Application of a Self-Adaptive Grid Method to Complex Flows

G. S. Deiwert, E. Venkatapathy, C. Davies, J. Djomehri, and K. Abrahamson

July 1989
Application of a Self-Adaptive Grid Method to Complex Flows

G. S. Deiwert, Ames Research Center, Moffett Field, California
E. Venkatapathy, Eloret Institute, Sunnyvale, California
C. Davies, Sterling Software, Palo Alto, California
J. Djomehri, Ames Research Center, Moffett Field, California
K. Abrahamson, Boeing Military Airplane Co., Wichita, Kansas

July 1989
APPLICATION OF A SELF-ADAPTIVE GRID METHOD TO COMPLEX FLOWS

G. S. Deiwert, E. Venkatapathy,1 C. Davies,2 J. Djomehri,3 and K. Abrahamson 4

Ames Research Center

ABSTRACT

A directional-split, modular, user-friendly grid point distribution code is applied to several test problems. The code is self-adaptive in the sense that grid point spacing is determined by user-specified constants denoting maximum and minimum grid spacings and constants relating the relative influence of smoothness and orthogonality. Estimates of truncation error, in terms of flow-field gradients and/or geometric features, are used to determine the point distribution. Points are redistributed along grid lines in a specified direction in an elliptic manner over a user-specified subdomain, while orthogonality and smoothness are controlled in a parabolic (marching) manner in the remaining directions. Multidirectional adaption is achieved by sequential application of the method in each coordinate direction. The flow-field solution is redistributed onto the newly distributed grid points after each unidirectional adaption by a simple one-dimensional interpolation scheme. For time-accurate schemes such interpolation is not necessary and time-dependent metrics are carried in the fluid dynamic equations to account for grid movement.

THE METHOD

The grid adaption method used herein is a pointwise redistribution of node points based on an approximation of the three-dimensional variational method of Brackbill and Saltzman (1982). Their variational method is analogous to minimizing the energy of a set of grid points which can be imagined as being connected by tension springs that are coaligned with the grid lines. The strengths of the springs are proportional to the gradients of the flow-field solution and are, in some sense, related to the discretization error of the solution. The Brackbill and Saltzman scheme is rigorous and robust but is computationally intensive for multidimensional flow fields. To make the method more efficient, Nakahashi and Deiwert (1984,1985) introduced the concept of directional splitting proposed by Gnooffo (1982) and Dwyer (1982). To keep the method truly multidimensional, torsion springs along the grid lines were introduced at each grid point to bring in the influence of the other directions. The strengths of these torsion springs were adjusted to control both the smoothness and the degree of orthogonality of the grid. To eliminate the problems of excessive computer time associated with multidimensional ellipticity, the torsion springs are constrained to have one-sided influence. Hence, the grid points are redistributed elliptically in a single split direction according to tension springs while grid smoothness and orthogonality control are maintained in a one-sided manner in which spacewise marching techniques can be used. The redistribution of the flow-field solution is also simplified in that one-dimensional interpolation can be used to determine the solution on the redistributed

1 Eloret Institute.
2 Sterling Software.
3 NRC Fellow.
4 Boeing Military Airplane Co.
set of points. Multidirectional adaption can be achieved by a sequence of split one-dimensional adaptions. The only significant penalty to using unidirectional splitting to achieve computational efficiency is the loss of direct control of computational cell volumes as a constraint on the variational scheme. The effect of constraining the influence of torsion springs to a single side should not be considered a serious penalty, since the additional precision realized by two-sided influence is within the uncertainty of optimal grid determination. The advantages of computational efficiency and ease of use outweigh this loss of precision. It should be noted, however, that the redistribution of grid points with a directionally split method and one-sided torsion control does not result in a unique solution. Rather the solution is dependent both on the order of directional splitting and on the direction of the one-sided torsion control.

To make the method user friendly and robust, Nakahashi and Deiwert (1985) introduced the concept of "self adaption." In this approach the principal control parameters specified by the user are: 1) the minimum and maximum grid spacing in the split coordinate direction, 2) the magnitude of the torsional influence, and 3) the relative importance of grid smoothness and orthogonality. The effect of each of these parameters will be illustrated in this paper. Other control parameters, such as the adaption domain, the sequence of splitting for multidirectional adaption, and the direction for the one-sided torsion springs, are also user-specified but are less critical in the determination of an acceptable grid.

CONTROL PARAMETERS

The parameters $\Delta s_{\text{min}}$ and $\Delta s_{\text{max}}$ control the relative density of the redistributed points and are the maximum and minimum allowable grid spacings. They are input proportioned to the average spacing over the adapted domain, e.g., $\Delta s_{\text{max}} = 2.0$ will constrain a converged redistribution to maintain a maximum grid spacing equal to twice the average step size. Note that because of additional constraints, such as smoothness and orthogonality, this control of grid spacing need not be precisely maintained. A torsion control parameter, $\lambda$, determines the torsion spring coefficient and controls the relative influence of the torsion parameter with respect to the tension parameter. Values for this parameter are required for each direction other than the principal split direction. A value of zero will turn off the torsion control, the effect of which may result in oscillatory grid lines which have been adapted to the "noise" in the solution. Large values of $\lambda$ will diminish the influence of the flow-field gradients (tension spring control) and minimize alignment to high gradient regions. The direction of the torsion control is determined by $C_t$ (whereas $\lambda$ determines magnitude). Values of $C_t$ at or near zero favor orthogonality and values at or near unity favor smoothness.

The default values of the principal control parameters set in the computer code are as follows:

1. $\Delta s_{\text{min}} = 0.5$ and $\Delta s_{\text{max}} = 2.0$ (0.01 < $\Delta s_{\text{min}}$ < 1.0 and 1.0 < $\Delta s_{\text{max}}$ < 10.0 are typical ranges)
2. $\lambda = 0.001$ (where 0.00001 < $\lambda$ < 0.5 is a typical range)
3. $C_t = 0.5$ (where 0 < $C_t$ < 1.0)
ADAPTATION PARAMETERS

As mentioned previously, gradients of the flow-field solution are typically used to determine the tension spring constants for redistribution of grid points. On a uniform grid these gradients are directly proportional to the discretization error, and redistribution of points based on these gradients results in a minimization of the discretization error under the imposed constraints. In practical applications, such functions as density gradient, pressure gradient, and momentum flux gradient are typically used to determine the tension spring constants. Weighted combinations of these functions can be used as well. It is at the user's discretion to select the most appropriate combination of flow-field gradients to be used in grid point redistribution.

An additional feature of the current method is the option to use body surface curvature and other functions to distribute points based on geometry definition and preferred stretching functions. In this manner the method can be used as an initial grid generator and cluster points in regions of high curvature, near body walls, and near singular points. An example of this application is given later.

Finally, it is a straightforward application of the method to add grid lines in regions where gradients are large and grid distribution is sparse. This method of grid enrichment is comparable to other methods commonly called grid-adaption methods.

EXAMPLE APPLICATIONS

To illustrate the effect of the principal control parameters (grid spacing, torsion strength, and smoothness/orthogonality control) an example of supersonic channel flow with shock reflection and expansion is considered. Mach 2.5 viscous flow through a variable-width channel with one side contracted inward in a straight segment is simulated using the thin-layer Navier-Stokes equations. The flow simulations were performed by Abrahamson (1988) using the PARC2D code. Shown in figure 1 are the initial grid (fig. 1a) and resulting isopycnics (fig. 1b). The grid is composed of 101 points in the streamwise direction and 81 across the channel. Clustering has been imposed in the streamwise direction at the corners of the upper wall and in the transverse direction near the channel walls where viscous effects are important. Flow is from left to right and a shock emanates from the upstream upper wall corner and is seen to reflect off the lower wall (fig. 1b). An expansion fan emanates from the downstream upper wall corner and interacts with the reflected shock.

The grid is first adapted to the solution (density gradient) in the streamwise direction only. Shown in figure 2 is the adapted grid (fig. 2a) and resulting computed isopycnics (fig. 2b) for parameter values of $\Delta s_{\text{max}} = 2.0$, $\Delta s_{\text{min}} = 0.5$, $\lambda = 0.001$, and $C_r = 0.5$. The improvement in resolution of the incident and reflected shock is obvious. Furthermore, the pressure rise behind the reflected shock is significantly improved over the unadapted solution.

The effect of increasing and decreasing the torsion spring force is shown in figures 3 and 4 for values of $\lambda = 0.01$ and 0.0001, respectively. It can be seen in figure 3a that for large values of the torsion parameter there is very little alignment with the oblique shocks and expansion waves, and hence, little improvement in resolution of these features. Conversely, for smaller values of the torsion parameter the clustering and
alignment and resulting resolution of the solution is increased. The torsion parameter cannot, however, be taken to the zero limit or the control of grid smoothness and orthogonality would be lost. Also observable from these results is the fact that grid alignment is clearly an important attribute of this scheme for which grid enrichment (additional points) is an unsatisfactory substitute.

When the smoothness parameter, $C_t$, is set equal to unity, a strongly aligned grid with high gradient regions results, as shown in figure 5, and when the parameter is set equal to zero to enhance orthogonality a grid results that is significantly less clustered in these regions, as shown in figure 6. Here the orthogonality control is one-sided (from the top down), corresponding to the one-sided tension control.

The effect of changing the maximum and minimum grid spacing is shown in figure 7 where $\Delta s_{\text{max}} = 4.0$ and $\Delta s_{\text{min}} = 0.25$. The results here are markedly similar to those where the smoothness parameter, $C_t$, was set to unity (fig. 5). Clearly, alignment with additional clustering improves the resolution of the high-gradient features of the flow. When both the maximum and minimum grid spacings are set to unity, a uniformly spaced grid with no alignment to the high gradient regions results.

To assess the effect of direction, the uniformly spaced streamwise grid is adapted in the cross-flow direction, between the two boundary layers, to gradients of the streamwise momentum. Typical results are shown in figure 8. Here the principal control parameters were set at the default values. Compared to streamwise adapted results (fig. 2b) the isopycnics in figure 8b show a crisper incident shock, but with less definition in the reflection region where there is a nearly vertical Mach stem. The decrease of resolution in the shock-reflection region is due to the lack of clustering in the streamwise direction. Hence, when the high-gradient regions are obliquely aligned with the primary grid direction, there is no preferred direction for adapting. In these cases both grid alignment and clustering are important to achieve improved resolution. However, when the gradients are essentially coaligned with one of the principal grid directions, adaption across that direction is important. Finally, if two-directional adaption is used across obliquely oriented discontinuities, it is necessary to assign a dominant role to one direction or the two families of grid lines will be competing to both align and cluster to the discontinuity. This observation carries over to three-dimensional adaption schemes as well.

To assess the method for cases where there are multiple discontinuities which may be coaligned with more than one principal grid coordinate direction, it is necessary to consider multidirectional adaption. Examples of two-directional adaption are shown for complex plume flow fields in figures 9 and 10 and for a shock impingement on a cowl lip in figure 11. Each of these examples have been previously published (Venkatapathy and Feiereisen, 1989; Ruffin and Venkatapathy, 1989; Klopfer and Lee, 1988), where they have been discussed in some detail. Shown in figure 9a is the initial- and solution- (Mach and temperature gradients) adapted grids for a supersonic jet in a quiescent or very low subsonic stream (Venkatapathy and Feiereisen, 1989). Computed Mach contours (on the adapted grid) and an experimental shadowgraph are shown in figure 9b. It would not be possible to obtain such good agreement with experiment with the number of grid points used unless they were judiciously placed by such an adaptive scheme. Shown in figure 10a is the adapted grid for a hypersonic one-sided nozzle and the adaption was based on Mach and temperature gradients. Computed Mach contours on the adapted grid are shown in figure 10b (Ruffin and Venkatapathy, 1989). An experimental program is under way at NASA Ames to corroborate these results. The third example is the cowl/lip shock interaction problem described by Klopfer and Yee (1988). In this case
the initial grid was stretched from the lip surface to the outer boundary and straight radial grid lines were uniformly distributed around the lip. Shown in figure 11a is the grid which has been adapted in two directions to density gradient, and in figure 11b is shown the computed Mach contours.

Finally, two examples of three-dimensional adaptions reported by Djomehri and Deiwert (1988) are presented here. In the first example, the three-dimensional grid for the two-nozzle afterbody flow, shown as Mach contours in figure 12a, is adapted based on Mach gradients. The adapted grid is shown in figure 12b. The application of the present adaptive grid scheme as an initial grid generator is best illustrated in figure 13, where the geometry of the bump is used to control grid clustering. These results are discussed in the above-mentioned paper and will not be described here.

CONCLUDING REMARKS

A self-adaptive grid procedure for multidimensional flows has been identified and discussed. Several illustrative examples which show the influence of user-specified control parameters and the application to complex two- and three-dimensional flows have been presented and discussed. A fully documented, user-friendly version of a two-dimensional adaptive grid code (SAGE) is available from the authors (Davies and Venkatapathy, 1989). A three-dimensional version of this code is in the final stages of preparation. The method has been applied to unsteady flows and to solutions determined by finite-volume algorithms. It is straightforward to add or delete grid lines when adapting with this method. Large and/or complex flow fields can be divided into subsets and the method can be applied to these subsets separately. The method has also been applied to patched and overlapping grid structures and has been shown to be ideally suited for block/pencil data structures that have been used in vectorized codes for algorithms using directional splitting concepts.
REFERENCES


Figure 1a.– Initial grid for the supersonic channel flow.

Figure 1b.– Isopycnics from the computed solution using the initial grid shown in figure 1a.
Figure 2a.– Adapted grid with default control parameters.

Figure 2b.– Isopycnics from the computed solution using the adapted grid shown in figure 2a.
Figure 3a.– Adapted grid with $\lambda = 0.01$ and other control parameters set to default values.

Figure 3b.– Isopycnics from the computed solution using the adapted grid shown in figure 3a.
Figure 4a.— Adapted grid with $\lambda = 0.0001$ and other control parameters set to default values.

Figure 4b.— Isopycnics from the computed solution using the adapted grid shown in figure 4a.
Figure 5.— Adapted grid with $C_t = 1.0$ and other control parameters set to default values.

Figure 6.— Adapted grid with $C_t = 0.0$ and other control parameters set to default values.
Figure 7. Adapted grid with $\Delta s_{\text{max}} = 4.0$ and $\Delta s_{\text{min}} = 0.25$ and other control parameters set to default values.
Figure 8a.— Cross-flow-adapted grid with control parameter set to default values.

Figure 8b.— Isopycnics from the computed solution using the adapted grid shown in figure 8a.
Figure 9a.— Initial (top half) and adapted (bottom half) grids for the plume flow problem (Venkatapathy and Feiereisen, 1989).

Figure 9b.— Computed Mach contours (top half) on the adapted grid and the experimental shadowgraph (bottom half).
Figure 10a.— Adapted grid for the hypersonic one-sided nozzle problem (Ruffin and Venkatapathy, 1989).

Figure 10b.— Mach contours from the adapted solution for the hypersonic one-sided nozzle (Ruffin and Venkatapathy, 1989).
Figure 11a.— Adapted grid for the cowl/lip shock interaction problem (Klopfer and Yee, 1988).

Figure 11b.— Mach contours (initial solution) for the cowl/lip shock interaction problem (Klopfer and Yee, 1988).
Figure 12a.— Mach contours (initial solution) for the two-nozzle plume flow (Djomehri and Deiwert, 1988).

Figure 12b.— Three-dimensional adapted grid for the two-nozzle plume flow (Djomehri and Deiwert, 1988).
Figure 13.—Initial grid around a three-dimensional supersonic bump adapted based on geometric constraints (Djomehri and Deiwer, 1988).
A directional-split, modular, user-friendly grid point distribution code is applied to several test problems. The code is self-adaptive in the sense that grid point spacing is determined by user-specified constants denoting maximum and minimum grid spacings and constants relating the relative influence of smoothness and orthogonality. Estimates of truncation error, in terms of flow-field gradients and/or geometric features, are used to determine the point distribution. Points are redistributed along grid lines in a specified direction in an elliptic manner over a user-specified subdomain while orthogonality and smoothness are controlled in a parabolic (marching) manner in the remaining directions. Multidirectional adaption is achieved by sequential application of the method in each coordinate direction. The flow-field solution is redistributed onto the newly distributed grid points after each unidirectional adaption by a simple one-dimensional interpolation scheme. For time-accurate schemes such interpolation is not necessary and time-dependent metrics are carried in the fluid dynamic equations to account for grid movement.