DEVELOPMENT OF AN INTEGRATED BEM FOR HOT
FLUID-STRUCTURE INTERACTION

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Summary

One very difficult problem in the engine structural component durability analysis is the determination of the temperatures and fluxes in the structural components directly in contact with the hot gas flow path. Currently there exists no rational analytical or numerical technique which can effectively deal with this problem. The analysts involved in the hot fluid dynamics who use the finite difference method very rarely interact with those engaged in the thermal analysis of the structural components where the dominant numerical method is the finite element method. Since the temperature distribution in the structural components is strongly influenced by both the fluid flow and the deformation as well as the cooling system in the structure, the only effective way to deal with this problem is to develop an integrated solid mechanics, fluid mechanics and heat transfer analysis for this problem.

In the present work, BEM is chosen as a basic analysis tool principally because the definition of quantities like fluxes, temperature, displacements, velocities is very precise on a boundary base discretization scheme. One fundamental difficulty is, of course, that the entire analysis requires a very considerable amount of analytical work which is not present in other numerical methods. During the last 18 months all of this analytical work has been completed and a two-dimensional, general purpose code has been written. Some of the early results are described. It is anticipated that within the next two to three months almost all two-dimensional idealizations will be examined. It should be noted that the analytical work for the three-dimensional case has also been done and numerical implementation will begin next year.
DEVELOPMENT OF AN INTEGRATED BEM FOR HOT FLUID-STRUCTURE INTERACTION

* INTRODUCTION

* REVIEW OF CURRENT PRACTICE

* FUTURE ANALYTICAL REQUIREMENTS

* BEM DEVELOPMENT
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  * STATE-OF-THE-ART
  * FORMULATION
  * IMPLEMENTATION
  * EXAMPLES
  * FUTURE DIRECTION

* SUMMARY
VIEW GRAPH 3

A typical fluid structure interaction problem relevant to the present discussion is shown. The passage of hot fluid is modified by the structure which itself is being cooled. The temperature distribution in the structure is affected by the hot fluid dynamics, the heat conduction through the structure and convection of fluid inside the structure. Without a rational analysis of these interactions, one needs a very large amount of empirical data of convection coefficients which will have to be varied spatially and in time. On the other hand, in a rational analysis, the behavior is controlled by measurable physical parameters.
FUTURE ANALYTICAL REQUIREMENTS

* CONTINUOUS NEED TO ENHANCE CAPABILITIES

* FLUID AND STRUCTURE ARE NOT SEPARATE PROBLEMS

* SIGNIFICANT INTERACTION

* THERMAL GRADIENTS

* DISTORTION
VIEW GRAPH 4

The need for an integrated analysis will be even greater in the future because of increased demand for more efficient and powerful engines leading to higher thermal gradients and severe structural deformation. Fluid and structure cannot be independently analyzed.
BEM DEVELOPMENT

OBJECTIVE (1986-1989)

Conduct a pilot study to assess the appropriateness of BEM for the coupled fluid-structure problem

STATE-OF-THE-ART

* SOLID

  * ELASTIC
  * INELASTIC (PLASTICITY, VISCOPLASTICITY)
  * TRANSIENT DYNAMICS

* FLUID

  * INVISCID (POTENTIAL) FLOW
  * Viscous flow
VIEW GRAPH 5

The BEM has reached a very high level of performance in the solid mechanics where elastic, elastoplastic, viscoplastic, thermoelastic, thermoplastic and large deformation analyses can now be carried for any large two-dimensional, axisymmetric and three-dimensional problems both under steady-state, as well as transient loading. (BEST3D, BEST2D and GPBEST were developed with funding from NASA, Pratt and Whitney and other large industrial corporations, respectively.)

The development in the fluid mechanics applications, however, is many years behind. Almost all of the available work is in the area of inviscid potential flow. Only a few have attempted viscous flow problems.

In the early phases of this work, the fundamental solution necessary for the solid part of the problem was derived from Nowaki's work. This involves the determination of transient displacements due to a set of unit forces and a temperature source in an infinite solid. For fluid, a similar solution has never been attempted. Without this fundamental solution, BEM cannot be developed.
BEM DEVELOPMENT
FORMULATION AND IMPLEMENTATION

* SOLID

* UNCOUPLED THERMOELASTICITY
  * TRANSIENT HEAT TRANSFER ANALYSIS
  * STRESS ANALYSIS
  * THERMOPLASTICITY

* FLUID

* THERMALLY-SENSITIVE, COMPRESSIBLE, VISCOUS FLOW
* VORTICITY-DILATATION-TEMPERATURE FORMULATION
* VELOCITY-TEMPERATURE FORMULATION
VIEW GRAPH 6

As mentioned earlier, solid mechanics formulations for the present problem are at a very well developed stage. For the fluid mechanics part of the problem, three formulations are feasible. From some preliminary studies, it was apparent that for the entire flow regime (low Reynolds number to high Reynolds number) different formulations are needed. Of these, the velocity-temperature formulation was adopted primarily because it provides precise definitions at boundary points.
COMPRESSIBLE VISCOUS FLOW

GOVERNING EQUATIONS

Momentum Balance

\[(\lambda + \mu) \frac{\partial^2 u_j}{\partial x_i \partial x_j} + \mu \frac{\partial^2 v_i}{\partial x_j \partial x_i} - \frac{\partial p}{\partial x_i} = \rho \frac{\partial v_i}{\partial t} + \rho \nu_j \frac{\partial v_i}{\partial x_j}\]

Energy Balance

\[k \frac{\partial^2 T}{\partial x_j \partial x_j} - \rho c_v \frac{\partial T}{\partial t} = -\psi\]

Conservation of Mass

\[\frac{\partial \rho}{\partial t} + v_j \frac{\partial \rho}{\partial x_j} + \rho \frac{\partial v_j}{\partial x_j} = 0\]

Equation of State

\[p = \rho RT\]

BOUNDARY INTEGRAL EQUATION

\[c_{\alpha \beta} v_\beta = \int_{\partial \Omega} (G_{\alpha \beta} \cdot t_\beta - F_{\alpha \beta} \cdot v_\beta) \, dS + \int_{\Omega} (G_{\alpha \beta} \cdot f_\beta) \, dV\]
VIEW GRAPH 7

One of the biggest disadvantages of BEM is that it cannot be developed if the fundamental solution for the set of governing differential equations of the problem does not exist. Unfortunately, this is true for the coupled set of governing equations for the present problem.

Unlike the finite element or the finite difference method where very little mathematical (analytical) work is necessary to derive the shape functions, the fundamental solutions (which play the same role as the shape functions do in FEM) require a great deal of mathematical work.

By separating the density into a reference density and a variable part and adding the variable part times the time derivative of velocity with the convective derivative (second term on the right hand side of the momentum balance equation), a separate right hand side nonlinear body force term can be developed. Similarly the energy balance equation can be modified to include the convective term in a source term on the right hand side. For this coupled set of transient equations, a fundamental solution can now be constructed. For the present work, both two and three-dimensional solutions have been derived. These solutions give the velocity and temperature due to an impulsive body force in the interior of an infinite fluid mass of reference density. It also provides the velocity and temperature due to a unit heat source within an infinite fluid mass of reference density. These fundamental solutions can now be used to develop an exact boundary integral equation in which the changes in the density, as well as convective body force terms appear in the volume integral. This is indeed the basis of all nonlinear analysis by BEM where unknown nonlinear quantities are taken as a volume integral and they are needed only where nonlinearities exist.
BEM DEVELOPMENT
FORMULATION AND IMPLEMENTATION

FORMULATION

* INTEGRAL EQUATION FOR MOMENTUM AND ENERGY BALANCE

* CONTINUITY VIA GLOBAL SHAPE FUNCTION

IMPLEMENTATION

* TWO DIMENSIONAL

* GENERALIZED DEGREES-OF-FREEDOM

* STATE-OF-THE-ART INFRASTRUCTURE
In numerical implementation, the integral representation for the momentum and energy balance are taken care of by the discretization of the exact boundary integral equation. The density changes are then considered via volume integral with the aid of global shape functions in the continuity equation.

The computer program has been developed in a very general manner to admit solutions of any two-dimensional hot fluid structure interaction problem.
VIEW GRAPH 9
As a test problem, the simplest problem of couette flow is shown. It is possible to get a very accurate solution with just 4 quadratic boundary elements.
FIGURE 1 - Driven Cavity Problem Definition

1. ITERATIVE
2. VARIABLE STIFFNESS TYPE
3. SUBSTRUCTURED
VIEW GRAPH 10

As a more challenging nonlinear problem, the driven cavity flow is analyzed. Two of the three nonlinear algorithms (namely iterative and variable stiffness type) have been implemented. The program also allows extensive substructuring or multizoning. Note that the boundary conditions for this problem at both top corners are ambiguous. While the lack of precision at boundaries, both in FEM and FDM, allows one to overlook this problem, in BEM the ambiguity is eliminated by applying a parabolic velocity input near the corners.
DRIVEN CAVITY (INCOMPRESSIBLE VISCOUS FLOW)

Boundary Element Model

- Corner node
- Midnode
- Interior point
DRIVEN CAVITY (INCOMPRESSIBLE VISCOUS FLOW)

VELOCITY DISTRIBUTION (Re=0)
DRIVEN CAVITY (INCOMPRESSIBLE VISCOUS FLOW)

VELOCITY DISTRIBUTION (Re=100)
DRIVEN CAVITY (INCOMPRESSIBLE VISCOUS FLOW)

VELOCITY DISTRIBUTION (Re=250)
VIEW GRAPHS 12-14

These show the developing flow starting with nearly zero Reynolds number to one about 250. Note that circulation center moves slightly.
FIGURE 5 - Driven Cavity (Three Region, Thirty-six Cell Model)
Same problem was also analyzed using 3 subregions to check out the program.
VIEW GRAPH 16
Shows the horizontal velocity profile through the depth at different Reynolds numbers.
VIEW GRAPH 17
To check the thermal coupling in the fluid flow, the nonlinear problem of buoyancy driven cavity problem is examined. As a result of a steady-state heat flow from left to right, the fluid circulates and the resulting temperature profile is no longer uniform over the height.
BEM DEVELOPMENT
FUTURE DIRECTION

* SUMMER 1987

* CONTINUE VERIFICATION OF EXISTING FORMULATION

* DEVELOP FUNDAMENTAL SOLUTION INCLUDING INERTIA TERMS IN MOMENTUM EQUATIONS

* FALL-WINTER 1987

* IMPLEMENT AND VERIFY NEW KERNELS

* CONDUCT CONVERGENCE STUDIES

* 1988

* EXAMINE MORE REALISTIC PROBLEMS

* DEVELOP AND IMPLEMENT FLUID-STRUCTURE INTERACTION CODING
We have achieved the objective of bringing the BEM development of fluid mechanics to nearly the same level of development which currently exists in solid mechanics. Within the next few months, it will be possible to examine the hot-fluid structure interaction problem which hopefully will establish that empirical convection coefficients are not necessary to determine the temperature and fluxes at solid boundaries.