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BOUNDARY-LAYER RECEPTIVITY AND LAMINAR-FLOW AIRFOIL DESIGN

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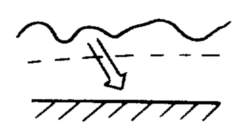
BOUNDARY-LAYER RECEPTIVITY

Boundary-layer receptivity examines the processes by which unsteady disturbances in the free-stream flow enter the boundary layer. In contrast, classical stability theory examines the evolution of disturbances that are already present in the boundary layer. Unsteady environmental disturbances of importance include free-stream turbulence, sound waves, and body vibration. Experimental evidence suggests that the receptivity process and the initial growth of the instability waves are well described by linear equations. Hence we consider linear, time-harmonic disturbances to the steady boundary-layer flow. The mathematical description of the receptivity process has the form of a boundary value problem, since the free-stream disturbances are specified. In contrast, classical stability theory leads to an eigenvalue problem in which the growth or decay rate of the disturbance is found, but in which the actual amplitude of the instability wave cannot be determined.

RECEPTIVITY

- Examines the mechanisms by which external disturbances enter the boundary layer.

- Boundary value problem

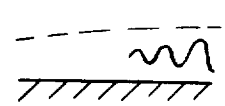


As Contrasted To

STABILITY THEORY

- Examines the evolution of disturbances that are already present in the boundary layer.

- Eigenvalue problem



IMPORTANCE OF RECEPTIVITY IN TRANSITION PREDICTION

Conceptually, the phenomenon of boundary-layer transition can be separated into three stages. These stages are the receptivity process, the linear growth of the instability wave, and the nonlinear breakdown into turbulence. The nonlinear breakdown is a violent process which occurs over a fairly short streamwise distance. Most of the distance between the airfoil leading edge and the transition point is covered by the receptivity and linear growth stages of the transition process. Hence, the details of the first two stages are most critical for prediction of the transition point.

Current transition prediction methods are based on linear stability theory, and hence consider only the second stage of the transition process. Linear stability theory cannot determine the amplitude of the instability waves, and hence the e^N criterion examines the ratio of the instability wave amplitude to its (unknown) amplitude at the neutral stability point. The amplitude ratio exponent N must be determined empirically by comparison with experiments and is found to be a strong function of the disturbance environment. A modified transition prediction method which combines receptivity with linear stability theory would have several advantages. The amplitude ratio criterion could be replaced by a critical amplitude criterion, the environmental disturbances would be directly accounted for, and the influence of the boundary-layer characteristics upstream of the neutral stability point would be included.

- Transition Involves Three Stages
  (1) Receptivity
  (2) Linear growth of instability
  (3) Nonlinear breakdown

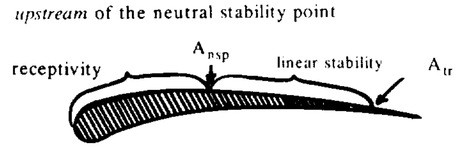
- First Two Stages Most Critical for Transition Prediction

- Current e^N Methods Consider Only Stage (2)

  $$e^N = \frac{A_{tr}}{A_{nsp}} = \text{amplitude ratio}$$

  N = empirical function of disturbance environment

- Method Combining Receptivity and Linear Growth
  - utilizes amplitude criterion (A_{tr})
  - directly accounts for disturbance environment
  - includes influence of boundary layer characteristics *upstream* of the neutral stability point



FUNDAMENTAL CONCEPTS OF RECEPTIVITY THEORY

While the importance of free-stream disturbances for the transition process has been recognized for many years, an appropriate mathematical description has been developed only recently (refs. 1, 2, 3 and 4). The fundamental ideas of this receptivity theory can be described as follows. The evolution of instability waves is governed by the Orr-Sommerfeld equation of linear stability theory. This equation assumes that, compared to the instability wave, the steady boundary-layer flow changes slowly in the streamwise direction. Boundary conditions representing free-stream disturbances may be imposed on the Orr-Sommerfeld equation, but these generate only particular solutions that are unrelated to the instability wave eigensolutions. This leads to the conclusion that the generation of the instability waves, or equivalently the receptivity process, must occur in regions where the boundary layer changes so rapidly that the Orr-Sommerfeld equation is invalid. The instability wave amplitude is then found by asymptotic matching of the receptivity region with the evolution region.

- Evolution of Instability Waves Governed by Orr-Sommerfeld Equation
  - assumes

$$\frac{1}{\delta}\frac{d\delta}{dx} << \frac{1}{\lambda_{TS}}$$

  - free-stream disturbances produce particular solutions that are unrelated to instability waves

- Generation of Instability Waves Occurs in Regions where O.S. Equation is Invalid

$$\frac{1}{\delta}\frac{d\delta}{dx} = O(\frac{1}{\lambda_{TS}})$$

- Amplitudes of Instability Waves Found by Asymptotic Matching of Generation and Evolution Regions

REGIONS WHERE RECEPTIVITY OCCURS

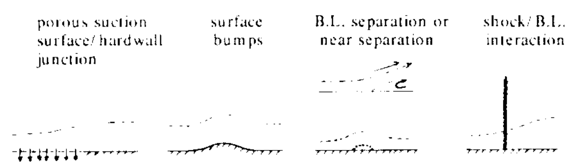
There are two classes of regions where receptivity occurs. The first is near leading edges, where the boundary layer is thin and growing rapidly. Since the boundary layer is thin, the pressure may be assumed constant across it and the disturbances are governed by the linearized, unsteady boundary-layer equation rather than by the Orr-Sommerfeld equation. In contrast to the O-S equation, the mean flow divergence enters at leading order in the unsteady boundary-layer equation. The interaction of the mean flow divergence with the unsteady disturbances imposed on the boundary layer by the free stream results in the generation of instability waves. The second class corresponds to regions further downstream where the boundary layer is forced to adjust on a streamwise length scale which is short compared to the body length. Examples of this class are wall suction - hardwall junctions, surface bumps and shock-boundary-layer interactions.. In these situations both the mean flow and the unsteady flow exhibit triple deck structures (refs. 5 and 6). The unsteady flow in the lower deck adjacent to the wall is again governed by the unsteady boundary layer equation, and the instability wave is generated by interaction between the unsteady motion and the mean flow divergence.

- Near Leading Edges
  - Boundary layer thin and growing rapidly
  - Disturbances governed by unsteady boundary layer equation

$$\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} + V\frac{\partial u'}{\partial y} + \underbrace{u'\frac{\partial U}{\partial x}}_{} + v'\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + \nu\frac{\partial^2 u'}{\partial y^2}$$

terms not present in O.S. equation

- In Regions of Rapid Boundary Layer Adjustment

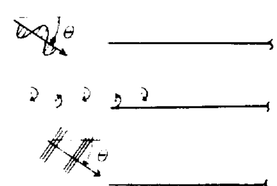


  - Mean flow and disturbance flow exhibit triple-deck structures
  - Disturbance flow in lower deck governed by unsteady boundary layer equation

LEADING-EDGE RECEPTIVITY

In order to assess the relative importance of various leading-edge receptivity mechanisms, we have examined the case of the Blasius boundary layer at low Mach numbers. Three types of free-stream disturbances have been considered: convected gusts, which are the linear representation of turbulence, the von Kármán vortex street which is produced by a vibrating ribbon, and oblique acoustic waves. An oblique acoustic wave at $\theta = 90'$ also represents plate transverse vibration. For each of these free-stream disturbances, a closed form solution for the inviscid interaction with the semi-infinite flat plate is first determined. The slip velocity on the plate surface is then calculated. This slip velocity has two distinct components: the slip velocity that would occur for the interaction of the free-stream disturbance with an infinite plate and a cylindrical acoustic wave generated by interaction with the leading edge. The slip velocity then provides the boundary condition for the numerical solution of the unsteady boundary-layer equation, and the receptivity coefficient is extracted from the large x behavior of the solution.

- Flat Plate Geometry, $M_\infty << 1$
- Free Stream Disturbances
  - convected gust (turbulence)
  - Von Karman vortex street
  - acoustic wave
    - $\theta = 90^\circ \rightarrow$ plate transverse vibration



- Inviscid Interaction with Semi-Infinite Plate
  - analytical solutions
  - slip velocity on plate surface contains two components
    - (1) infinite plate component
    - (2) diffracted acoustic wave from leading edge
- Slip Velocity Provides Boundary Condition for Numerical Solution of Unsteady Boundary Layer Equation
- Receptivity Coefficient Extracted from Large x Behavior of Unsteady Boundary Layer

EXTRACTION OF RECEPTIVITY COEFFICIENT

The asymptotic structure of the unsteady boundary-layer equation for large x consists of a particular solution plus an infinite set of asymptotic eigensolutions. The eigensolutions are asymptotic in the sense that they do not satisfy the unsteady boundary-layer equation for all x, but only for large x. Hence the coefficients B_n are not arbitrary, but rather are determined by the full solution for all x. The asymptotic matching of the unsteady boundary layer and Orr-Sommerfeld regions shows that the n = 1 asymptotic eigensolution of the unsteady boundary-layer equation matches with the unstable Tollmien-Schlichting wave. Hence the amplitude of the T-S wave is linearly proportional to B_1, and we call B_1 the receptivity coefficient. We determine B_1 by solving the unsteady boundary-layer equation numerically, and then examining the solution for large x. The asymptotic eigenvalues λ_n are ordered such that the n = 1 eigensolution is exponentially small for large x, making direct extraction of its coefficient difficult. In order to overcome this, the unsteady boundary-layer equation is solved along a ray in the "complex x plane", where the n = 1 eigensolution grows exponentially with downstream distance.
The receptivity coefficient B_1 is found simply by examining the ratio of the numerical solution to the n = 1 asymptotic eigensolution.

- Analytical Structure of Unsteady Boundary Layer as $x \to \infty$

$$u' = u_p' + \sum_{n=1}^{n=\infty} B_n g_n(x,\eta) e^{(1-i)\lambda_n x^{1/2}}$$

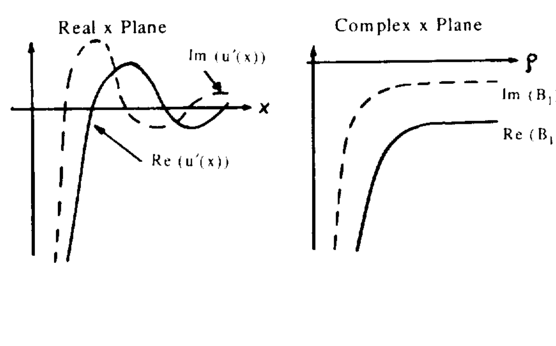
  - n = 1 eigensolution matches with T.S. wave

    B_1 = receptivity coefficient $\rightarrow A_{nsp}$

  - $\lambda_1 > \lambda_2 > \lambda_3 \ldots \rightarrow e^{-(1-i)\lambda_1 x^{1/2}}$ exponentially small as $x \to \infty$

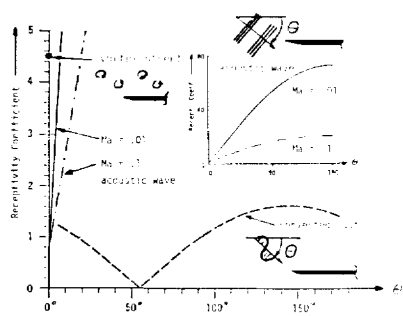
  - Numerical integration performed in "complex x" plane ($x = -i\rho$) where first eigensolution is exponentially large

$$B_1 = \lim_{\rho \to \infty} \left(\frac{\text{numerical solution}}{g_1 e^{\sqrt{2}\lambda_1 \rho^{1/2}}} \right)$$

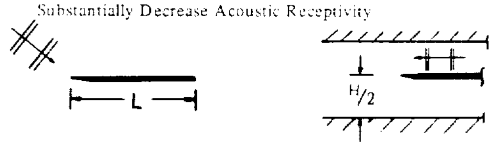


RESULTS FOR LEADING-EDGE RECEPTIVITY

The leading-edge receptivity coefficients for various free-stream disturbances are compared in the figure below. The disturbance characteristics are chosen such that the instability wave has the same frequency in all cases, and the amplitude of the velocity fluctuations at the location of the plate, but in its absence, is identical. Both the convected gust and acoustic wave receptivities have a strong dependence on disturbance orientation θ. The null at θ = 55' for the convected gust results from destructive interference between the instability waves generated by the infinite plate and leading-edge slip velocity components. The von Karman vortex street produces a receptivity value approximately 4 times the convected gust result. The parallel acoustic wave (θ = 0) receptivity is on the same order as that for the convected gust, but as θ increases the acoustic wave receptivity rises rapidly, by as much as two orders of magnitude for M = 0.01. The explanation for this behavior is in the strength of the cylindrical acoustic wave generated by the interaction of the free-stream disturbance with the leading edge. At low Mach numbers the strength of this scattered wave varies as $M^{-1/2}$. However, it should be noted that this behavior occurs only for the case of an isolated semi-infinite flat plate. We are presently investigating the influences of finite plate length and wind tunnel walls on leading-edge receptivity to acoustic waves at low Mach numbers.



- Convected Gust Receptivity Strong Function of Gust Angle
  - θ dependence due to relative phase of diffracted and infinite plate components
- Acoustic Wave Receptivity Very Strong at Low M_∞
  - dominated by diffracted wave, strength $\sim\lambda_{ac}^{1/2} \sim M^{-1/2}$
- Finite Plate Length or Wind-Tunnel Walls Should Substantially Decrease Acoustic Receptivity



RECEPTIVITY TO ACOUSTIC WAVES AT A SUCTION SURFACE - HARD WALL JUNCTION

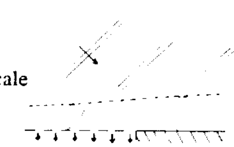
We are presently addressing one problem in the second class of receptivity mechanisms, namely the receptivity to free-stream acoustic waves which occurs at a junction between a suction surface and a hard wall. This particular problem is clearly relevant to hybrid laminar-flow design. In addition, since the instability waves decay exponentially upstream of the neutral stability point, even a weak receptivity mechanism close to the neutral stability point may be more important than a much stronger mechanism which occurs near the leading edge.

There are two receptivity mechanisms at the junction. The first is associated with the mean flow adjustment in the vicinity of the junction. This mean flow adjustment occurs over the triple deck length scale $L/Re^{3/8}$, where L is the body length and Re the Reynolds number. The T-S wavelength near the lower branch of the neutral stability curve is comparable, leading to efficient coupling. In addition to the mean flow adjustment, the change in wall admittance produces a diffracted acoustic wave whose short local length scale couples into the T-S wave. This second mechanism does not require wall suction. Since the phenomenon is linear, the two mechanisms can be linearly superposed.

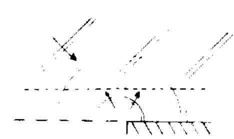
- Motivations
  - relevant to hybrid laminar flow design
  - exponential decay upstream of neutral stability point
    → receptivity mechanisms closest to nsp dominate

- Two Receptivity Mechanisms at Junction

  (1) Mean Flow Adjustment
  - adjustment occurs over triple deck scale ($L/Re^{3/8}$)
  - T.S. wavelength comparable



  (2) Wall Admittance Change
  - produces diffracted acoustic wave with short local scale
  - wall suction not required



- Mechanisms Can Be Linearly Superposed

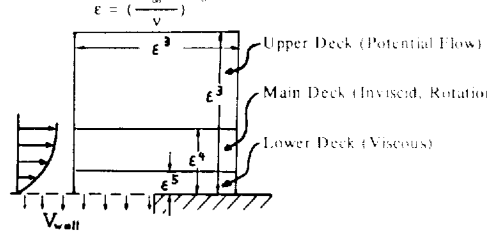
TRIPLE DECK STRUCTURE

Near the suction surface - hard wall junction, both the mean flow and the unsteady flow exhibit a triple deck structure. An important characteristic of the triple-deck theory is that the pressure field is not imposed on the boundary layer, but is determined by an interactive relationship between the upper and lower deck solutions. The basic physics of the triple deck is as follows. The short streamwise length scale causes the original boundary layer or main deck to respond inviscidly. A new, thinner boundary layer or lower deck is then necessary to satisfy the no-slip wall boundary condition. Finally, the rapid variation in boundary-layer displacement thickness induces irrotational motion with this same scale outside the boundary layer. This latter region is called the upper deck. The streamwise length scale of $L/Re^{3/8}$ is necessary for consistency between the decks.

We consider wall suction velocities of the same order as the standard boundary-layer scaling. The mean flow adjustment then satisfies a linear set of equations, and the solution can be found in closed form by Fourier transform techniques. The unsteady flow is found to be a small perturbation of a Stokes shear wave. This perturbation satisfies an inhomogeneous equation with a source term involving interaction between the mean flow adjustment and the Stokes wave. The wall admittance variation enters as an inhomogeneous boundary condition. The solution for the unsteady flow adjustment (u_1', v_1') is found in terms of a Fourier transform, and the amplitude of the T-S wave generated by the interaction is given by the residue of the appropriate pole of the transform.

- Interactive Pressure-Displacement Relationship
- Asymptotic Description of Mean Flow, Unsteady Flow, T.S. Waves

$$\varepsilon = \left(\frac{U_\infty L}{\nu}\right)^{-\frac{1}{8}}$$



- $\frac{V_{wall}}{U_\infty \varepsilon^4} = O(1) \rightarrow$ mean flow adjustment (U_1, V_1) satisfies linear eqs

- Unsteady Flow
  - main and upper decks quasi-steady
  - lower deck small perturbation to Stokes wave ($u' = u_{st}' + \varepsilon u_1'$)

$$L(u_1', v_1') = u_{st}' \frac{\partial U_1}{\partial x} + V_1 \frac{\partial u_{st}'}{\partial y}$$

unsteady b.l. eq — mean flow adjustment

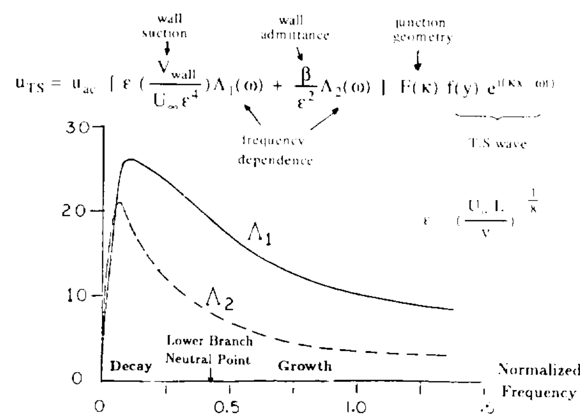
$$u'(x,0) = 0 \qquad v'(x,0) = \beta(x)p'$$

wall admittance change

  - T.S. wave = pole of Fourier Transform solution
  - Receptivity Coefficient = residue

SUCTION SURFACE JUNCTION RECEPTIVITY

The general structure of the T-S wave generated by the unsteady interaction at the junction is illustrated in the figure below. The amplitude of the instability wave is linear with respect to the free-stream acoustic wave and also with respect to the wall impedance and suction velocity. The two factors Λ_1 and Λ_2 are frequency dependent, while the junction geometry appears in the multiplicative factor $F(\kappa)$. The frequency dependence in Λ_1 and Λ_2 is fairly mild, with a maximum amplitude generated by frequencies corresponding to positions quite far upstream of the neutral stability point. In terms of proposed laminar-flow wall suction designs, the parameter $V_{wall}/U_\infty\varepsilon^4$ is on the order of 0.1 for distributed suction systems, and on the order of 1 for strip suction systems. We have not yet explored the influence of wall admittance in detail, but it is interesting to note that the wall admittance β is divided by ε^2, while the wall suction parameter is multiplied by ε. Thus, even small admittances may be important in the receptivity process.



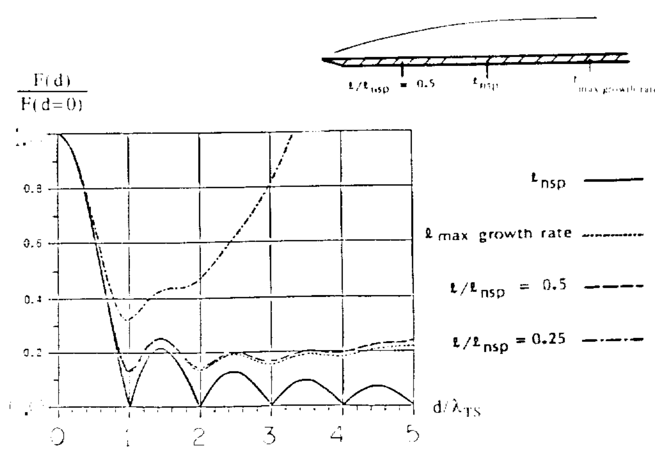
- Laminar Flow Designs
  - distributed suction $\frac{V_{wall}}{U_\infty \varepsilon^4} = O(0.1)$
  - suction strips $\frac{V_{wall}}{U_\infty \varepsilon^4} = O(1)$
- Even Small Wall Admittance May be Important

  $\beta = O(\varepsilon^3)$ corresponds to $\frac{V_{wall}}{U_\infty \varepsilon^4} = O(1)$

INFLUENCE OF SUCTION STRIP WIDTH

The figure below illustrates the influence of suction strip width on receptivity to free-stream acoustic waves. The total mass flow through the strip is held constant as strip width is varied, and hence the wall suction velocity is inversely proportional to the strip width. The Blasius profile is used for the undisturbed boundary layer. Results are shown for four strip locations. The quantity plotted is the receptivity coefficient divided by its narrow slot limit. It is seen that the maximum receptivity generally occurs for very narrow slots. For the slot located at the neutral stability point, a slot width equal to an integer number of T-S wavelengths produces a zero receptivity coefficient. Essentially, the instability wave generated at the front edge of the slot is cancelled by that generated at the rear edge.

Almost identical results are found for the slot located at 1/2 the distance from the leading edge to the neutral stability point and the slot located at the point of maximum instability growth rate. The general shape of these curves is similar to that for the neutral wave, but the minima at integer values of d/λ are nonzero. Basically, the growth or decay of the instability wave modifies the perfect cancellation between front and rear edges of the slot. However it can be seen that, by choosing a slot width equal to the instability wavelength, the receptivity can be reduced to 12% of the narrow slot limit. At the 1/4 point location closer to the leading edge the receptivity is not reduced as much by choosing $d/\lambda = 1$. Note also that, for sufficiently wide slots, the receptivity is actually higher than the narrow slot limit. This behavior is caused by the downstream displacement of the slot rear edge with increase in slot width.



- $\dfrac{F(d)}{F(d=0)} = \dfrac{\text{Receptivity Coefficient}}{\text{Narrow Slot Limit}}$ 
- Maximum Receptivity for Narrow Strips
- Receptivity Minimized for $\dfrac{d}{\lambda_{TS}} = 1$
  - reduction most significant for near neutral waves

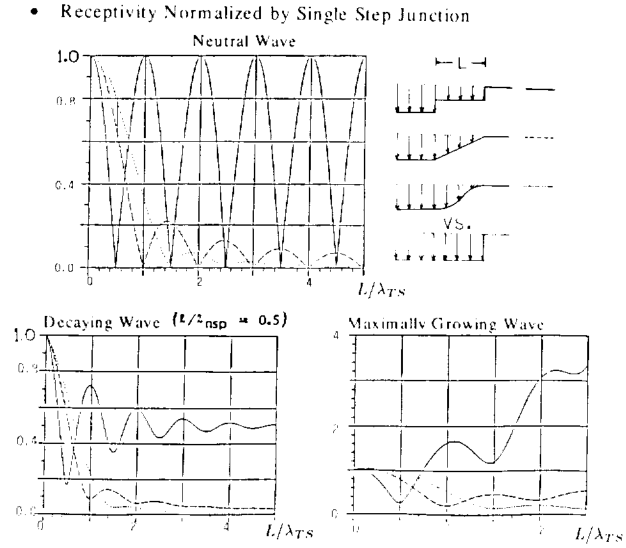
INFLUENCE OF JUNCTION SMOOTHING

For hybrid laminar-flow designs with distributed suction surfaces, the receptivity coefficient depends on the details of the transition from the suction surface to the downstream hard wall. To examine this dependence, we have compared the receptivity for a number of suction transition profiles. The case of a step discontinuity has been taken as the baseline. The additional profiles considered are a double step discontinuity, a linear variation and a cosine variation, as shown in the figure below. Three junction locations are considered: at the neutral stability point, at 1/2 the distance from the leading edge to the neutral stability point, and at the point of maximum growth rate. In all cases the ordinate is the receptivity coefficient normalized by the receptivity coefficient for the baseline case of the step discontinuity, and the abscissa is the transition length normalized by the instability wavelength.

The double step discontinuity generally has the largest receptivity coefficient and the cosine variation generally has the smallest. The results for the linear profile are surprisingly close to those for the cosine profile. A choice of transition profile length approximately equal to two instability wavelengths appears near optimum in most cases. Profile smoothing is less effective in reducing the receptivity coefficient for growing waves as compared to the neutral wave or decaying wave cases. In fact, for the maximally growing wave the double step discontinuity generally increases the receptivity coefficient as compared to the single step baseline case. Essentially, the double step junction has a discontinuity farther upstream, and the additional growth of this upstream generated wave negates the beneficial effects of spreading out the discontinuity in wall suction.

DISTRIBUTED SUCTION-HARD WALL JUNCTION

- Receptivity Normalized by Single Step Junction



- Transition Lengths Greater Than λ_{TS} Significantly Reduce Receptivity
- Smooth Transition Generally Beneficial
- Multiple Step Transition Increases Receptivity in Growing Wave Case

SUMMARY

In summary, receptivity examines the way in which external disturbances generate instability waves in boundary layers. Receptivity theory is complementary to stability theory, which studies the evolution of disturbances that are already present in the boundary layer. A transition prediction method which combines receptivity with linear stability theory would directly account for the influence of free-stream disturbances and also consider the characteristics of the boundary layer upstream of the neutral stability point. The current e^N transition prediction methods require empirical correlations for the influence of environmental disturbances, and totally ignore the boundary layer characteristics upstream of the neutral stability point.

The regions where boundary-layer receptivity occurs can be separated into two classes, one near leading edges and the other at downstream points where the boundary layer undergoes rapid streamwise adjustments. Analyses have been developed for both types of regions, and parametric studies which examine the relative importance of different mechanisms have been carried out. The work presented here has focused on the low Mach number case. Extensions to high subsonic and supersonic conditions are presently under way.

- Instability Waves Generated
  - in leading edge region
  - in regions of rapid boundary layer adjustment
- Leading Edge Receptivity
  - strongly dependent on disturbance type
  - oblique acoustic waves dominant at low M_∞
- Junction Receptivity Dependent on Geometry
- Suction Slot Receptivity
  - maximum for narrow slots
  - minimum for $\frac{d}{\lambda_{TS}} = 1$
- Distributed Suction - Hard Wall Junction Receptivity
  - minimized by transition lengths greater than λ_{TS}
  - smaller for smooth transitions
- Theories Currently Being Extended to Supersonic Conditions

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