This paper is primarily concerned with the phase 1 activity; namely, the rigorous mathematical optimization of a helicopter rotor system to minimize a combination of horsepower required at various flight conditions and hub shear transmitted from the rotor to the fuselage. The design will satisfy a set of design requirements (constraints) including those on blade frequencies, autorotational inertia, aerodynamic performance, and blade structural constraints. Additionally, the design is required to satisfy constraints imposed by response of the fuselage and also those constraints related to acoustics requirements.

**GENERAL APPROACH AND SCOPE**

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The general approach for the activity is illustrated in figure 1. In phase 1, the blade aerodynamic analysis, blade dynamics, and blade structural analysis are coupled and driven by the optimizer. The optimization of the blade aerodynamic geometry as well as the internal structure (spar, leading and trailing edge, ballast, etc.) takes place inside the box in figure 1. The influences of the airframe dynamics and acoustics are accounted for in terms of design requirements (constraints) on the blade design. These requirements are described in the next section of the paper. For a check on the efficacy of representing the acoustics requirements indirectly, the "final" design will be input to an acoustics analysis. The acoustics analysis calculates the acoustic constraints and derivatives of these constraints with respect to the design variables. This information will be used to determine how well the design was able to satisfy the actual acoustics design requirements.

The phase 2 procedure, wherein acoustics is fully integrated with the blade aerodynamics, blade dynamics, and blade structural analysis, is also illustrated in figure 1. The design produced in phase 2 (when converged) will satisfy acoustics goals. Airframe dynamics in phase 2, as in phase 1, is accounted for by effective constraints on the blade dynamics, aerodynamics, and structural behavior. Finally,
in phase 3 airframe dynamics is integrated and the result is a fully integrated optimization strategy.

This section of the paper consists of details of the integrated rotorcraft optimization problem. Included are descriptions of the following: the objective function (the quantity to be minimized for obtaining an optimum design); the design variables (dimensions and other parameters of the design); constraints (a set of behavioral or characteristic limitations required to assure acceptable and safe performance); and definitions of the interactions among the disciplines.

Objective Function

The objective function will consist of a combination of the main rotor horsepower at five flight conditions plus a measure of vibratory shear transmitted from the rotor to the hub. Although several multiple objective function techniques are available (ref. 11) one leading candidate is a linear combination whereby

\[ F = k_1 HP_1 + k_2 HP_2 + k_3 HP_3 + k_4 HP_4 + k_5 HP_5 + k_6 S \]  

where \( F \) is the objective function

\( k_1 \) through \( k_6 \) are weighting factors

\( HP_1 \) through \( HP_5 \) are required horsepower at various flight conditions

\( S \) is the vertical hub shear

A candidate set of flight conditions would be:

<table>
<thead>
<tr>
<th>Flight condition</th>
<th>Description</th>
<th>Velocity (kts)</th>
<th>Load factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hover</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>Cruise</td>
<td>140</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>High speed</td>
<td>200</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>Maneuver</td>
<td>120</td>
<td>3.5</td>
</tr>
<tr>
<td>5</td>
<td>Climb</td>
<td>1000 fpm (VROC)</td>
<td>-</td>
</tr>
</tbody>
</table>
Blade Model and Design Variables

Figure 2 is a depiction of the rotor blade model to be used in the phase 1 optimization activity. Also shown in figure 2 are the design variables which are defined in table 1. The blade model may be tapered in both chord and depth. The depth is linearly tapered from root to tip. The chord is constant from the root to a spanwise location (referred to as the point of taper initiation) and may be linearly tapered thereafter to the tip. Design variables which characterize the overall shape of the blade include the blade radius, point of taper initiation, taper ratios for chord and depth, the root chord, the blade depth at the root, the flap hinge offset, and the blade maximum twist. Tuning masses located along the blade span are characterized by the mass values and locations. Design variables which characterize the spar box beam cross section include the wall thicknesses at each spanwise segment and the ply thickness at 0° and ±45°. Additional design variables include the number of rotor blades, the rotor angular speed, and the distribution of airfoils.

Constraints

As previously described, the phase 1 activity is based on integrating the blade aerodynamic, dynamic, and structural analyses within the optimization procedure. The acoustics and airframe dynamics analyses are decoupled from the first three disciplines and their influences are expressed in terms of constraints. Accordingly, the total set of constraints is made up of two subsets. The first subset consists of constraints which are evaluated directly from the first three disciplinary analyses and are a direct measure of the degree of acceptability of the aerodynamic, dynamic, and structural behavior. The second subset represents indirect measures of the satisfaction of constraints on the acoustics behavior and the requirement of avoiding excessive vibratory excitation of the airframe by the rotor.

The constraints are summarized in table 2. The first two constraints are for aerodynamic performance and require that for all flight conditions, main rotor
horsepower not exceed available horsepower and that airfoil section stall not occur at any azimuthal location. The next nine constraints address blade dynamics. The first requires that the blade natural frequencies be bounded to avoid approaching any multiples of rotor speed. The next five impose upper limits on the blade vertical and inplane loads, transmitted hub shear, hub pitching, and rolling moments. The next three dynamic constraints are an upper limit on blade response amplitude, a lower limit on blade autorotational inertia, and finally, the aeroelastic stability requirement. The structural constraints consist of upper limits on box beam stresses, blade static deflection, and blade twist deformation. The acoustic constraints are expressed as an upper bound on the tip Mach number and an upper bound on the blade thickness to limit thickness noise; and an upper bound on the gradient of the lift distribution to limit blade vortex interaction (BVI) and loading noise. The effective airframe constraints are expressed first as a separation of the fundamental blade inplane natural frequency in the fixed system from the fundamental pitching and rolling frequency of the fuselage. Second is a bounding of the blade passage frequency to avoid the proximity to any fuselage frequency. The final constraint is an upper limit on the blade mass which will avoid any designs which satisfy the constraints at the expense of large mass increases.

Interdisciplinary Coupling

Phase 1 of the effort will utilize several design variables which have historically been significant drivers of disciplinary phenomena. In addition, other variables are being included to provide other unexplored design opportunities. Table 3 shows an attempt to quantify the interactions among the disciplines through the design variables. For example, rotor tip speed has driven past rotor designs based solely on acoustics, performance, or dynamics. This variable also influences blade structural integrity and fixed system response to transmitted loads. This provides the strong interdisciplinary coupling for tip speed shown in table 3. There
are variables, such as blade twist, which can strongly influence some disciplines, such as aerodynamics, while not perturbing others (e.g., structures) and other variables such as a hinge offset which, heretofore, have not greatly influenced conventional rotor design.

A significant part of the current effort will explore not only the obvious strong design variable couplings, but will also address those variables which may provide design synergism for multidisciplinary design goals. This may provide a design key for missions which have not been accomplished with today's rotorcraft.

Organization of System

The overall organization of the system to optimize a blade design for aerodynamics, dynamics, and structural requirements is shown schematically in figure 3. In order to perform the aerodynamic, dynamic, and structural analyses indicated in the blocks in figure 3, it is first necessary to transform or "pre-process" the design variables into quantities needed in the various analyses. For example, the dynamic and structural analyses both need stiffnesses $E_I$ and $GJ$, and laminate properties. The aerodynamic analysis needs lift and drag coefficients for the airfoils used. The above information is obtained by the design variable pre-processors which act as translators of the global design variables into local variables needed in the analyses. The output of each analysis block, in general, serves two purposes. First, response-type output may be transmitted to another analysis block (e.g., airloads from aerodynamics to dynamics); second, information entering into the objective function or constraints is supplied to the objective function and constraints block (e.g., stress constraints from the structural analysis). A key part of the procedure is the sensitivity analysis. This block corresponds to the calculation of derivatives of the constraints and objective function with respect to the design variables. The derivatives quantify the effects of each design variable on the design and,
thereby, identify the most important design changes to make enroute to the optimum design.

The sensitivity data are passed to the optimizer along with the current values of the design variables, constraints, and objective function. The optimizer uses the information to generate a new set of design variables, and the entire procedure is repeated until a converged design is obtained. For our purposes, a design is converged when all constraints are satisfied and the objective function has reached a value which has not changed for a specified number of cycles.

Optimization Algorithm

The basic optimization algorithm to be used in this work is a combination of the general-purpose optimization program CONMIN (ref. 12) and piecewise linear approximate analyses for computing the objective function and constraints. Since the optimization process requires many evaluations of the objective function and constraints before an optimum design is obtained, the process can be very expensive if complete analyses are made for each function evaluation. However, as Miura (ref. 3) points out, the optimization process primarily uses analysis results to move in the direction of the optimum design; therefore, a complete analysis needs to be made only occasionally during the design process and always at the end to check the final design. Thus, various approximation techniques can be used during the optimization to reduce costs. In the present work, the objective function and constraints will be approximated using piecewise linear analyses that consist of linear Taylor series expansions.

CONMIN.- CONMIN is a general-purpose optimization program that performs constrained minimization using a usable-feasible directions search algorithm. In the search for new design variable values, CONMIN requires derivatives of the objective function and constraints. The user has the option of either letting CONMIN determine the derivatives by finite differences or supplying such derivatives to CONMIN. The
second option will be used in this work. Analytical derivatives will be used whenever possible - for example for vibration frequencies, mode shapes, and modal shear. Eventual incorporation of the Global Sensitivity Equation (GSE) approach is planned. As described in reference 13, the GSE approach is potentially very effective for integrated problems such as a helicopter rotor. Finite difference schemes will be used for derivative calculations where analytical approaches are unavailable.

**Piecewise linear approximation.** In the approximate analysis method, derivatives of the objective function and constraint functions with respect to the design variables are used for linear extrapolation of these functions. The assumption of linearity is valid over suitably small changes in the design variable values and will not introduce a large error into the analysis provided the changes remain small.

Specifically, the objective function $F_0$, the constraints $g_0$, and their respective derivatives are calculated for the design variables $V_{o,k}$ using an accurate analysis. For example the aerodynamic performance constraints are supplied by CAMRAD (ref. 14). The first-order Taylor series approximations for the new objective function and the constraint values are as follows:

$$F = F_0 + \sum_{k=1}^{NDV} \frac{\partial F}{\partial V_k} (V_k - V_{o,k})$$

and

$$g = g_0 + \sum_{k=1}^{NDV} \frac{\partial g}{\partial V_k} (V_k - V_{o,k})$$

where $NDV$ is the number of design variables, $F$ is the extrapolated value of the objective function, $g$ is the extrapolated value of the constraint, and $V_k$ is the updated design variable value determined by CONMIN.

Errors introduced by the piecewise linear approach are controlled by imposing "move limits" on each design variable. Move limits are specified as fractional
This section of the paper deals with the aerodynamic performance aspects of rotor blade design. Design considerations, aerodynamic constraints and design variables are described.

Design Considerations

An important aspect of aerodynamic design of a helicopter rotor blade is the selection of the airfoils which could be applied over various regions of the blade radius. The choice of airfoils is controlled by the need to avoid exceeding the section drag divergence Mach number on the advancing side of the rotor disc, avoid exceeding the maximum section lift coefficients on the retreating side of the rotor disc, and avoid high oscillatory pitching moments on either side of the rotor disc. Since airfoils with high maximum lift coefficients are advantageous in high speed forward flight and pull-up maneuvers, high lift sections are generally used from the rotor blade root out to the radial station where the advancing side drag divergence Mach number precludes the use of the section. From that station outward, other airfoil sections which have higher drag rise Mach numbers are used.

Once the airfoils and an initial airfoil distribution are selected, the induced and profile power components become functions of twist, taper ratio, point of taper initiation, and blade root chord (ref. 16). For the hover condition, the majority of the power is induced power and the remainder is profile power. Rotor blade designs which minimize both induced and profile power are desirable. The induced power is a function of blade radius, chord, and section lift coefficient. The profile power is a function of blade radius, chord, and section drag coefficient. The induced and profile power can be reduced (provided the aerodynamics of all retreating blade