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**Electric Quadrupole Excitations in the Interactions of
⁸⁹Y with Relativistic Nuclei**

by

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Abstract

The first complete calculations of electric quadrupole excitations in relativistic nucleus-nucleus collisions are presented herein. Neutron emission from ⁸⁹Y is studied and quadrupole effects are found to be a significant fraction of the cross section.

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Nucleus-nucleus collisions proceed predominantly via the Strong and Electromagnetic (EM) forces, both of which have been studied extensively.^{1,2)} The EM interaction consists of many multipoles such as electric dipole (E1), electric quadrupole (E2), magnetic dipole (M1) etc. The electric dipole is the most important of these and this has been the only EM multipole for which calculations have been made and compared to experiment in relativistic nucleus-nucleus collisions. In this paper I present the very first accurate calculations of the electric quadrupole effect.

In nucleus-nucleus collisions the Strong interaction dominates the cross section at impact parameters approximately less than the sum of the nuclear radii, ie. for impact parameters smaller than

$$b_{\min} = R_{0.1}(T) + R_{0.1}(P) \quad (1)$$

where $R_{0.1}$ represents the 10-percent charge density radius^{3, 4)} of the target or projectile. (Other expressions for b_{\min} are possible,^{2, 5)} but for the sake of simplicity they are not discussed here. For impact parameters larger than b_{\min} , the interaction occurs via the EM force and the EM cross section is calculated via

$$\begin{aligned} \sigma &= \sigma_{E1} + \sigma_{E2} \\ &= \int [N_{E1}(E) \sigma_{E1}(E) + N_{E2}(E) \sigma_{E2}(E)] dE \end{aligned} \quad (2)$$

where $N_{Ei}(E)$ is the virtual photon spectrum (of energy E) of a particular multipolarity due to the projectile nucleus and $\sigma_{Ei}(E)$ is the photonuclear reaction cross section of the target nucleus. (In principle the above equation should include other EM multipoles, but their effect is much less important.)

All previous comparisons between theory and experiment⁵⁻⁹⁾ have only included the electric dipole effect using $N_{E1}(E)$ calculated from Weizsacker-

Williams (WW) theory ^{2, 10} where

$$\sigma_{\text{WW}} = \int N_{\text{WW}}(E) \sigma(E) dE \quad (3)$$

where $\sigma(E)$ is the experimentally measured photonuclear reaction cross section. $N_{\text{WW}}(E)$ is equal to $N_{\text{E1}}(E)$ ²⁾ so that WW theory does not include the quadrupole component.

There are difficulties in evaluating σ in equation (2). To be as accurate as possible, one should use experimental values for $\sigma_{\text{E1}}(E)$ and $\sigma_{\text{E2}}(E)$, fold them into the energy dependent spectra $N_{\text{E1}}(E)$ and $N_{\text{E2}}(E)$ and integrate the whole expression numerically. Bertulani and Baur ²⁾ have made a crude estimate of the EM cross section using equation (2). However they pulled N_{E1} and N_{E2} outside of the integral and evaluated them only at a single energy corresponding to a theoretical estimate of the peak in the E1 and E2 cross section. The remaining $\int \sigma(E) dE$ for E1 and E2 were evaluated using theoretical sum rules. This procedure led, for example, to a total cross section σ of 839 mb and 266 mb for the reactions $^{197}\text{Au} (^{56}\text{Fe}, X) ^{196}\text{Au}$ and $^{89}\text{Y} (^{56}\text{Fe}, X) ^{88}\text{Y}$ respectively (at 1.7 GeV/N), whereas the measured cross sections are 601 ± 54 mb and 217 ± 20 mb respectively ⁵⁾. Given such a discrepancy, I decided to retain the energy dependence in $N(E)$ by doing a numerical integration, as described above, using, where possible experimental photonuclear cross sections, as detailed below.

In the present work, results are presented for the reaction $^{89}\text{Y} (\text{Projectile}, X) ^{88}\text{Y}$ only; the major point being simply to illustrate the importance of E2 effects using an accurate calculation. Results for other nuclei such as ^{12}C , ^{16}O , ^{18}O , ^{59}Co and ^{197}Au and detailed comparisons to data will be presented elsewhere.

For best accuracy I have followed the suggestions of Berman et al. ¹¹⁾ concerning which photonutron reaction data to use for ^{89}Y . Following their suggestion I have used the Saclay ¹²⁾ data but multiplied by a factor of 0.82. (The data actually stops at 27 MeV and a smooth extrapolation was used to estimate the small amount of remaining data beyond this energy.) However, all experimental photoneutron data consists of $\sigma_{\text{E1}}(E)$

plus $\sigma_{E2}(E)$ and a way must be found to separate out these components so that they can be inserted into equation (2) and numerically integrated. This separation was achieved by using a theoretical calculation¹³⁾ of the isoscalar component of the electric giant quadrupole resonance (GQR)

$$\sigma_{E2}(E) = \frac{\sigma_{EWSR} E^2}{1 + (E^2 - E_{GQR}^2)^2 / E^2 \Gamma^2} \quad (4)$$

with the energy-weighted sum rule cross section

$$\sigma_{EWSR} = f \frac{0.22 Z A^{2/3} \mu b \text{ MeV}^{-1}}{\pi \Gamma/2} \quad (5)$$

The parameters in the above expressions were taken from Bertrand.¹⁴⁾ For ^{89}Y the width Γ is 3.2 MeV, the energy of the giant quadrupole resonance E_{GQR} is 13.8 MeV and the fractional exhaustion of the EWSR f is 55%. (Note that the ^{89}Y nucleus is approximately spherical, thus justifying the use of a single Lorentzian in equation (4).) The expressions for $\sigma_{E2}(E)$ in equations (4) and (5) were used in equation (2). The dipole cross section was determined by subtracting $\sigma_{E2}(E)$, as given above, from the experimental cross section $\sigma_{\text{expt.}}(E)$ of Lepretre¹²⁾ as in

$$\sigma_{E1}(E) = \sigma_{\text{expt.}}(E) - \sigma_{E2}(E) \quad (6)$$

where $\sigma_{\text{expt.}}$ is 0.82 times the Lepretre¹²⁾ cross section. Then $\sigma_{E2}(E)$ and $\sigma_{E1}(E)$ were inserted into equation (2) and $N_{E1}(E)$ and $N_{E2}(E)$ were taken from expressions derived by Bertulani and Baur.²⁾ The integrals in equation (2) were performed numerically to give the EM nucleus-nucleus cross sections. Because of the use of equation (6), uncertainties in the GQR parameters (even if they were as large as $\pm 2\text{MeV}$ in Γ and E_{GQR} and $\pm 20\%$ in f) do not change the total calculated EM cross section $\sigma_{E1} + \sigma_{E2}$ (which is compared to data in Table 1) by more than 4%. Thus the calculations presented herein are expected to be very accurate even if the quadrupole parameters are uncertain.

Results for the reaction ^{89}Y (projectile, X) ^{88}Y are presented in Table 1 and compared to the experimental measurements of Mercier et al. ⁵⁾ Both individual dipole σ_{E1} and quadrupole σ_{E2} cross sections are presented as well as their sum σ which is to be compared to the data. Also presented are results obtained using WW theory. (Note that the EM calculations using WW theory in reference 5 are not correct. ^{15, 16)} In all calculations b_{\min} from equation (1) are used. The 10-percent charge radii ^{3, 4)} are also listed in Table 1.

One can see that WW theory agrees with experiment for the ^{12}C and ^{20}Ne projectiles and is reasonably close for the ^{40}Ar and ^{56}Fe projectiles. Agreement could be reached by using a different expression for b_{\min} . However a detailed study of treating b_{\min} as an adjustable parameter will be reported elsewhere ¹⁷⁾. The quadrupole cross sections σ_{E2} are all seen to be about 10% of the dipole cross sections σ_{E1} , and for all reactions $\sigma_{E1} + \sigma_{E2}$, is about 7% bigger than σ_{WW} . This is because $N_{E2}(E)$ is always larger than $N_{E1}(E)$ so that the quadrupole photonuclear component $\sigma_{E2}(E)$ is enhanced over the dipole. Adding the quadrupole now gives improved agreement between theory and experiment.

In summary, the first accurate calculations of electric quadrupole effects in relativistic nucleus-nucleus collisions are reported. For the reaction ^{89}Y (Projectile, X) ^{88}Y the quadrupole cross section is about 10% of the dipole cross section, and thus I conclude that electric quadrupole effects are an important consideration in the analysis of nucleus-nucleus collisions.

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Table 1 EM Cross Sections for the Reaction ^{89}Y (Projectile, X) ^{88}Y

The Photoneutron Cross Section measured by Lepretre et al ¹²⁾ (but multiplied ¹¹⁾ by 0.82) was used in the analysis described in the text. The 10 percent charge radius used for ^{89}Y is 6.02 fm and the GQR parameters (see text) are $f = 0.55$, $\Gamma = 3.2$ MeV, $E_{\text{GQR}} = 13.8$ MeV. The ^{89}Y (γ, n) threshold is at 11.0 MeV. Calculations are made for Weizacker-Williams theory (σ_{WW}) and also individual E1 & E2 multipole cross sections are calculated. The total cross section $\sigma_{\text{E1}} + \sigma_{\text{E2}}$ is to be compared to experiment. All calculations use the minimum impact parameter given by $b_{\text{min}} = R_{0.1}(\text{P}) + R_{0.1}(\text{T})$.

Projectile	$R_{0.1}(\text{P})$ (fm)	Energy (GeV/N)	$\sigma_{\text{expt.}}$ (mb) (ref. 5)	σ_{WW} (mb)	σ_{E1} (mb)	σ_{E2} (mb)	$\sigma_{\text{E1}} + \sigma_{\text{E2}}$ (mb)
^{12}C	3.30	2.1	9 ± 12	12	12	1	13
^{20}Ne	4.00	2.1	43 ± 12	32	31	3	34
^{40}Ar	4.72	1.8	132 ± 17	90	88	9	97
^{56}Fe	5.24	1.7	217 ± 20	175	171	16	187

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