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FINAL REPORT ON NASA GRANT NO. NAG-8-755, OPTIMAL TRAJECTORIES FOR THE AEROASSISTED FLIGHT EXPERIMENT, 1988-89

by

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RICE UNIVERSITY

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Abstract. This paper summarizes the research on optimal trajectories for the aeroassisted flight experiment, performed by the Aero-Astronautics Group of Rice University during the period 1988-89. This research includes the following topics: equations of motion in an Earth-fixed system; equations of motion in an inertial system; formulation of the optimal trajectory problem; results on the optimal trajectory problem; guidance implications.

Key Words. Flight mechanics, equations of motion, hypervelocity flight, atmospheric flight, aeroassisted flight experiment, aeroassisted orbital transfer, guidance.
1. **Introduction**

The objective of this research is to study optimal trajectories for the aeroassisted flight experiment (AFE). This flight experiment simulates a GEO-to-LEO transfer by an aeroassisted orbital transfer vehicle (AOTV), to be flown on board of the space shuttle in 1994.

During the atmospheric pass, the angle of attack of the AFE vehicle is kept constant, \( \alpha = 17 \) deg, and the angle of bank \( \mu \) is controlled in such a way that the total characteristic velocity is minimized, subject to two major constraints: (a) after the atmospheric pass, the AFE spacecraft must ascend to a specified low Earth orbit \( (h = 178 \) NM), where the motion is circularized; and (b) the exit orbital plane of the AFE spacecraft is identical with the entry orbital plane, which in turn is identical with the shuttle orbital plane. With reference to (a), two possible maneuvers are considered: (IA) indirect ascent to a 178 NM orbit via a 197 NM apogee; and (DA) direct ascent to a 178 NM orbit.

The methods of optimal control theory have been employed in conjunction with the sequential gradient-restoration algorithm (SGRA) in order to minimize the total characteristic velocity, subject to the specified constraints.
2. Research Results

The research undertaken with the present grant has led to the publication of four Aero-Astronautics Reports (Refs. 1-4) and one paper, presented at the 40th Congress of the International Astronautical Federation, Malaga, Spain, 1989 (Ref. 5).

References 1-2 are introductory in nature and deal with the derivation of the equations of motion for the AFE spacecraft in both an Earth-fixed system (Ref. 1) and an inertial system (Ref. 2).

References 3-5 deal with the problem of the optimal trajectories of the AFE spacecraft. The major findings are summarized below.

As explained, two possible transfers are considered: (IA) indirect ascent to a 178 NM perigee via a 197 NM apogee; and (DA) direct ascent to a 178 NM apogee. For both transfers, two cases are investigated: (i) the bank angle is continuously variable; and (ii) the trajectory is divided into segments along which the bank angle is constant. For case (ii), the following subcases are studied: two segments, three segments, four segments, and five segments; because the time duration of each segment is optimized, the above subcases involve four, six, eight, and ten parameters, respectively. For comparison purposes and only for Transfer (IA), a reference trajectory is also considered: this is a five-segment trajectory, close to the nominal trajectory given in Ref. 6.

From the extensive numerical computations, the following conclusions arise:
(i) The optimal trajectories are two-subarc trajectories, with the bank angle constant in each subarc; hence, the control is bang-bang.

(ii) In the atmospheric entry phase, the bank angle is near 180 deg, yielding a positive projected lift $L_p$, which in turn causes the path inclination to increase gradually from the entry negative value to nearly zero value.

(iii) In the atmospheric exit phase, the bank angle is near 0 deg, yielding a negative projected lift $L_p$, which offsets the centrifugal force effects due to the curvature of the Earth, so as to ensure exit conditions compatible with the desired apogee requirement.

(iv) The lateral component of the lift during the atmospheric entry phase and the lateral component of the lift during the atmospheric exit phase have the same sign and the same order of magnitude; they are directed in such a way that they nearly offset the effects due to the Earth's rotation. In this way, the instantaneous orbital plane is almost identical with the initial orbital plane, meaning that the wedge angle $\eta$ is nearly zero during the atmospheric pass. This means that, for efficient flight, the motion of the AFE spacecraft is nearly planar in an inertial space; in other words, one must avoid energy dissipation associated with the lateral motion.

(v) Comparison of the optimal trajectories and the reference trajectory shows that the OTs are superior to the RT in terms of the main quantities of interest, namely, the characteristic
velocity, the peak dynamic pressure, the peak heating rate, and the peak wedge angle. In particular, for Transfer (IA), the characteristic velocity is $\Delta V = 98.0 \text{ m/sec}$ for the RT and $\Delta V = 81.9 \text{ m/sec}$ for the OT. For Transfer (DA), the characteristic velocity of the OT is 72.0 m/sec.

(vi) The results on optimal trajectories have important guidance implications. They suggest the idea of developing a new guidance scheme based on the following precepts: to control both the angle of attack (near 17 deg) and the angle of bank; and to utilize a single switch of the bank angle, so as to have $L_p > 0$ in the entry phase and $L_p < 0$ in the exit phase. This new guidance system is promising, and it appears to be an improvement of the existing guidance scheme in the control of the lateral motion of the AFE spacecraft.

For the abstracts of Refs. 1-5, see Section 3.

For more information on the guidance implications of the results on optimal trajectories, see Section 4. These guidance implications are not only of interest within the frame of the AFE vehicle, but are also important within the frame of Mars penetration vehicles, Mars return vehicles, and lunar return vehicles.
3. Abstracts of Publications


Abstract. This report is the first of a series dealing with the determination of optimal trajectories for the aeroassisted flight experiment (AFE). The AFE refers to the study of the free flight of an autonomous spacecraft, shuttle-launched and shuttle-recovered. Its purpose is to gather atmospheric entry environmental data for use in designing aeroassisted orbital transfer vehicles (AOTV).

It is assumed that: the spacecraft is a particle of constant mass; the Earth is rotating with constant angular velocity; the Earth is an oblate planet, and the gravitational potential depends on both the radial distance and the latitude; however, harmonics of order higher than four are ignored; the atmosphere is at rest with respect to the Earth.

Under the above assumptions, the equations of motion for hypervelocity atmospheric flight (which can be used not only for AFE problems, but also for AOT problems and space shuttle problems) are derived in an Earth-fixed system. Transformation relations are supplied which allow one to pass from quantities computed in an Earth-fixed system to quantities computed in an inertial system, and vice versa.

Abstract. This report is the second of a series dealing with the determination of optimal trajectories for the aeroassisted flight experiment (AFE). The AFE refers to the study of the free flight of an autonomous spacecraft, shuttle-launched and shuttle-recovered. Its purpose is to gather atmospheric entry environmental data for use in designing aeroassisted orbital transfer vehicles (AOTV).

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Under the above assumptions, the equations of motion for hypervelocity atmospheric flight (which can be used not only for AFE problems, but also for AOT problems and space shuttle problems) are derived in an inertial system. Transformation relations are supplied which allow one to pass from quantities computed in an inertial system to quantities computed in an Earth-fixed system, and vice versa.

Abstract. This report is the third of a series dealing with the determination of optimal trajectories for the aeroassisted flight experiment (AFE). The intent of this experiment is to simulate a GEO-to-LEO transfer, where GEO denotes a geosynchronous Earth orbit and LEO denotes a low Earth orbit. Specifically, the AFE spacecraft is released from the space shuttle and is accelerated by means of a solid rocket motor toward Earth, so as to achieve atmospheric entry conditions identical with those of a spacecraft returning from GEO. During the atmospheric pass, the angle of attack is kept constant, and the angle of bank is controlled in such a way that the following conditions are satisfied: (a) the atmospheric velocity depletion is such that, after exiting, the AFE spacecraft first ascends to a specified apogee and then descends to a specified perigee; and (b) the exit orbital plane is identical with the entry orbital plane. The final maneuver, not analyzed here, includes the rendezvous with and the capture by the space shuttle.

In this report, the trajectories of an AFE spacecraft are analyzed in a 3D-space, employing the full system of 6 ODEs describing the atmospheric pass. The atmospheric entry conditions are given, and the atmospheric exit conditions are adjusted in such a way that requirements (a) and (b) are met, while simultaneously minimizing the total characteristic velocity, hence the propellant consumption
required for orbital transfer. Two possible transfers are considered: (IA) indirect ascent to a 178 NM perigee via a 197 NM apogee; and (DA) direct ascent to a 178 NM apogee.

For both transfers, two cases are investigated: (i) the bank angle is continuously variable; and (ii) the trajectory is divided into segments along which the bank angle is constant. For case (ii), the following subcases are studied: two segments, three segments, four segments, and five segments; because the time duration of each segment is optimized, the above subcases involve four, six, eight, and ten parameters, respectively.

A surprising result of the analysis is that the optimal trajectories of cases (i) and (ii) coalesce into a single trajectory: a two -subarc trajectory, with the bank angle constant in each subarc (bang-bang control). Specifically, the bank angle is near 180 deg in the atmospheric entry phase (positive lift projection phase) and is near 0 deg in the atmospheric exit phase (negative lift projection phase). Another surprising result is that, during the atmospheric pass, the peak values of the changes of the orbital inclination and the longitude of the ascending node are nearly zero; hence, the peak value of the wedge angle (angle between the instantaneous orbital plane and the initial orbital plane) is nearly zero. This means that the motion of the spacecraft is nearly planar in an inertial space.

The guidance implications of the above results are discussed.
3.4. MIELE, A., WANG, T., LEE, W.Y., WANG, H., and WU, G. D.,
Optimal Trajectories for the Aeroassisted Flight Experiment,
Part 4, Data, Tables, and Graphs, Rice University, Aero-

Abstract. This report is the fourth of a series dealing
with the determination of optimal trajectories for the aero-
assisted flight experiment (AFE). It presents data, tables, and
graphs relative to the following transfers: (IA) indirect ascent
to a 178 NM perigee via a 197 NM apogee; and (DA) direct ascent
to a 178 NM apogee.

For both transfers, two cases are investigated: (i) the
bank angle is continuously variable; and (ii) the trajectory is
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For case (ii), the following subcases are studied: two segments,
three segments, four segments, and five segments; because the
time duration of each segment is optimized, the above subcases
involve four, six, eight, and ten parameters, respectively.

To sum up, this report presents systematic data on ten
optimal trajectories (OT), five for Transfer (IA) and five for
Transfer (DA). For comparison purposes and only for Transfer
(IA), a reference trajectory (RT) is also considered: this is a
five-segment trajectory, close to the nominal trajectory given
in Ref. 6.
Abstract. This paper deals with the determination of optimal trajectories for the aeroassisted flight experiment (AFE). The intent of this experiment is to simulate a GEO-to-LEO transfer, where GEO denotes a geosynchronous Earth orbit and LEO denotes a low Earth orbit. Specifically, the AFE spacecraft is released from the space shuttle and is accelerated by means of a solid rocket motor toward Earth, so as to achieve atmospheric entry conditions identical with those of a spacecraft returning from GEO. During the atmospheric pass, the angle of attack is kept constant, and the angle of bank is controlled in such a way that the following conditions are satisfied: (a) the atmospheric velocity depletion is such that, after exiting, the AFE spacecraft first ascends to a specified apogee and then descends to a specified perigee; and (b) the exit orbital plane is identical with the entry orbital plane. The final maneuver, not analyzed here, includes the rendezvous with and the capture by the space shuttle.

In this paper, the trajectories of an AFE spacecraft are analyzed in a 3D-space, employing the full system of 6 ODEs describing the atmospheric pass. The atmospheric entry conditions are given, and the atmospheric exit conditions are adjusted in such a way that requirements (a) and (b) are met, while
simultaneously minimizing the total characteristic velocity, hence the propellant consumption required for orbital transfer. Two possible transfers are considered: (IA) indirect ascent to a 178 NM perigee via a 197 NM apogee; and (DA) direct ascent to a 178 NM apogee.

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It is shown that the optimal trajectories of cases (i) and (ii) coalesce into a single trajectory: a two-subarc trajectory, with the bank angle constant in each subarc (bang-bang control). Specifically, the bank angle is near 180 deg in the atmospheric entry phase (positive lift projection phase) and is near 0 deg in the atmospheric exit phase (negative lift projection phase). It is also shown that, during the atmospheric pass, the peak values of the changes of the orbital inclination and the longitude of the ascending node are nearly zero; hence, the peak value of the wedge angle (angle between the instantaneous orbital plane and the initial orbital plane) is nearly zero. This means that the motion of the spacecraft is nearly planar in an inertial space.
4. **Guidance Implications**

The results of Section 2 provide a foundation for developing a more efficient guidance scheme for the AFE spacecraft. For information concerning the existing guidance scheme, see Refs. 6-7.

First, we recall the basic idea of the existing guidance scheme. For the AFE spacecraft, the angle of attack varies from 7 to 27 deg, and the lift coefficient varies from -0.47 to -0.21. Therefore, if the AFE spacecraft were to be controlled via only the angle of attack, the lift coefficient range would be $|\Delta C_L| = 0.26$. To offset the above difficulty, the existing guidance scheme employs a constant angle of attack $\alpha = 17$ deg, corresponding to $C_L = -0.37$, coupled with a variable angle of bank.

Let $L_P = L \cos \mu$ denote the projection of the lift vector in the vertical direction; let $C_{LP} = C_L \cos \mu$ denote the projected lift coefficient. Because $C_L$ is constant and $\cos \mu$ varies from -1 to +1, the projected lift coefficient range is $|\Delta C_{LP}| = 0.74$. This is nearly triple the lift coefficient range $|\Delta C_L| = 0.26$.

To sum up, for the AFE spacecraft, the existing guidance scheme employs constant angle of attack and variable angle of bank in order to achieve the following goals: for the longitudinal motion, to dissipate excess velocity so that the spacecraft, upon exiting the atmosphere, can ascend to a specified apogee; and for the lateral motion, to keep the instantaneous orbital plane close to the initial orbital plane, hence to keep the wedge angle $\eta$ close to zero. These goals are achieved as follows: for
the longitudinal motion, one controls the modulus of the bank angle; for the lateral motion, one controls the sign of the bank angle; thus, whenever the wedge angle exceeds a specified bound, the sign of the bank angle is reversed.

Unquestionably, the existing guidance scheme has obvious merits: (a) simplicity; and (b) the fact that the projected lift coefficient range is three times the lift coefficient range. However, there are weaknesses associated with the control of the lateral motion:

(A) There is no equilibrium point for the control of the lateral motion. When \( \eta = 0 \), it would be desirable to have \( \dot{\eta} = 0 \), so that the wedge angle continues to be zero. However, \( \mu \) is determined from the requirement of longitudinal motion control; hence, generally speaking, \( \dot{\eta} \neq 0 \) when \( \eta = 0 \). This means that the system is not stable around \( \eta = 0 \).

(B) There is a bank angle error due to noninstantaneous switches. The implementation of the existing guidance scheme ideally requires instantaneous switches in \( \text{sign}(\mu) \), so as to keep \( \cos \mu \) unchanged. In practice, this is not possible, since \( |\dot{\mu}| < 20 \text{ deg/sec} \). For example, a switch from \( \mu = +170 \) to \( \mu = -170 \) deg requires \( \Delta t = 17 \text{ sec} \). If the correct \( \mu \)-values are \( \mu = +170 \) and \( \mu = -170 \) deg, this means that, during the time interval \( \Delta t = 17 \text{ sec} \), there is an error in the value of \( \mu \) which is required for the control of the longitudinal motion.

(C) There is a contradiction between accuracy and stability. For accurate control of the lateral motion, the wedge angle tolerance
should be small. On the other hand, if the wedge angle tolerance is small, the number of switches in sign(\(\mu\)) increases; therefore, the system stability becomes worse.

To offset the above difficulties, consideration should be given to developing a new guidance scheme, based on the properties of the optimal trajectories. The basic ideas of the new guidance scheme are: to control both the angle of attack (near 17 deg) and the angle of bank; and to utilize a single switch of the bank angle, consistently with the optimal trajectory properties, so as to have \(L_p > 0\) in the entry phase and \(L_p < 0\) in the exit phase; here, \(L_p\) is the projected lift. The new guidance scheme retains the basic advantage of the existing guidance scheme, in that the projected lift coefficient range is three times the lift coefficient range. In addition, it has the following advantages:

(A) There is an equilibrium point for the control of the lateral motion, due to the fact that both the angle of attack and the angle of bank are being varied. Hence, when \(\eta = 0\), it is possible to have \(\dot{\eta} = 0\), which means that the system is stable around \(\eta = 0\).

(B) There is less bank angle error due to noninstantaneous switches. For the existing guidance scheme, the effects due to the noninstantaneous switch from \(+\mu\) to \(-\mu\) can be serious, since there are multiple switches and they occur randomly. For the new guidance scheme, the effects due to the noninstantaneous switch from \(+\mu\) to \(-\mu\) are less serious, owing to the fact that there
is only one switch, which occurs whenever a preselected velocity is achieved.

(C) There is improved accuracy and stability in the control of the lateral motion. This is because the lateral motion is no longer controlled by the sign of the bank angle, but by the values of both the bank angle and the angle of attack.

(D) The new guidance scheme is based on the properties of the optimal trajectories. Hence, it preserves the good properties of the optimal trajectories concerning the characteristic velocity, the peak dynamic pressure, the peak heating rate, and the peak wedge angle.

(E) The new guidance scheme uses the same hardware (AFE configuration, measurements, sensors, and reaction control system) as the existing guidance scheme. The only difference is in the software (computer code). Therefore, if acceptable, a change from the existing guidance scheme to the new guidance scheme would be of limited cost.

Finally, it must be stressed that the development of a new guidance scheme is not only of interest within the frame of the AFE vehicle, but is also important within the frame of Mars penetration vehicles, Mars return vehicles, and lunar return vehicles.
References


