USE OF THE QUASILINEARIZATION ALGORITHM FOR THE SIMULATION OF LSS SLEWING

By

Feiyue Li and P. M. Bainum
Howard University
Washington, DC 20059

ABSTRACT

The use of the Maximum Principle for the large angle slewing of LSS usually results in the so-called two-point boundary-value problem, in which many requirements (e.g., minimum time, small amplitude, and limited control power, etc.) must be satisfied simultaneously. The successful solution of this problem depends largely on the use of an efficient numerical algorithm. There are many candidate algorithms available for this problem (e.g., quasilinearization, gradient, etc.). Here we discuss only the quasilinearization method which has been used for several cases of large angle slewing of LSS. The basic idea of this algorithm is to make a series of successive approximations of the solution from a particular solvable case (linear or nonlinear) to a more general practical case.

For the rigid spacecraft slewing problem with no constraints on the controls, the solution procedure can be found in the literature. This procedure needs to be modified if a minimum time for the slewing problem is desired with control limits given. Recently, an indirect method for finding the minimum time was developed to meet all these requirements.

For the general mixed (including both rigid and flexible parts) problem, an additional constraint of small vibrational amplitude on the flexible parts is imposed. To solve this problem several steps in which the complexity increases gradually are needed, i.e., from a linearized version to a final nonlinear problem, from a less constrained case for the control to a more constrained one, from a nonminimum-time level to a near-minimum-time slewing in which a trade-off needs to be made between minimum time and small flexural amplitude requirements. Some examples of these algorithms are presented for planar slewing maneuvers of the SCOLE configuration.

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P. M. BAINUM

PROFESSOR OF AEROSPACE ENGINEERING

FEIYUE LI

GRADUATE RESEARCH ASSISTANT

DEPARTMENT OF MECHANICAL ENGINEERING

HOWARD UNIVERSITY, WASHINGTON, D.C. 20059

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Feiyue Li
Graduate Research Assistant, (202)636-7124
and
P. M. Bainum
Professor of Aerospace Engineering, (202)636-6612
Department of Mechanical Engineering
Howard University, Washington, D.C. 20059

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The use of the Maximum Principle for the large angle slewing of LSS usually results in the so-called two-point boundary-value problem, in which many requirements (e.g., minimum time, small amplitude, and limited control power, etc) must be satisfied simultaneously. The successful solution of this problem depends largely on the use of an efficient numerical algorithm. There are many candidate algorithms available for this problem (e.g., quasilinearization, gradient, etc.). Here we discuss only the quasilinearization method which has been used for several cases of large angle slewing of LSS. The basic idea of this algorithm is to make a series of successive approximations of the solution from a particular solvable case (linear or nonlinear) to a more general practical case.

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INTRODUCTION

MAXIMUM PRINCIPLE IS APPLIED TO
THE ATTITUDE MANEUVER AND VIBRATION CONTROL
OF LARGE SPACE STRUCTURES

(A) PERFORMANCE INDICES
(B) BOUNDARY CONDITIONS
(C) CONTROL REQUIREMENTS

THIS LEADS TO THE TWO-POINT BOUNDARY-VALUE PROBLEM (TPBVP)

ONE OF THE METHODS OF SOLVING TPBVP IS THE QUASILINEARIZATION ALGORITHM

MAXIMUM PRINCIPLE

STATE EQUATIONS

$$\dot{x} = f(x) + B(x)u, \quad x(0)=x_0, \; x(t_f)=x_f \tag{1}$$

PERPORMANCE INDICES

$$J_1=(1/2)\int_0^{t_f} (x^TQx + u^TRu)dt \tag{2}$$

$$J_2= \int_0^{t_f} (1)dt= t_f \qquad |u_i| \leq u_{ib}, \; i=1 \ldots n \tag{3}$$

NECESSARY CONDITIONS

$$H_1=(1/2)(x^TQx + u^TRu) + \lambda^T(f(x) + Bu) \tag{4}$$

$$\dot{\lambda} = -(\partial H_1/\partial x), \qquad \lambda(0) \text{ unknown} \tag{5}$$

$$(\partial H_1/\partial u)=0, \qquad Ru=-B^T\lambda \tag{6}$$

$$H_2= 1 + \lambda^T(f(x) + Bu) \tag{7}$$

$$\dot{\lambda} = -(\partial H_2/\partial x), \qquad \lambda(0) \text{ unknown} \tag{8}$$

$$u_i= - u_{ib} \, \mathrm{sign}(B^T\lambda), \quad i=1 \ldots n \tag{9}$$

TPBVP

$$\dot{z} = g(z), \qquad z=[x, \lambda]^T = [z_1, z_2]^T \tag{10}$$

$z_1(0)$, $z_1(t_f)$ known;

$z_2(0)$, $z_2(t_f)$ unknown.

$z_2(0)$ to be determined.

QUASILINEARIZATION ALGORITHM

(A) LINEAR DIFFERENTIAL EQUATION:

Nonhomogeneous: $\dot{z} = Az + B, \quad z=[z_1, z_2]^T,$ (11)

$z_1(0), \; z_1(t_f)$ known, $z_2(0)$ to be determined

Homogeneous: $\dot{z} = Az$ (12)

(a) n solns. of (12) + 1 particular soln. of (11)

(b) n + 1 particular solns. of (11)

(B) NONLINEAR CASE:

Linearized equation of (10):

$$\dot{z}^{(k+1)} = (\partial g/\partial z)\, z^{(k+1)} + h(\, z^{(k)}\,) \tag{13}$$

where

$z^{(k)}$ is the k^{th} approximate solution of the nonlinear equation (10),

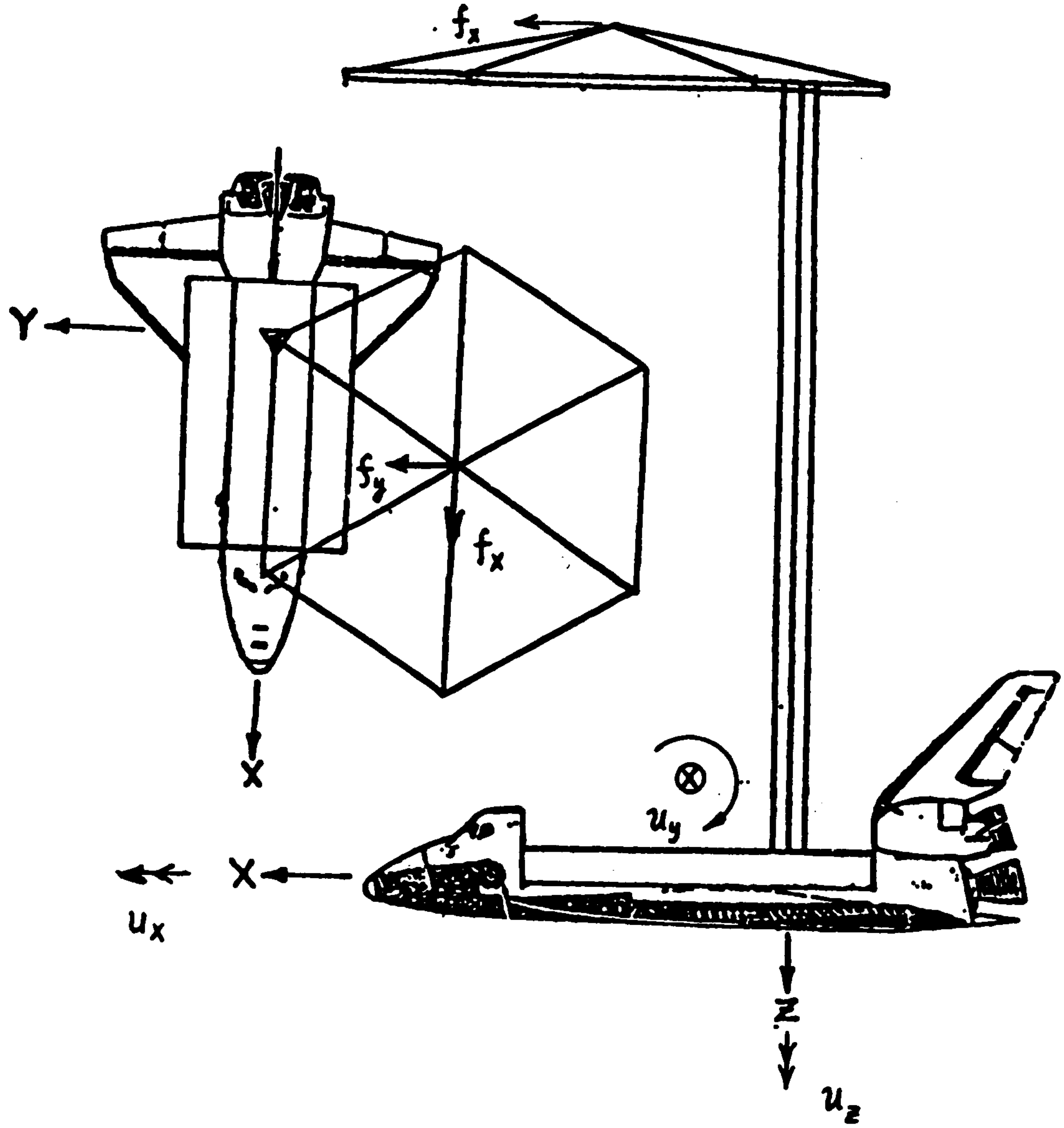
$z^{(k+1)} = z^{(k)} + \Delta z^{(k)}$

$z=[z_1, z_2]^T,$

$z_1^{(k+1)}(0), \; z_1^{(k+1)}(t_f)$, known

$z_2^{(k+1)}(0)$ to be determined

SPACECRAFT CONTROL LAB EXPERIMENT
(SCOLE)



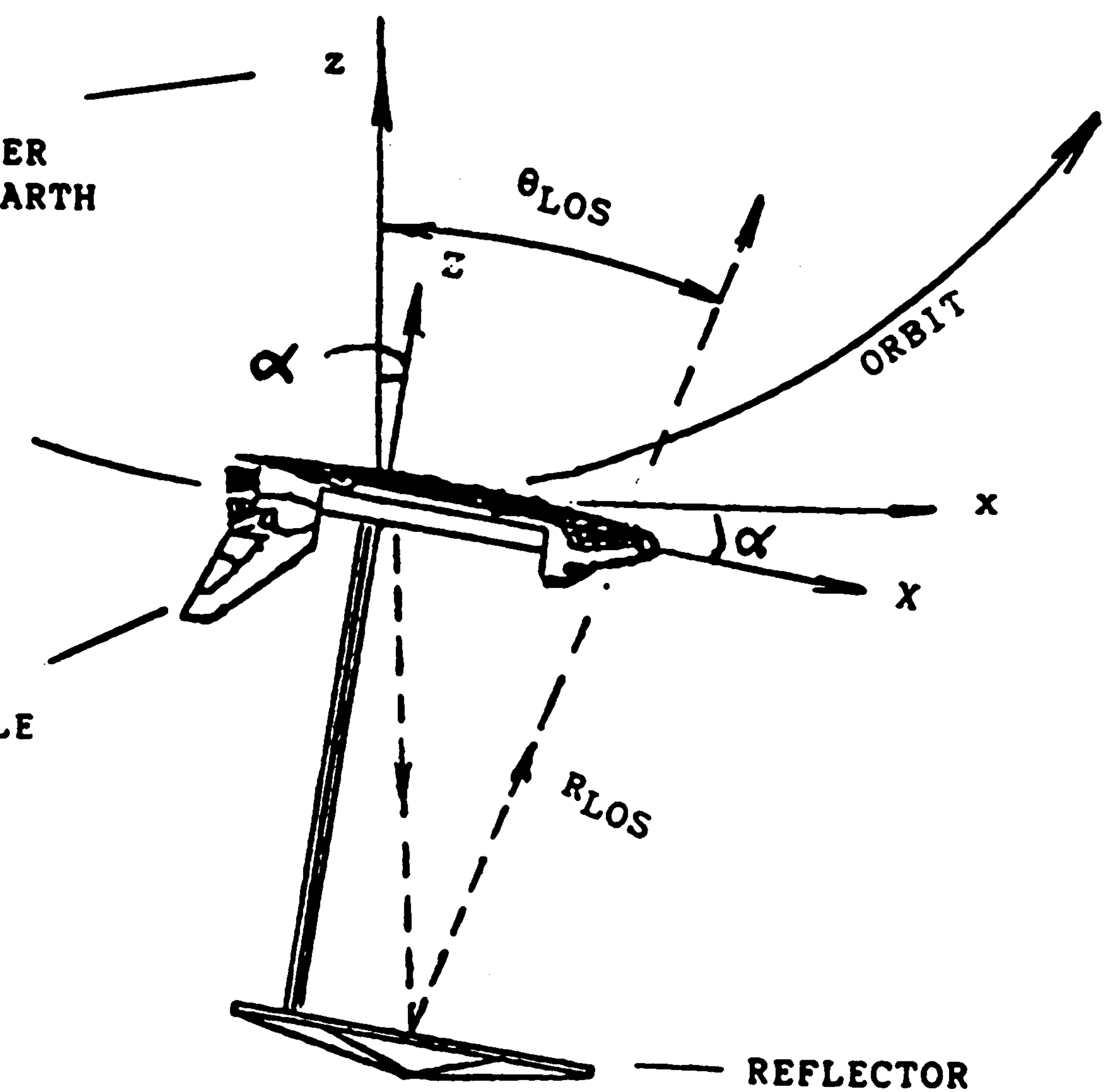
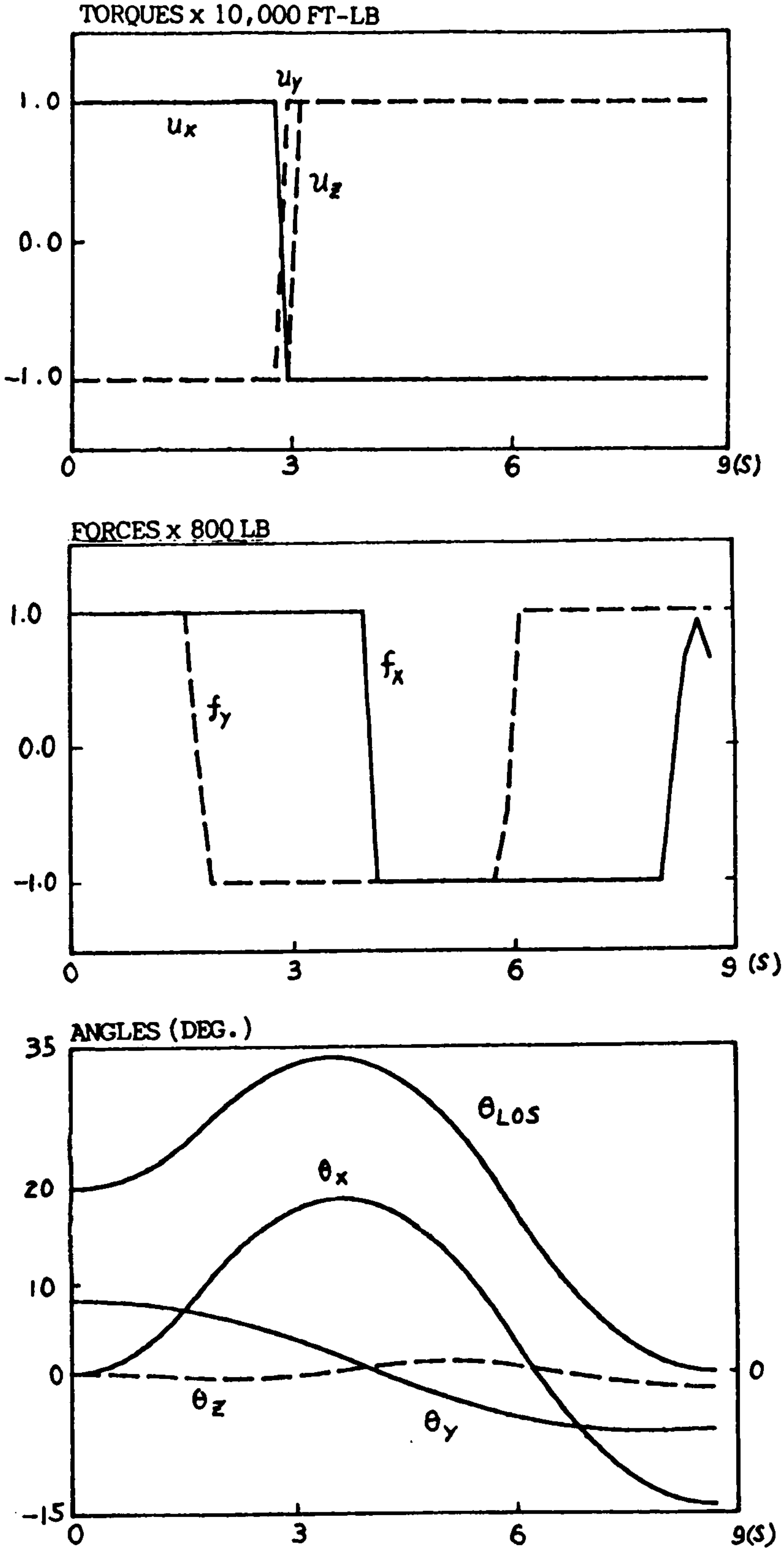


Fig. 1b. Attitude of the SCOLE Showing Antenna Line of Sight



SCOLE (rigid) - Example Slewing, $t_f = 8.69434$ (s)

PLANAR SLEWING OF FLEXIBLE SCOLE

LINEARIZED EQUATION OF MOTION:

$$\begin{bmatrix} \mathbf{I} & \mathbf{m}^T \\ \mathbf{m} & \mathbf{M} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\eta} \end{bmatrix} + \begin{bmatrix} 0 & \underline{0}^T \\ \underline{0} & \mathbf{K} \end{bmatrix} \begin{bmatrix} \theta \\ \eta \end{bmatrix} = \begin{bmatrix} 1 & z_1 & z_2 & L \\ \underline{0} & \Phi_1 & \Phi_2 & \Phi_L \end{bmatrix} \begin{bmatrix} u_s \\ u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

where

θ is the angle of rotation,

η_{nx1} is the amplitude vector of the flexible modes,

n is the number of modes used,

$\mathbf{I}$ is the moment of inertia about the axis of rotation

$\mathbf{m}$, $\mathbf{M}$ are the inertia parameter vector, matrix.

$\mathbf{K}$ is the stiffness matrix,

$\Phi(z)$ is the mode shape function vector,

$\Phi_i = \Phi(z_i)$, z_i is the coordinate along z axis,

L is the length of the beam,

u_S is the control torque on the Shuttle,

u_i are the control actuators on the beam and the reflector.

STATE EQUATIONS

$$\dot{s} = As + Bu$$

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}, \quad s1 = \begin{bmatrix} \theta \\ \eta \end{bmatrix}, \quad s2 = \begin{bmatrix} \dot{\theta} \\ \dot{\eta} \end{bmatrix}$$

BOUNDARY CONDITIONS FOR s

$$s(0)=\begin{bmatrix} \theta_f \\ \underline{0} \\ -- \\ 0 \\ \underline{0} \end{bmatrix}, \quad s(t_f)=\begin{bmatrix} 0 \\ \underline{0} \\ -- \\ 0 \\ \underline{0} \end{bmatrix}_{2(n+1)x1}$$

where n is the number of mode shapes used.

PERFORMANCE INDEX

$$J = (1/2) \int_0^{t_f} (x^T Q x + u^T R u) dt \quad , \quad x = s$$

TPBVP

$$\dot{z} = Cz , \quad z = [s , \lambda]^T = [z_1 , z_2]^T$$

λ is the costate vector,

$z_1(0)$, $z_1(t_f)$ known;

$z_2(0)$ to be determined.

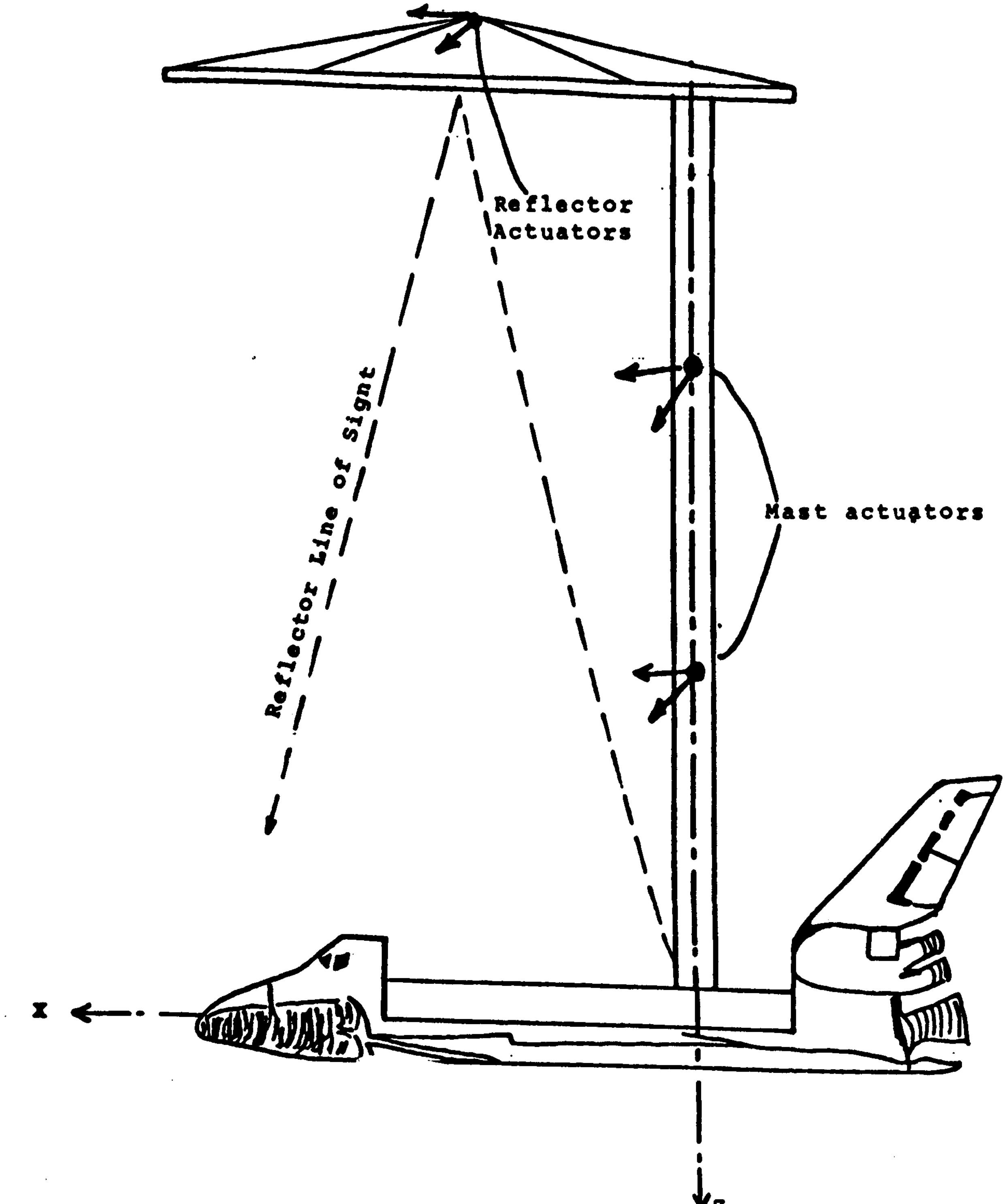


FIGURE I.1: DRAWING OF THE SCOLE CONFIGURATION

NUMERICAL RESULTS

(A) SLEWING ABOUT X-AXIS (ROLL)

Only η_1 (first mode shape) is used.

$\theta_f = 20$ (deg), $t_f = 40$ (s)

$J = \frac{1}{2}\int_0^{t_f} (s^T Q s + u^T R u)\, dt$, $s = [\theta, \eta_1, \dot{\theta}, \dot{\eta}_1]^T$

$$R = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

CASE 1: u_1 is used,

$$Q_1 = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

CASE 2: u_1 is used

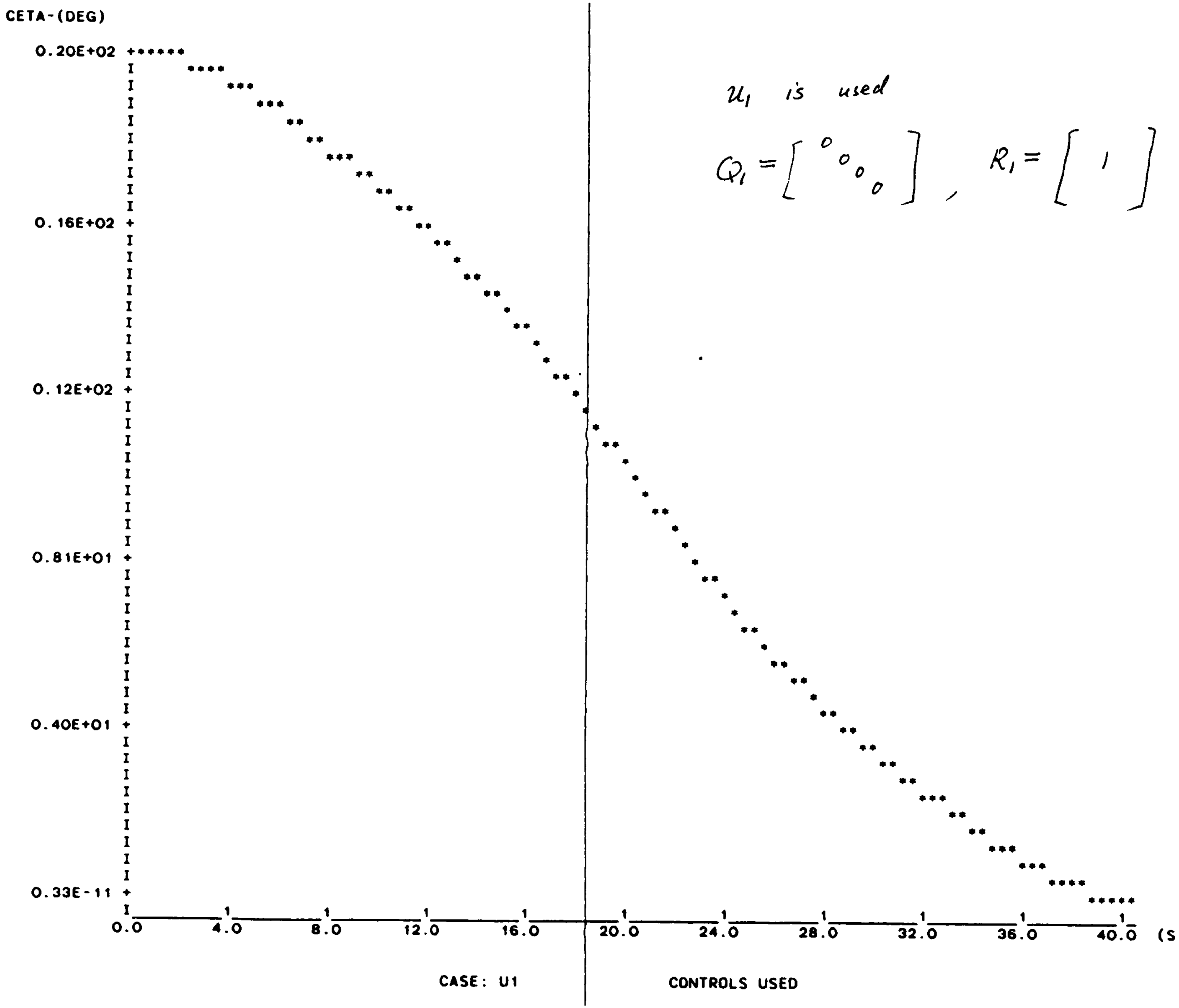
$$Q_2 = \begin{bmatrix} 0 & & & \\ & 0.001 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

CASE 3: u_1, u_2 are used

$$Q_3 = \begin{bmatrix} 0 & & & \\ & 0.01 & & \\ & & 0.01 & \\ & & & 0.01 \end{bmatrix}$$

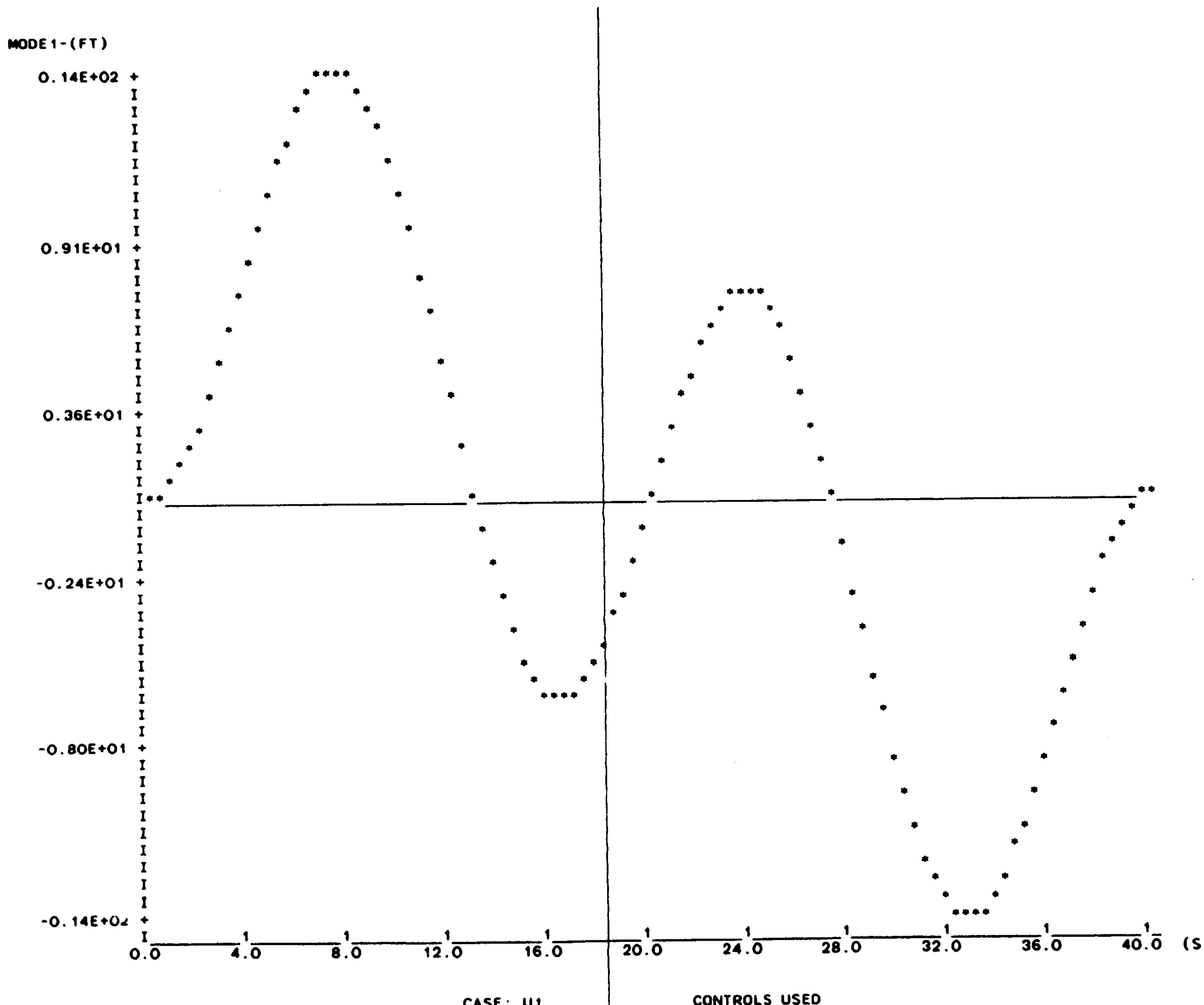
CASE 3: u_1, u_3 are used

$$Q_4 = \begin{bmatrix} 0 & & & \\ & 0.01 & & \\ & & 0.01 & \\ & & & 0.01 \end{bmatrix}$$



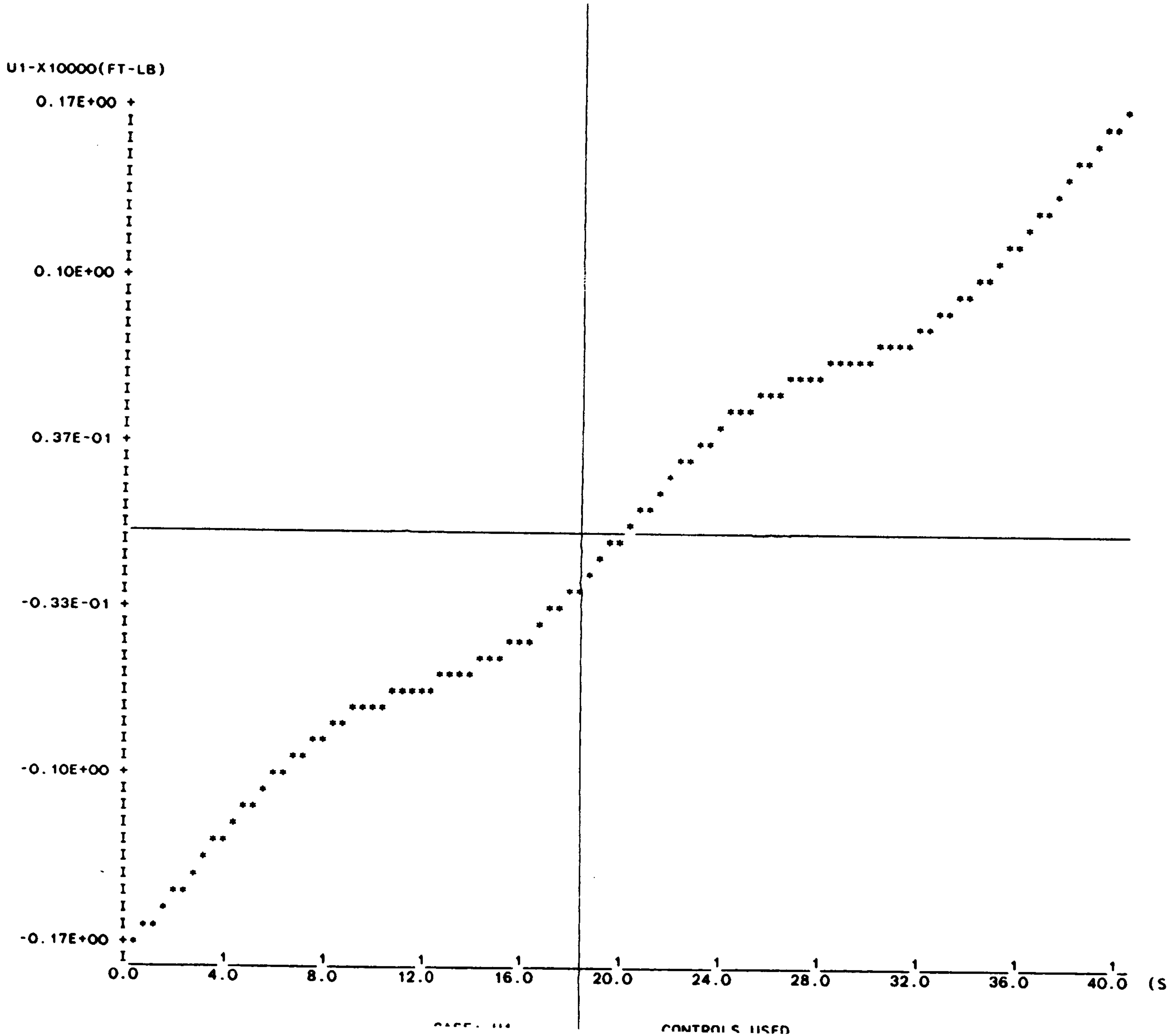
CETA-(DEG)
0.20E+02
0.16E+02
0.12E+02
0.81E+01
0.40E+01
0.33E-11
0.0
4.0
8.0
12.0
16.0
20.0
24.0
28.0
32.0
36.0
40.0
(S)
CASE: U1
CONTROLS USED
u_1 is used
$Q_1 = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$, $R_1 = \begin{bmatrix} 1 \end{bmatrix}$

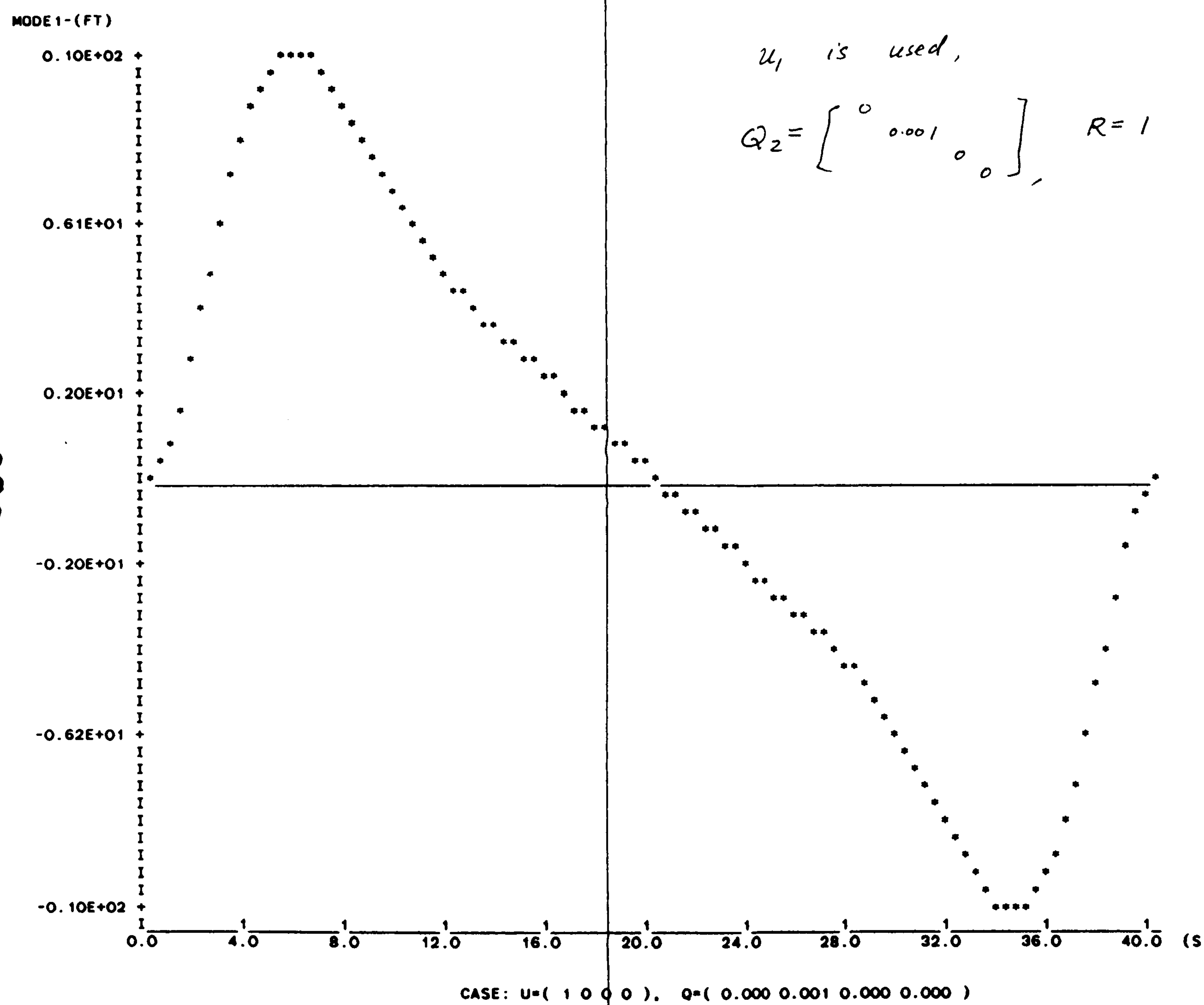
680



CASE: U1

CONTROLS USED

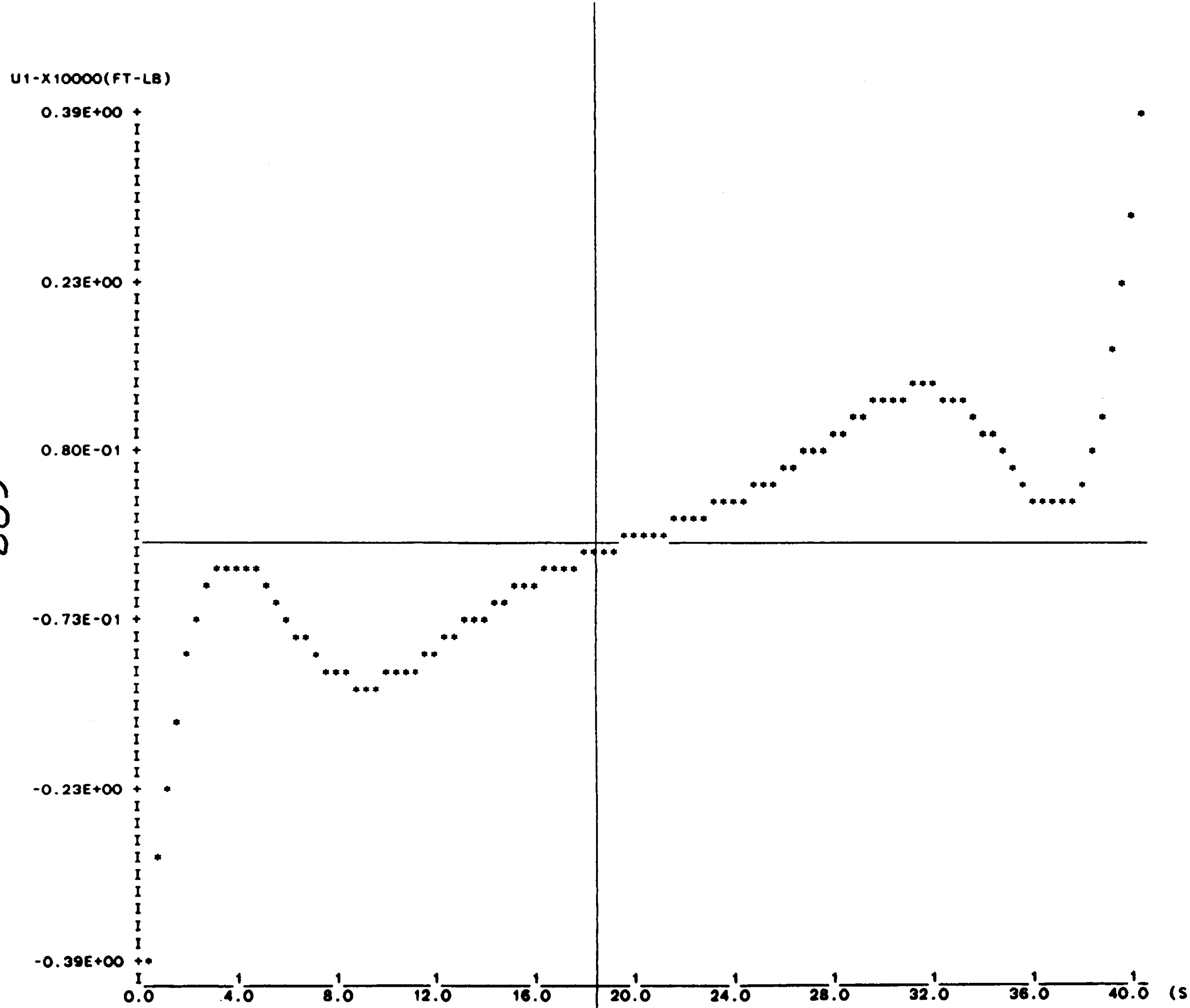

U1-X10000(FT-LB)
0.17E+00
0.10E+00
0.37E-01
-0.33E-01
-0.10E+00
-0.17E+00
0.0
4.0
8.0
12.0
16.0
20.0
24.0
28.0
32.0
36.0
40.0
(S)

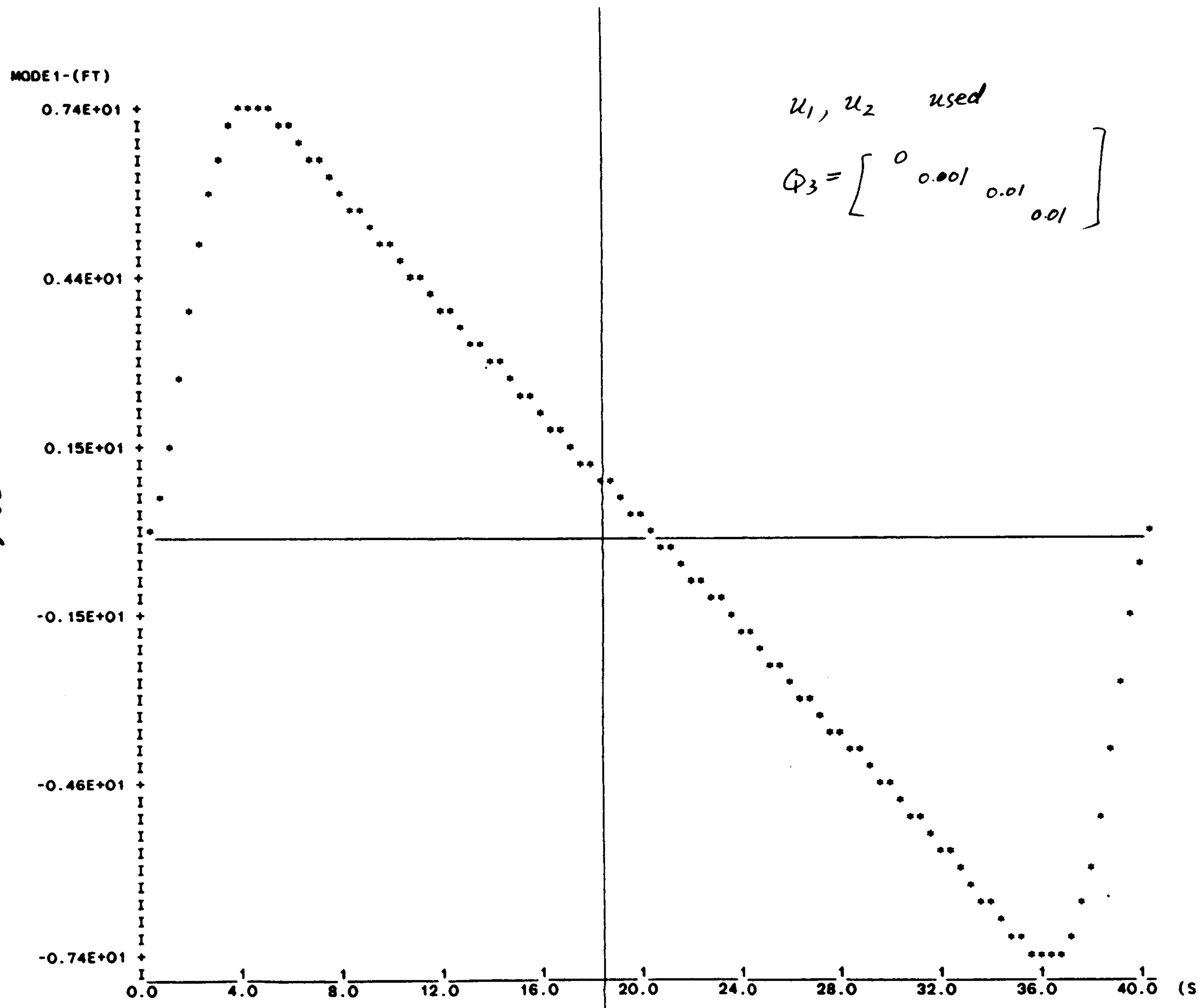


u_1 is used,

$Q_2 = \begin{bmatrix} 0 & & & \\ & 0.001 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$, $R = 1$

682

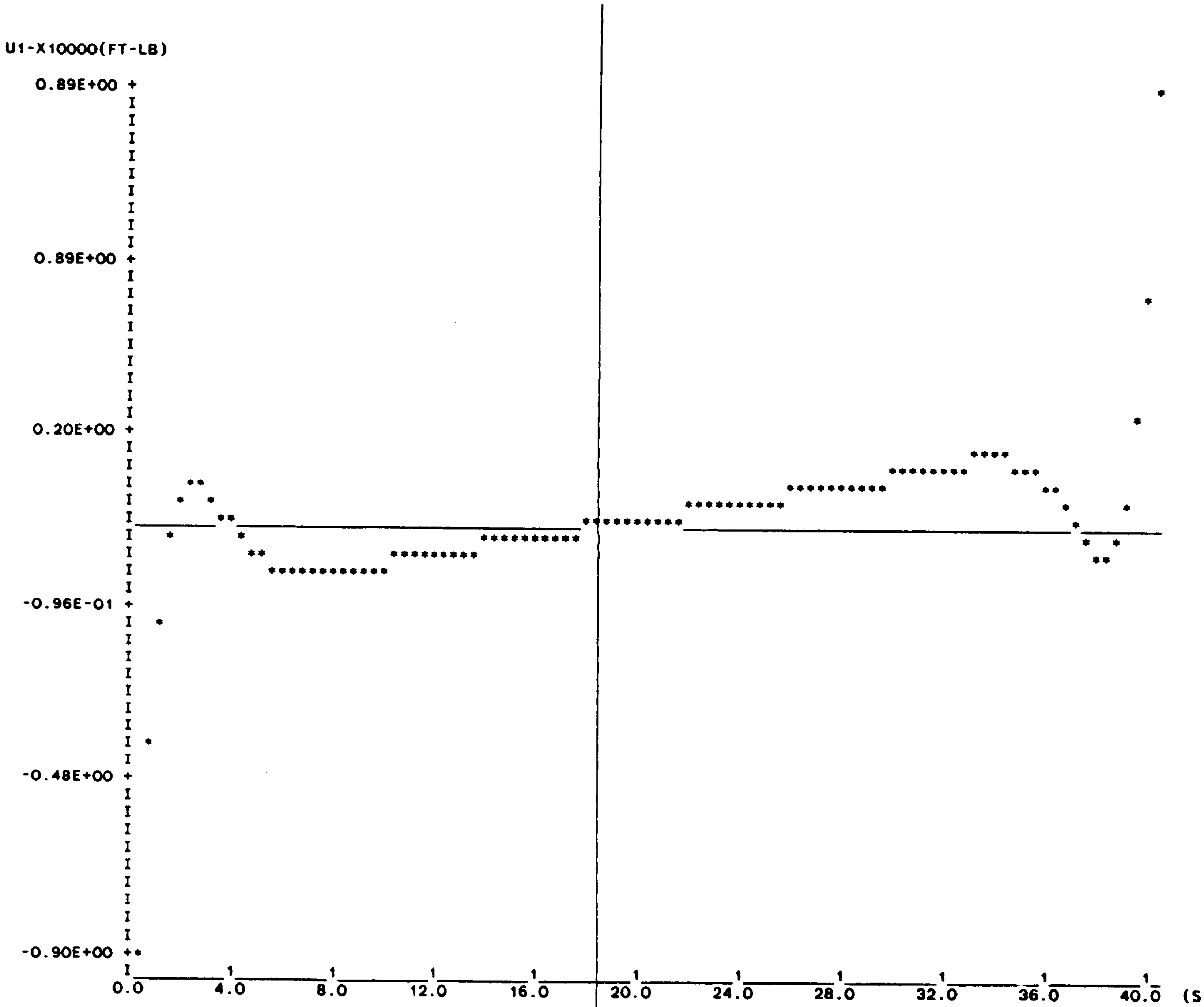

U1-X10000(FT-LB)
0.39E+00
0.23E+00
0.80E-01
-0.73E-01
-0.23E+00
-0.39E+00
0.0
4.0
8.0
12.0
16.0
20.0
24.0
28.0
32.0
36.0
40.0
(S)
CASE: U=(1 0 0 0), Q=(0.000 0.001 0.000 0.000)



u_1, u_2 used

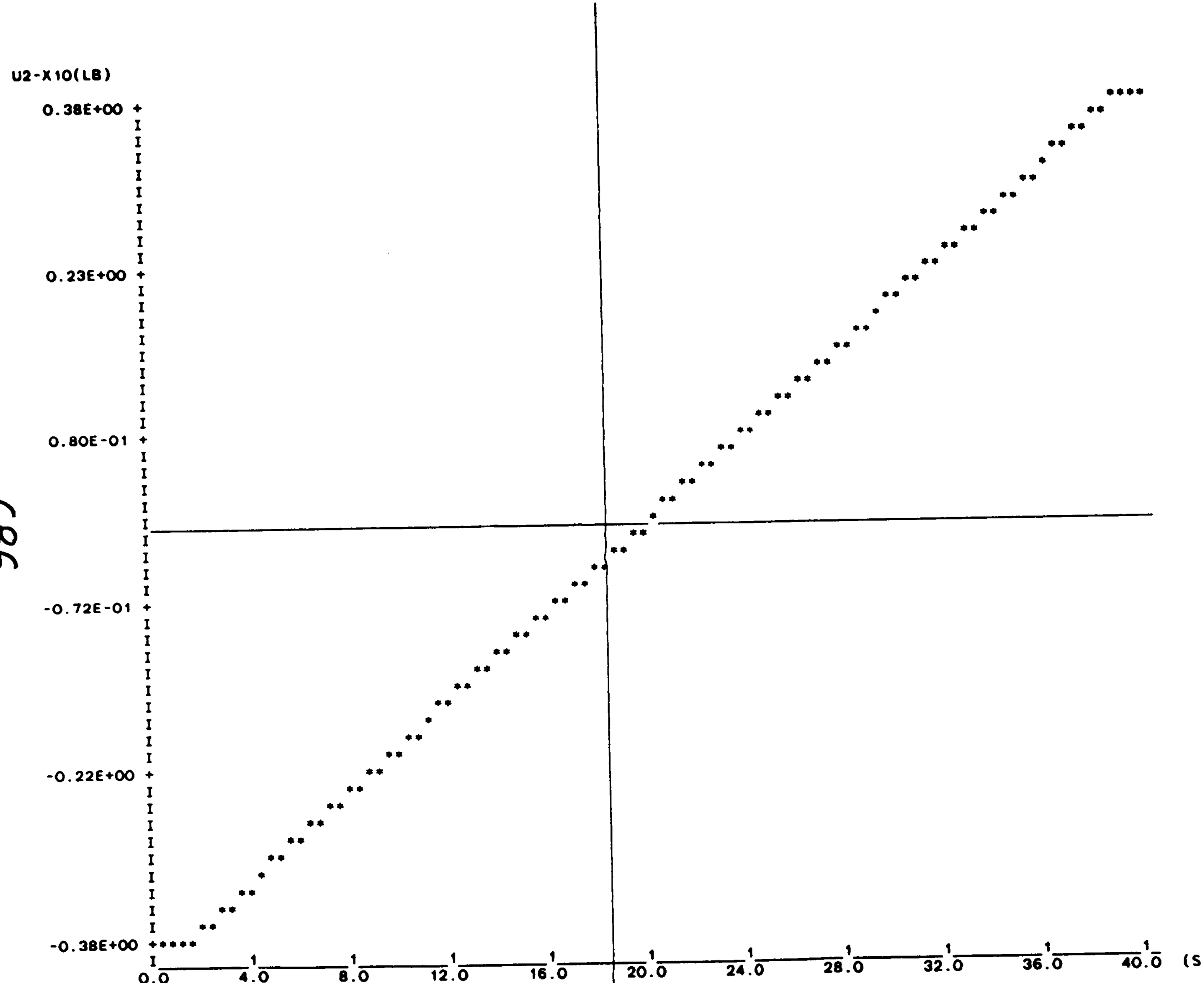
$$Q_3 = \begin{bmatrix} 0 & & & \\ & 0.001 & & \\ & & 0.01 & \\ & & & 0.01 \end{bmatrix}$$

CASE: U=(1 2 0 0), Q=(0.000 0.010 0.010 0.010)



CASE: U=(1 2 0 0), Q=(0.000 0.010 0.010 0.010)

685

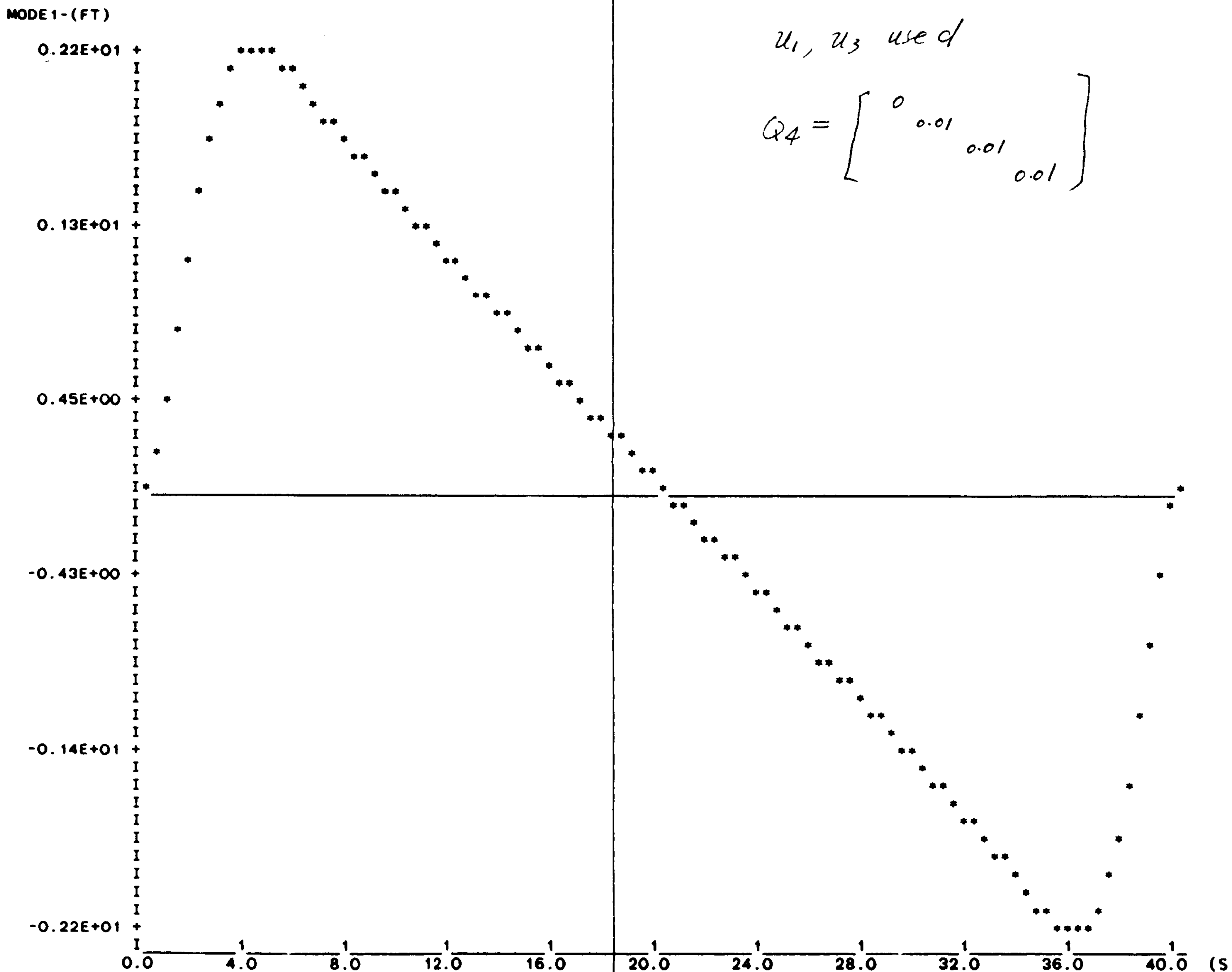


CASE , U=(1 2 0 0), Q=(0.000 0.010 0.010 0.010)

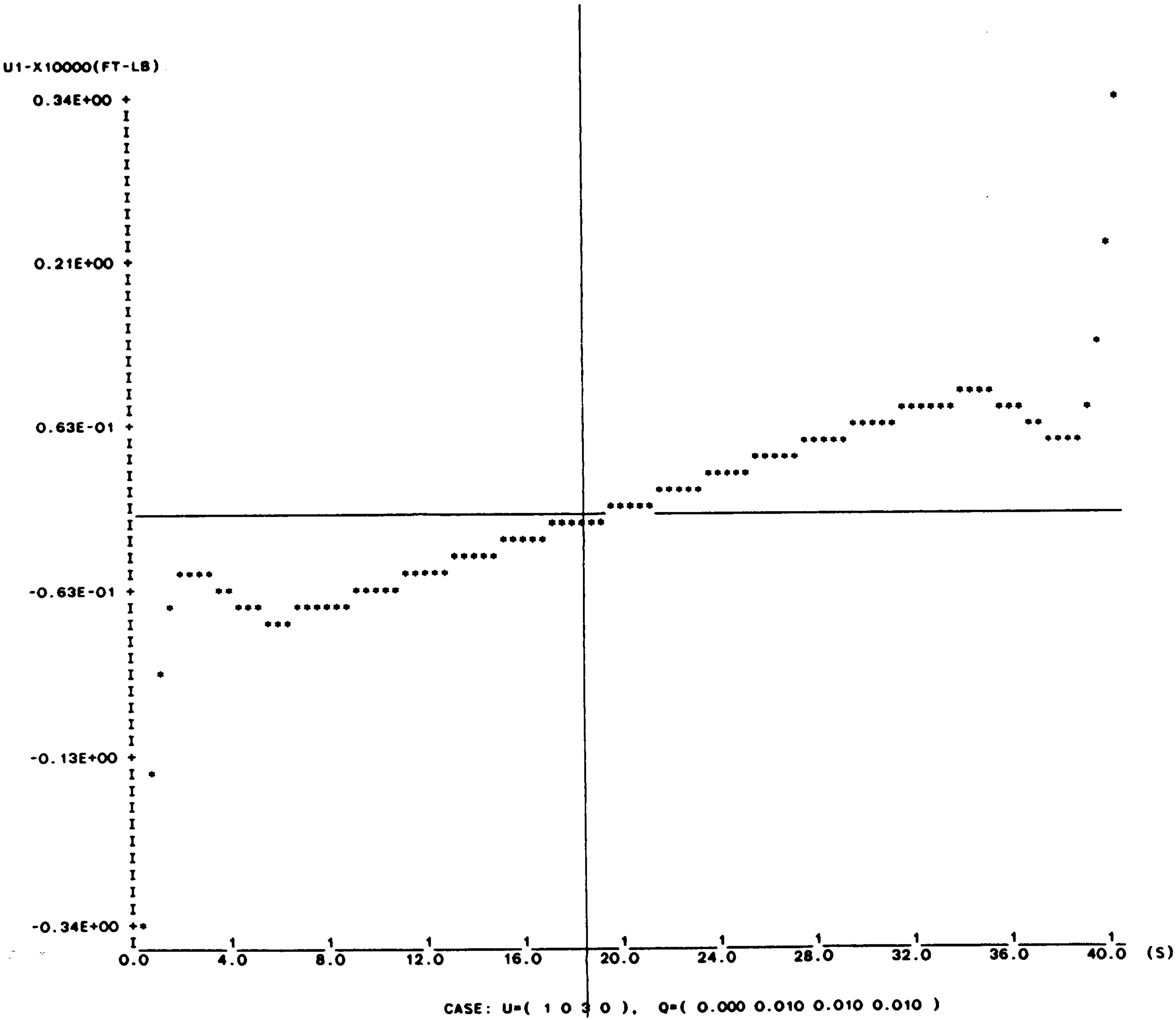
686

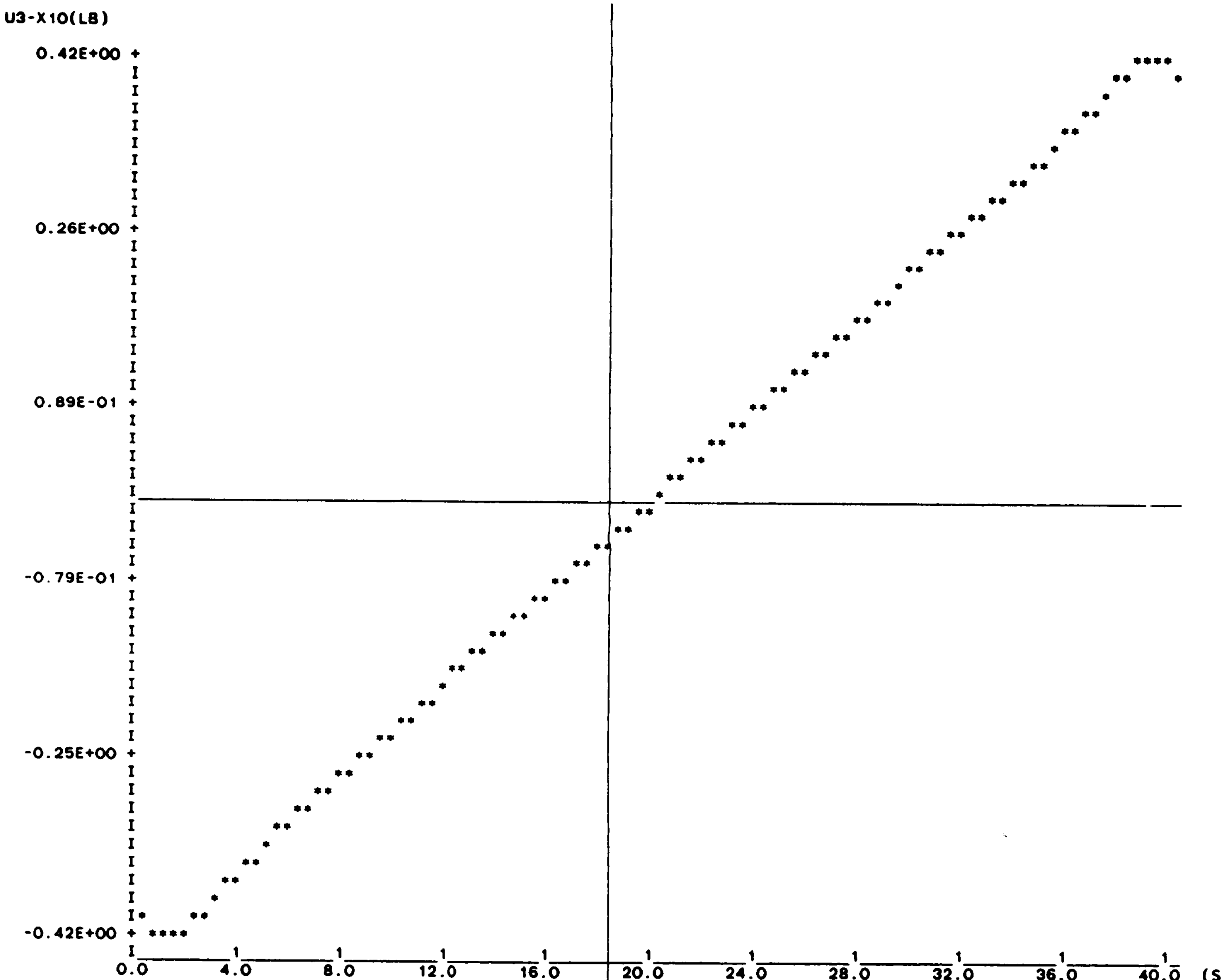
u_1, u_3 used

$$Q_4 = \begin{bmatrix} 0 & & & \\ & 0.01 & & \\ & & 0.01 & \\ & & & 0.01 \end{bmatrix}$$



CASE: U=(1 0 3 0), Q=(0.000 0.010 0.010 0.010)


U1-X10000(FT-LB)
0.34E+00
0.21E+00
0.63E-01
-0.63E-01
-0.13E+00
-0.34E+00
0.0
4.0
8.0
12.0
16.0
20.0
24.0
28.0
32.0
36.0
40.0
(S)
CASE: U=(1 0 3 0), Q=(0.000 0.010 0.010 0.010)



CASE: U=(1 0 3 0), Q=(0.000 0.010 0.010 0.010)

CONCLUDING REMARKS

1) Solution has been obtained for nonlinear rigid spacecraft attitude maneuver (including the rigidized SCOLE).

2) Use of the Maximum Principle can make the states satisfy the boundary conditions very well.

3) Due the fact that the costates must be used in the method, the dimension of equations of the system is doubled, and higher computational ability is needed in this method.

4) Further work on more complicated models (nonlinear differential equation) is needed.

5) Need to consider different cost functions and perform parametric studies.