HIERARCHICAL PARALLEL COMPUTER
ARCHITECTURE DEFINED BY
COMPUTATIONAL MULTIDISCIPLINARY
MECHANICS*

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*Work supported by NASA Lewis and monitored by Chris Chamis
GOAL

Develop Architecture for Parallel Processor Enabling Optimal Handling of Multidisciplinary Computation of Fluid-Solid Simulations Employing Finite Element and Difference Schemes
Paper Outline

1. Goals
2. Paper Overview
3. Philosophical Directions
4. Modeling Directions
5. Static Poly tree
6. Dynamic Poly tree
7. Example Problems
8. Interpolative Reduction
9. Impact on Solvers
10. Summary
11. Future Directions
Philosophical Thrusts

1. Reduce Load Per Processor
2. Reduce Number of Processors
3. Reduce I/O Between Processors
4. Provide for Most Natural Route of I/O Flow Between Processors
5. Enable Optimal Handling of Model Topology
6. Enable Optimal Handling of Automatic Mesh Refinement
7. Provide Logical Framework to Implement Generalized Saint Venants Type Model Reduction
Modelling Directions

1. Static Single Level Models
   (Traditional Simulation)

   • Modelling Requirements Defined Initially

   • No Changes Occur During Computation
2. Dynamic Multilevel Models

• First Level of Model Refinement
  Established by User

• Multilevels of Refinement
  Established Via Automatic
  Physical Criteria ie.
  Cavitation
  Plasticity (Inelasticity)
  Shock Formation
  Flow Separation
  High Stress and Strain
  Gradients
  Etc.
1. Static Model Organized into Convenient Substructural Components
2. Each Substructural Component is Partitioned into Optimal Number of 2nd Level Substructures
3. The Various 2nd Level Substructures May Themselves be Partitioned into a 3rd Level
4. The Process May be Repeated to Yield a Multilevel Poly Tree
DEVELOPMENT OF STATIC POLY TREE

STEP 1

1st Level

\[ \{ \ldots \} \]

\[ k \]

\[ L \]

\[ 1 \]

2nd Level

GLOBAL

GLOBAL

GLOBAL

\[ \{ \ldots \} \]

\[ k \]

\[ L \]

\[ 1 \]

\[ \ldots \]
DEVELOPMENT OF STATIC POLY TREE

Global Level

1st Level

2nd Level

HIGHER LEVELS
Choice of Levels and Associated Partitions: Static Model

1. Number of Boundary and Internal Variables at Each Level/Partition Balanced to Yield
   - Optimal Hierarchy of Bandwidths
   - Minimum I/O Between Levels

2. Choice of Levels Contingent on
   - Reducing Load Per Processor
   - Minimize Number of Processors for a Given Level of Speed Enhancement
DYNAMIC POLY TREE PARALLELISM

Steps

1. First Level Organized into Convenient Substructural Components (Optimal in Static Sense)

2. Each Substructural Component Refined as Per Local Physics

3. To Maintain Optimality, Refinements May Require Several Levels of Processors
Choice of Numbers of Level &

Associated: Dynamic Model

1. First Level Defined as Per Static Tree

2. Choice of Additional Levels and Associated Partitions Contingent on

- Maintaining Optimality of a Given Branch of Poly Tree
- Reducing Load Per Processor
- Minimize Number of Processors
- Maintain Balance Between Internal and External Variables
- Minimize I/O Between Levels
OPTIMAL PARALLEL COMPUTER ARCHITECTURE
FOR INTERDISCIPLINARY MECHANICS SIMULATIONS

Densifying Solid-Fluid Model

Dynamic Poly Tree Arrangement of Processors

Level 1

 Fluid Processors

Solid Processors

Level 2

 Fluid Processors

Solid Processors

Level 3

 Fluid Processors

Solid Processors
Example Problem

Given:
Consider Square Region With Fine Uniform Mesh

Problem:
Define Optimal Poly Tree

\[(N_g)^2 - \text{Total Mesh Points}\]
**TWO LEVEL POLY TREE**

\[ N = \frac{N_g}{K} \]

- Global Level
- 1st Level

\[ K^2 + 1 \] - Total Number of Processors
ASYMPTOTIC COMPUTATIONAL EFFORT:

TWO LEVEL

- STRAIGHT FULL SIMULATION

\[ C_g \sim \frac{1}{4} (N_g)^4 \]

- TWO LEVEL POLY TREE

\[ C_0 \sim \frac{9}{4} K(N_g)^3 \]
\[ C_1 \sim \frac{9}{2} \left( \frac{N_g}{K} \right)^4 \]

- COMMUNICATIONS

\[ C_c \sim B_r (8(N_g)^2 + 8K N_g) \]
ASYMPTOTIC COMPUTATIONAL EFFORTS:

TWO LEVEL

- RATIO COMPARISON

\[ \frac{R_p}{g} \sim \psi \left( \frac{C_0 + C_1}{C_g} \right) + \frac{\Omega}{N_c} \frac{C_c}{C_g} \]

\[ \frac{R_p}{g} \sim \psi \left( \frac{9}{(K)^4} + 4.5 \frac{K}{N_g} \right) \]

\[ \frac{\Omega B_r}{N_c} \left( \frac{K}{(N_g)^3} + \frac{1}{(N_g)^2} \right) \]
OPTIMAL SOLUTION

- GLOBALLY OPTIMIZED

\[
\frac{d}{dk} \left( \frac{R_p}{g} \right) = 0 \rightarrow \\

k \sim \\

\left[ \frac{8 \ N_g}{1 + \frac{16}{9\psi} \frac{\Omega B_R}{N_c (N_g)^2}} \right]^{1/5}
\]
<table>
<thead>
<tr>
<th>((N_g)^2)</th>
<th>(K)</th>
<th>(R_{p/g})</th>
<th>(R_{0/g})</th>
<th>(R_{1/g})</th>
<th>(\text{SPEED UP})</th>
<th>(\text{NUMBER PROCESSORS})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2.5 \times 10^5)</td>
<td>5</td>
<td>0.045</td>
<td>0.0144</td>
<td>0.045</td>
<td>17</td>
<td>24</td>
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<tr>
<td>(2.5 \times 10^7)</td>
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<td>0.0072</td>
<td>0.00219</td>
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<td>(2.5 \times 10^9)</td>
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<td>0.00117</td>
<td>0.000315</td>
<td>0.00117</td>
<td>673</td>
<td>169</td>
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THREE LEVEL POLY TREE

Global Level

2nd Level

1st Level

K1

K2

Poly Tree

Global Level

1st Level

2nd Level
ASYMPTOTIC COMPUTATIONAL EFFORT:

THREE LEVEL

\[ C_0 = \frac{9}{4} K_1 (N_g)^3 \]

\[ C_1 \sim \frac{49}{4} \frac{K_2 (N_g)^3}{(K_1)^3} \]

\[ C_2 \sim \frac{9}{4} \left( \frac{N_g}{K_1 K_2} \right)^4 \]

\[ \frac{R_p}{g} \sim \left( \frac{9}{(K_1 K_2)^4} + \frac{49}{2} \frac{K_2}{(K_1)^3 N_g} + 4.5 \frac{K_1}{N_g} \right) \]

2nd Level 1st Level 0th Level
SUBOPTIMAL TRENDS: TWO LEVELS

\[ \frac{R_p}{g} \sim \frac{9}{(K)^4} + 4.5 \frac{K}{N_g} \]

\( K \rightarrow \text{Large; } K \sim O(N_g) \)

\[ \frac{R_p}{g} \sim 4.5 \ (450\%) \]
<table>
<thead>
<tr>
<th>$K_1/K_2$</th>
<th>$R_0/g$</th>
<th>$R_1/g$</th>
<th>$R_2/g$</th>
<th>$R_p/g$</th>
<th>SPEED UP</th>
<th>NO. PROCESSORS</th>
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<tr>
<td>2/6</td>
<td>$0.18 \times 10^{-2}$</td>
<td>$0.37 \times 10^{-2}$</td>
<td>$0.4 \times 10^{-3}$</td>
<td>$0.58 \times 10^{-2}$</td>
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<td>144</td>
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<td>$0.91 \times 10^{-3}$</td>
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<td>255</td>
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<td>10/4</td>
<td>$0.9 \times 10^{-2}$</td>
<td>$0.19 \times 10^{-4}$</td>
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<td>$0.9 \times 10^{-2}$</td>
<td>110</td>
<td>1600</td>
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OPTIMAL THREE LEVEL POLY TREE; \( N_g = 50000 \)

<table>
<thead>
<tr>
<th>( K_1/K_2 )</th>
<th>( R_0/g )</th>
<th>( R_1/g )</th>
<th>( R_2/g )</th>
<th>( R_P/g )</th>
<th>SPEED UP</th>
<th>NO. PROCESSEORS</th>
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<tr>
<td>2/8</td>
<td>.18x10^{-3}</td>
<td>.49x10^{-3}</td>
<td>.13x10^{-3}</td>
<td>.81x10^{-3}</td>
<td>1239</td>
<td>256</td>
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<tr>
<td>4/7</td>
<td>.36x10^{-3}</td>
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<td>10/6</td>
<td>.9x10^{-3}</td>
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<td>3600</td>
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INTERPOLATIVE REDUCTION : 3 LEVEL

<table>
<thead>
<tr>
<th>MESH LEVEL</th>
<th>REDUCTION</th>
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<tbody>
<tr>
<td>Global</td>
<td>I₀</td>
</tr>
<tr>
<td>1st</td>
<td>I₁</td>
</tr>
<tr>
<td>2nd</td>
<td>I₀</td>
</tr>
</tbody>
</table>

379
INTERPOLATIVE REDUCTION:  3 LEVEL

\[ C_0 \sim \frac{9}{4} K_1 (N_g)^3 (I_1I_0)^3 \]

\[ C_1 \sim \frac{49}{4} \frac{K_2}{(K_1)^3(N_g)} (I_1)^3 \]

\[ C_2 \sim \frac{9}{(K_1K_2)^4} \]

\[ R_{P/g} \sim \]

\[ \frac{9}{(K_1K_2)^4} + \frac{49}{2} \frac{K_2}{(K_1)^3N_g} (I_1)^3 + 4.5 \frac{K_1}{N_g} (I_1I_0)^3 \]

2nd Level  1st Level  0th Level
REDUCTION EFFECTS: THREE LEVEL POLY TREE:

\[ N_g = 5000 \]

\[ I_1 = \frac{1}{2}, I_0 = \frac{1}{4} \]

<table>
<thead>
<tr>
<th>( K_1/K_2 )</th>
<th>( R_o/g )</th>
<th>( R_1/g )</th>
<th>( R_2/g )</th>
<th>( K_1 )</th>
<th>( R_1 )</th>
<th>( R_0 )</th>
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<tbody>
<tr>
<td>3/5</td>
<td>5.3x10^-6</td>
<td>1.1x10^-4</td>
<td>1.18x10^-3</td>
<td>264</td>
<td>3386</td>
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<td>10/4</td>
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<td>2.3x10^-6</td>
<td>335x10^-5</td>
<td>110</td>
<td>42920</td>
<td>1600</td>
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REDUCTION EFFECTS: THREE LEVEL POLY TREE;

\[ N_g = 50000 \]

\[ I_1 = \frac{1}{2}, \quad I_0 = \frac{1}{4} \]

<table>
<thead>
<tr>
<th>( K_1/K_2 )</th>
<th>( R_0/g )</th>
<th>( R_1/g )</th>
<th>( R_2/g )</th>
<th>SPEED UP</th>
<th>NUMBER PROCESSORS</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>STRAIGHT</td>
<td>REDUCED</td>
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<tr>
<td>4/7</td>
<td>( 7 \times 10^{-7} )</td>
<td>( 6.7 \times 10^{-6} )</td>
<td>( 1.5 \times 10^{-4} )</td>
<td>2335</td>
<td>44,640</td>
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<tr>
<td>10/6</td>
<td>( 1.7 \times 10^{-6} )</td>
<td>( 3.6 \times 10^{-8} )</td>
<td>( 7 \times 10^{-6} )</td>
<td>1110</td>
<td>402,250</td>
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Impact on Solvers

Static/Dynamic Poly Tree Architecture

Provides a Logical Framework for

- Direct Solvers
- Iterative Solvers
- Mixed (Direct/Iterative) Solvers
- Multi Time Scale Transient Solver
- Local/Global Constrained Nonlinear Solvers
- Mesh Refinement Procedures
- Interpolative Reduction (Saint Venants)
- Etc.
Summary

The Poly Tree Arrangement Yields

- Optimal Choice of Number of Processors Required for Given Problem
- Reduces Load Per Processor
- Reduces I/O Between Processors
- Enables Optimal Handling of Automatic Mesh Refinement
- Provide Most Natural Route for I/O Flow
- Enables an Orderly Way to Perform Interpolative Mesh Refinement
Future Directions

1. Continue Refinement of Scheme
2. Develop Associated Parallel Solution Procedure
   - Direct
   - Iterative
   - Mixed
   - Steady State
   - Transient
3. Establish requirements of Data Based Management System Required for Overall Control