\]

Now reset IEROPT to the user’s value:

\[
\begin{align*}
\text{CALL ORACES(LEROPT, NEROPT)}
\end{align*}
\]

This suppresses printout after the first OMEGA. (Printout for OMGTAB(1) is determined by user initialization of ISOPTA(3).) Thus:

\[
\begin{align*}
\text{ISOPTA(3)} = 0
\end{align*}
\]

If any error was detected, update JERR. (By this buffering of error codes, the nonfatal error code will be reported to the user if a nonfatal eigenvalue calculation error is detected on the first OMEGA and no other error is found. If a fatal error is found later, it will override the nonfatal error code in JERR.) Thus,

\[
\begin{align*}
\text{IF (LERR .NE. 0)} & \text{ JERR = LERR}
\end{align*}
\]

If a fatal error was detected, abandon the calculation:

\[
\begin{align*}
\text{IF (JERR .NE. 0 .AND. JERR .NE. 2 .AND. JERR .NE. 6 .AND.} \\
+ \text{ JERR .NE. 26) GO TO 40}
\end{align*}
\]

Tabulate the maximum and minimum singular values. Any other “once per OMEGA” code could go here (e.g., brief printout, tabulation of other quantities, etc.). This is the end of the tabulation loop:

\[
\begin{align*}
\text{SVMAX(I)} &= \text{SIGMA(1)} \\
\text{SVMIN(I)} &= \text{SIGMA(NSIG)}
\end{align*}
\]

20 CONTINUE
If execution reaches this point, the singular values have been calculated for all OMEGA's in OMGTAB without any error being detected. Generate a “normal” return:

```
KERR = 0
RETURN
```

This is where control passes if any fatal error is detected in the loop that works through the values in OMGTAB. A “fatal error” message will be written and, depending on the user’s setting of IEROPT, either a “FREQ ERROR STOP” will be generated or control will return to the calling program:

```
40 CONTINUE
   WRITE (MSG,50) KERR,OMEGA
50 FORMAT(’ERROR IN SDTAB, KERR = ’,I7,’ AT OMEGA = ’,1PE13.6)
   CALL ORCERP (3,MSG)
   RETURN
END
```
Appendix B

Selected Subroutine Preambles

This appendix contains the program preambles from the five user interface routines from FREQ (i.e., SIGCAL, SIGDYN, SIGTAB, SDTAB, and TABGEN). These are not being included in the COSMIC distribution tapes for reasons of economy.

SUBROUTINE SIGCAL(NG,LG,MG,AG,BG,CG,DG,DGNUL,IGOPTA,FIRSTG,SAVEG, 1 EVRG,EVIG,IGERRA,FIRSTG,SAVEG, 2 OMEGA,RCONDS,ISOPTA,G,SIGMA,U,V,INFO, 3 RDUM,CDUM,IDUM,JERR)

COMPLEX
+ CDUM(*), G(MG,*), U(MG,*), V(LG,*)
INTEGER
+ JERR, LG, MG, NG, IDUM(*), IGERRA(2), IGOPTA(31, INFO, ISOPTA(3)
LOGICAL
+ DGNUL, FIRSTG
REAL
+ OMEGA, RCONDS, RCONDS, AG(*), BG(*), CG(*), DG(*), EVIG(*),
+ EVRG(*), RDUM(*), SAVEG(*), SIGMA(*)
COMMON /ORACIO/ NIN, NOUT, NERR
INTEGER
+ NERR, NIN, NOUT
SAVE /ORACIO/

C SIGCAL GENERATES A VARIETY OF MATRICES USED FOR THE
C FREQUENCY DOMAIN ANALYSIS OF MULTIVARIABLE CONTINUOUS OR DISCRETE
C CONTROL SYSTEMS. OPTIONS ARE PROVIDED TO COMPUTE SINGULAR VALUES
C AND VECTORS.

C +++++
C + I I
C >-----O-----I GO I------->
C - I I I
C - I ++------ I
C I I
C +-------+
C
C FIRST, SIGCAL REQUIRES THE TRANSFER MATRIX GO EVALUATED AT
C S = SQRT(-1)*OMEGA (FOR THE CONTINUOUS CASE) OR AT
C S = EXP(SQRT(-1)*OMEGA) (FOR THE DISCRETE CASE).
C THE USER HAS THE OPTION OF CALCULATING THIS MATRIX AND PASSING IT
C TO SIGCAL IN ARRAY G. IN THE AUTHORS' VIEW, THE "NORMAL"
C OPERATING MODE OF FREQ WILL BE TO HAVE SIGCAL CALCULATE
GO = CG*((S*(IDENTITY MATRIX) - AG)INVERSE)*BG + DG
FROM USER-SUPPLIED LINEAR STATE-SPACE REALIZATION MATRICES
(AG, BG, CG, DG).

THE FOLLOWING MATRICES CAN THEN BE COMPUTED.

(1) LOOP-GAIN TRANSFER MATRIX.
   \[ G = Go \]

(2) FEEDBACK TRANSFER MATRIX (GO SQUARE).
   \[ G = Go((I + Go)INVERSE) \\
   = (I + (Go INVERSE))INVERSE \text{ WHEN GO IS NONSINGULAR} \]

(3) SENSITIVITY MATRIX (GO SQUARE).
   \[ G = (I + Go)INVERSE \]

(4) INVERSE RETURN DIFFERENCE MATRIX (GO SQUARE).
   \[ G = I + (Go INVERSE) \text{ WHEN GO IS NONSINGULAR} \]

(5) RETURN DIFFERENCE MATRIX (GO SQUARE).
   \[ G = I + Go \]

WHEN THE SINGULAR VALUE DECOMPOSITION IS CALCULATED
\[ G = U*(\text{DIAG}(\text{SIGMA}(I)))*(\text{V CONJUGATE TRANSPOSE}) \]

THE WORKING SYSTEM MATRIX

ONE MODULE OF THIS CODE, SUBROUTINE SFRMG, ACCEPTS AS INPUT OMEGA, AG, BG, AND CG, AND COMPUTES THE LOOP-GAIN TRANSFER MATRIX GO. WHEN DG = 0, AN ALTERNATE FORMULA FOR THE FEEDBACK TRANSFER MATRIX (2) IS
\[ CG((S1-AG+BG*CG)INVERSE)BG. \]
THIS CODE USES THE LOOP-GAIN TRANSFER MATRIX MODULE WITH AG REPLACED BY AG-BG*CG TO CALCULATE THE FEEDBACK TRANSFER MATRIX. ACCORDINGLY, IF ISOPTA(1) = 2 OR 12 AND DG = 0, THE WORKING SYSTEM MATRIX IS AG-BG*CG; OTHERWISE IT IS AG.

PARAMETERS

\[ \text{NG INTEGER} \]
\[ \text{IF ISOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN ARGUMENT IN THE CALLING SEQUENCE. IF ISOPTA(3) = 0, USED AS FOLLOWS:} \]
\[ \text{ON ENTRY, NUMBER OF STATES} \]
ON RETURN, UNCHANGED

LG INTEGER
ON ENTRY, NUMBER OF INPUTS
ON RETURN, UNCHANGED

MG INTEGER
ON ENTRY, NUMBER OF OUTPUTS
ON RETURN, UNCHANGED

AG REAL NGXNG MATRIX STORED AS PACKED ONE-DIMENSIONAL ARRAY
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS FOLLOWS:
ON ENTRY, SYSTEM MATRIX
ON RETURN, UNCHANGED

BG REAL NGXLG MATRIX STORED AS PACKED ONE-DIMENSIONAL ARRAY
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS FOLLOWS:
ON ENTRY, INPUT INFLUENCE MATRIX
ON RETURN, UNCHANGED

CG REAL MGXNG MATRIX STORED AS PACKED ONE-DIMENSIONAL ARRAY
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS FOLLOWS:
ON ENTRY, SYSTEM OUTPUT MATRIX
ON RETURN, UNCHANGED

DG REAL MGXLG MATRIX STORED AS PACKED ONE-DIMENSIONAL ARRAY
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS FOLLOWS:
ON ENTRY, IF DGNUL = .FALSE., SYSTEM FEEDFORWARD MATRIX
ON RETURN, UNCHANGED

DGNUL LOGICAL VARIABLE
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS FOLLOWS:
ON ENTRY, IF DGNUL = .TRUE., NOT USED, BUT MUST APPEAR AS
AN ARGUMENT OF THE CALLING SEQUENCE
ON RETURN, UNCHANGED

ON RETURN, UNCHANGED

IF DGNUL = .FALSE., SIGCAL ASSUMES THAT THE
SYSTEM FEEDFORWARD MATRIX IS NULL AND IGNORES THE
CONTENTS OF DG

IF DGNUL = .TRUE., SIGCAL TAKES THE SYSTEM
FEEDFORWARD MATRIX FROM ARRAY DG
ON RETURN, UNCHANGED

IGOPTA INTEGER ARRAY OF DIMENSION 3
ON ENTRY, CONTROLS OPTIONS AVAILABLE DURING CALCULATION
OF THE LOOP-GAIN TRANSFER MATRIX GO
ON RETURN, UNCHANGED

IGOPTA(1)
IGOPTA(1) IS IGNORED IF FIRST = .FALSE.
WHEN FIRST = .TRUE. AND IGOPTA(1) .GE. 0, THE WORKING
SYSTEM MATRIX SUBMITTED TO SFRMG WILL BE BALANCED AND
THE EIGENVALUES WILL BE COMPUTED. IF IGOPTA(1) .LT. 0,
THE WORKING SYSTEM MATRIX WILL NOT BE BALANCED AND THE
EIGENVALUES WILL NOT BE COMPUTED.

IGOPTA(2)
LET H = SI-A' WHERE S = IS AS ABOVE AND A' IS THE
RESULT OF REDUCING THE WORKING SYSTEM MATRIX TO UPPER
HESSENBERG FORM. IF IGOPTA(2) .LT. 0, THE CONDITION
NUMBER OF H WILL BE ESTIMATED AND ITS RECIPROCAL
Returned in RCONDG. IF IGOPTA(2) .GE. 0, THE
CONDITION NUMBER OF H WITH RESPECT TO INVERSION WILL
NOT BE ESTIMATED.

IGOPTA(3)
IF IGOPTA(3) = 0, SIGCAL CALCULATES
GO = CG((SI-AG)INVERSE)BG + DG EVALUATED AT
S = SQRT(-1)*OMEGA OR EXP(SQRT(-1)*OMEGA).
IF IGOPTA(3) = 1, SIGCAL ASSUMES THAT THE USER
HAS CALCULATED GO AND PLACED IT IN ARRAY G.
OTHER VALUES OF IGOPTA(3) GENERATE AN ERROR RETURN.

FIRSTG LOGICAL
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS FOLLOWS:
ON ENTRY, IF FIRST = .TRUE., SIGCAL STORES INTERMEDIATE
RESULTS IN ARRAY SAVEG WHICH CAN BE REUSED WITH NEW
OMEGA VALUES.
IF FIRSTG = .FALSE., SIGCAL ASSUMES THAT NG,
RG, MG, AG, BG, CG, SAVEG, AND THE WORKING SYSTEM
MATRIX (SEE ABOVE) ARE UNCHANGED SINCE IT WAS CALLED
WITH FIRSTG = .TRUE. AND RECOVERS INTERMEDIATE VALUES
FROM SAVEG INSTEAD OF RECALCULATING THEM.
ON RETURN, SIGCAL SETS FIRSTG = .FALSE.

SAVEG REAL ARRAY OF DIMENSION AT LEAST NG*(MG+NG+LG)
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0, USED AS FOLLOWS:

ON ENTRY, IF FIRSTG = .TRUE., NOT USED (NO INPUT DATA REQUIRED, RESERVED FOR OUTPUT STORAGE).

IF FIRSTG = .FALSE., MUST CONTAIN VALUES PLACED THERE BY PRIOR ENTRY INTO SIGCAL WITH FIRSTG = .TRUE.

ON RETURN, IF FIRSTG = .TRUE., CONTAINS VALUES WHICH SIGCAL CAN USE ON SUBSEQUENT CALLS WITH FIRSTG = .FALSE.

IF FIRSTG = .FALSE., UNCHANGED.

EVRG REAL ARRAY OF DIMENSION AT LEAST NG.

IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0, USED AS FOLLOWS:

NOT REQUIRED IF IGOPTA(1) .LT. 0 OR FIRSTG = .FALSE., BUT STILL MUST APPEAR IN THE CALLING SEQUENCE.

ON ENTRY, NOT USED.

ON RETURN, IF FIRSTG = .TRUE. AND IGOPTA(1) .GE. 0, EVRG CONTAINS THE REAL PARTS OF THE EIGENVALUES OF THE WORKING SYSTEM MATRIX. OTHERWISE, EVRG IS UNCHANGED.

EVIG REAL ARRAY OF DIMENSION AT LEAST NG.

IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0, USED AS FOLLOWS:

ALL REMARKS ABOUT EVRG (PRECEDING) APPLY EXCEPT THAT, IF FIRSTG = .TRUE. AND IGOPTA(1) .GE. 0, EVIG IS SET TO THE IMAGINARY PARTS OF EIGENVALUES CORRESPONDING TO REAL PARTS IN EVRG.

IGERRA INTEGER ARRAY OF DIMENSION 2

ON ENTRY, NOT USED.

ON RETURN, ERROR CODES FOR INDIVIDUAL POTENTIAL PROBLEMS IN THE MODULE THAT CALCULATES THE LOOP-GAIN TRANSFER MATRIX.

IGERRA(1)
ON RETURN, IF FIRSTG = .TRUE. AND IGOPTA(1) .GE. 0 ON ENTRY, THEN IGERRA(1) IS RETURNED AS THE IERR PARAMETER OF THE EISPACK SUBROUTINE HQR -- NORMAL RETURN IS 0. OTHERWISE, IGERRA(1) IS UNCHANGED.

IGERRA(2)
ON RETURN, IF IGOPTA(2) .GE. 0 ON ENTRY, IGERRA(2) IS RETURNED AS THE INFO PARAMETER OF SUBROUTINE CHEFA.
Otherwise, if \( 1.0 + \text{RCONDG} \cdot \text{EQ. 1.0} \), \( (H \text{ defined under 'IGOPT(2)'} \) above is singular to working precision), IGERRA(2) is returned set to -1. Normal return in either case is 0.

\textbf{RCONDG} \text{ REAL SCALAR}

If IGOPTA(3) \( .NE. 0 \), unused, but must appear as an argument in the calling sequence. If IGOPTA(3) = 0, used as follows:

On entry, not used.

On return, if IGOPTA(2) \( .LT. 0 \), the reciprocal of the condition number of the \( H \) matrix defined in the discussion of IGOPTA(2) above is returned in RCONDG. If IGOPTA(2) \( .GE. 0 \), RCONDG is unchanged.

\textbf{OMEGA} \text{ REAL SCALAR}

On entry, frequency (in the continuous case, should be positive).

Normalized frequency (in the discrete case, should be angular frequency times sampling rate).

On return, unchanged.

\textbf{RCONDS} \text{ REAL SCALAR}

On entry, not used.

On return, if ISOPTA(1) = 2 or 12 and DGNUL = .FALSE., or if ISOPTA(1) = 3 or 13, or if ISOPTA(1) = 4 or 14, \( G \) is calculated from \( G0 \) by solving the matrix equation 

\( (I + G0)G = G0, \ (I + G0)G = I, \text{ or } G0(G - I) = I \), respectively, for \( G \). The subroutine, CSOLVE, which solves these equations is a driver program for standard LINPACK modules of the CGE-- family. The estimated reciprocal condition number of the coefficient matrix calculated by CGECO is returned in RCONDS. In this case, RCONDS is between 0.0 and 1.0. If RCONDS is so small that \( 1.0 + \text{RCONDS} \cdot \text{EQ. 1.0} \) to within machine precision, the equation is too ill-conditioned to solve. This error condition is also signaled by setting JERR = 11.

If CSOLVE is not used, RCONDS is set to 2.0.

\textbf{ISOPTA} \text{ INTEGER ARRAY OF DIMENSION 3}

On entry, transmits user selected options to SIGCAL -- which \( G \) matrix to compute and whether to use continuous or discrete formulation; whether to compute singular values or singular vectors; and whether to use the built-in printout feature.

On return, unchanged.

31
ISOPTA(1)

-EQ. 1, CONTINUOUS LOOP-GAIN TRANSFER MATRIX RETURNED IN G.
-EQ. 2, CONTINUOUS FEEDBACK TRANSFER MATRIX RETURNED IN G.
-EQ. 3, CONTINUOUS SENSITIVITY MATRIX RETURNED IN G.
-EQ. 4, CONTINUOUS INVERSE RETURN DIFFERENCE MATRIX RETURNED IN G.
-EQ. 5, CONTINUOUS RETURN DIFFERENCE MATRIX RETURNED IN G.
-EQ. 11, DISCRETE LOOP-GAIN TRANSFER MATRIX RETURNED IN G.
-EQ. 12, DISCRETE FEEDBACK TRANSFER MATRIX RETURNED IN G.
-EQ. 13, DISCRETE SENSITIVITY MATRIX RETURNED IN G.
-EQ. 14, DISCRETE INVERSE RETURN DIFFERENCE MATRIX RETURNED IN G.
-EQ. 15, DISCRETE RETURN DIFFERENCE MATRIX RETURNED IN G.

ISOPTA(2)

-EQ. 1, DO NOT COMPUTE SINGULAR VALUES OR VECTORS OF G. RETURN AFTER COMPUTING G.
-EQ. 2, COMPUTE SINGULAR VALUES BUT NO SINGULAR VECTORS OF G.
-EQ. 3, COMPUTE BOTH SINGULAR VALUES AND VECTORS OF G.

ISOPTA(3)

-NE. 0, PRINT INPUT, PROBLEM STATEMENT, AND RESULTS OF CALCULATIONS AS THEY PROCEED.
-EQ. 0, NO PRINTING EXCEPT FOR ERROR MESSAGES.

G COMPLEX MGXLG MATRIX STORED AS A PACKED ONE-DIMENSIONAL COMPLEX ARRAY
ON ENTRY, IF ISOPTA(3) = 0, NOT USED.
ON RETURN, CONTAINS THE G-MATRIX SELECTED BY ISOPTA(1).

SIGMA REAL ARRAY OF DIMENSION AT LEAST MIN(MG,LG).
ON ENTRY, NOT USED.
ON NORMAL RETURN, SIGMA CONTAINS THE SINGULAR VALUES OF G ARRANGED IN DESCENDING ORDER OF MAGNITUDE.
ON ERROR RETURN, FIRST MIN(M,R)-INFO ENTRIES OF SIGMA ARE
SINGULAR VALUES, REMAINING ENTRIES ARE UNDEFINED.

U COMPLEX MG X MIN(LG,MG) MATRIX STORED AS A PACKED
ONE-DIMENSIONAL COMPLEX ARRAY
NOT REQUIRED IF ISOPTA(2) = 1 OR 2, BUT A DUMMY
ARGUMENT MUST APPEAR IN THIS POSITION OF THE CALLING
SEQUENCE.
IF ISOPTA(2) = 3, U SERVES THE FOLLOWING FUNCTION.
ON ENTRY, NOT USED.
ON NORMAL RETURN, CONTAINS THE LEFT SINGULAR VECTORS
OF G.
ON ERROR RETURN, SEE THE DISCUSSION OF INFO BELOW.

V COMPLEX LG X MIN(LG,MG) MATRIX STORED AS A PACKED
ONE-DIMENSIONAL COMPLEX ARRAY
NOT REQUIRED IF ISOPTA(2) = 1 OR 2, BUT A DUMMY
ARGUMENT MUST APPEAR IN THIS POSITION OF THE CALLING
SEQUENCE.
IF ISOPTA(2) = 3, V SERVES THE FOLLOWING FUNCTION.
ON ENTRY, NOT USED.
ON NORMAL RETURN, CONTAINS THE RIGHT SINGULAR VECTORS
OF G.
ON ERROR RETURN, SEE THE DISCUSSION OF INFO BELOW.

INFO INTEGER
ON ENTRY, NOT USED.
ON RETURN, ERROR CODE FOR THE G-MATRIX SINGULAR VALUES
CALCULATION. IF ISOPTA(2) = 2 OR 3, INFO IS SET TO THE
"INFO" PARAMETER RETURNED FROM SINGULAR VALUE
DECOMPOSITION SUBROUTINE CSVDC. OTHERWISE INFO IS SET
TO 0. NORMAL RETURN IS 0. IN THE EVENT OF AN ERROR
RETURN FROM CSVDC, REFER TO THE DOCUMENTATION OF
LINPACK SUBROUTINE CSVDC FOR THE MEANING OF THE
CONTENTS OF MATRICES U AND V. IN THAT DOCUMENTATION,
REFERENCE IS MADE TO COMPLEX VECTORS S AND E. IN THE
EVENT OF AN ERROR RETURN, THE CONTENTS OF S AND E ARE
PRESERVED AS, RESPECTIVELY, THE FIRST MIN(MG+1, LG) AND
THE NEXT MIN(MG, LG) ENTRIES IN ARRAY CDUM (SEE BELOW).

RDUM REAL SCRATCH ARRAY OF DIMENSION AT LEAST NG.

CDUM COMPLEX SCRATCH ARRAY.
DIMENSIONAL NEEDS OF CDUM VARY WITH APPLICATION. AN
UPPER BOUND ON SPACE NEEDED IS 2*K*(K+1)+1 LOCATIONS
OF TYPE COMPLEX, WHERE K = MAX(MG, NG, LG). EXACT
STORAGE NEEDED BY CDUM IS CALCULATED AS FOLLOWS. LET
I1 = NG*(NG+LG+1). IF ISOPTA(1) = 2 OR 12 AND DGNUL =
.FALSE., OR IF ISOPTA(1) = 3, 4, 13, OR 14, LET I2 =
IDUM INTEGER SCRATCH ARRAY OF DIMENSION AT LEAST MAX(MG,NG)

JERR INTEGER

IF NO ERROR IS DETECTED BY SIGNAL, JERR IS SET TO 0.

IF AN ERROR IS DETECTED, JERR IS SET TO THE FIRST APPLICABLE OF THE FOLLOWING ERROR CODES.

1 MATRIX DIMENSIONS NONPOSITIVE OR INCONSISTENT WITH CALCULATION CHOSEN BY ISOPTA(1).

2 HQR ERROR OCCURRED, SEE IGERRA(1) ABOVE.

3 INFO PARAMETER OF CHEFA INDICATES AN ERROR, SEE DISCUSSION OF IGERRA(2) ABOVE.

4 1. EQ. 1. + RCONDG, H IS (NEARLY) SINGULAR, SEE DISCUSSION OF RCONDG ABOVE.

5 ISOPTA(3) WAS NEITHER 0 NOR 1.

11 CSOLVE ERROR OCCURRED, SEE RCONDS ABOVE.

12 CSVDC ERROR OCCURRED, SEE INFO ABOVE.
SIGDYN GENERATES A VARIETY OF MATRICES USED FOR THE
FREQUENCY DOMAIN ANALYSIS OF MULTIVARIABLE CONTINUOUS OR DISCRETE
CONTROL SYSTEMS.

SIGDYN FIRST DETERMINES IF THE USER HAS SUPPLIED EITHER OR BOTH
OF THE SYSTEM AND COMPENSATOR DYNAMICS MATRICES, GO AND KO.
ANY WHICH HAS NOT BEEN USER-SUPPLIED IS CALCULATED USING
THE FORMULAS

\[ \text{GO} = \text{CG}((\text{SI} - \text{AG})^{-1})\text{BG} + \text{DG} \]
\[ \text{KO} = \text{CK}((\text{SI} - \text{AK})^{-1})\text{BK} + \text{DK} \]

EVALUATED AT
\[ S = \exp(\text{SQRT}(-1)\ast \text{OMEGA}) \] (DISCRETE CASE).

IN THE FOLLOWING, GL STANDS FOR THE USER'S CHOICE OF ONE OF
EIGHT MATRICES COMPUTED FROM GO AND KO BY GL = G' * K' OR
K' * G' WHERE G' = GO OR (GO)^{-1} AND K' = KO OR
(KO)^{-1}.
THE FOLLOWING MATRICES CAN THEN BE COMPUTED.

1. LOOP-GAIN TRANSFER MATRIX.
   \[ G = GL \]

2. FEEDBACK TRANSFER MATRIX (GL SQUARE).
   \[ G = GL((I + GL)^{-1})^{-1} \] WHEN GL IS NONSINGULAR

3. SENSITIVITY MATRIX (GL SQUARE).
   \[ G = (I + GL)^{-1} \]

4. INVERSE RETURN DIFFERENCE MATRIX (GL SQUARE).
   \[ G = I + (GL)^{-1} \] WHEN GL IS NONSINGULAR

5. RETURN DIFFERENCE MATRIX (GL SQUARE).
   \[ G = I + GL \]

OPTIONS ARE PROVIDED TO COMPUTE SINGULAR VALUES AND VECTORS.

WHEN THE SINGULAR VALUE DECOMPOSITION IS CALCULATED

\[ G = U\text{(Diag} \text{(SIGMA(I)))}\text{)} \text{(V CONJUGATE TRANSPOSE)} \]

PARAMETERS. ***NOTE*** THE FIVE INTEGER VARIABLES IKG, IGINV, IKINV, M, AND L ARE DEFINED IN TERMS OF THE PARAMETERS ISOPTA, MG, LG, MK, AND LK FROM THE CALLING SEQUENCE OF SIGDYN AND ARE USED THROUGHOUT THE FOLLOWING DISCUSSION. THEY ARE DEFINED AS FOLLOWS:

- **ISOPTA(4) IS RESTRICTED TO BE BETWEEN 0 AND 7 INCLUSIVE,**
- **IKG, IGINV, AND IKINV ARE UNIQUELY DEFINED BY THE REQUIREMENTS THAT EACH BE EITHER 0 OR 1 AND**
- **ISOPTA(4) = IKG + 2*IGINV + 4*IKINV.**
- **IF IKG = 0, THEN M IS DEFINED TO BE MG AND L TO BE LK.**
- **IF IKG = 1, THEN M IS DEFINED TO BE MK AND L TO BE LG.**

\[ NG \text{ INTEGER} \]
\[ \text{IF ISOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN} \]
\[ \text{ARGUMENT IN THE CALLING SEQUENCE. IF ISOPTA(3) = 0,} \]
\[ \text{USED AS FOLLOWS:} \]
\[ \text{ON ENTRY, NUMBER OF SYSTEM STATES} \]
\[ \text{ON RETURN, UNCHANGED} \]

\[ LG \text{ INTEGER} \]
\[ \text{ON ENTRY, NUMBER OF SYSTEM INPUTS} \]
\[ \text{ON RETURN, UNCHANGED} \]
INTEGER
ON ENTRY, NUMBER OF SYSTEM OUTPUTS
ON RETURN, UNCHANGED

REAL NXNG MATRIX STORED AS PACKED ONE-DIMENSIONAL ARRAY
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS follows:
ON ENTRY, SYSTEM MATRIX
ON RETURN, UNCHANGED

REAL NXLG MATRIX STORED AS PACKED ONE-DIMENSIONAL ARRAY
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS follows:
ON ENTRY, INPUT INFLUENCE MATRIX
ON RETURN, UNCHANGED

REAL MGXNG MATRIX STORED AS PACKED ONE-DIMENSIONAL ARRAY
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS follows:
ON ENTRY, SYSTEM OUTPUT MATRIX
ON RETURN, UNCHANGED

REAL MGXLG MATRIX STORED AS PACKED ONE-DIMENSIONAL ARRAY
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS follows:
ON ENTRY, IF DGNUL = .FALSE., SYSTEM FEEDFORWARD MATRIX
    IF DGNUL = .TRUE., NOT USED, BUT MUST APPEAR AS
    AN ARGUMENT OF THE CALLING SEQUENCE
ON RETURN, UNCHANGED

LOGICAL VARIABLE
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS follows:
ON ENTRY, IF DGNUL = .TRUE., SIGDYN ASSUMES THAT THE
    SYSTEM FEEDFORWARD MATRIX IS NULL AND IgNORES THE
    CONTENTS OF DG
    IF DGNUL = .FALSE., SIGDYN TAKES THE SYSTEM
    FEEDFORWARD MATRIX FROM ARRAY DG
ON RETURN, UNCHANGED

INTEGER ARRAY OF DIMENSION 3
ON ENTRY, CONTROLS OPTIONS AVAILABLE DURING CALCULATION
    OF THE SYSTEM DYNAMICS MATRIX GO
ON RETURN, UNCHANGED

37
IGOPTA(1)

IGOPTA(1) is ignored if FIRSTG = .FALSE. when FIRSTG = .TRUE. and IGOPTA(1) .GE. 0, AG will be balanced and its eigenvalues will be computed. If IGOPTA(1) .LT. 0, AG will not be balanced and its eigenvalues will not be computed.

IGOPTA(2)

Let \( H = S^{-1}A' \) where \( S \) is as above and \( A' \) is the result of reducing AG to upper Hessenberg form. If IGOPTA(2) .LT. 0, the condition number of \( H \) with respect to inversion will be estimated and its reciprocal returned in \( RCONDG \). If IGOPTA(2) .GE. 0, the condition number of \( H \) with respect to inversion will not be estimated.

IGOPTA(3)

If IGOPTA(3) = 0, SIGCAL calculates
\[
Go = C((S1 - A)^{-1})B + D \text{ evaluated at } S = \sqrt{-1} \times \Omega \text{ or } \exp(\sqrt{-1} \times \Omega).
\]
If IGOPTA(3) = 1, SIGCAL assumes that the user has calculated Go (Q. V.). Other values of IGOPTA(3) generate an error return.

FIRSTG

Logical

If IGOPTA(3) .NE. 0, unused, but must appear as an argument in the calling sequence. If IGOPTA(3) = 0, used as follows:

On entry, if FIRSTG = .TRUE., SIGDYN stores intermediate results in array SAVEG which can be reused with new omega values.

If FIRSTG = .FALSE., SIGDYN assumes that NG, LG, MG, AG, BG, CG, and SAVEG are unchanged since it was called with FIRSTG = .TRUE. and recovers intermediate values from SAVEG instead of recalculating them.

On return, SIGDYN sets FIRSTG = .FALSE.

SAVEG

Real array of dimension at least NG*(MG+NG+LG)

If IGOPTA(3) .NE. 0, unused, but must appear as an argument in the calling sequence. If IGOPTA(3) = 0, used as follows:

On entry, if FIRSTG = .TRUE., not used (no input data required, reserved for output storage).

If FIRSTG = .FALSE., must contain values placed there by prior entry into SIGDYN with FIRSTG = .TRUE.

On return, if FIRSTG = .TRUE., contains values which SIGDYN can use on subsequent calls with FIRSTG =
FALSE.

IF FIRSTG = .FALSE., UNCHANGED.

EVRG REAL ARRAY OF DIMENSION AT LEAST NG
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS FOLLOWS:
NOT REQUIRED IF IGOPTA(1) .LT. 0 OR FIRSTG = .FALSE.,
BUT STILL MUST APPEAR IN THE CALLING SEQUENCE.
ON ENTRY, NOT USED.
ON RETURN, IF FIRSTG = .TRUE. AND IGOPTA(1) .GE. 0,
EVRG CONTAINS THE REAL PARTS OF THE EIGENVALUES
OF AG. OTHERWISE, EVRG IS UNCHANGED.

EVIG REAL ARRAY OF DIMENSION AT LEAST NG
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN
ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0,
USED AS FOLLOWS:
ALL REMARKS ABOUT EVRG (PRECEDING) APPLY EXCEPT THAT,
IF FIRSTG = .TRUE. AND IGOPTA(1) .GE. 0,
EVIG IS SET TO THE IMAGINARY PARTS OF EIGENVALUES
CORRESPONDING TO REAL PARTS IN EVRG.

GO COMPLEX ARRAY OF DIMENSION AT LEAST MG*LG
ON ENTRY, IF IGOPTA(3) = 0, NOT USED.
IF IGOPTA(3) = 1, CONTAINS THE SYSTEM DYNAMICS
MATRIX EVALUATED AT S = SQRT(-1)*OMEGA OR
S = EXP(SQRT(-1)*OMEGA).
ON RETURN, IF IGOPTA(3) = 0, CONTAINS
CG((S1-AG)INVERSE)BG + DG
FOR S = SQRT(-1)*OMEGA OR
S = EXP(SQRT(-1)*OMEGA).
IF IGOPTA(3) = 1, UNCHANGED.

IGERRA INTEGER ARRAY OF DIMENSION 2
ON ENTRY, NOT USED.
ON RETURN, ERROR CODES FOR INDIVIDUAL POTENTIAL PROBLEMS
IN THE MODULE THAT CALCULATES GO.

IGERRA(1)
ON RETURN, IF FIRSTG = .TRUE. AND IGOPTA(1) .GE. 0 ON
ENTRY, THEN IGERRA(1) IS RETURNED AS THE IERR
PARAMETER OF THE EISPACK SUBROUTINE HQR -- NORMAL
RETURN IS 0. OTHERWISE, IGERRA(1) IS UNCHANGED.

IGERRA(2)
ON RETURN, IF IGOPTA(2) .GE. 0 ON ENTRY, IGERRA(2) IS
RETURNED AS THE INFO PARAMETER OF SUBROUTINE ZHEFA.
OTHERWISE, IF 1.0 + RCONDG .EQ. 1.0, (H DEFINED UNDER 'IGOPTA(2)' ABOVE IS SINGULAR TO WORKING PRECISION), IGERRA(2) IS RETURNED SET TO -1. NORMAL RETURN IN EITHER CASE IS 0.

RCONDG REAL SCALAR
IF IGOPTA(3) .NE. 0, UNUSED, BUT MUST APPEAR AS AN ARGUMENT IN THE CALLING SEQUENCE. IF IGOPTA(3) = 0, USED AS FOLLOWS:
ON ENTRY, NOT USED.
ON RETURN, IF IGOPTA(2) .LT. 0, THE RECIPROCAL OF THE CONDITION NUMBER OF THE H MATRIX DEFINED IN THE DISCUSSION OF IGOPTA(2) ABOVE IS RETURNED IN RCONDG.
IF IGOPTA(2) .GE. 0, RCONDG IS UNCHANGED.

NK,LK,MK,AK,BK,CK,DK,DKNUL,IKOPTA,FIRSTK,SAVEK, EVRK,EVIK,KO,IKERRA,RCONDG
USED FOR THE DYNAMIC COMPENSATOR IN EXACTLY THE SAME WAY AS
NG,LG,MG,AG,BG,CG,DG,DGNUL,IGOPTA,FIRSTG,SAVEG, EVRG,EVIG,GO,IGERRA,RCONDG
RESPECTIVELY, ARE USED FOR THE BASIC SYSTEM.

OMEGA REAL SCALAR
ON ENTRY, FREQUENCY (IN THE CONTINUOUS CASE, SHOULD BE POSITIVE).
NORMALIZED FREQUENCY (IN THE DISCRETE CASE, SHOULD BE ANGULAR FREQUENCY TIMES SAMPLING RATE).
ON RETURN, UNCHANGED.

RCONDS REAL SCALAR
ON ENTRY, NOT USED.
ON RETURN, IF ISOPTA(4) = 2, 3, 4, 5, 6, OR 7, GL IS CALCULATED BY SOLVING THE EQUATION GO*GL = KO,
GO'*GL' = KO', KG*GL' = GO', KO*GL = GO,
(KO*GO)*GL = I, OR (KO*GO)*GL = I, RESPECTIVELY, FOR GL. IF ISOPTA(1) = 2 OR 12; OR 3 OR 13; OR 4 OR 14; G IS CALCULATED FROM GL BY SOLVING THE MATRIX EQUATION (I + GL)G = GL, (I + GL)G = I, OR
GL(G - I) = I, RESPECTIVELY, FOR G. THE SUBROUTINE, CSOLVE, WHICH SOLVES THESE EQUATIONS IS A DRIVER PROGRAM FOR STANDARD LINPACK MODULES OF THE CGE--FAMILY. EACH CALL TO CSOLVE RESULTS IN AN ESTIMATE OF THE RECIPROCAL OF THE CONDITION NUMBER OF THE COEFFICIENT MATRIX BEING CALCULATED BY CGECO. THE MINIMUM OF THESE OVER ALL CALLS TO CSOLVE IS RETURNED.

40
IN RCONDS. IN THIS CASE, RCONDS IS BETWEEN 0.0 AND 1.0. IF RCONDS IS SO SMALL THAT 1.0 + RCONDS .EQ. 1.0 TO WITHIN MACHINE PRECISION, THE EQUATION IS TOO ILL-CONDITIONED TO SOLVE. THIS ERROR CONDITION IS ALSO SIGNALED BY SETTING JERR = 10 OR 11.

IF CSOLVE IS NOT USED, RCONDS IS SET TO 2.0.

ISOPTA INTEGER ARRAY OF DIMENSION 4
ON ENTRY, TRANSMITS USER-SELECTED OPTIONS TO SIGDYN — WHICH G MATRIX TO COMPUTE; WHETHER TO COMPUTE SINGULAR VALUES OR SINGULAR VECTORS; WHETHER TO USE THE BUILT-IN PRINTOUT FEATURE; AND WHAT GL IS.
ON RETURN, UNCHANGED.

ISOPTA(1)
.EQ. 1, CONTINUOUS LOOP-GAIN TRANSFER MATRIX RETURNED IN G.
.EQ. 2, CONTINUOUS FEEDBACK TRANSFER MATRIX RETURNED IN G.
.EQ. 3, CONTINUOUS SENSITIVITY MATRIX RETURNED IN G.
.EQ. 4, CONTINUOUS INVERSE RETURN DIFFERENCE MATRIX RETURNED IN G.
.EQ. 5, CONTINUOUS RETURN DIFFERENCE MATRIX RETURNED IN G.
.EQ. 11, DISCRETE LOOP-GAIN TRANSFER MATRIX RETURNED IN G.
.EQ. 12, DISCRETE FEEDBACK TRANSFER MATRIX RETURNED IN G.
.EQ. 13, DISCRETE SENSITIVITY MATRIX RETURNED IN G.
.EQ. 14, DISCRETE INVERSE RETURN DIFFERENCE MATRIX RETURNED IN G.
.EQ. 15, DISCRETE RETURN DIFFERENCE MATRIX RETURNED IN G.

ISOPTA(2)
.EQ. 1, DO NOT COMPUTE SINGULAR VALUES OR VECTORS OF G. RETURN AFTER COMPUTING G.
.EQ. 2, COMPUTE SINGULAR VALUES BUT NO SINGULAR VECTORS OF G.
.EQ. 3, COMPUTE BOTH SINGULAR VALUES AND VECTORS OF G.

ISOPTA(3)
.NE. 0, PRINT INPUT, PROBLEM STATEMENT, AND RESULTS OF CALCULATIONS AS THEY PROCEED.
.EQ. 0, NO PRINTING EXCEPT FOR ERROR MESSAGES.

ISOPTA(4) MUST BE BETWEEN 0 AND 7. DETERMINES HOW GL WILL BE CALCULATED FROM GO AND KO.
WITH IKG, IGINV, AND IKINV DEFINED AS IN THE NOTE ABOVE,
SET $GL = G' \times K'$ IF $IKG = 0$,
$GL = K' \times G'$ IF $IKG = 1$;
WHERE $G' = GO$ IF $IGINV = 0$,
$G' = (GO)\text{INVERSE}$ IF $IGINV = 1$,
$K' = KO$ IF $IKINV = 0$, AND
$K' = (KO)\text{INVERSE}$ IF $IKINV = 1$.

$G$ COMPLEX $M \times R$ MATRIX STORED AS A PACKED ONE-DIMENSIONAL
COMPLEX ARRAY
ON ENTRY, NOT USED.
ON RETURN, CONTAINS THE G-MATRIX SELECTED BY ISOPTA(1).

$SIGMA$ REAL ARRAY OF DIMENSION AT LEAST $\min(M,R)$.
NOT REQUIRED IF ISOPTA(2) = 1, BUT STILL MUST APPEAR AS
AN ARGUMENT OF CALLING SEQUENCE.
ON ENTRY, NOT USED.
ON NORMAL RETURN, SIGMA CONTAINS THE SINGULAR VALUES OF
$G$ ARRANGED IN DESCENDING ORDER OF MAGNITUDE.
ON ERROR RETURN, FIRST $\min(M,R)-INFO$ ENTRIES OF SIGMA
ARE SINGULAR VALUES; REMAINING ENTRIES ARE UNDEFINED.

$U$ COMPLEX $M \times \min(R,M)$ MATRIX STORED AS A PACKED
ONE-DIMENSIONAL COMPLEX ARRAY
NOT REQUIRED IF ISOPTA(2) = 1 OR 2, BUT A DUMMY
ARGUMENT MUST APPEAR IN THIS POSITION OF THE CALLING
SEQUENCE.
IF ISOPTA(2) = 3, $U$ SERVES THE FOLLOWING FUNCTION.
ON ENTRY, NOT USED.
ON NORMAL RETURN, CONTAINS THE LEFT SINGULAR VECTORS
OF $G$.
ON ERROR RETURN, SEE THE DISCUSSION OF $INFO$ BELOW.

$V$ COMPLEX $L \times \min(R,M)$ MATRIX STORED AS A PACKED
ONE-DIMENSIONAL COMPLEX ARRAY
NOT REQUIRED IF ISOPTA(2) = 1 OR 2, BUT A DUMMY
ARGUMENT MUST APPEAR IN THIS POSITION OF THE CALLING
SEQUENCE.
IF ISOPTA(2) = 3, $V$ SERVES THE FOLLOWING FUNCTION.
ON ENTRY, NOT USED.
ON NORMAL RETURN, CONTAINS THE RIGHT SINGULAR VECTORS
OF $G$.
ON ERROR RETURN, SEE THE DISCUSSION OF $INFO$ BELOW.

$INFO$ INTEGER
ON ENTRY, NOT USED.
ON RETURN, ERROR CODE FOR THE G-MATRIX SINGULAR VALUES
CALCULATION. IF ISOPTA(2) = 2 OR 3, $INFO$ IS SET TO THE
"INFO" PARAMETER RETURNED FROM SINGULAR VALUE
DECOMPOSITION SUBROUTINE CSVDC. OTHERWISE INFO IS SET TO 0. NORMAL RETURN IS 0. IN THE EVENT OF AN ERROR RETURN FROM CSVDC, REFER TO THE DOCUMENTATION OF LINPACK SUBROUTINE CSVDC FOR THE MEANING OF THE CONTENTS OF MATRICES U AND V. IN THAT DOCUMENTATION, REFERENCE IS MADE TO COMPLEX VECTORS S AND E. IN THE EVENT OF AN ERROR RETURN, THE CONTENTS OF S AND E ARE PRESERVED AS, RESPECTIVELY, THE FIRST MIN(M+1,L) AND THE NEXT MIN(M,L) ENTRIES IN ARRAY CDUM (SEE BELOW).

RDUM REAL SCRATCH ARRAY OF DIMENSION AT LEAST MAX(NG,NK).

CDUM COMPLEX SCRATCH ARRAY.

DIMENSIONAL NEEDS OF CDUM VARY WITH APPLICATION. AN UPPER BOUND ON SPACE NEEDED IS J*(3*J+2) LOCATIONS OF TYPE COMPLEX, WHERE J = MAX(MG,NG,LG,MK,NK,LK). EXACT STORAGE NEEDED BY CDUM IS CALCULATED AS FOLLOWS.

LET I1 = NG*(NG+LG+1). LET I2 = NK*(NK+LK+1).

IF ISOPTA(1) IS 2, 3, 4, 12, 13, OR 14, LET I3 = M*(2*M+1); OTHERWISE I3 = 0. IF ISOPTA(2) IS 2 OR 3 AND M < L, LET I4 = (M+1)*(L+2)-1; OTHERWISE, I4 = 0. IF ISOPTA(2) IS 2 OR 3 AND M >= L, LET I5 = L*(M+2)+M; OTHERWISE I5 = 0. DETERMINE I6 FROM THE FOLLOWING TABLE:

<table>
<thead>
<tr>
<th>ISOPTA(4)</th>
<th>I6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 OR 1</td>
<td>0</td>
</tr>
<tr>
<td>2 OR 5</td>
<td>M*(M+1)</td>
</tr>
<tr>
<td>3 OR 4</td>
<td>L*(2*L+M+1)</td>
</tr>
<tr>
<td>6 OR 7</td>
<td>M*(2*M+1)</td>
</tr>
</tbody>
</table>

THEN CDUM NEEDS MAX(I1,I2,I3,I4,I5,I6) TYPE COMPLEX LOCATIONS.

IDUM INTEGER SCRATCH ARRAY OF DIMENSION AT LEAST MAX(M,NG,NK) UNLESS ISOPTA(4) = 6 OR 7. THEN IDUM MUST HAVE AT LEAST MAX(M,L,NG,NK) LOCATIONS.

JERR INTEGER

IF NO ERROR IS DETECTED BY SIGDYN, JERR IS SET TO 0. IF AN ERROR IS DETECTED, JERR IS SET TO THE FIRST APPLICABLE OF THE FOLLOWING ERROR CODES.

1 MATRIX DIMENSIONS NONPOSITIVE OR INCONSISTENT WITH CALCULATION CHOSEN BY ISOPTA(1) AND/OR ISOPTA(4), OR ISOPTA(4) OUT OF RANGE.

2 HQR ERROR OCCURRED IN GO CALCULATION; SEE IGERRA(1) ABOVE.
INFO PARAMETER OF ZHEFA INDICATES AN ERROR IN
GO CALCULATION; SEE DISCUSSION OF IGERRA(2).
1. .EQ. 1. + RCONDG; H IS (NEARLY) SINGULAR;
SEE DISCUSSION OF RCONDG ABOVE.
IGOPTA(3) WAS NEITHER 0 NOR 1.
HQR ERROR OCCURRED IN KO CALCULATION, LIKE
JERR = 2 FOR GO CALCULATION.
INFO PARAMETER OF ZHEFA INDICATES AN ERROR IN
KO CALCULATION, LIKE JERR = 3 FOR GO CASE.
1. .EQ. 1. + RCONDK, LIKE JERR = 4 FOR GO CASE.
IKOPTA(3) WAS NEITHER 0 NOR 1.
CSOLVE ERROR OCCURRED IN CALCULATING GL FROM GO
AND KO; SEE RCONDS ABOVE.
CSOLVE ERROR OCCURRED IN CALCULATING G FROM GL;
SEE RCONDS ABOVE.
CSVDC ERROR OCCURRED; SEE INFO ABOVE.
BOTH ERRORS 2 AND 6 ABOVE OCCURRED.
THE PURPOSE OF SIGTAB IS TO REPEATEDLY CALL SIGCAL IN ITS "GO = CG*(SQRT(-1)*OMEGA*I - AG)INVERSE*BG + DG" MODE WHILE VARYING THE VALUE OF THE FREQUENCY PARAMETER "OMEGA" SO AS TO TABULATE THE MAXIMUM AND MINIMUM SINGULAR VALUES OF THE "G" MATRIX WHICH SIGCAL CHOSES TO CALCULATE BASED ON THE VALUE OF THE PARAMETER "ISOPTA(1)". THESE COMMENTS ASSUME THAT THE READER IS FAMILIAR WITH THE PROGRAM PRELUDE DOCUMENTATION OF SIGCAL.

TWO OF THE 27 PARAMETERS TO SIGCAL ARE COMPLEX MATRICES, IDENTIFIED IN SIGCAL DOCUMENTATION AS "U" AND "V". THESE ARE USED WHEN SIGCAL IS CALLED WITH "ISOPTA(2)" SET EQUAL TO 3 TO RETURN LEFT AND RIGHT SINGULAR VECTORS OF "G". SINCE SIGTAB CALLS SIGCAL WITH "ISOPTA(2)" SET TO 2, SIGTAB DOES NOT NEED "U" AND "V". THE FIRST 25 PARAMETERS OF SIGTAB, I.E., "NG" THROUGH "JERR" ARE IDENTICAL IN ORDER AND FUNCTION (AND IN NAME IN THIS DOCUMENT) WITH THE REMAINING 25 PARAMETERS TO SIGCAL WITH THE EXCEPTION THAT ONLY THE FIRST TWO ENTRIES OF IGOPTA ARE FUNCTIONAL; EVEN IF THE THIRD ENTRY IS PRESENT, IT IS IGNORED AND SIGCAL IS DRIVEN IN ITS "COMPUTE GO INTERNALLY" MODE.

SIGTAB SETS THE VALUE OF ISOPTA(3) TO 0 AFTER THE FIRST CALL TO SIGCAL SO THAT DETAIL PRINTOUT OCCURS ONLY WHEN SIGCAL IS CALLED WITH OMEGA = OMGTAB(1), AND THAT ONLY IF THE USER SET ISOPTA(3) .NE. 0 ON ENTRY TO SIGTAB.

FIRST 25 PARAMETERS
ON ENTRY, THE USER MUST INITIALIZE PARAMETERS NG, RG, MG, AG, BG, CG, DG, DGNUL, ISOPTA(1), ISOPTA(2), FIRSTG,
ISOPTA(1), AND ISOPTA(3) IN ACCORDANCE WITH SIGCAL DOCUMENTATION.

ON RETURN,
  IF KERR = -1, UNCHANGED
  IF KERR = 0, AS RETURNED BY SIGCAL WITH
    OMEGA = OMGTAB(NO) EXCEPT ISOPTA(2) HAS BEEN SET TO
    2 AND ISOPTA(3) HAS BEEN SET TO 0
  IF KERR > 0, AS RETURNED BY SIGCAL WITH
    OMEGA = OMGTAB(KERR) EXCEPT ISOPTA(2) HAS BEEN SET
    TO 2 AND ISOPTA(3) HAS BEEN SET TO 0

JOP INTEGER, OPTION ON INPUT OMEGAS
  ON ENTRY,
    JOP = 0, USER HAS PRESET THE ENTIRE OMGTAB ARRAY
    JOP = 1, USER HAS PRESET OMGTAB(1) AND OMGTAB(NO).
    SIGTAB WILL FILL IN INTERMEDIATE VALUES
    EQUITLY SPACED
    JOP = 2, USER HAS PRESET OMGTAB(1) AND OMGTAB(NO)
    TO NONZERO VALUES OF THE SAME SIGN. SIGTAB
    WILL FILL IN INTERMEDIATE VALUES SO THAT CONSECUTIVE
    RATIOS ARE THE SAME (EQUALLY SPACED ON A LOG SCALE).
  ON RETURN, JOP = 0

NO INTEGER
  ON ENTRY, NUMBER OF ENTRIES IN THE OMGTAB, SVMAX, AND
  SVMIN ARRAYS.
  ON RETURN, UNCHANGED

OMGTAB REAL ARRAY OF LENGTH AT LEAST NO
  ON ENTRY, PRESET AS DESCRIBED UNDER JOP
  ON RETURN, FULLY SET ACCORDING TO JOP OPTION

SVMAX, SVMIN REAL ARRAYS OF LENGTH AT LEAST NO
  ON ENTRY, NOT USED
  ON RETURN, SVMAX(I) AND SVMIN(I) ARE SET TO, RESPECTIVELY,
    THE MAXIMUM AND MINIMUM SINGULAR VALUES OF THE
    G MATRIX CALCULATED BY SIGCAL WHEN OMEGA IS
    SET TO OMGTAB(I).

KERR INTEGER, ERROR PARAMETER OF SIGTAB
  ON ENTRY, NOT USED
  ON RETURN,
    KERR = 0, NO ERROR DETECTED
    KERR = -1, ERROR DETECTED IN INPUT VALUES OF
      NO, JOP, OR OMGTAB
    KERR > 0, SIGCAL DETECTED AN ERROR WHILE CALCULATING
      WITH OMEGA = OMGTAB(KERR). IN THIS CASE, THE
      FIRST 23 PARAMETERS HAVE THE VALUES RETURNED
BY SIGCAL WHEN THE ERROR OCCURRED. IF KERR .GT. 1, THE FIRST KERR - 1 ENTRIES OF SVMAX AND SVMIN WERE CALCULATED WITHOUT ANY ERROR BEING DETECTED.
THE PURPOSE OF SDTAB IS TO REPEATEDLY CALL SIGDYN IN ITS "CALCULATE BOTH GO AND KO INTERNALLY" MODE WHILE VARYING THE VALUE OF THE FREQUENCY PARAMETER "OMEGA" SO AS TO TABULATE THE MAXIMUM AND MINIMUM SINGULAR VALUES OF THE "G" MATRIX WHICH SIGDYN Chooses TO CALCULATE BASED ON THE VALUE OF THE PARAMETER "ISOPTA(1)".

THESE COMMENTS ASSUME THAT THE READER IS FAMILIAR WITH THE PROGRAM PRELUDE DOCUMENTATION OF SIGDYN.

COMMENTS ARE INTERSPERSED THROUGHOUT THE CODE TO AID THE USER WHO WISHES TO WRITE A ROUTINE TO TABULATE SINGULAR VALUES FOR A SYSTEM WITH ONE OR BOTH OF THE GO AND KO MATRICES CALCULATED EXTERNALLY. SEE APPENDIX A (P. 22).

TWO OF THE 44 PARAMETERS TO SIGDYN ARE COMPLEX MATRICES, IDENTIFIED IN SIGDYN DOCUMENTATION AS "U" AND "V". THESE ARE USED WHEN SIGDYN IS CALLED WITH "ISOPTA(2)" SET EQUAL TO 3 TO RETURN LEFT AND RIGHT SINGULAR VECTORS OF "G". SINCE SDTAB CALLS SIGDYN WITH "ISOPTA(2)" SET TO 2, SDTAB DOES NOT NEED "U" AND "V". THE FIRST 42 PARAMETERS OF SDTAB, I.E., "NG" THROUGH "JERR", ARE IDENTICAL IN ORDER AND FUNCTION (AND IN NAME IN THIS DOCUMENT) WITH THE REMAINING 42 PARAMETERS TO SIGDYN WITH THE EXCEPTION THAT ONLY THE FIRST TWO ENTRIES OF IGOPTA AND IKOPTA ARE FUNCTIONAL; EVEN IF THE THIRD ENTRIES ARE PRESENT THEY ARE IGNORED AND SIGCAL IS DRIVEN IN ITS "COMPUTE GO AND KO INTERNALLY" MODE.

SDTAB SETS THE VALUE OF ISOPTA(3) TO 0 AFTER THE FIRST CALL TO SIGDYN SO THAT DETAIL PRINTOUT OCCURS ONLY WHEN SIGDYN IS CALLED WITH OMEGA = OMGTAB(1), AND THAT ONLY IF THE USER SET ISOPTA(3) .NE. 0 ON ENTRY TO SDTAB.
FIRST 42 PARAMETERS

ON ENTRY, THE USER MUST INITIALIZE PARAMETERS NG, LG, MG, AG, BG, CG, DG, DGNUL, ISOPTA(1), ISOPTA(2), FIRSTG, NK, LK, MK, AK, BK, CK, DK, DKNULL, IKOPTA(1), IKOPTA(2), FIRSTK, ISOPTA(1), ISOPTA(3), AND ISOPTA(4) IN ACCORDANCE WITH SIGDYN DOCUMENTATION.

ON RETURN,
IF KERR = -1, UNCHANGED
IF KERR = 0, AS RETURNED BY SIGDYN WITH
OMEGA = OMGTAB(NO) EXCEPT ISOPTA(2) HAS BEEN SET TO 2 AND ISOPTA(3) HAS BEEN SET TO 0
IF KERR > 0, AS RETURNED BY SIGDYN WITH
OMEGA = OMGTAB(KERR) EXCEPT ISOPTA(2) HAS BEEN SET TO 2 AND ISOPTA(3) HAS BEEN SET TO 0

JOP INTEGER, OPTION ON INPUT OMEGAS
ON ENTRY,
JOP = 0, USER HAS PRESET THE ENTIRE OMGTAB ARRAY
JOP = 1, USER HAS PRESET OMGTAB(1) AND OMGTAB(NO).
SDTAB WILL FILL IN INTERMEDIATE VALUES EQUALLY SPACED
JOP = 2, USER HAS PRESET OMGTAB(1) AND OMGTAB(NO) TO NONZERO VALUES OF THE SAME SIGN. SDTAB WILL FILL IN INTERMEDIATE VALUES SO THAT CONSECUTIVE RATIOS ARE THE SAME (EQUALLY SPACED ON A LOG SCALE).
ON RETURN, JOP = 0

NO INTEGER
ON ENTRY, NUMBER OF ENTRIES IN THE OMGTAB, SVMAX, AND SVMIN ARRAYS.
ON RETURN, UNCHANGED

OMGTAB REAL ARRAY OF LENGTH AT LEAST NO
ON ENTRY, PRESET AS DESCRIBED UNDER JOP
ON RETURN, FULLY SET ACCORDING TO JOP OPTION

SVMAX, SVMIN REAL ARRAYS OF LENGTH AT LEAST NO
ON ENTRY, NOT USED
ON RETURN, SVMAX(I) AND SVMIN(I) ARE SET TO, RESPECTIVELY, THE MAXIMUM AND MINIMUM SINGULAR VALUES OF THE G MATRIX CALCULATED BY SIGDYN WHEN OMEGA IS SET TO OMGTAB(I).

KERR INTEGER, ERROR PARAMETER OF SDTAB
ON ENTRY, NOT USED
ON RETURN,
KERR = 0, NO ERROR DETECTED
KERR = -1, ERROR DETECTED IN INPUT VALUES OF
NO, JOP, OR OMGTAB

KERR > 0, SIGDYN DETECTED AN ERROR WHILE CALCULATING
WITH OMEGA = OMGTAB(KERR). IN THIS CASE, THE
FIRST 38 PARAMETERS HAVE THE VALUES RETURNED
BY SIGDYN WHEN THE ERROR OCCURRED. IF
KERR .GT. 1, THE FIRST KERR - 1 ENTRIES OF
SVMAX AND SVMIN WERE CALCULATED WITHOUT ANY
ERROR BEING DETECTED.
SUBROUTINE TABGEN(NO,OMGTAB,JOP,KERR)

INTEGER + JOP, KERR, NO
REAL + OMGTAB(NO)

SUBROUTINE TABGEN GENERATES A TABLE OF NUMBERS SPACED WITH THE
USER'S CHOICE OF EQUAL INCREMENTS OR EQUAL CONSECUTIVE RATIOS.

PARAMETERS

NO INTEGER
ON ENTRY, NUMBER OF LOCATIONS IN OMGTAB.
ON RETURN, UNCHANGED.

OMGTAB REAL ARRAY OF AT LEAST NO LOCATIONS.
ON ENTRY, OMGTAB(1) AND OMGTAB(NO) MUST BE SET TO THE
FIRST AND LAST ENTRIES OF THE DESIRED TABLE.
ON NORMAL RETURN, THE REMAINDER OF OMGTAB IS FILLED IN
ACCORDING TO THE OPTION REQUESTED BY PARAMETER JOP.

JOP INTEGER
ON ENTRY,
IF JOP = 0, CHECK THAT NO IS POSITIVE, DO NOT ALTER
OMGTAB.
IF JOP = 1, FILL IN OMGTAB SO THAT CONSECUTIVE
DIFFERENCES ARE EQUAL.
IF JOP = 2, FILL IN OMGTAB SO THAT CONSECUTIVE
RATIOS ARE EQUAL.
ON NORMAL RETURN, JOP = 0.

KERR INTEGER
ON ENTRY, NOT USED.
ON RETURN,
KERR = 0, NORMAL RETURN, NO ERROR DETECTED.
KERR = -1, ERROR RETURN, OMGTAB AND JOP UNCHANGED.
POSSIBLE CAUSES: NO NONPOSITIVE; JOP NOT EQUAL TO
0, 1, OR 2; JOP = 2 BUT OMGTAB(1) AND OMGTAB(NO)
DO NOT HAVE THE SAME SIGN.
## Appendix C

### Symbolic Names of Global Entities

In this appendix the global names used in FREQ are listed. The user's application program that uses FREQ must not use any of these names for global entities (main program name, subroutine name, function name, common block name, etc.). We also tell where each of the FREQ global entities belongs in the software structure of figures 2 and 3.

The single-precision version of FREQ uses the global names:

<table>
<thead>
<tr>
<th>BALANC</th>
<th>CHECO</th>
<th>CSCAL</th>
<th>CUNITY</th>
<th>MULT</th>
<th>SCL1</th>
<th>SIGCAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CADD</td>
<td>CHEFA</td>
<td>CSOLVE</td>
<td>EQUIATE</td>
<td>ORACCH</td>
<td>SCNRM2</td>
<td>SIGDYN</td>
</tr>
<tr>
<td>CAXPY</td>
<td>CHESL</td>
<td>CSROT</td>
<td>FRDTIT</td>
<td>ORACES</td>
<td>SDTAB</td>
<td>SIGTAB</td>
</tr>
<tr>
<td>CDOTC</td>
<td>CL1NRM</td>
<td>CSSCAL</td>
<td>FREQNT</td>
<td>ORACIO</td>
<td>SFRMG</td>
<td>SROTG</td>
</tr>
<tr>
<td>CEQUAT</td>
<td>CMMULT</td>
<td>CSUBT</td>
<td>FREQNV</td>
<td>ORACPC</td>
<td>SHETR</td>
<td>SUBT</td>
</tr>
<tr>
<td>CGECO</td>
<td>CPRNT</td>
<td>CSWAP</td>
<td>ICAMAX</td>
<td>PRNT</td>
<td>SIGCA2</td>
<td></td>
</tr>
<tr>
<td>CGESL</td>
<td>CRADD</td>
<td>CTRANP</td>
<td>LNCNT</td>
<td>SCASUM</td>
<td>SIGCA3</td>
<td></td>
</tr>
</tbody>
</table>

The double-precision version of FREQ uses the global names:

<table>
<thead>
<tr>
<th>BALANC</th>
<th>CSUBT</th>
<th>DZASUM</th>
<th>LNCNT</th>
<th>SDTAB</th>
<th>TABGEN</th>
<th>ZHECO</th>
</tr>
</thead>
<tbody>
<tr>
<td>CADD</td>
<td>CTRANP</td>
<td>DZNRM2</td>
<td>MULT</td>
<td>SIGCA1</td>
<td>ZAXPY</td>
<td>ZHEFA</td>
</tr>
<tr>
<td>CEQUAT</td>
<td>CUNITY</td>
<td>EQUIATE</td>
<td>ORACCH</td>
<td>SIGCA2</td>
<td>ZDTC</td>
<td>ZHESL</td>
</tr>
<tr>
<td>CMMULT</td>
<td>DCABSI</td>
<td>FRDTIT</td>
<td>ORACES</td>
<td>SIGCA3</td>
<td>ZDROT</td>
<td>ZLRNM</td>
</tr>
<tr>
<td>CMULT</td>
<td>DCLI</td>
<td>FREQNT</td>
<td>ORACIO</td>
<td>SIGCAL</td>
<td>ZDSCAL</td>
<td>ZCAL</td>
</tr>
<tr>
<td>CPRNT</td>
<td>DFRMG</td>
<td>FREQNV</td>
<td>ORACPC</td>
<td>SIGDYN</td>
<td>ZGECO</td>
<td>ZSVDC</td>
</tr>
<tr>
<td>CRADD</td>
<td>DHETR</td>
<td>HQR</td>
<td>ORCERP</td>
<td>SIGTAB</td>
<td>ZGEFA</td>
<td>ZSWAP</td>
</tr>
<tr>
<td>CSOLVE</td>
<td>DROTG</td>
<td>IZAMAX</td>
<td>PRNT</td>
<td>SUBT</td>
<td>ZGESL</td>
<td></td>
</tr>
</tbody>
</table>

These global entities break down in the following manner. With reference to figure 2, the top three boxes show the initialization routines. They are FREQNT, FRDTIT, and FREQNV. The output routines are ORACES, LNCNT, ORCERP, PRNT, and CPRNT. The common blocks are/ORACIO/, /ORACPC/, and /ORACCH/.

Everything else falls in the category “Computational routines.” With reference to figure 3, the top five boxes show the user interface routines. They are SIGCAL, SIGDYN, SIGTAB, SDTAB, and TABGEN. The subordinate routines are SIGCA1, SIGCA2, SIGCA3, CSOLVE, CRADD, CEQUAT, CTRANP, CUNITY, CADD, CSUBT, CMULT, CMMULT, EQUIATE, SUBT, and MULT.

In the single-precision version of FREQ, Laub's frequency-response subroutines are CHECO, CHEFA, CHESL, CL1NRM, SCL1, SFRMG, and SHETR; whereas in the double-precision version, they are DFRMG, DHETR, ZHECO, ZHEFA, ZHESL, ZL1NRM, and DCL1.

The EISPACK subroutines are BALANC and HQR. In single precision, the LINPACK subroutines are CGECO, CGEFA, CGESL, CSVDC, CAXPY, CDOTC, CSCAL, CSROT, CSSCAL, CSWAP, ICAMAX, SCASUM, SCNRM2, and SROTG; whereas in double precision, they are ZGECO, ZGEFA, ZGESL, ZSVDC, DCABSI, DROTG, DZASUM, DZNRM2, ZDROT, IZAMAX, ZAXPY, ZDTC, ZDSCAL, ZSCAL, and ZSWAP.

The global names LNCNT and PRNT from the output group and EQUIATE, SUBT, and MULT from the subordinate routines group also occur in the software package ORACLS. The FREQ versions have the same function and (except that the fourth parameter of PRNT is now of type CHARACTER) the same calling sequences as their ORACLS counterparts. They have been modified to be compatible with FREQ output and error-processing protocols.
Appendix D

Example Program Listing

This appendix presents a listing of the program that gives rise to the example presented previously.

```
PROGRAM HOOPCA

C
C**** SPECIFY VARIABLES AND ARRAYS
C
C PARAMETER TO SIZE THE FREQUENCY TABLE
C
INTEGER
+ NMAX
PARAMETER (NOMAX=400)
C
DATA ARRAYS TO BUILD PLANT AND COMPENSATOR MATRICES AND
VARIABLES AND ARRAYS NEEDED FOR FREQ
C
CHARACTER*80
+ TITLE
COMPLEX
+ CDUM(780), GC(3,3), GO(3,3), KO(3,3)
INTEGER
+ I, IDUM(26), IEROPT, IGERRA(2), IKERRA(2), IKOPTA(2),
+ INFO, ISOPTA(4), IUL, J, JERR, JOP, JUL, KERR, LD, LG, LK, LF,
+ MD, MG, MK, MP, NCL, ND, ND1(2), NERR, NG, NIN, NK, NLP, NO,
+ NOUT, NP, NPLT, NVC(2), N102(2), N1212(2), N123(2), N1414(2),
+ N143(2), N2020(2), N203(2), N22(2), N2626(2), N263(2), N312(2),
+ N314(2), N326(2), N33(2)
LOGICAL
+ DDNUL, DGNUL, DKNUL, DPNUL, FIRSTD, FIRSTG, FIRSTK, FIRSTP
REAL
+ AD(14,14), AE(2,2), AG(26,26), AK(12,12), AP(12,12), BD(14,3),
+ BG(26,3), BK(12,3), BP(12,3), CD(3,14), CG(3,26), CK(3,12),
+ CP(3,12), DD, DG(1), DK(1), DP, D1(144), EIG(26), EVIK(12),
+ EVRG(26), EVRK(12), ES(3,3), FNDR(10,2), G(3,12), GI(12,3),
+ H(12,3), OMEGA, OMGTAB(NOMAX), RCOUNTG, RCOUNTK, RCOUNTS, RDUM(26),
+ RINV(3,3), SAVED(280), SAVEEG(832), SAVEK(216), SAVEP(216),
+ SIGMA(26), SVMAX(NOMAX,11), SVMIN(NOMAX,11), U(20,20), UI(2,2),
+ VB(20,3), VC(3,20)
C
C**** INITIALIZATION OF FILES AND COMMON BLOCKS IN FREQ
C
C OPEN FILE 'HCAEGD' FOR USE AS FREQ INPUT FILE AND FILE 'HCAEGO'
```
FOR USE AS FREQ OUTPUT AND ERROR MESSAGE FILE

OPEN(5,FILE='HCAEGD',STATUS='OLD')
OPEN(6,FILE='HCAEGO',STATUS='UNKNOWN')
REWIND 5
REWIND 6

SET FREQ PARAMETERS TO DEFAULT VALUES

CALL FREQNT

ENTER THOSE DEFAULT VALUES INTO USER VARIABLES

CALL FREQNV(-1,NIN,NOUT,NERR,NLP,NCL,IEROPT,TITLE)

IN THE USER VARIABLES, REPLACE THE DEFAULT VALUES BY USER-
SELECTED VALUES

NLP = 55
NCL = 80
TITLE = 'DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN '
1 // 'NASA TP-2560'

INSTRUCT FREQ TO USE THE NEW VALUES

CALL FREQNV(+1,NIN,NOUT,NERR,NLP,NCL,IEROPT,TITLE)

***** READ IN DATA MATRICES

THE CALL TO LNCNT TELLS THE FREQ OUTPUT LINE COUNTER HOW MANY
LINES THE USER IS PUTTING ON THE FREQ OUTPUT FILE

CALL LNCNT(12)
WRITE (6,100)
100 FORMAT(/1X,'DATA FROM NASA TP-2560 USED TO GENERATE LINEAR SYSTEM'
+ ', ' MATRICES FOR',/,
+ 6X,'THE 26TH-ORDER PLANT',/,
+ 6X,'THE 12TH-ORDER DESIGN MODEL',/,
+ 6X,'THE 14TH-ORDER ERROR (DELTA G) SYSTEM',/,
+ 6X,'THE 12TH-ORDER ATTITUDE FEEDBACK COMPENSATOR',/,
+ 6X,'"FNDR" CONTAINS THE FREQUENCIES (HZ) AND THE DAMPING RATIOS'
+ ',/6X,
+ ' "VB" CONTAINS MODE SLOPE INFORMATION FOR "B" AND "C" MATRICES'
+ ',/6X,"H" IS THE KALMAN FILTER GAINS MATRIX',/,
+ 6X,"GT" IS THE TRANSPOSE OF THE FEEDBACK GAINS MATRIX',/) CALL READ(5,FNDR,N102,RINV,N33,VB,N203,H,N123,GT,N123)

***** BUILD PLANT MATRICES AS IN THE APPENDIX OF NASA TP-2560

54
E3 IS THE 3 BY 3 IDENTITY MATRIX

CALL UNITY(E3,N33)

INITIALIZE AG

N2626(1) = 26
N2626(2) = 26
CALL NULL(AG,N2626)

INSERT IDENTITY IN AG (LOC. CIT., EQUATIONS (A3) AND (A4))

CALL MERGE(E3,N33,AG,N2626,1,4)

INSERT FLEXIBLE MODES IN AG
(LOC. CIT., EQUATIONS (A3), (A5), (A6))

AE(1,1) = 0.0E+00
AE(1,2) = 1.0E+00
N22(1) = 2
N22(2) = 2
DO 10 I = 1,10
AE(2,1) = -FNDR(I,1)**2
AE(2,2) = -2.0E+00*FNDR(I,1)*FNDR(I,2)
IUL = 2*I + 5
JUL = IUL
CALL MERGE(AE,N22,AG,N2626,IUL,JUL)

10 CONTINUE

CALL UNITY(E3,N33)

INITIALIZE BG

N263(1) = 26
N263(2) = 3
CALL NULL(BG,N263)

INSERT INVERSE OF INERTIA MATRIX (LOC. CIT., EQN (A7))

CALL MERGE(RINV,N33,BG,N263,4,1)

INSERT ACTUATOR EFFECTS ON FLEXIBLE MODES (IBID)

CALL MERGE(VB,N203,BG,N263,7,1)

INITIALIZE CG

N326(1) = 3
N326(2) = 26
CALL NULL(CG,N326)
C INSERT IDENTITY IN CG (LOC. CIT., EQN (A11))

CALL MERGE(E3,N33,CG,N326,1,1)

FORM VC (LOC. CIT., EQNS (A12), (A13), (A14))

UI(1,1) = 0.0E+00
UI(1,2) = 1.0E+00
UI(2,1) = 1.0E+00
UI(2,2) = 0.0E+00

N2020(1) = 20
N2020(2) = 20

CALL NULL(U,N2020)

DO 20 I = 1,10
IUL = 2*I-1
JUL = IUL

CALL MERGE(UI,N22,U,N2020,IUL,JUL)

20 CONTINUE

C**** BUILD COMPENSATOR MATRICES AS IN NASA TP-2560, SECTIONS 5 AND 6

C DESIGN MODEL IS THE FIRST 12 STATES OF THE PLANT

C COMPENSATOR USES ATTITUDE FEEDBACK

C COMPENSATOR GAINS FROM LOC. CIT., TABLE II

C GENERATE AP, BP, AND CP MATRICES

N1212(1) = 12
N1212(2) = 12
N312(1) = 3
N312(2) = 12

CALL EXTRACT(AG,N2626,AP,N1212,1,1)
CALL EXTRACT(BG,N263,BP,N123,1,1)
CALL EXTRACT(CG,N326,CP,N312,1,1)

C GENERATE G FROM THE G TRANSPOSED WHICH WAS INPUT

CALL TRANP(GT,N123,G,N312)

C BUILD AK AS AP-BP*G-H*CP (FROM LOC. CIT., P5, LAST EQUATION)

CALL MULT(BP,N123,G,N312,D1,ND1)
CALL SUBT(AP,N1212,D1,ND1,AK,ND1)
CALL MULT(H,N123,CP,N312,D1,ND1)
CALL SUBT(AK,N1212,D1,ND1,AK,N1212)
BK AND CK ARE H AND G, RESPECTIVELY (IBID.)

CALL EQUATE(H,N123,BK,N123)
CALL EQUATE(G,N312,CK,N312)

DETERMINE THE MATRICES FOR THE DELTA G OF LOC. CIT., FIGURE 17

N1414(1) = 14
N1414(2) = 14
N143(1) = 14
N143(2) = 3
N314(1) = 3
N314(2) = 14
CALL EXTRACT(AG,N2626,AD,N1414,13,13)
CALL EXTRACT(BG,N263,BD,N143,13,1)
CALL EXTRACT(CG,N326,CD,N314,1,13)

**** FIRST WE WILL DO THE OPEN-LOOP SINGULAR VALUE CALCULATIONS
FOR SEVERAL OF THESE SYSTEMS. HERE WE SET PARAMETERS WHICH
WILL BE THE SAME FOR SEVERAL CALLS TO SIGTAB.

BALANCE SYSTEM MATRIX AND CALCULATE EIGENVALUES
IGOPTA(1) = 1

ESTIMATE RECIPROCAL CONDITION NUMBER WHEN CALCULATING LOOP-GAIN
TRANSFER MATRIX
IGOPTA(2) = -1

CALCULATE LOOP-GAIN TRANSFER MATRIX
ISOPTA(1) = 1

CALCULATE SINGULAR VALUES AND NO SINGULAR VECTORS
(SINCE WE WILL CALL SIGTAB, THIS CHOICE WOULD BE FORCED
ON US ANYWAY)
ISOPTA(2) = 2

USE A PROBLEM-DEPENDENT SUBROUTINE TO SET NO AND OMGTAB.
IN THIS CASE, WE MERGE THE GRID USED FOR THE PLOTS IN TP-2560 WITH
301 POINTS DISTRIBUTED OVER THE 3 DECADES WE ARE USING (FROM .01 HZ
TO 10 HZ) SO THAT RATIOS OF CONSECUTIVE GRID VALUES ARE ALL THE
SAME. THESE POINTS WILL BE EQUALLY SPACED ALONG THE LOG AXIS USED
FOR PLOTTING SINGULAR VALUES AS A FUNCTION OF FREQUENCY.
CALL OMGEN(NOMAX,NO,OMGTAB)

C
C TELL SIGTAB THAT THE OMGTAB ARRAY IS PRESET
C
JOP = 0
C
C***** NOW SET THE PARAMETERS NEEDED BY SIGTAB TO CALCULATE THE OPEN-
C LOOP FREQUENCY RESPONSE OF THE PLANT
C
          NG = 26
          LG = 3
          MG = 3
          DGNUL = .TRUE.
          FIRSTG = .TRUE.
          ISOPTA(3) = 1
          CALL LNCNT(4)
          WRITE (6,130)
          FORMAT(//,lX,'NEXT CALCULATE THE OPEN-LOOP RESPONSE OF THE ',
              + 'FULL 26TH-ORDER PLANT',/)
          CALL SIGTAB(NG,LG,MG,AG,BG,CG,DG,DGNUL,IGOPTA,FIRSTG,SAVEG,
            1 EVRG, EVIG, IGERRA, RCONDG,
            2 OMEGA, RCONDS, ISOPTA, GC, SIGMA, INFO,
            3 RDUM, CDUM, IDUM, JERR,
            4 JOP, NO,OMGTAB, SVMAX(1,1), SVMIN(1,1), KERR)
C
C***** NOW SET THE PARAMETERS NEEDED BY SIGTAB TO CALCULATE THE OPEN-
C LOOP FREQUENCY RESPONSE OF THE DESIGN MODEL
C
          NP = 12
          LP = 3
          MP = 3
          DPNUL = .TRUE.
          FIRSTP = .TRUE.
          ISOPTA(3) = 1
          CALL LNCNT(4)
          WRITE (6,110)
          FORMAT(//,lX,'NEXT CALCULATE THE OPEN-LOOP RESPONSE OF THE ',
              + '12TH-ORDER DESIGN MODEL',/)
          CALL SIGTAB(NP,LP,MP,AP,BP,CP,DP,DPNUL,IGOPTA,FIRSTP,SAVEP,
            1 EVRG, EVIG, IGERRA, RCONDG,
            2 OMEGA, RCONDS, ISOPTA, GC, SIGMA, INFO,
            3 RDUM, CDUM, IDUM, JERR,
            4 JOP, NO,OMGTAB, SVMAX(1,2), SVMIN(1,2), KERR)
C
C***** NOW SET THE PARAMETERS NEEDED BY SIGTAB TO CALCULATE THE OPEN-
C LOOP FREQUENCY RESPONSE OF THE "DELTA G" (ADDITIVE ERROR) SYSTEM
C
C LINEAR SYSTEM SIZING PARAMETERS

58
ND = 14
LD = 3
MD = 3
DNNUL = .TRUE.
FIRSTD = .TRUE.
ISOPTA(3) = 1
CALL LNCNT(4)
WRITE (6,120)
120 FORMAT(//,1X,'NEXT CALCULATE THE SINGULAR VALUE PLOTS OF THE', + ' 14TH-ORDER ERROR SYSTEM',/)
CALL SIGTAB(ND,LD,MD,AD,BD,CD,DD,DNNUL,ISOPTA,FIRSTD,SAVED,1
   EVRG,EVIG,IGERRA,RCONDG,
2   OMEGA,RCONDS,ISOPTA,GC,SIGMA,INFO,
3   RDUM,CDUM,IDUM,JERR,
4   JOP,NO,OMGTAB,SVMAX(1,3),SVMIN(1,3),KERR)

C**** NOW SET THE PARAMETERS NEEDED BY SIGTAB TO CALCULATE THE OPEN-
C LOOP FREQUENCY RESPONSE OF THE COMPENSATOR
C
NK = 12
LK = 3
MK = 3
DKNUL = .TRUE.
IKOPTA(1) = 1
IKOPTA(2) = -1
FIRSTK = .TRUE.
ISOPTA(3) = 1
CALL LNCNT(4)
WRITE (6,140)
140 FORMAT(//,1X,'NEXT CALCULATE THE OPEN-LOOP RESPONSE OF THE ', + 'COMPENSATOR',/)
CALL SIGTAB(NK,LK,MK,AK,BK,CK,DK,DKNUL,IKOPTA,FIRSTK,SAVEK,1
   EVRK,EVIK,IKERR,KCONDK,
2   OMEGA,RCONDS,ISOPTA,GC,SIGMA,INFO,
3   RDUM,CDUM,IDUM,JERR,
4   JOP,NO,OMGTAB,SVMAX(1,4),SVMIN(1,4),KERR)

C**** SET THE PARAMETERS (NOT ALREADY SET) NEEDED BY SDTAB TO ANALYZE
C THE DESIGN MODEL WITH COMPENSATOR.
C
C TELL FREQ TO USE GO*KO FOR LOOP TRANSFER MATRIX
C
ISOPTA(4) = 0
C TURN ON PRINT (WAS TURNED OFF BY PREVIOUS CALL TO SIGTAB OR SDTAB)
ISOPTA(3) = 1
C TABULATE THE MAXIMUM AND MINIMUM SINGULAR VALUES OF THE OPEN- AND
C CLOSED-LOOP DESIGN MODEL WITH COMPENSATOR. NOTE THAT EACH CASE
C HAS ITS OWN COLUMN IN OUTPUT ARRAYS. ONLY THE OPEN-LOOP
C CALCULATION WILL GENERATE PRINTOUT.
DO 30 I = 1, 2
C
C FIRST TIME THROUGH THE DO LOOP, SINGULAR VALUE ANALYSIS WILL BE
C APPLIED TO THE LOOP-GAIN TRANSFER MATRIX; SECOND TIME, THE
C FEEDBACK TRANSFER MATRIX WILL BE USED.
C
ISOPTA(1) = I
IF (I .EQ. 1) CALL LNCNT(5)
IF (I .EQ. 1) WRITE (6,150)
150 FORMAT(//, 1X, "NEXT CALCULATE THE OPEN-LOOP RESPONSE OF THE ",
+ 3X, "CASCADE OF COMPENSATOR AND 12TH-ORDER DESIGN MODEL",//)
CALL SDTAB(NP, LP, MP, AP, BP, CP, DP, DPNUL, IGOPTA, FIRSTP, SAVEP,
1 EVRG, EVIG, GO, IGERRA, RCONDG,
2 NK, LK, MK, AK, BK, CK, DK, DKNUL, IKOPTA, FIRSTK, SAVEK,
3 EVRK, EVIK, KO, IKERRA, RCONDK,
4 OMEGA, RCONDS, ISOPTA, GC, SIGMA, INFO,
5 RDUM, CDUM, IDUM, JERR,
6 JOP, NO, OMTAB, SVMAX(1, 4+1), SVMIN(1, 4+1), KERR)
C**** SET THE PARAMETERS (NOT ALREADY SET) NEEDED BY SDTAB TO ANALYZE
C THE PLANT WITH COMPENSATOR. BY EXECUTING A LOOP WHICH ALTERS THE
C VALUE OF ISOPTA(1), ANALYSIS WILL BE PERFORMED USING THE
C (1) LOOP-GAIN TRANSFER MATRIX
C (2) FEEDBACK TRANSFER MATRIX
C (3) SENSITIVITY MATRIX
C (4) INVERSE RETURN DIFFERENCE MATRIX
C (5) RETURN DIFFERENCE MATRIX
C OF THE TRANSFER FUNCTION GO*KO WHERE GO AND KO ARE, RESPECTIVELY,
C THE PLANT AND COMPENSATOR TRANSFER MATRICES. PARAMETERS WHICH
C CAN BE ALTERED BY CALLING SDTAB WILL BE SET INSIDE THE LOOP,
C OTHERS BEFORE ENTERING THE LOOP.
C
START LOOP
C
THE NEXT LINE FORCES THE SUBSEQUENT WRITE TO THE FREQ OUTPUT
UNIT TO START AT THE TOP OF A NEW PAGE
C
CALL LNCNT(-1)
CALL LNCNT(6)
C
THE FOLLOWING PRINTOUT INFORMS THE USER ABOUT THE LACK OF DETAILED
OUTPUT FOR THE SECOND TRIP THROUGH THE PREVIOUS "DO 30" LOOP AND
THE 5 TRIPS THROUGH THE FOLLOWING "DO 40" LOOP
C
WRITE (6, 160)
160 FORMAT(//, 1X,
+ 'PRINTOUT HAS BEEN TURNED OFF FOR THE NEXT 6 ANALYSES.',//,
 THESE ARE THE FEEDBACK TRANSFER MATRIX FOR THE DESIGN 'MODEL WITH' 'COMPENSATOR; ' AND ALL 5 ANALYSES OFFERED BY "FREQ" USING THE FULL' '26TH-ORDER PLANT WITH THE COMPENSATOR')

DO 40 I = 1,5

SELECT MATRIX TO ANALYZE

ISOPTA(I) = I

TABULATE THE MAXIMUM AND MINIMUM SINGULAR VALUES OF THE SELECTED MATRIX AS A FUNCTION OF FREQUENCY. NOTE THAT EACH CASE HAS ITS OWN COLUMN IN OUTPUT ARRAYS.

CALL SDTAB(NG, LG, MG, AG, BG, CG, DG, DGNUL, IGOTA, FIRSTG, SAVEG,
1 EVRG, EVIG, GO, IGERRA, RCONDG,
2 NK, LK, MK, AK, BK, CK, DK, DKNUL, IKOPTA, FIRSTK, SAVEK,
3 EVRK, EVIK, KO, IKERRA, RCONDK,
4 OMEGA, RCONDS, ISOPTA, GC, SIGMA, INFO,
5 RDUM, CDUM, IDUM, JERR,
6 JOP, NO, OMGTAB, SVMAX(1, I+6), SVMIN(1, I+6), KERR)

CONTINUE

WRITE OUT A DATA FILE CONTAINING THE SINGULAR VALUE INFORMATION

OPEN(10, FILE='HCAEGS', STATUS='UNKNOWN')
REWIND 10
NPLOT = 11
WRITE (10, '(2I20)') NO, NPLOT
DO 50 I = 1, NO
WRITE (10, '(4E20.14)') OMGTAB(I), (SVMAX(I, J), SVMIN(I, J), J = 1, NPLOT)
CONTINUE
END
SUBROUTINE OMGEN(NOMAX, NO, OMGTAB)
  INTEGER
    + NO, NOMAX
  REAL
    + OMGTAB(*)
  INTEGER
    + NOL, NXTRA
  PARAMETER (NOL = 70, NXTRA=301)

C**** PROBLEM-DEPENDENT SUBROUTINE TO GENERATE THE ARRAY OF FREQUENCY
CVALUES AT WHICH THE ANALYSES ARE TO BE PERFORMED.
CTHIS IS A MERGING OF THE GRID USED FOR THE PLOTS IN TP-2560
CWITH A 301-POINT GRID GEOMETRICALLY SPACED OVER THE SAME
C3 DECADE INTERVAL FROM .01 Hz TO 10 Hz.
C
CHARACTER*80
  + MSG
  INTEGER
    + I, JOP, KERR
  REAL
    + OM(NOL)
  DATA OM/.01,.03,.05,.07,.082,.09,.1,.15,.22,.27,.3,.4,.5,.6,.7,
1 .75,.8,.9,1.,
1 1.1,1.2,1.3,1.35,1.4,1.5,1.6,1.7,1.8,1.9,2.,2.5,3.,3.1,3.18,
1 3.2,3.3,3.4,3.6,4.,4.3,4.5,4.7,5.,5.1,5.3,5.4,5.5,5.6,
1 5.7,5.8,5.9,6.,6.4,6.6,6.7,6.8,6.9,7.,7.2,7.3,7.4,
1 7.5,7.7,7.9,8.,8.6,8.78,8.9,9.,10./

CIF THERE IS ENOUGH ROOM IN THE OMGTAB ARRAY
CIF (NOMAX .GE. NOL + NXTRA) THEN
C
CSET NO AND ENTER THE FREQUENCY VALUES INTO OMGTAB
C
NO = NOL + NXTRA
DO 10 I = 1, NO
  OMGTAB(I) = OM(I)
10 CONTINUE
OMGTAB(NOL+1) = OM(1)
OMGTAB(NO) = OM(NOL)
JOP = 2
CALL TABGEN(NXTRA, OMGTAB(NOL+1), JOP, KERR)
IF (KERR .NE. 0) THEN
  WRITE (MSG,40) KERR
40 FORMAT('IN OMGEN, TABGEN KERR = ',I20)
  CALL ORCERP(3,MSG)
  STOP 'OMGEN FAILURE AT TABGEN'
END IF
CALL SRTNOD(NO,OMGTAB)
ELSE
  IF OMGTAB IS TOO SMALL, WRITE ERROR MESSAGE AND STOP
  WRITE (MSG,20) NOL
  FORMAT('OMGEN CANNOT PUT ',I10,' ENTRIES IN OMGTAB')
  CALL ORCERP(1,MSG)
  WRITE (MSG,30) NOMAX
  FORMAT('ONLY ',I10,' SPACES AVAILABLE')
  CALL ORCERP(2,MSG)
  STOP 'OMGEN FAILURE'
END IF
RETURN
END
SUBROUTINE SRTNOD(N, A)
INTEGER + N
REAL + A(*)
INTEGER + I, J, K
REAL + T
C
C**** SORT A(1),... ,A(N) INTO INCREASING ORDER
C IF THERE ARE DUPLICATES, ELIMINATE THEM AND REDUCE N ACCORDINGLY
C
IF (N .LE. 1) RETURN
K = 2
10 CONTINUE
   IF (K .GT. N) RETURN
   T = A(K)
   DO 20 J = 1, K - 1
      IF (T .LE. A(J)) GO TO 30
   20 CONTINUE
   K = K + 1
   GO TO 10
30 CONTINUE
   IF (T .LT. A(J)) GO TO 40
   A(K) = A(N)
   N = N - 1
   GO TO 10
40 CONTINUE
   DO 50 I = K - 1, J - 1
      A(I+1) = A(I)
   50 CONTINUE
   A(J) = T
   GO TO 10
END
SUBROUTINE READ(I, A, NA, B, NB, C, NC, D, ND, E, NE)
INTEGER
+ I, NA(2), NB(2), NC(2), ND(2), NE(2)
REAL
+ A(*), B(*), C(*), D(*), E(*)
COMMON /ORACIO/ NIN, NOUT, NERR
INTEGER
+ NERR, NIN, NOUT
SAVE /ORACIO/
CHARACTER*4
+ LAB
INTEGER
+ NZ(2)
IF (I .LT. 1) GO TO 999
READ(NIN,100,END=10000) LAB, NZ(1), NZ(2)
CALL READ1(A, NA, NZ, LAB)
IF (I .EQ. 1) GO TO 999
READ(NIN,100,END=10000) LAB, NZ(1), NZ(2)
CALL READ1(B, NB, NZ, LAB)
IF (I .EQ. 2) GO TO 999
READ(NIN,100,END=10000) LAB, NZ(1), NZ(2)
CALL READ1(C, NC, NZ, LAB)
IF (I .EQ. 3) GO TO 999
READ(NIN,100,END=10000) LAB, NZ(1), NZ(2)
CALL READ1(D, ND, NZ, LAB)
IF (I .EQ. 4) GO TO 999
READ(NIN,100,END=10000) LAB, NZ(1), NZ(2)
CALL READ1(E, NE, NZ, LAB)
100 FORMAT(BZ,A4,4X,214)
999 RETURN
10000 CALL ORCERP(3,'END OF FILE IN READ')
END
SUBROUTINE TRANP(A,NA,B,NB)
  INTEGER + NA(2), NB(2)
  REAL + A(*), B(*)
  CHARACTER*42 + MSG
  INTEGER + I, IJ, IR, J, L, NC, NR
  NR=NA(1)
  NC=NA(2)
  L=NR*NC
  IF( NR .LT. 1 .OR. NC .LT. 1 ) GO TO 999
  NB(1)=NC
  NB(2)=NR
  L=NR*NC
  IR=0
  DO 300 I=1,NR
  IJ=I-NR
  DO 300 J=1,NC
  IJ=IJ+NR
  IR=IR+1
  300 B(IR)=A(IJ)
  RETURN
999 CONTINUE
  WRITE (MSG,50) NA
  CALL ORCERP(3,MSG)
50 FORMAT ('DIMENSION ERROR IN TRANP  NA=',2(1X,I5))
  RETURN
END
SUBROUTINE UNITY(A, NA)
INTEGER
+ NA(2)
REAL
+ A(*)
CHARACTER*42
+ MSG
INTEGER
+ I, IT, J, L, NAX
IF(NA(1).NE.NA(2) .OR. NA(1).LT.1) GO TO 999
L=NA(1)*NA(2)
DO 100 IT=1,L
100 A(IT)=0.0E+00
J = - NA(1)
NAX = NA(1)
DO 300 I=1,NAX
J=NAX +J+1
300 A(J)=1.0E+00
RETURN
999 CONTINUE
WRITE (MSG,50) NA
CALL ORCERP(3,MSG)
50 FORMAT ('DIMENSION ERROR IN UNITY NA=',2(1X,I5))
END
SUBROUTINE NULL(A,NA)
INTEGER
+ NA(2)
REAL
+ A(*)
CHARACTER*42
+ MSG
INTEGER
+ I, N
IF( NA(1) .LT. 1 .OR. NA(2) .LT. 1 ) GO TO 999
N=NA(1)*NA(2)
DO 10 I=1,N
10 A(I) = 0.0E+00
RETURN
999 CONTINUE
WRITE (MSG,50) NA
CALL ORCERP(3,MSG)
50 FORMAT ('DIMENSION ERROR IN NULL NA=',2(I5))
RETURN
END
SUBROUTINE READ1 (A, NA, NZ, NAM)
CHARACTER*(*)
+ NAM
INTEGER
+ NA(2), NZ(2)
REAL
+ A(*)
COMMON /ORACIO/ NIN, NOUT, NERR
INTEGER
+ NERR, NIN, NOUT
SAVE /ORACIO/
CHARACTER*50
+ MSG
INTEGER
+ I, J, NC, NLST, NR
IF (NZ(1).EQ.0) GO TO 410
NR=NZ(1)
NC=NZ(2)
NLST=NR*NC
IF (NLST .LT. 1 .OR. NR .LT. 1 ) GO TO 999
DO 400 I = 1, NR
READ (NIN,101,END=998) (A(J), J = I,NLST,NR)
400 CONTINUE
NA(1)=NR
NA(2)=NC
410 CALL PRNT (A,NA,NAM,1)
101 FORMAT(BZ,6E10.2)
RETURN
999 CONTINUE
WRITE (MSG,916) NAM,NR,NC
CALL ORCERP(3,MSG)
916 FORMAT('ERROR IN READ1 MATRIX ',A4,' HAS NA=',2(I5))
RETURN
998 CONTINUE
WRITE (MSG,917) NAM
CALL ORCERP(3,MSG)
917 FORMAT('ERROR IN READ1, END OF FILE READING MATRIX ',A4)
RETURN
END
SUBROUTINE EXTRCT(A, NA, B, NB, IUL, JUL)
INTEGER
+ IUL, JUL, NA(2), NB(2)
REAL
+ A(*), B(*)

THIS SUBROUTINE EXTRACTS A SUBMATRIX FROM MATRIX A AND PLACES IT IN B. DATA MAY BE STORED IN A IN EITHER PACKED (ORACLCS) FORMAT OR IN FORTRAN FORMAT. DATA ARE PLACED IN B IN PACKED (ORACLCS) FORMAT.

INPUT PARAMETERS (RETURNED UNCHANGED)
A - MATRIX WITH DATA STORED EITHER PACKED BY COLUMNS (ORACLCS FORMAT) OR AS FORTRAN NATURALLY STORES IT.
NA - INTEGER ARRAY OF 2 ENTRIES. IF A IS STORED IN ORACLCS FORMAT, NA(1) AND NA(2) MUST BE SET TO THE ACTUAL ROW AND COLUMN COUNT OF A. IF A IS IN FORTRAN FORMAT, NA(1) MUST BE SET TO THE ROW DIMENSION OF A AS DECLARED IN THE DIMENSION STATEMENT IN THE CALLING PROGRAM. NA(2) IS USED ONLY IN ERROR CHECKING. IT SHOULD BE SET TO THE NUMBER OF COLUMNS OF DATA WHICH ARE ACTUALLY IN A.
NB - 2 ENTRY INTEGER ARRAY. NB(1) AND NB(2) MUST BE SET TO THE ACTUAL ROW AND COLUMN COUNTS OF MATRIX B. THIS DEFINES THE SIZE OF THE SUBMATRIX TO BE MOVED.
IUL, JUL - INTEGERS SET TO THE ROW AND COLUMN INDICES OF THE UPPER LEFT CORNER OF THE SUBMATRIX OF A TO BE MOVED.

OUTPUT PARAMETERS
B - MATRIX IN WHICH THE DESIGNATED SUBMATRIX OF A IS STORED IN ORACLCS FORMAT.

NOTES -
(A) EXTRCT CAN BE USED TO PACK A FORTRAN ARRAY FOR USE BY ORACLCS BY SETTING IUL = JUL = 1, AND ENTERING IN NB THE NUMBER OF ROWS AND COLUMNS OF ACTUAL DATA IN A TO BE PACKED.
(B) THE ARRAYS A AND B CAN OCCUPY NONOVERLAPPING AREAS OF STORAGE OR CAN BE THE SAME. IF EXTRCT IS USED ON ARRAYS A AND B WHICH OVERLAP WITH OFFSET STARTING POINTS, RESULTS MAY BE WRONG.

CHARACTER*42
+ MSG
INTEGER
+ I, IA, J, MA, MB, NA1, NA2, NB1, NB2

NA1 = NA(1)
NA2 = NA(2)
NB1 = NB(1)
NB2 = NB(2)
IF (IUL .LT. 1) GO TO 100
IF (NB1 .LT. 1) GO TO 100
IF (NA1 .LT. IUL + NB1 - 1) GO TO 100
IF (JUL .LT. 1) GO TO 100
IF (NB2 .LT. 1) GO TO 100
IF (NA2 .LT. JUL + NB2 - 1) GO TO 100
MA=(JUL-1)*NA1+IUL
MB=1
IA=NA1-NB1
DO 20 I=1,NB2
DO 10 J=1,NB1
B(MB)=A(MA)
MB=MB+1
MA=MA+1
10 CONTINUE
MA=MA+IA
20 CONTINUE
RETURN
100 CONTINUE
CALL ORCERP(1,
+ 'DIMENSION AND/OR INDEXING ERROR IN SUBROUTINE EXTRCT')
WRITE(MSG,110) NA
110 FORMAT(3X,5HNA = ,2I20)
CALL ORCERP(0,MSG)
WRITE(MSG,120) NB
120 FORMAT(3X,5HNB = ,2I20)
CALL ORCERP(0,MSG)
WRITE (MSG,130) IUL,JUL
130 FORMAT(3X,6HIUL = ,I20,8H, JUL = ,I20)
CALL ORCERP(2,MSG)
RETURN
END
SUBROUTINE MERGE(B,NB,A,NA,IUL,JUL)
    INTEGER
    + IUL, JUL, NA(2), NB(2)
    REAL
    + A(*), B(*)

THIS SUBROUTINE WRITES MATRIX B OVER A SUBMATRIX OF MATRIX A.
DATA MAY BE STORED IN A IN EITHER PACKED (ORACL) FORMAT OR
IN FORTRAN FORMAT. DATA IN B MUST BE IN PACKED (ORACL) FORMAT

INPUT PARAMETERS (RETURNED UNCHANGED)

B - MATRIX WITH DATA STORED PACKED BY COLUMNS (ORACL FORMAT).
NB - INTEGER ARRAY OF 2 ENTRIES. NB(1) AND NB(2) MUST BE SET
    TO THE ACTUAL ROW AND COLUMN COUNT OF B.
NA - INTEGER ARRAY OF 2 ENTRIES. IF A IS IN ORACL
    FORMAT, NA(1) AND NA(2) MUST BE SET TO THE ACTUAL ROW AND
    COLUMN COUNT OF A. IF A IS IN FORTRAN FORMAT, NA(1)
    AND NA(2) MUST BE SET TO THE ROW AND COLUMN DIMENSIONS OF A
    AS DECLARED IN THE DIMENSION STATEMENT IN THE CALLING PROGRAM.
IUL, JUL - INTEGERS SET TO THE ROW AND COLUMN INDICES OF THE
    UPPER LEFT CORNER OF THE SUBMATRIX OF A TO BE OVERWRITTEN;
    I.E., THE MATRIX B WILL BE MOVED ONTO THE SUBMATRIX OF A
    WITH UPPER LEFT CORNER A(IUL,JUL).

OUTPUT PARAMETER

A - MATRIX IN WHICH THE DESIGNATED SUBMATRIX WILL BE REPLACED BY
    THE CONTENTS OF MATRIX B. OTHER ELEMENTS OF A ARE NOT
    AFFECTED.

NOTES -
(A) MERGE CAN BE USED TO UNPACK AN ORACL ARRAY FOR USE BY
    FORTRAN BY SETTING IUL = JUL = 1.
(B) THE ARRAYS A AND B CAN OCCUPY NONOVERLAPPING AREAS OF
    STORAGE OR CAN BE THE SAME. IF MERGE IS USED ON ARRAYS A AND
    B WHICH OVERLAP WITH OFFSET STARTING POINTS, RESULTS MAY BE
    WRONG.

CHARACTER=60
    + MSG
    INTEGER
    + I, IA, J, NA, MB, NA1, NA2, NB1, NB2
    NA1=NA(1)
    NA2=NA(2)
    NB1=NB(1)
    NB2=NB(2)
IF (IUL .LT. 1) GO TO 100
IF (NB1 .LT. 1) GO TO 100
IF (NA1 .LT. IUL+NB1-1) GO TO 100
IF (JUL .LT. 1) GO TO 100
IF (NB2 .LT. 1) GO TO 100
IF (NA2 .LT. JUL+NB2-1) GO TO 100
MA=(JUL+NB2-2)*NA1+IUL+NB1-1
MB=NB1*NB2
IA=NA1-NB1
DO 20 I=1,NB2
   DO 10 J=1,NB1
      A(MA)=B(MB)
      MB=MB-1
      MA=MA-1
   10 CONTINUE
MA=MA-IA
20 CONTINUE
RETURN
100 CONTINUE
   CALL LNCNT(6)
   CALL ORCERP(1,
   + 'DIMENSION AND/OR INDEXING ERROR IN SUBROUTINE MERGE')
   WRITE(MSG,110) NA
110 FORMAT(3X,5HNA = ,2I20)
   CALL ORCERP(0,MSG)
   WRITE(MSG,120) NB
120 FORMAT(3X,5HNB = ,2I20)
   CALL ORCERP(0,MSG)
   WRITE (MSG,130) IUL,JUL
130 FORMAT(3X,6HIUL = ,120,8H, JUL = ,120)
   CALL ORCERP(2,MSG)
RETURN
END
Appendix E

Output From Example Computation

The following pages give the output of the example program as described in the section "Example" and listed in appendix D.

The format shown was achieved by setting the page length parameter (NLP) to 55, the line length parameter (NCL) to 80, and the page title parameter (TITLE) to

'DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560'

Column 1 of the output file contains carriage control information ('1' to indicate the first line of a new page, ' ' to print the line after a single-line space advance) and is not printed here; instead, the effect of using the carriage control information is shown.
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

DATA FROM NASA TP-2560 USED TO GENERATE LINEAR SYSTEM MATRICES FOR
THE 26TH-ORDER PLANT
THE 12TH-ORDER DESIGN MODEL
THE 14TH-ORDER ERROR (DELTA G) SYSTEM
THE 12TH-ORDER ATTITUDE FEEDBACK COMPENSATOR

"FNDR" CONTAINS THE FREQUENCIES (HZ) AND THE DAMPING RATIOS
"VB" CONTAINS MODE SLOPE INFORMATION FOR "B" AND "C" MATRICES
"H" IS THE KALMAN FILTER GAINS MATRIX
"GT" IS THE TRANSPOSE OF THE FEEDBACK GAINS MATRIX

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75
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

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GT MATRIX 12 ROWS 3 COLUMNS

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NEXT CALCULATE THE OPEN-LOOP RESPONSE OF THE FULL 26TH-ORDER PLANT
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

PROGRAM TO COMPUTE MATRIX G EVALUATED AT

\[ S = \sqrt{-1} \times OMEGA, \ OMEGA = .1000000E-01 \]

\[ \text{GO = CG( (SI - AG)INVERSE )BG + DG} \]

**CG MATRIX**

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DG IS A NULL MATRIX

G = G0, LOOP-GAIN TRANSFER MATRIX

SINGULAR VALUES BUT NO SINGULAR VECTORS OF G ARE COMPUTED

SYSTEM MATRIX IS INPUT AG

RCOND = 6.1791092E-06
RCOND = ESTIMATE OF RECIPROCAL OF CONDITION NUMBER FOR
H = SQRT(-1)*OMEGA*I - SYSTEM MATRIX
DEMONSTRATION OF FREq, BASED ON EXAMPLE IN NASA TP-2560

EIGENVALUES, (REAL PART, IMAG. PART), OF SYSTEM MATRIX

\[
\begin{pmatrix}
-6.840000E-02, & 6.839658E+00 \\
-6.840000E-02, & 6.839658E+00 \\
-7.400000E-02, & 7.399630E+00 \\
-7.400000E-02, & 7.399630E+00 \\
-8.780000E-02, & 8.779561E+00 \\
-8.780000E-02, & 8.779561E+00 \\
-7.500000E-03, & 7.499625E-01 \\
-7.500000E-03, & 7.499625E-01 \\
-1.350000E-02, & 1.349932E+00 \\
-1.350000E-02, & 1.349932E+00 \\
-1.700000E-02, & 1.699915E+00 \\
-1.700000E-02, & 1.699915E+00 \\
-3.180000E-02, & 3.179841E+00 \\
-3.180000E-02, & 3.179841E+00 \\
-4.530000E-02, & 4.529773E+00 \\
-4.530000E-02, & 4.529773E+00 \\
-5.590000E-02, & 5.589720E+00 \\
-5.590000E-02, & 5.589720E+00 \\
-5.780000E-02, & 5.779711E+00 \\
-5.780000E-02, & 5.779711E+00 \\
.000000E+00, & .000000E+00 \\
.000000E+00, & .000000E+00 \\
.000000E+00, & .000000E+00 \\
.000000E+00, & .000000E+00 \\
.000000E+00, & .000000E+00 \\
.000000E+00, & .000000E+00 \\
\end{pmatrix}
\]

G COMPLEX MATRIX 3 ROWS 3 COLUMNS

\[
\begin{pmatrix}
-2.3651E-03, & -6.6587E-10 \\
.0000E+00, & .0000E+00 \\
1.2817E-07, & 6.8422E-11 \\
\end{pmatrix}
\]
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

FREQ PROGRAM TO COMPUTE MATRIX G EVALUATED AT
S = SQRT(-1)XOMEGA, OMEGA = .1000000E-01
GO = CG((SI - AG)INVERSE)BG + DG

<table>
<thead>
<tr>
<th>CG MATRIX</th>
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<th>12 COLUMNS</th>
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<td>.0000E+00</td>
</tr>
<tr>
<td>6.3000E-05</td>
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<tr>
<td>.0000E+00</td>
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<td>2.3700E-07</td>
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<td>.0000E+00</td>
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DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

DG IS A NULL MATRIX

G = Go, LOOP-GAIN TRANSFER MATRIX

SINGULAR VALUES BUT NO SINGULAR VECTORS OF G ARE COMPUTED

SYSTEM MATRIX IS INPUT AG

RCOND = 3.4482494E-05
RCOND = ESTIMATE OF RECIPROCAL OF CONDITION NUMBER FOR

\[ H = \sqrt{-1} \cdot \Omega \cdot i \]

EIGENVALUES, (REAL PART, IMAG. PART), OF SYSTEM MATRIX

<table>
<thead>
<tr>
<th>Real Part</th>
<th>Imaginary Part</th>
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<tbody>
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<tr>
<td>-7.500000E-03</td>
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G COMPLEX MATRIX

<table>
<thead>
<tr>
<th>Real Part</th>
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<tbody>
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VECTORS SIGMA OF COMPUTED SINGULAR VALUES OF G

SIGMA(1) = 3044731E-02
SIGMA(2) = 2365353E-02
SIGMA(3) = 2365230E-02

NEXT CALCULATE THE SINGULAR VALUE PLOTS OF THE 14TH-ORDER ERROR SYSTEM
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

PROGRAM TO COMPUTE MATRIX G EVALUATED AT
S = SQRT(-1)XOMEGA, OMEGA = .1000000E-01

GO = CG((SI - AG)INVERSE)BG + DG

<table>
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DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

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<tr>
<td>-7.7088E+01</td>
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</tr>
</tbody>
</table>

BG MATRIX 14 ROWS 3 COLUMNS

| .0000E+00 | .0000E+00 | .0000E+00 |
| .0000E+00 | .0000E+00 | -7.4900E-07 |
| .0000E+00 | .0000E+00 | -6.2100E-04 |
| .0000E+00 | .0000E+00 | 5.5500E-06 |
| .0000E+00 | .0000E+00 | .0000E+00 |
| .0000E+00 | .0000E+00 | .0000E+00 |
| .0000E+00 | .0000E+00 | .0000E+00 |
| .0000E+00 | .0000E+00 | .0000E+00 |
| .0000E+00 | .0000E+00 | -3.0400E-03 |
| .0000E+00 | .0000E+00 | 2.4300E-03 |

DG IS A NULL MATRIX

G = GO, LOOP-GAIN TRANSFER MATRIX

SINGULAR VALUES BUT NO SINGULAR VECTORS OF G ARE COMPUTED

SYSTEM MATRIX IS INPUT AG

RCOND = 1.0618094E-01

RCOND = ESTIMATE OF RECIPROCAL OF CONDITION NUMBER FOR

H = SQRT(-1) * OMEGA*I - SYSTEM MATRIX

EIGENVALUES, (REAL PART, IMAG. PART), OF SYSTEM MATRIX

( -3.180000E-02 , 3.179841E+00 )
( -3.180000E-02 , -3.179841E+00 )
( -4.530000E-02 , 4.529773E+00 )
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

FREQ PROGRAM

( -4.530000E-02, -4.529773E+00 )
( -5.590000E-02, 5.589720E+00 )
( -5.780000E-02, -5.789720E+00 )
( -5.780000E-02, 5.779711E+00 )
( -6.840000E-02, 6.839658E+00 )
( -6.840000E-02, -6.839658E+00 )
( -7.400000E-02, 7.399630E+00 )
( -7.400000E-02, -7.399630E+00 )
( -8.780000E-02, 8.779561E+00 )
( -8.780000E-02, -8.779561E+00 )

G COMPLEX MATRIX

( 1.2126E-07, -3.3308E-12 )
( -6.9143E-09, 1.5587E-13 )
( 1.7625E-07, -4.8202E-12 )
( -6.9143E-09, 1.5587E-13 )

VECTOR SIGMA OF COMPUTED SINGULAR VALUES OF G

SIGMA( 1) = .1762490E-06
SIGMA( 2) = .1216641E-06
SIGMA( 3) = .4213911E-08

NEXT CALCULATE THE OPEN-LOOP RESPONSE OF THE COMPENSATOR
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

PROGRAM TO COMPUTE MATRIX G EVALUATED AT
\[ S = \sqrt{-1} \times \Omega, \quad \Omega = 0.1000000E-01 \]
\[ G = CG \left( (SI - AG)^{-1} \right) BG + DG \]

<table>
<thead>
<tr>
<th>CG MATRIX</th>
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<td>7.6100E+02</td>
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DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

FREQ PROGRAM

-2.5900E-06 -5.1600E-06 1.1900E-10
-3.2900E-06 5.6900E-07 8.9200E-11
-1.6500E-06 -3.3100E-06 1.2600E-10

DG IS A NULL MATRIX

G = GO, LOOP-GAIN TRANSFER MATRIX

SINGULAR VALUES BUT NO SINGULAR VECTORS OF G ARE COMPUTED

SYSTEM MATRIX IS INPUT AG

RCOND = 6.4820193E-03

RCOND = ESTIMATE OF RECIPROCAL OF CONDITION NUMBER FOR

H = SQRT(-1)*OMEGA*I - SYSTEM MATRIX

EIGENVALUES, (REAL PART, IMAG. PART), OF SYSTEM MATRIX

\[
\begin{align*}
( & -6.699942E-01 , & 1.781728E+00 ) \\
( & -6.699942E-01 , & -1.781728E+00 ) \\
( & -1.320700E+00 , & 1.529557E+00 ) \\
( & -1.320700E+00 , & -1.529557E+00 ) \\
( & -5.792836E-01 , & 1.423450E+00 ) \\
( & -5.792836E-01 , & -1.423450E+00 ) \\
( & 7.261809E-02 , & 8.830831E-02 ) \\
( & 7.261809E-02 , & -8.830831E-02 ) \\
( & -3.336587E-02 , & 3.660153E-01 ) \\
( & -3.336587E-02 , & -3.660153E-01 ) \\
( & -3.161989E-02 , & 3.637067E-01 ) \\
( & -3.161989E-02 , & -3.637067E-01 )
\end{align*}
\]

G COMPLEX MATRIX 3 ROWS 3 COLUMNS

\[
\begin{align*}
( & 5.8832E+04 , & 6.5483E+03 ) ( & 1.0497E-03 , & 6.3063E-05 ) ( & 4.9397E+02 , & 1.1730E+02 ) \\
( & 4.7052E+02 , & 1.1765E+02 ) ( & -3.8366E-03 , & -4.3116E-04 ) ( & 1.0462E+05 , & 1.9395E+04 )
\end{align*}
\]

VECTOR SIGMA OF COMPUTED SINGULAR VALUES OF G

\[
\begin{align*}
\text{SIGMA}( 1) = & \quad 0.1064070E+06 \\
\text{SIGMA}( 2) = & \quad 0.5919041E+05 \\
\text{SIGMA}( 3) = & \quad 0.5835953E+05
\end{align*}
\]
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

FREQ PROGRAM

NEXT CALCULATE THE OPEN-LOOP RESPONSE OF THE
CASCADE OF COMPENSATOR AND 12TH-ORDER DESIGN MODEL
DEMONSTRATION OF FREq, BASED ON EXAMPLE IN NASA TP-2560

PROGRAM TO COMPUTE MATRIX G EVALUATED AT
S = SQRT(-1)XOMEGA, OMEGA = .1000000E-01
GO = CG( (SI - AG)INVERSE )BG + DG

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90
DEMONSTRATION OF FREq, BASED ON EXAMPLE IN NASA TP-2560

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DG IS A NULL MATRIX

KO = CK((SI - AK)INVERSE)BK + DK

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DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

FREQ PROGRAM

-5.1200E-06  1.4300E-06  -3.3400E-11
-2.5900E-06  -5.1600E-06  1.1900E-10
-3.2900E-06  5.6900E-07  8.9200E-11
-1.6500E-06  -3.3100E-06  1.2600E-10

DK IS A NULL MATRIX

GL = GO * KO

G = GL, LOOP-GAIN TRANSFER MATRIX

SINGULAR VALUES BUT NO SINGULAR VECTORS OF G ARE COMPUTED

GO CALCULATION

RCOND = 3.4482494E-05
RCOND = ESTIMATE OF RECIPROCAL OF CONDITION NUMBER FOR
H = SQRT(-1)*OMEGA*I - SYSTEM MATRIX

GO COMPLEX MATRIX  3 ROWS  3 COLUMNS
(-2.3652E-03, -5.6223E-10)  (.0000E+00,   .0000E+00)  (1.3508E-07, 6.8266E-11)
(.0000E+00,   .0000E+00)  (-2.3654E-03, -6.8843E-10)  (.0000E+00,   .0000E+00)
(1.3508E-07, 6.8266E-11)  (.0000E+00,   .0000E+00)  (-3.0447E-03, -1.2059E-08)

KO CALCULATION

RCOND = 6.4820193E-03
RCOND = ESTIMATE OF RECIPROCAL OF CONDITION NUMBER FOR
H = SQRT(-1)*OMEGA*I - SYSTEM MATRIX

KO COMPLEX MATRIX  3 ROWS  3 COLUMNS
( 5.8832E+04,  6.5483E+03)  (1.0497E-03,  6.3063E-05)  (4.9397E+02, 1.1730E+02)
( 6.4537E-04, -6.1292E-06)  (5.8002E+04,  6.4479E+03) (-1.7400E-02, -2.2371E-03)
( 4.7052E+02,  1.1765E+02) (-3.8365E-03, -4.3116E-04) (1.0462E+05, 1.9395E+04)

G COMPLEX MATRIX  3 ROWS  3 COLUMNS
(-1.3915E+02, -1.5488E+01) (-2.4834E-06, -1.4922E-07) (-1.1542E+00, -2.7482E-01)
(-1.5265E-06,  1.4497E-08) (-1.3720E+02, -1.5252E+01) ( 4.1157E-05,  5.2914E-06)
(-1.4247E+00, -3.5732E-01) ( 1.1681E-05,  1.3128E-06) (-3.1854E+02, -5.9053E+01)
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

VECTOR SIGMA OF COMPUTED SINGULAR VALUES OF G

SIGMA( 1) = .3239745E+03
SIGMA( 2) = .1400017E+03
SIGMA( 3) = .1380409E+03
DEMONSTRATION OF FREQ, BASED ON EXAMPLE IN NASA TP-2560

PRINTOUT HAS BEEN TURNED OFF FOR THE NEXT 6 ANALYSES.
THESE ARE THE FEEDBACK TRANSFER MATRIX FOR THE DESIGN MODEL WITH
COMPENSATOR; AND ALL 5 ANALYSES OFFERED BY "FREQ" USING THE FULL
26TH-ORDER PLANT WITH THE COMPENSATOR.
References


Dongarra, J. J.; Moler, C. G.; Bunch, J. R.; and Stewart, G. W. 1979: LINPACK Users' Guide. SIAM.


Figure 1. Equivalent forms of the standard feedback configuration.
Figure 2. Initialization and output structure of FREQ.

Figure 3. Computational routines structure of FREQ.
Figure 4. Loop-gain transfer matrix (open-loop response) for the 26th-order model of the hoop/column antenna (G).

Figure 5. Loop-gain transfer matrix (open-loop response) for the 12th-order design model (Gp).
Figure 6. Loop-gain transfer matrix (open-loop response) for the additive-error model ($\Delta G$).

Figure 7. Loop-gain transfer matrix (open-loop response) for the compensator ($G_c$).
Figure 8. Loop-gain transfer matrix (open-loop response) for the design model with compensator \((G_p G_c)\).

Figure 9. Feedback transfer matrix (closed-loop response) for the design model with compensator \((G_p G_c(I + G_p G_c)^{-1})\).
Figure 10. Loop-gain transfer matrix (open-loop response) for the hoop/column antenna with compensator $(\mathbf{GG}_c)$.

Figure 11. Feedback transfer matrix (closed-loop response) for the hoop/column antenna with compensator $(\mathbf{GG}_c(\mathbf{I} + \mathbf{GG}_c)^{-1})$. 
Figure 12. Sensitivity matrix for the hoop/column antenna with compensator \((I + GG_c)^{-1}\).

Figure 13. Inverse return difference matrix for the hoop/column antenna with compensator \((I + GG_c)^{-1}\).
Figure 14. Return difference matrix for the hoop/column antenna with compensator \((I + GG_c)\).
FREQ: A Computational Package for Multivariable System Loop-Shaping Procedures

Daniel P. Giese and Ernest S. Armstrong

NASA Langley Research Center
Hampton, VA 23665-5225

Many approaches in the field of linear, multivariable time-invariant systems analysis and controller synthesis employ loop-shaping procedures wherein design parameters are chosen to shape frequency-response singular value plots of selected transfer matrices. This report documents a software package, FREQ, for computing within one unified framework many of the most used multivariable transfer matrices for both continuous and discrete systems. The matrices are evaluated at user-selected frequency-response values, and singular values and vectors are computed. FREQ also tabulates maximum and minimum singular values against frequency. Example computations are presented to demonstrate the use of the FREQ code.