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THE GENERALIZED DRIFT FLUX APPROACH: IDENTIFICATION OF THE VOID-DRIFT CLOSURE LAW

J. A. Bouré

Commissariat à l'Energie Atomique
Centre d'Etudes Nucléaires de Grenoble
Service d'Etudes Thermohydrauliques
85 X, F 38041, Grenoble Cedex, France

ABSTRACT

The main characteristics and the potential advantages of generalized drift flux models are recalled. In particular it is stressed that the issue on the propagation properties and on the mathematical nature (hyperbolic or not) of the model and the problem of closure are easier to tackle than in two-fluid models.

The problem of identifying the differential void-drift closure law inherent to generalized drift flux models is then addressed. Such a void-drift closure, based on wave properties, is proposed for bubbly flows. It involves a drift relaxation time which is of the order of 0.25 s.

It is observed that, although wave properties provide essential closure validity tests, they do not represent an easily usable source of quantitative information on the closure laws.

I. INTRODUCTION

Generalized drift flux models were recently shown (Bouré, 1988b) to be attractive alternatives for current 1-D two-fluid models. Drift flux models are characterized by the uses of a single momentum balance (the mixture balance instead of two phasic balances) and of a void-drift closure law. In classical drift flux models the void-drift closure law is expressed through an algebraic equation, which amounts to ignoring nonequilibrium drift effects. In generalized drift flux models, the void-drift closure equation is a partial differential equation.

Generalized drift flux models and two-fluid models are compared in the next section and in table 1. The comparison brings out the drawbacks of two-fluid models. However, both kinds of model must be complemented by closure laws. In particular, generalized drift flux models need a void-drift closure law which remains to be specified.

Since generalized drift flux models were introduced to account for the properties of kinematic waves (Bouré, 1988a), it seems logical to use these properties to identify (i.e. to evaluate the coefficients of) the void-drift closure equation. The purpose of the present paper is to discuss the identification problem, assuming that the void-drift closure equation may be approached by a quasi linear differential equation of the first order.

II. A REMINDER ON GENERALIZED DRIFT FLUX MODELS AND TWO-FLUID MODELS

The comparison between generalized drift flux models and two-fluid models is summarized in table 1 which, like must of the substance of this section, is taken from Bouré (1988b). The mass and energy balances are parts of both kinds of models, and they are not discussed further hereafter. It is only noted for completeness that they require a few closure equations, in particular for the mass and energy transfers at the walls and at the interfaces.

The momentum balances contain the two phasic averaged pressures P_G and P_L, the subscripts G and L corresponding to the two phases. For the following discussion, it is convenient to express P_G and P_L in terms of the average pressure P and the pressure difference P_{LG}, with :

$$P \triangleq \alpha P_G + (1-\alpha) P_L \ , \qquad P_{LG} \triangleq P_G - P_L \tag{2.1}$$

α being the void fraction. Now, in the set of balance equations, the two phasic momentum balances are equivalent to a subset of two equations, namely the *mixture momentum balance* and the *pressure difference equation*, obtained on eliminating P between the two phasic balances.

The mixture momentum balance may be written :

$$\frac{\partial P}{\partial z} + \alpha \rho_G \left(\frac{\partial W_G}{\partial t} + W_G \frac{\partial W_G}{\partial z}\right) + (1-\alpha) \rho_L \left(\frac{\partial W_L}{\partial t} + W_L \frac{\partial W_L}{\partial z}\right)$$
$$+ \frac{\partial R}{\partial z} + \frac{\partial \tau}{\partial z} + M_{LG} (W_G - W_L) - I_\sigma + F_W + \rho g = 0 \tag{2.2}$$

the terms of the second line representing respectively :

- a term accounting for fluctuation and transverse distribution effects
- a term accounting for longitudinal stress variations
- an interfacial mass transfer term
- a surface tension term (interfacial)
- a friction term (walls)
- a gravity term.

t and z are respectively the time and the space variables, ρ_G and ρ_L are the phase densities, W_G and W_L are the phase average velocities, g is the gravity acceleration, and :

$$\rho \triangleq \alpha \rho_G + (1-\alpha) \rho_L \tag{2.3}$$

w being the local instantaneous velocity along Oz and $\underline{T}$ the local instantaneous deviatoric stress tensor, $\overline{}^{k}$ indicating the conditional time or ensemble averaging operator and $\langle \ \rangle$ the space averaging operator, and $\underline{n}_z$ being the unit vector of the Oz axis, R and τ are defined as :

$$R \triangleq \left\langle \alpha \; \rho_G \; \overline{(w_G - W_G)^2}^{G} + (1-\alpha) \; \rho_L \; \overline{(w_L - W_L)^2}^{L} \right\rangle \qquad (2.4)$$

$$\tau \triangleq - \left\langle \left[\alpha \; \overline{\underline{\underline{T}}}_G^{G} + (1-\alpha) \; \overline{\underline{\underline{T}}}_L^{L} \right] \cdot \underline{n}_z \right\rangle \cdot \underline{n}_z \qquad (2.5)$$

Independently of the mass transfer, already present in the mass balances, eq. *2.2* requires *four* closure laws for R, τ, I_σ and F_W. Turbulence effects are present through R.

The pressure difference equation may be written :

$$\alpha \; (1-\alpha) \left[\rho_G \left(\frac{\partial \; W_G}{\partial t} + W_G \; \frac{\partial \; W_G}{\partial z} \right) - \rho_L \left(\frac{\partial \; W_L}{\partial t} + W_L \; \frac{\partial \; W_L}{\partial z} \right) + \frac{\partial \; P_{LG}}{\partial z} \right]$$

$$+ L_R + L_\tau + M_{LG} \; [(1-\alpha) \; (W_G - W_{GI}) + \alpha \; (W_L - W_{LI})] \qquad (2.6)$$

$$+ L_I + L_{FW} + L_{FI} - \alpha \; (1-\alpha) \; \rho_{GL} \; g = 0$$

the terms of the second and third lines representing respectively :

- a term accounting for fluctuation and transverse distribution effects
- a term accounting for longitudinal stress variations
- an interfacial mass transfer term (W_{GI} and W_{LI} are interfacial averages of the phasic velocities)
- an induced inertia ("added mass") term
- two friction terms (respectively wall and interfaces)
- a gravity term, with

$$\rho_{GL} \triangleq \rho_L - \rho_G \qquad (2.7)$$

The above terms are defined in Bouré (1988b). For instance :

$$L_R \triangleq (1-\alpha)\, \frac{\partial}{\partial z} \left\langle \alpha\, \overline{\rho_G (w_G - W_G)^2}^{G} \right\rangle - \alpha\, \frac{\partial}{\partial z} \left\langle (1-\alpha)\, \overline{\rho_L (w_L - W_L)^2}^{L} \right\rangle \qquad (2.8)$$

$$L_T \triangleq -\, (1-\alpha)\, \frac{\partial}{\partial z} \left\langle \alpha\, \bar{\underline{\underline{T}}}_G^{G} \cdot \underline{n}_z \right\rangle \cdot \underline{n}_z + \alpha\, \frac{\partial}{\partial z} \left\langle (1-\alpha)\, \bar{\underline{\underline{T}}}_L^{L} \cdot \underline{n}_z \right\rangle \cdot \underline{n}_z \qquad (2.9)$$

Equation (2.6) requires *seven* closure laws for L_R, L_T, W_{GI}, W_{LI}, L_I, L_{FW}, L_{FI}. Moreover, retaining eq. (2.6) implies, at least in theory, retaining also the interfacial momentum balance, which requires one supplementary closure law.

In two-fluid models, eqs (2.2) and (2.6) are both implicitly written. A crucial point is that eq. (2.6) is *stiff*. This means that, due to the relative orders of magnitude of its terms, small variations on the closure laws (and in particular on the induced inertia and interfacial friction closure laws) induce large variations on P_{LG} (when P_{LG} is not merely ignored) and/or W_G. Hence the difficulty to adjust the very badly known closure laws of eq. (2.6) to avoid unrealistic values of the light phase velocity.

A second crucial point is that, whenever eq. (2.6) is closed by algebraic laws only, a non-hyperbolic set results : two characteristic velocities are complex conjugate, with the consequence that the model is unconditionally unstable.

In drift flux models, eq. (2.6) is not written, which is acceptable since the value of P_{LG} does not really matter in practice. The two foregoing difficulties are not encountered.

Besides the closure laws already mentioned, current two-fluid models are closed through an assumption on P_{LG} (the set of 6 phasic balance equations involves 7 dependent variables, namely two pressures, two velocities, two enthalpies and the void fraction). Such an assumption imposes an artificial constraint on the pressures and pressure gradients. It disturbs the description of the corresponding propagation phenomena.

Generalized drift flux models are closed through the *direct description of the void-drift dynamic dependency*. Assuming a quasi-linear, partial differential relationship of the first order, and using the convenient variables :

$$W \triangleq \alpha W_G + (1 - \alpha) W_L , \quad \delta \triangleq \alpha (1 - \alpha) (W_G - W_L) = \alpha (W_G - W) \quad (2.10)$$

(center of volume velocity and drift), it may be approached by :

$$\eta \left(\frac{\partial W}{\partial t} + W_2 \frac{\partial W}{\partial z}\right) + (W + W_4 - \Sigma) \frac{\partial \alpha}{\partial t} + (WW_4 - \Pi) \frac{\partial \alpha}{\partial z} + \frac{\partial \delta}{\partial t} + W_4 \frac{\partial \delta}{\partial z} = \frac{1}{\theta} (f - \delta) \quad (2.11)$$

Equation (2.11) requires *seven* closure laws, i.e. the same number as eq. (2.6) but with simpler physical significances and more straightforward consequences : f is the fully-developed drift value, θ a relaxation time, Σ and Π are respectively the sum and product of the two characteristic velocities corresponding to the kinematic waves, η expresses inertia effects, W_2 and W_4 are averaged velocities close to W.

It can be concluded that developing a closure set for eq. (2.11) and using generalized drift flux models appear as less hazardous and hopefully easier than developing a closure set for eq. (2.6) and using current two-fluid models.

III. INFLUENCE OF THE VOID-DRIFT CLOSURE ON THE PROPERTIES OF KINEMATIC WAVES ("Direct problem")

As long as the kinematic wave velocities are small with respect to the sonic velocity, the properties of small harmonic kinematic waves of the form

$$\bar{x} = \bar{x}_0 \; e^{i\,(\omega t - kz)} \tag{3.1}$$

where $\bar{x}_0$ (constant) ω and k are real or complex quantities, result from the approximate dispersion equation (Bouré, 1988b)

$$\omega - C_\alpha \, k + i \, \theta \, (\omega^2 - \Sigma \, \omega k + \Pi \, k^2) = 0 \tag{3.2}$$

where :

$$C_\alpha \triangleq W + f'_\alpha, \qquad \left(f'_\alpha \triangleq \frac{\partial f}{\partial \alpha}\right) \tag{3.3}$$

A first consequence is that the kinematic wave properties, which do not significantly depend on η, W_2, W_4, cannot be used to evaluate these quantities (η is related to the sonic velocity, W_2 and W_4 have only weak influences).

In the exploitable experiments (Tournaire, 1987, Bouré, 1988a) ω is imposed and the kinematic wave velocities V and their spatial amplification coefficients k_i result from the data processing. Eqs. (3.1) and (3.2) must therefore be used with ω real and :

$$k \triangleq k_r + i \, k_i \tag{3.4}$$

from which

$$\bar{x} = \bar{x}_0 \; e^{k_i \, z} \; e^{i\,(\omega t - k_r z)} \tag{3.5}$$

$$V = \frac{\omega}{k_r} \tag{3.6}$$

k_r, which does not depend on the frame of reference, can be used instead of ω, which does, to characterize a wave.

Introducing eqs. (3.4) and (3.6) in the dispersion equation (3.2) and separating the real and imaginary parts lead to :

$$k_r \; [V - C_\alpha + \theta \, \Sigma \, V \, k_i \; - 2 \, \theta \, \Pi \, k_i] = 0 \qquad (3.7)$$

$$- C_\alpha \, k_i \; + \theta \, V^2 \; k_r^2 - \theta \, \Sigma \, V \, k_r^2 + \theta \, \Pi \left(k_r^2 - k_i^2\right) = 0 \qquad (3.8)$$

Equations (3.7) and (3.8) provide the solutions to the direct problem, viz. computing the properties of the kinematic waves of wave number k_r when C_α, θ, Σ and Π are known. There are two solutions corresponding to two modes (noted with the subscripts 3 and 4). In the experiments it was found that modes 3 and 4 are respectively predominant at low void fractions ($0 < \alpha < 0.25$) and at "large" void fractions ($\alpha > 0.30$)

In particular for $k_r = 0$, the two solutions are :

$$k_i = 0 \qquad \text{with } \frac{k_i}{k_r^2} = \frac{\theta \; (C_\alpha - C_3) \; (C_\alpha - C_4)}{C_\alpha}, \qquad V = C_\alpha \qquad (3.9)$$

and

$$k_i = - \frac{C_\alpha}{\theta \; \Pi}, \quad V = C'_\alpha \qquad \text{with } \frac{1}{C'_\alpha} \triangleq \frac{1}{C_3} + \frac{1}{C_4} - \frac{1}{C_\alpha} \qquad (3.10)$$

C_3 and C_4 being defined by :

$$\Sigma \triangleq C_3 + C_4 \qquad \Pi \triangleq C_3 \; C_4 \qquad (3.11)$$

For $k_r \to \infty$, the two solutions are :

$$V = C_3 \qquad k_i = - \frac{C_\alpha - C_3}{\theta \; C_3 \; (C_4 - C_3)} \qquad (3.12)$$

$$V = C_4 \qquad k_i = - \frac{C_4 - C_\alpha}{\theta \; C_4 \; (C_4 - C_3)} \qquad (3.13)$$

In equations (3.7) and (3.8) the three quantities θ k_r, k_i are present only through the two products $(\theta\ k_r)$ and $(\theta\ k_i)$. In actual experimental runs, k_r is never zero and the equations may be written :

$$V - C_\alpha + (C_3 + C_4)\ V\ (\theta\ k_i) - 2\ C_3\ C_4\ (\theta\ k_i) = 0 \qquad (3.14)$$

$$- C_\alpha \frac{\theta\ k_i}{(\theta\ k_r)^2} + V^2 - (C_3 + C_4)\ V + C_3\ C_4 - C_3\ C_4 \frac{(\theta\ k_i)^2}{(\theta\ k_r)^2} = 0 \quad (3.15)$$

Since in the exploitable experimental data, V does not significantly depend on k_r (no significant dispersion), it is convenient to eliminate V between eqs. (3.14) and (3.15). Eq. (3.14) yields :

$$V = \frac{C_\alpha + 2\ C_3 C_4\ (\theta\ k_i)}{1 + (C_3 + C_4)\ (\theta\ k_i)} \qquad (3.16)$$

and eq. (3.15) yields :

$$V = \frac{C_3 + C_4}{2} \pm \sqrt{\frac{(C_4 - C_3)^2}{4} + C_\alpha \frac{\theta\ k_i}{(\theta\ k_r)^2} + C_3\ C_4 \frac{(\theta\ k_i)^2}{(\theta\ k_r)^2}} \qquad (3.17)$$

The solutions for θ k_i are then the real solutions of the equation

$$\frac{2\ C_\alpha - (C_3 + C_4) - (C_4 - C_3)^2\ (\theta\ k_i)}{1 + (C_3 + C_4)\ \theta\ k_i} =$$

$$\pm \sqrt{(C_4 - C_3)^2 + 4\ C_\alpha \frac{\theta\ k_i}{(\theta\ k_r)^2} + 4\ C_3\ C_4 \frac{(\theta\ k_i)^2}{(\theta\ k_r)^2}} \qquad (3.18)$$

Computing directly θ k_i as a function of θ k_r from eq. (3.18) is not straightforward. It is more convenient to transform eq. (3.18) to express θ k_r as a function of θ k_i

$$\frac{(\theta\ k_r)^2}{\theta\ k_i} = \frac{[C_\alpha + C_3\ C_4\ \theta\ k_i]\ [1 + (C_3 + C_4)\ \theta\ k_i]^2}{(C_\alpha - C_3)\ (C_\alpha - C_4) - (C_4 - C_3)^2 \theta\ k_i\ [C_\alpha + C_3 C_4\ \theta\ k_i]} \quad (3.19)$$

For a given value of k_i, eq. (3.19) yields zero or one real positive value of k_r.

IV. IDENTIFICATION OF THE VOID-DRIFT CLOSURE EQUATION FROM KINEMATIC WAVE PROPERTIES ("Inverse problem")

The problem posed in this paper is the determination of C_α, θ, Σ and Π, using the experimental data on kinematic waves. It is the inverse of the problem of section 3.

The principle of the method is to write eqs. (3.7) and (3.8), for instance, for several sets of experimental conditions for which k_r, V and k_i are known and to use the resulting equations to compute C_α, θ, Σ and Π.

In the exploitable data, as already noted, V does not depend significantly on k_r. Accordingly, when a single mode is predominant, V may be expected to be close to both C_3 (or C_4) and C_α (or C'_α). This has two consequences :

1. Whenever a single mode is predominant, the experimental correlation for V should be a correlation for C_α as well. This is corroborated by the fact that is satisfies the definition (3.3). In the experimental conditions of Tournaire (1987) (upward vertical flow, low pressure), it leads to (Bouré, 1988a, f and C_α - W in m/s) :

For $0 < \alpha < 0.2$ (mode 3 predominant)

$$\left.\begin{aligned} f &= 0.22\ \alpha\ (1-\alpha)\ [1 - 1.25\ \alpha\ (1-\alpha)] \\ C_\alpha - W &= 0.22\ (1 - 2\alpha)\ [1 - 2.5\ \alpha\ (1-\alpha)] \end{aligned}\right\} \quad (4.1)$$

For $0.3 < \alpha < 0.41$ (mode 4 predominant)

$$\left.\begin{array}{l} f = 0.22\,\alpha - 0.028 \\ C_\alpha - W = 0.22 \end{array}\right\} \qquad (4.2)$$

For $0.2 < \alpha < 0.3$ (the two modes coexist) the values of f and $C_\alpha - W$ may be interpolated between (4.1) and (4.2) with, from the experimental data :

$$C_\alpha - W \simeq 0.08 \qquad (4.3)$$

for $\alpha = 0.25$

2. When mode 3 (respectively mode 4) is predominant, $C_\alpha - C_3$ (respectively $C_4 - C_\alpha$) should be "small".

C_α being known and V eliminated, the problem may now be reformulated, eqs. (3.7) and (3.8) being replaced by eq. (3.18) or (3.19) to be solved for θ, C_3, C_4. Only mode 3 results are exploitable since mode 4 results for k_i are too few in number. In view of the forms of eqs. (3.18) and (3.19), the foregoing problem is far from simple. This is confirmed by fig. 1 in which, as suggested by eq. (3.19), the experimental results for $-k_r^2/k_i$ are plotted as a function of $-k_i$, and which exhibits an important scatter (in the representation of fig. 1, the points corresponding to $|k_i| < 0.1$ are subject to large errors and therefore meaningless. They were not plotted in the figure).

In the domain in which mode 3 is predominant, the conditions :

$$C_3 < C_\alpha < C'_\alpha < C_4 \qquad (4.4)$$

may be expected to hold (see Bouré, 1988a). They entail

$$\frac{C_\alpha - C_3}{\theta\, C_3\, (C_4 - C_3)} < \frac{1}{\theta\, (C_3 + C_4)} < \frac{C_4 - C_\alpha}{\theta\, C_4\, (C_4 - C_3)} < \frac{C_\alpha}{\theta\, C_3\, C_4} \qquad (4.5)$$

Then, as k_i decreases from zero, k_r^2/k_i resulting from eq. (3.19) starts from the value corresponding to eq. (3.9) with a slope which may be of either sign. However, when $|k_i|$ is sufficient, k_r^2/k_i also decreases. It tends towards $-\infty$ for the value corresponding to eq. (3.12). For the values of k_i comprised between those given by eqs. (3.12) and (3.13), there is no physical solution $\left(k_r^2 \leqslant 0\right)$. Finally, for the values of k_i comprised between those given by eqs. (3.13) and (3.10), there is a solution again corresponding to mode 4.

By trial and error, θ, C_3 and C_4 may be adjusted to fit the curve representing eq. (3.19) to mode 3 data. In view of the absence of dispersion C_3 may be expected to be close to C_α.

For $0 < \alpha \leqslant 0.20$: the following set of values is acceptable

$$\left.\begin{aligned} \theta &= 0.25 \text{ s} \\ C_\alpha - C_3 &= 0.02 \text{ m/s} \\ C_4 - C_\alpha &= 0.08 \text{ m/s} \end{aligned}\right\} \qquad (4.6)$$

Precise adjustement would need more accurate data for the wave velocities *and* especially for the damping/amplification coefficients. Such data does not exist and cannot be expected to be obtained soon in view of the available instrumentation. For $\alpha > 0.25$, no sufficiently accurate data is available.

Equations (4.1) and (4.6) confirm that, even at low pressure, the relevant velocity differences corresponding to fully developed conditions are fairly small in bubbly flows. They are negligible as soon as W is large enough (say 3 m/s). Accordingly the correlations for $C_\alpha - W$, $C_3 - W$, $C_4 - W$ are probably not crucial. On the other hand, the drift relaxation time θ is an essential parameter of the generalized drift flux model.

V. CONCLUSIONS

After a reminder on the main characteristics and the potential advantages of generalized drift flux models, the problem of the identification of the differential void-drift closure law they imply has been addressed.

Two advantages of generalized drift flux models versus complete two-fluid models are :

1. The correct description of the kinematic wave phenomena and the straightforward control and interpretation of the mathematical nature (hyperbolic or not) of the model set of partial differential equations that they enable.

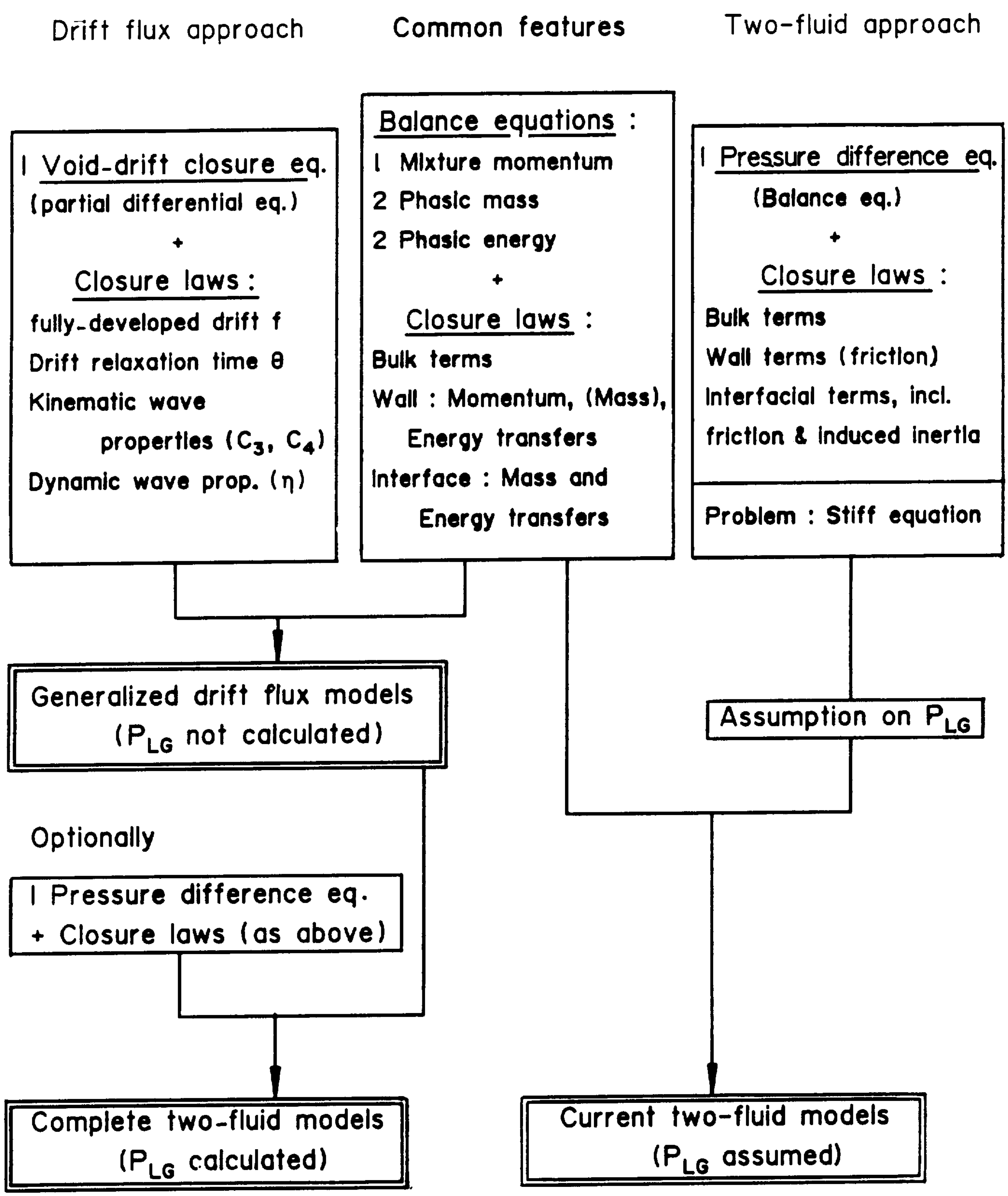
2. The simplification of the closure problem, involving only closure laws of simple physical significance and easy to assess.

On the other hand, it has been found that the available data on kinematic wave properties is not quite adequate to enable the identification of the void-drift closure law. Such an identification would need a very good accuracy (difficult to reach in practice) on the wave damping or amplification coefficients. Accordingly kinematic wave properties seem to be more useful as a closure validity test than as a source of quantitative information on the closure laws.

However, a void-drift closure, suitable for bubbly flows, has been adjusted on the available kinematic wave data. It involves several velocities which differ only slightly from each other and from the average fluid velocity but whichare necessary to the description of the kinematic wave properties. It also involves a drift relaxation time which is an essential parameter of generalized drift flux models and which was tentatively found to be of the order of 0.25 s for the upward flow of air-water mixtures at low pressure.

Table 1

Simplified comparison of modeling strategies



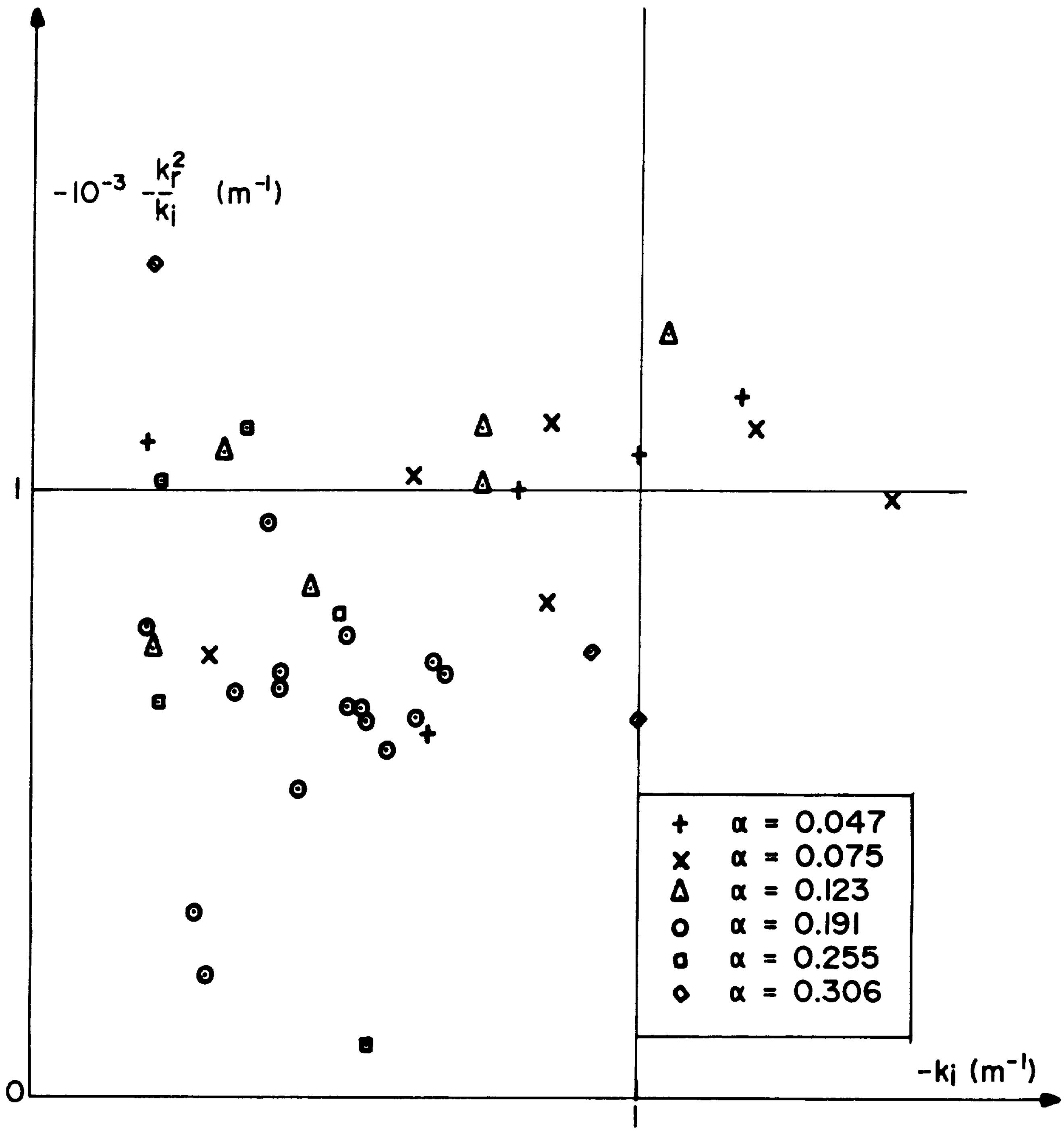


Fig. 1 : Experimental data (Mode 3)

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