Algorithm for Atmospheric Corrections of Aircraft and Satellite Imagery

Robert S. Fraser, Richard A. Ferrare, Yoram J. Kaufman, and Shana Mattoo

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Robert S. Fraser
Goddard Space Flight Center
Greenbelt, Maryland

Yoram J. Kaufman
University of Maryland
College Park, Maryland

Richard A. Ferrare
University Space Research Association
Greenbelt, Maryland

Shana Mattoo
Applied Research Corporation
Landover, Maryland

NASA
National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, MD
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Abstract

This report describes a simple and fast atmospheric correction algorithm used to correct radiances of scattered sunlight measured by aircraft and/or satellite above a uniform surface. The atmospheric effect, the basic equations, a description of the computational procedure, and a sensitivity study are discussed. The program is designed to take the measured radiances, view and illumination directions, and the aerosol and gaseous absorption optical thicknesses to compute the radiance just above the surface, the irradiance on the surface, and surface reflectance. Alternatively, the program will compute the upward radiance at a specific altitude for a given surface reflectance, view and illumination directions, and aerosol and gaseous absorption optical thicknesses. The algorithm can be applied for any view and illumination directions and any wavelength in the range 0.48 \( \mu m \) - 2.2 \( \mu m \). The relation between the measured radiance and surface reflectance, which is expressed as a function of atmospheric properties and measurement geometry, is computed using a radiative transfer routine. The results of the computations are tabulated in a look-up table which forms the basis of the correction algorithm. The algorithm can be used for atmospheric corrections in the presence of a rural aerosol. The sensitivity of the derived surface reflectance to uncertainties in the model and input data is discussed.
5.3 Output
5.4 Summary of Error Messages

6.0 Sensitivity Study
6.1 Model Errors
   6.1.1 Aerosol Single Scattering Phase Function
   6.1.2 Polarization
   6.1.3 Water Vapor and Ozone Absorption
   6.1.4 Aerosol Absorption
   6.1.5 Vertical Distribution of Aerosols
   6.1.6 Bidirectional Reflectance
6.2 Interpolation Errors
6.3 Input Errors

7.0 Conclusion
8.0 References
9.0 Listing of FORTRAN program
List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Spectral bands, aerosol indices of refraction, and optical thicknesses</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>Aerosol size distribution parameters</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>Variables in the FIFEWAV program and associated subroutines and their equivalent in the text</td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td>Examples of input data</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>Examples of output results for the examples in Table 4</td>
<td>34</td>
</tr>
<tr>
<td>6</td>
<td>An example of output for interpolated radiation parameters</td>
<td>39</td>
</tr>
<tr>
<td>7</td>
<td>Examples of error messages for various errors in the input data</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>Case numbers assigned to sensitivity tests for the atmospheric correction algorithm</td>
<td>43</td>
</tr>
<tr>
<td>9</td>
<td>Summary of sensitivity results</td>
<td>48</td>
</tr>
<tr>
<td>10</td>
<td>Sensitivity results</td>
<td>52</td>
</tr>
</tbody>
</table>
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Surface reflectance and corresponding radiance above the atmosphere for a typical vegetation</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Angular coordinates</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Surface reflectance at the FIFE site</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>Aerosol particle number density altitude profiles</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>Aerosol single scattering phase functions</td>
<td>67</td>
</tr>
</tbody>
</table>
1.0 Introduction

1.1 Background

The purpose of this report is to describe a fast and simple atmospheric correction algorithm to derive the surface reflectance, and other parameters, from radiances measured by satellite and/or aircraft in the visible and near IR parts of the spectrum. The original version of this algorithm was developed to correct the radiances measured during FIFE (First ISLSCP Field Experiment - Sellers et al., 1988) which is taking place at the Konza Prairie in Kansas. While the algorithm has been designed to correct radiances measured over a rural site for the wavelength range 0.48 μm ≤ λ ≤ 2.2 μm, a sensitivity study has shown that the algorithm can be a practical tool for many applications of remote sensing, for which a uniform surface can be assumed, and for which the optical characteristics of the aerosol do not differ significantly from the rural aerosol (the algorithm should not be applied for correcting for the effect of desert dust or fog). As a result we would like to bring the algorithm to the attention of the scientific and engineering community.

1.2 The Intervening Atmosphere

Atmospheric aerosols, which are liquid or solid particles suspended in the air, have a significant importance in evaluating satellite imagery for remote sensing of the earth's surface. The atmospheric aerosol results from natural sources (e.g. desert dust, condensation and oxidation of gases released from the biosphere and oceans) and anthropogenic sources (e.g. biomass burning and the industrial emission of gases which participate in atmospheric chemical reactions and condense into liquid particles). In the Southern Hemisphere, near Australia, the aerosol concentration is usually very low (aerosol optical thickness in the visible is less than 0.10) due to the low population, large ocean areas, and low humidity. In desert areas, dust storms can increase the optical thickness (τ) to τ = 2.0 and above (hiding the sun). In the Northern Hemisphere the concentration may be rather large during long period of times, (e.g. the aerosol optical thickness is around 0.6 in the Eastern part of the United States during July and August - Kaufman and Fraser, 1983; Peterson et al., 1981), due to industrial pollution. In the state of Rondonia, Brazil, biomass burning due to deforestation generates dense smoke (optical thickness 1.0-3.0) that covers the area during most of the dry season. For an aerosol optical thickness over land larger than 0.20, aerosols affect a significant share of the outgoing visible radiation for a cloudless sky.
Therefore, since aerosols affect satellite imagery of the earth's surface, attempts for its correction should be taken.

Satellite images of the Earth's surface in the solar spectrum are contaminated by sunlight scattered towards the sensor by atmospheric molecules, aerosols, and clouds (path radiance). In addition, solar energy that is reflected from the Earth's surface and serves as the remote sensing signal, is attenuated by the atmosphere. This combined atmospheric effect is wavelength dependent, varies in time and space, and depends on the surface reflectance and its spatial variation. Correction for this atmospheric effect can produce remote sensing signals that are more closely related to the surface characteristics. Molecular scattering and absorption in the atmosphere can be accounted for satisfactorily. Gaseous absorption is minimized by choosing sensor bands in atmospheric windows. Therefore, aerosol scattering and absorption, and the presence of subpixel clouds, are the main variables in the atmospheric effect on satellite imagery.

For a cloudless sky, aerosol scattering is the major variable component of the atmospheric effect for dark surfaces, while aerosol absorption is important for bright surfaces (Fraser and Kaufman, 1985). In order to perform atmospheric corrections of remotely sensed data, the optical characteristics of the atmosphere must be estimated. These characteristics may be given in varying levels of detail—from considerable detail (profile of the extinction coefficient, the single-scattering albedo and the scattering phase function), to less detail (the vertical optical thickness, the average aerosol scattering phase function and single-scattering albedo).

In order to demonstrate the effect of the atmosphere on remote sensing we shall discuss vegetation as an example. We may distinguish among the following ecological regions:

- Over remote land areas, with no substantial anthropogenic aerosol contribution, and no dust, the aerosol optical thickness may vary between 0.02-0.10. For this variation and for typical aerosol characteristics (single scattering albedo $\omega_o = 0.96$), the reflectance of the atmosphere alone will increase about 0.01 (Fraser and Kaufman, 1985), and a vegetation index (ratio between the difference in reflectances between the near IR and the visible and the sum - NDVI) of NDVI=0.60, as measured by satellite, will change to 0.58 (Holben, 1986). These changes are relatively small and, therefore, atmospheric correction in this case is not important for most applications. (Remote sensing of ocean color is affected by even a small aerosol optical thickness (Gordon et al., 1983).)
- Over typical land areas, anthropogenic aerosols and/or dust may generate optical thicknesses in the range 0.05-0.25. The corresponding atmospheric effect would change a surface reflectance of $\rho = 0.02$ to 0.04, and vegetation index of NDVI = 0.60 to 0.55. These are significant errors which necessitate atmospheric correction.

- Over polluted areas, with anthropogenic aerosols from industrial sources (e.g. all of Eastern U.S. and Europe during the summer) or areas affected by dust, fog or smoke (tropical regions, regions in the far east and Sahel), the aerosol optical thickness may vary in the range 0.1-1.0. In this range of variation the atmospheric effects are very large. The surface reflectance would vary from 0.02 to 0.08 and the vegetation index would decrease from 0.6 to 0.45.

1.3 Atmospheric Corrections

The correction procedure requires information about the atmospheric optical characteristics. Due to the difficulty in determining these characteristics, the only operational use of atmospheric corrections today is that of the ocean color (Gordon et al., 1983), where the corrections depend on the condition of the very low reflectance of the water in the red. Otherwise, information on the atmospheric optical characteristics can be obtained from three different sources:

Climatology: Documented information on the atmospheric characteristics and their variation can be used to estimate the expected atmospheric effect for a specific part of the world and a specific season. Such documentation can be obtained from the analysis of measurements taken from the ground, and partially from the analysis of satellite data (Fraser et al., 1984; Kaufman, 1987; Kaufman et al., 1988). This source of information will be used for optical characteristics that cannot be determined otherwise for the particular image being corrected.

Measurements from the ground: The aerosol optical thickness can be obtained from sun-photometer measurements (King et al., 1978; Kaufman and Fraser, 1983). The phase function can be determined from inversion of solar almucantar measurements, and the single-scattering albedo from the collection of particles on filters, preferably by aircraft sampling of the entire atmospheric boundary layer. The single-scattering albedo can also be determined by measurements of the diffuse and direct flux (Herman et al., 1975; King and Herman, 1979; King, 1979) and by lidar techniques (Spinhirne et al., 1980). The
application of such measurements for atmospheric corrections is useful for intense field measurements, or for establishing the climatology of a given area.

Determination from satellite imagery: For the purpose of atmospheric corrections, the path radiance and the corresponding aerosol optical thickness can be derived from radiances detected by the satellite over a dark surface. Examples include many land surfaces in the blue spectrum, dense dark vegetation in the visible channels (Kaufman and Sendra, 1988), and water in the red and near IR. The wavelength dependence of the derived aerosol optical thickness (when available) can be used to estimate the particle size and the scattering phase function. In the past satellite imagery has been used to determine the aerosol optical thickness and other aerosol characteristics. The aerosol optical thickness has been derived from satellite imagery of oceans (Griggs, 1975; Mekler et al., 1977; Carlson, 1979; Koepke and Quenzel, 1979; Takayama and Takashima, 1986), and recently, over dense dark vegetation (Kaufman and Sendra, 1988). By using the difference in the brightness between a clear and a hazy day, Fraser et al. (1984) demonstrated that the difference in the optical thickness can be derived where the surface reflectance is less than 0.1. Determination of the aerosol single-scattering albedo and particle size was suggested by Kaufman et al. (1988) and Kaufman (1987), and applied to trace the evolution of smoke from a large forest fire (Ferrare et al., 1988). This method is useful to determine the aerosol characteristics (from imagery that includes water-land interfaces) in areas that suffer from substantial aerosol outbreaks (e.g. desert dust storms, smoke from fires and concentrated anthropogenic aerosol).

Once the atmospheric characteristics are specified, an atmospheric correction can be performed with an equation relating measured radiance to the optical properties of the atmosphere and surface. The computation requires application of complex and time consuming radiative transfer programs (Dave, 1972 a,b,c,d; Ahmad and Fraser, 1982). This report presents an algorithm that simplifies the correction procedure by using an a priori prepared look-up table that is based on radiative transfer computations. In essence, the algorithm simplifies the atmospheric correction procedure to a desktop operation, by sacrificing the flexibility to select specific aerosol size distributions and refractive indexes, but not optical thickness.
1.4 The algorithm

The algorithm is designed to compute the upward radiance for a given surface reflectance or to compute the surface reflectance for a given measured radiance, for almost any wavelength in the visible and near-IR spectrum (with appropriately specified gaseous absorption), for a wide range of observation zenith angles, solar illumination angles and azimuth angles between the observer and the solar rays, as well as any height of the observer (aircraft or satellite). Any practical value of the aerosol optical thickness can be used, but the algorithm is restricted to a specific aerosol size distribution and refractive index.

The relation between the measured radiance and the surface reflectance is expressed as a function of the path radiance, downward flux at the ground, atmospheric transmission, and the atmospheric backscattering ratio. Using this relation, a look-up table is constructed which relates the measured upward radiance to surface reflectance for several aerosol optical thicknesses, solar zenith angles, measurement wavelengths, and a range of observation directions. This look-up table is based on the tabulation of the results of radiative transfer computations which are made using a Dave (1972 a,b,c,d) code. It is assumed that the atmosphere and surface are horizontally homogeneous, and the surface reflects light according to Lambert's law. The light scattered by the atmosphere and the surface is assumed to be unpolarized. The atmosphere is also assumed to be cloud-free.

Radiation properties of the cloudless atmosphere depend on both molecular and aerosol constituents. Molecular scattering and absorption, except for water vapor absorption, are easy to account for. Aerosol effects are more variable and are therefore more difficult to correct. The parameters used to describe these aerosol effects are: the optical thickness which determines the amount of extinction, the single scattering albedo which determines the fraction of light scattered from the total extinction, and the single scattering phase function which describes how the light is scattered as a function of direction. For the most part, aerosol extinction is the dominant parameter in the aerosol component of the atmospheric effect (Fraser and Kaufman, 1985). Thus, in this algorithm the aerosol optical thickness is the only variable aerosol parameter. The algorithm uses a constant aerosol single scattering phase function and scattering albedo chosen to represent a rural environment. Because these assumptions can introduce error in the derived surface reflectances, a sensitivity analysis is performed to estimate the uncertainty in in derived surface reflectances.
In the first part of this document, the atmospheric effect and its effects on the surface reflectance are discussed. Next, a description of the equations used in the algorithm and the construction of the look-up table is given. A sensitivity study is then performed to estimate the errors associated with the initial assumptions, interpolation algorithm, and uncertainties in the input data. Finally, the FORTRAN code for the algorithm is listed.

2.0 Atmospheric Effect

2.1 General Discussion

The atmospheric effect is caused by the scattering and absorption of solar radiation by molecules and aerosols. There are three components to this effect:

1) The downward solar radiation is absorbed and scattered by the atmosphere and it is diffused by forward scattering. Because this diffusion increases the angular distribution of the radiation, the downward radiation interacts with the surface in a wide range of directions. Thus the surface reflection coefficient for this radiation is different from the reflection coefficient for the direct solar beam (Lee and Kaufman, 1986).

2) Radiant energy reaching a remote sensor is reflected from the ground both within the instantaneous-field-of-view (ifov) and from the region outside of it. Part of the reflected energy within the ifov is transmitted directly to a sensor and can be considered signal; the remaining radiation is absorbed and scattered. Part of the radiation that is reflected from outside of the ifov passes through the column containing the ifov and is scattered there towards the sensor. This component is associated with the adjacency effect. It augments the measured radiance and is partially corrected for when deriving the surface reflectance.

3) Radiation is scattered by the atmosphere into the ifov without being reflected by the surface. This component is called the path radiance and increases the apparent reflection of the surface.

An example of the difference between the spectral surface reflectance for typical vegetation (assumed Lambertian) and the corresponding upward radiance above the atmosphere is shown in Figure 1. In this figure, the upward radiance is normalized by the incident solar flux to produce reflectance units. This normalized radiance is the apparent reflectance as seen from the sensor. The reflectance of the earth-atmosphere system can be greater than the surface reflectance, the same, or weaker. In the visible spectrum the
Figure 1. Surface reflectance (-----) and the corresponding radiance (- - -) above the atmosphere for a typical vegetation. The radiance is normalized by $F_0$, the incident solar flux, and by $\mu_0$, the cosine of the solar zenith angle (from Kaufman, 1987).
surface reflectance is weak, the path radiance is relatively strong, and the reflectance above the atmosphere exceeds that at the ground. In the near infrared (0.7<\lambda<1.6 \mu m) the surface reflectance is strong, and the reflectances at the surface and above the atmosphere are nearly the same. The loss of radiation from the surface by extinction is augmented at the same rate by atmospheric scattering. For wavelengths longer than 1.6 \mu m, atmospheric scattering does not compensate for the attenuation loss.

2.2 Mathematical Description

In order to correct for the atmospheric effects discussed in the previous section, a relation is developed between the upward spectral radiance \( L^m \) measured from satellite or aircraft and the surface reflectance \( \rho \): \( L^m = f(\rho) \). The radiance is equivalent to the specific intensity as defined by Chandrasekhar (1960, p.1), except that the radiance, as used here, is the radiant energy within a unit wavelength interval instead of the energy per unit frequency. The function \( f \) depends on the atmospheric and surface optical properties, observation and sun directions, and wavelength. The radiance \( L^m \) can be expressed explicitly as a function of the path radiance \( L_o \) (upward radiance for zero surface reflectance), the downward flux through a horizontal surface at the ground \( F_d \) (for zero surface reflectance), the total (direct + diffuse) transmission from the surface to the observer \( T \), and the atmospheric backscattering ratio \( s \). It is assumed that the atmosphere and the surface are horizontally homogeneous, but the atmospheric optical properties vary in the vertical direction. The surface is assumed to reflect light according to Lambert’s law. The light scattered by the cloud-free atmosphere and surface is assumed to be unpolarized. The relation between \( L^m \) and \( \rho \) is (Chandrasekhar, 1960)

\[
L^m = L_o + \frac{(\rho F_d T)}{\pi (1 - s \rho)}
\]

Here \( L^m \) is the spectral radiance measured from aircraft or satellite and is a function of \( \lambda, \theta_o, \tau_a, \tau_g, \omega_o, \tau_g, \theta, Z, \) and \( \phi \), where

\[
\begin{align*}
\lambda & \quad \text{is the wavelength of the radiation,} \\
\theta_o & \quad \text{is the solar zenith angle,} \\
\tau_a & \quad \text{is the aerosol optical thickness (used with base e),}
\end{align*}
\]
\( \tau_{gs} \) is the molecular scattering optical thickness,

\( \omega_o \) is the ratio of the aerosol scattering and extinction optical thicknesses,

\( \tau_g \) is the gaseous absorption optical thickness,

\( Z \) is the observation height,

\( \theta \) is the propagation direction zenith angle of the radiant energy at the ground,

\( \varphi \) is the azimuth angle (azimuthal angles are measured with respect to the principal plane through the sun; \( 0^\circ \) lies in the plane containing the direction of propagation of the direct sunlight).

Figure 2 shows the angular coordinates used in the algorithm. The functions \( L_O, T, F_d \) and \( s \) have the following functional dependances:

\[
L_O = L_O(\lambda, \theta_o, \tau_a, \tau_{gs}, \tau_g, \omega_o, Z, \theta, \varphi) \quad F_d = F_d(\lambda, \theta_o, \tau_a, \tau_{gs}, \tau_g, \omega_o) \\
T = T(\lambda, \tau_a, \tau_{gs}, \tau_g, \omega_o, Z, \theta) \quad s = s(\lambda, \tau_a, \tau_{gs}, \tau_g, \omega_o)
\]

The correction algorithm is based on the inverse of eq. (1), where the surface reflectance \( \rho \) can be expressed in terms of the measured radiance \( L^m \):  

\[
\rho = \frac{f}{(1 + s \ f)} \quad (2)
\]

where

\[
f = \frac{\pi (L^m - L^o)}{(F_d \ T)} \quad (3)
\]

The algorithm will compute \( L^m \) using (1) if \( \rho \) is given or will compute \( \rho \) using (2) and (3) if \( L^m \) is given. Other quantities computed are: the total irradiant flux \( F_g \) on a horizontal surface at the ground,

\[
F_g = \frac{F_d}{(1 - s \ \rho)} \quad (4)
\]
Figure 2. Angular coordinates used in the algorithm. The X-Y plane is a horizontal plane tangent to the earth's surface at the observation point. The solar zenith angle $\theta_0$, observation zenith angle $\theta$, observation scan angle $\theta'$, and observation azimuth angle $\phi$ are shown. In this particular representation of planar geometry $\theta = \theta'$; the general relationship is given by eq. (9).
and the upward radiance $L_g$ at the ground in the direction of observation.

$$L_g = \frac{\rho F_d}{\pi (1 - s \rho)}$$  \hspace{1cm} (5)

3.0 Correction Algorithm

The correction program is based on the tabulation of the results of radiative transfer computations of $L_0$, $F_d$, $s$, and $T/\pi$ (note that the transmission $T$ is not tabulated). The primed parameters are normalized flux and radiance rather than their absolute values. The normalized values are related to their corresponding absolute values by the following equations:

$$F_d' (\theta_o, \tau_a) = \frac{F_d}{F_0 \cos \theta_o} \hspace{1cm} L_o' (\theta_o, \theta, \phi, \tau_a, Z) = \frac{\pi L_o}{F_0 \cos \theta_o}$$  \hspace{1cm} (6)

The value of $F_0$ represents the solar spectral flux passing through a surface orthogonal to its propagation at the top of the atmosphere. In order to use the table for atmospheric correction, the measured absolute radiance $L^m$ is converted to $L^{m'}$ using

$$L^{m'} = \frac{\pi L^m}{F_0' \cos \theta_o}$$  \hspace{1cm} (7)

where

$$F_0' = \frac{F_0}{R^2} \hspace{1cm} R = \frac{d}{d'}$$  \hspace{1cm} (8)

and $d$ is earth-sun distance for the day of the year when measurements are made, $d'$ is the mean earth-sun distance, and $F_0$ is the solar flux now computed for each of the spectral bands using the solar spectral flux data from Neckel and Labs (1984). Since the look-up table tabulates observation zenith angle $\theta$ of the line-of-sight from the ground to the observing platform, the scan angle $\theta'$ measured by aircraft or satellite is converted to observation zenith angle $\theta$ using
\[ \theta = \sin^{-1} \left[ (1 + \frac{Z}{r_s}) \sin \theta' \right] \] (9)

where \( Z \) is the height of the sensor above the ground, and \( r_s \) is the radius of the earth.

The computations are performed by a Dave code (1972a, b, c, d) with a series of radiative transfer programs. These programs compute the flux and radiance of the scattered radiation emerging at any level of a plane-parallel atmosphere. Henceforth, primed fluxes and radiances indicate that they have been normalized as in eq. 6. Variables with the superscript \( m \) are measured.

3.1 Model Wavelengths

The look-up table is computed for the following wavelengths: 0.639, 0.845, 0.486, 0.587, 0.663, 0.837, 1.663, and 2.189 \( \mu \)m which correspond to the following sensors: NOAA-9 AVHRR band 1 (0.58 - 0.68 \( \mu \)m), band 2 (0.725 - 1.10 \( \mu \)m); Landsat-5 TM and NS-001 TMS band 1 (0.45 - 0.52 \( \mu \)m), band 2 (0.52 - 0.60 \( \mu \)m), band 3 (0.63 - 0.69 \( \mu \)m), band 4 (0.76 - 0.90 \( \mu \)m), band 5 (1.55 - 1.80 \( \mu \)m), and band 7 (2.10 - 2.35 \( \mu \)m). These wavelengths are listed in Table 1. The wavelength chosen to represent a particular band is computed by first calculating

\[ \lambda^* = \frac{\int \lambda L^m F_o \Psi d\lambda}{\int L^m F_o \Psi d\lambda} \] (10)

where

\( L^m = \) normalized radiance at the top of the atmosphere

\( F_o = \) extraterrestrial solar spectral flux

\( \Psi = \) response function of the sensor

The values for \( F_o \) are obtained from Neckel and Labs (1984) while the values for \( \Psi \) correspond to the specific sensor. For the NOAA-9 AVHRR and Landsat TM sensors, these values are obtained from Kidwell (1985) and Markham and Barker (1989), respectively. The effective wavelength \( \lambda^* \) in eq. 10 is a function of the normalized radiance, which is a function of the surface reflectance, aerosol optical thickness, and
Table 1. Spectral bands, aerosol refractive indices, and optical thicknesses.

<table>
<thead>
<tr>
<th>Sensor Wavelengths</th>
<th>AVHRR 1</th>
<th>AVHRR 2</th>
<th>TM 1</th>
<th>TM 2</th>
<th>TM 3</th>
<th>TM 4</th>
<th>TM 5</th>
<th>TM 6</th>
<th>TM 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% minimum response (nm)</td>
<td>569.8</td>
<td>714.3</td>
<td>452.4</td>
<td>528.0</td>
<td>626.4</td>
<td>776.4</td>
<td>1567.5</td>
<td>2097.2</td>
<td></td>
</tr>
<tr>
<td>50% maximum response (nm)</td>
<td>699.3</td>
<td>982.2</td>
<td>517.8</td>
<td>609.3</td>
<td>693.2</td>
<td>904.5</td>
<td>1784.1</td>
<td>2349.0</td>
<td></td>
</tr>
<tr>
<td>Peak response (nm)</td>
<td>680.0</td>
<td>760.0</td>
<td>503.0</td>
<td>594.0</td>
<td>677.0</td>
<td>800.0</td>
<td>1710.0</td>
<td>2200.0</td>
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<tr>
<td>Model (equation 7) (nm)</td>
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<td>844.6</td>
<td>486.2</td>
<td>586.9</td>
<td>662.7</td>
<td>837.3</td>
<td>1662.7</td>
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<tr>
<td>n' (accumulation mode)</td>
<td>1.43</td>
<td>1.43</td>
<td>1.43</td>
<td>1.43</td>
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<td>1.40</td>
<td>1.40</td>
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<tr>
<td>k (accumulation mode)</td>
<td>10^{-8}</td>
<td>10^{-8}</td>
<td>10^{-8}</td>
<td>10^{-8}</td>
<td>10^{-8}</td>
<td>10^{-8}</td>
<td>10^{-4}</td>
<td>10^{-4}</td>
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</tr>
<tr>
<td>n' (coarse particle mode)</td>
<td>1.53</td>
<td>1.53</td>
<td>1.53</td>
<td>1.53</td>
<td>1.53</td>
<td>1.53</td>
<td>1.53</td>
<td>1.40</td>
<td>1.35</td>
</tr>
<tr>
<td>k (coarse particle mode)</td>
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<td>10^{-7}</td>
<td>10^{-7}</td>
<td>10^{-7}</td>
<td>10^{-7}</td>
<td>10^{-7}</td>
<td>10^{-4}</td>
<td>0.00814</td>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecular Scattering (\tau_{gs})</td>
<td>0.0540</td>
<td>0.0180</td>
<td>0.159</td>
<td>0.0841</td>
<td>0.0449</td>
<td>0.0176</td>
<td>0.0012</td>
<td>0.0004</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gaseous Absorption</th>
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<th></th>
<th></th>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Ozone (\tau_{g}^{O_3})</td>
<td>0.0240</td>
<td>0.00064</td>
<td>0.00663</td>
<td>0.0317</td>
<td>0.0174</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Water Vapor (\tau_{g}^{H_2O})</td>
<td>0.00605</td>
<td>0.0933</td>
<td>0.00000</td>
<td>0.0002</td>
<td>0.0068</td>
<td>0.0410</td>
<td>0.0957</td>
<td>0.0741</td>
<td></td>
</tr>
<tr>
<td>Carbon Dioxide (\tau_{g}^{CO_2})</td>
<td>0.00071</td>
<td>0.0146</td>
<td>0.00000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0021</td>
<td>0.0077</td>
<td>0.0091</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Aerosol Absorption ((\tau_a = 0.25))</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_{a}^a)</td>
<td>0.0148</td>
<td>0.0223</td>
<td>0.0130</td>
<td>0.0142</td>
<td>0.0150</td>
<td>0.0218</td>
<td>0.0423</td>
<td>0.0189</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Composite (section 3.3)</th>
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<th></th>
<th></th>
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<th></th>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_{g}^H)</td>
<td>0.0247</td>
<td>0.0152</td>
<td>0.0066</td>
<td>0.0317</td>
<td>0.0174</td>
<td>0.00206</td>
<td>0.00771</td>
<td>0.00908</td>
<td></td>
</tr>
<tr>
<td>(\tau_{g}^L)</td>
<td>0.0208</td>
<td>0.1156</td>
<td>0.0130</td>
<td>0.0142</td>
<td>0.0218</td>
<td>0.0628</td>
<td>0.138</td>
<td>0.0930</td>
<td></td>
</tr>
<tr>
<td>(\tau_g)</td>
<td>0.0455</td>
<td>0.131</td>
<td>0.0196</td>
<td>0.0459</td>
<td>0.0392</td>
<td>0.0648</td>
<td>0.146</td>
<td>0.102</td>
<td></td>
</tr>
</tbody>
</table>

13
geometry. The normalized radiance is computed for different models of the earth-atmosphere system representative of surface and atmospheric conditions expected for the FIFE Konza Prairie site. Figure 3 shows the surface reflectance profiles used to model this site. A similar approach can be used to determine the wavelengths which correspond to other sensors.

The algorithm is generalized to accept any wavelength in the range 0.48-2.2 μm. For wavelengths for which there are no entries in the look-up table (see Table 1), the algorithm will interpolate the atmospheric functions (L_0, T, F_d, s) for the desired wavelength. The interpolation is performed assuming that aerosol parameters are proportional to the wavelength raised to a power:

\[ \ln \tau_{gs} \sim -\ln \lambda, \quad \ln \tau_a \sim -\ln \lambda, \quad \ln P \sim \ln \lambda, \]

where P is the scattering phase function. As a result, the radiances in the look-up table can be interpolated linearly between the wavelengths on a log-log scale. The only nonlinear relation is between the gaseous absorptions in the different wavelengths. Correction for the gaseous absorption is discussed in section 3.3.

3.2 Aerosol Properties

Because the absorption and scattering of light by atmospheric aerosols is highly variable, some assumptions regarding the size, shape, and composition of the aerosol must be made. In this model, the aerosols are assumed to be spheres so that Mie theory can be used to calculate the scattering by aerosols. Although in general the aerosol particles are not spherical, it is assumed that the sizes assigned to the aerosol particles are the sizes of spheres that have similar scattering properties to the measured aerosol distribution (Shettle and Fenn, 1979). This assumption has further basis because it has been found that the aerosol particles become more spherical as the relative humidity increases (Nilsson, 1979).

The algorithm uses a bimodal aerosol size distribution which combines the optically effective fraction of the accumulation mode (0.1 μm ≤ d ≤ 1.0 μm) and the coarse particle mode (d > 0.5 μm). For the accumulation mode, the dry particles are assumed to be composed of 80% water soluble sulfates and 20% water insoluble, dust-like material (Nilsson, 1979). At 70% relative humidity, water composes half of the volume of these aerosols. The coarse particle mode aerosols are assumed to be made of mostly water insoluble, dust-like particles. The size distribution of the aerosols is represented as the sum of two log-normal distributions; the two distributions represent the accumulation and coarse
Figure 3. Surface reflectance for senescent grass from a burned surface on the Konza Prairie at the FIFE site. The first profile (——) is for nadir view, measured at 14:38 CDT on 15 July 1986 while the second profile (- - -) is for an observation zenith angle of 45° measured at 15:13 CDT on 15 July 1986 (Asrar, 1986, private communication).
particle modes. The number density function of particles of radius \( r \) per cubic centimeter of air per micrometer of radius is (Shettle and Fenn, 1979)

\[
n(r) = \frac{dN(r)}{dr} = \sum_{i=1}^{2} \left[ \frac{N_i}{\ln(10) r \sigma_i \sqrt{2\pi}} \right] \exp \left[ -\frac{(\log r - \log r_n^i)^2}{2\sigma_i^2} \right]
\]

(11)

where

- \( N(r) \) = cumulative number density of particles of radius \( r \)
- \( \sigma_i \) = standard deviation of the logarithm of the radius
- \( r_n^i \) = geometric mean radius
- \( N_i \) = total number density in \( i^{th} \) mode

The values of \( N_i, r_n^i \), and \( \sigma_i \) used in the model correspond to the 70% relative humidity, rural aerosol model of Shettle and Fenn (1979); these values are shown in Table 2. The values of \( N_i \) shown in Table 2 are normalized such that \( N_1 + N_2 = 1 \) particle/cm\(^3\). This bimodal aerosol size distribution is assumed constant with height.

The composition of the aerosols is expressed in terms of the complex refractive index \( n = n' - i k \). The refractive index for both the accumulation and coarse particle modes is assumed to depend on the wavelength. Five different real refractive indices are chosen (Nilsson, 1979) and are listed in Table 1. The imaginary index of refraction is modeled assuming that most of the aerosol consists of weakly absorbing particles with \( k < 10^{-4} \), mixed with a small number of highly absorbing particles with \( k \approx 1.0 \). Although Shettle and Fenn (1979) choose to represent mixtures of this type with a composite imaginary refractive index in the range \( 0.001 < k < 0.01 \), which is close to the values obtained by various remote sensing and in-situ techniques (Patterson and Grams, 1984; Reagan et al., 1980), this procedure is not adopted here because as Bohren and Huffman (1983) point out, no common substances exist which have an imaginary index in this range. In the correction algorithm, the imaginary refractive index corresponding to the weakly absorbing particles is used. The imaginary refractive indices for the accumulation mode, which are listed in Table 1, correspond to water while the values for the coarse particle mode correspond to crystalline quartz, which is a constituent of atmospheric dust (Nilsson, 1979). The aerosol refractive index is assumed to be constant with height.
Table 2. Aerosol size distribution parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Accumulation mode</th>
<th>Coarse particle mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric mean radius $r_n$</td>
<td>0.0285</td>
<td>0.457</td>
</tr>
<tr>
<td>Standard deviation of the logarithm (base 10)</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>Number density $N_i$</td>
<td>0.999875</td>
<td>0.000125</td>
</tr>
</tbody>
</table>
The aerosol absorption needed to match the experimentally measured value is modeled by adding the necessary absorption to the gaseous absorption values. Thus, it is assumed that the absorbing particles are small compared to the wavelength and that they occur separately from the other particles (external mode); in this case their scattering effects are small relative to their absorptive effects (Fraser and Kaufman, 1985). The required additional aerosol absorption amount is determined by the aerosol single scattering albedo \( \omega_0 \) which is the ratio of scattering to extinction. Aerosol absorption is then given by \( 1 - \omega_0 \). The values of \( \omega_0 \) used in the algorithm are derived from the 70% relative humidity, rural aerosol model of Shettle and Fenn (1979). The single scattering albedos, shown in Table 1, range from 0.95 at 0.486 \( \mu m \) to 0.86 at 2.550 \( \mu m \). These values for the visible spectrum agree with those reported by Waggoner et al. (1981) which lie in the range \( 0.89 \leq \omega_0 \leq 1.0 \) for rural areas. The amount of additional aerosol absorption which is added to the gaseous absorption is

\[
\tau_a^a = \tau_a (\omega_0' - \omega_0)
\]

(12)

where

\( \tau_a = \) aerosol optical thickness

\( \omega_0' = \) single scattering albedo for chosen refractive index

\( \omega_0 = \) desired single scattering albedo

Since in general \( k << 1 \), then \( \omega_0' = 1 \), and

\[
\tau_a^a = \tau_a (1 - \omega_0)
\]

(13)

Should the algorithm be applied for an urban aerosol or smoke, where the single scattering albedo \( \omega_0 \) used in the formation of the look-up tables (see Table 1) may be smaller or larger than the new value \( \omega_0^* \), the difference in absorption can be corrected by adding (or subtracting) the excess absorption to the gaseous absorption:

\[
\Delta \tau_a^a = \tau_a (\omega_0 - \omega_0^*)
\]

(14)

3.3 Gaseous Absorption

For the most part, the Landsat TM and NOAA AVHRR visible and near IR bands have been selected to minimize gaseous absorption. However, in some cases, the sensor
channel is either relatively broad (as in the case of AVHRR band 2) or lies within a broad gaseous absorption band (as in the case of TM bands 2 and 3 which lie within the ozone continuum) so that the absorption by atmospheric gases can be both significant and variable. In the atmospheric correction algorithm, gaseous absorption was computed using the LOWTRAN 6 code (Kniezys et al., 1983) which computes atmospheric absorption from 0.250 µm to 28.5 µm due to water vapor, carbon dioxide (and other uniformly mixed gases), ozone, nitrogen continuum, oxygen, and HNO3. In the case of the AVHRR and TM bands, the absorbing gases are water vapor, carbon dioxide, and ozone. The gaseous absorption optical thickness in the vertical direction due to gaseous species X for each band is computed using

\[ \tau_g^X = -\frac{1}{m} \ln \left( \frac{\int T_x L'_m F_0 \Psi d\lambda}{\int L'_m F_0 \Psi d\lambda} \right) \]  

(15)

where

- \( m \) = air mass along inclined path
- \( T_x \) = transmittance due to gaseous species \( X \)

Since the gaseous absorption is quite variable, the algorithm uses a weighted average of the absorption values computed using the tropical, mid-latitude summer, and mid-latitude winter atmospheres given in the LOWTRAN 6 code. The weighted average of the gaseous absorption \( \tau_g^X \) used in the algorithm is given by

\[ \tau_g^{X*} = 0.25 \tau_g^{X1} + 0.5 \tau_g^{X2} + 0.25 \tau_g^{X3} \]  

(16)

where \( \tau_g^{X1}, \tau_g^{X2}, \) and \( \tau_g^{X3} \) are the gaseous absorption values computed using the mid-latitude winter, mid-latitude summer, and tropical values respectively. The absorption optical thicknesses due to water vapor, carbon dioxide, and ozone for each band are shown in Table 1.

The algorithm can be applied to any specified gaseous optical thickness \( \tau_g \). An approximate correction is applied to the radiances in the look-up table to account for the excess (or deficit) in the absorption (\( \Delta \tau_g \)). The user may compute the required value of \( \tau_g \) based on the sensor spectral response using the LOWTRAN program (Kniezys et al., 1983). For the correction of the look-up table we may distinguish between \( \Delta \tau_{gL} \) - excess
or deficit in the gaseous absorption in the lower part of the atmosphere (e.g. water vapor),
assumed to be mixed uniformly with the aerosol; and $\Delta \tau_{gH}$ excess or deficit in the gaseous
absorption in the upper part of the atmosphere, above the aerosol layer (e.g. $O_2$, $O_3$,
$CO_2$). The correction is performed by the following transformations in the look-up table:

$$L_o \rightarrow L_o \exp \left[ - \left( \frac{\Delta \tau_{gL}}{2} + \Delta \tau_{gH} \right) \left( \frac{1}{\mu} + \frac{1}{\mu_0} \right) \right]$$

(17a)

$$F_d \rightarrow F_d \exp \left[ - \frac{(\Delta \tau_{gL} + \Delta \tau_{gH})}{\mu_0} \right]$$

(17b)

$$T \rightarrow T \exp \left[ - \frac{(\Delta \tau_{gL} + \Delta \tau_{gH})}{\mu} \right]$$

(17c)

$$s \rightarrow s \exp \left[ - 2 \Delta \tau_{gL} \right]$$

(17d)

Here $\mu = \cos \theta$ and $\mu_0 = \cos \theta_0$. In these equations the effect of multiple scattering on
the path length through the atmosphere is neglected. It is also assumed that the path
radiance $L_o$ is generated above the middle of the boundary layer. As result, the additional
gaseous attenuation is made by half of the boundary layer, but all of the atmosphere above.
The parameter $s$ is the reflectance of the atmosphere for radiation entering its base. The
effective direction for reflection is $60^\circ$. Hence, the effective absorption optical thickness is
twice the vertical value. Theses assumptions are based on our physical understanding of
radiative transfer and are tested in Section 6.

3.4 Altitude Profiles

The radiative transfer computations require as input the altitude distributions of both
the absorbing gases and aerosols. These two distributions have separate profiles. The
altitude distribution of aerosols used in the algorithm is based on the 'average' distribution
of Braslau and Dave (1973) which is shown in Figure 4. The aerosol altitude distribution is
first scaled to obtain the desired aerosol optical thickness.
Figure 4. Aerosol particle number density profiles for: 1) 'average' distribution of Braslau and Dave (1973) used in the correction algorithm; 2) 50 km surface visibility, 3) 23 km surface visibility, and 4) 10 km surface visibility models from Shettle and Fenn (1979). Figure 4a is for altitudes between 10 and 70 km while figure 4b is for altitudes between 0 and 10 km. These profiles have been normalized for an aerosol optical thickness \( \tau_a = 0.25 \) at 0.55 \( \mu \text{m} \).
Because the various gaseous absorbers described in the previous section have different altitude distributions, a composite altitude distribution is computed which accounts for all the absorbing gases including the aerosol absorption described in section 2.2. In this method, the total gaseous absorption is divided into "low" and "high" components. The "low" component is composed of water vapor and aerosol absorption while the "high" component is composed of ozone and CO₂ absorption. The "low" component uses an altitude distribution based on the 'average' aerosol profile of Braslau and Dave (1973), while the "high" component uses an altitude distribution based on the mid-latitude ozone profile of McClatchey et al. (1971). The altitude distributions of the "low" and "high" components are normalized and combined to produce an altitude profile that retains the maximum aerosol and water vapor absorption near the surface and the maximum ozone absorption in the lower stratosphere. Because the aerosol absorption is a function of the aerosol optical thickness (see eq. 13), the gaseous absorption profile is a function of aerosol optical thickness as well as wavelength. The atmospheric pressure profile used in the algorithm is adapted from the mid-latitude summer profile of McClatchey et al. (1971).

3.5 Molecular and Aerosol Optical Thickness

The molecular scattering (or Rayleigh) optical thickness τ₆s is computed for sea level from (Hansen and Travis, 1974)

\[
τ₆s = 0.008569 \lambda^{-4} (1 + 0.0113 \lambda^{-2} + 0.00013 \lambda^{-4})
\]

where the wavelength is in micrometers. This expression assumes the surface sea-level pressure is 1013 mb. In the case the surface is not at sea level. The value of the molecular scattering optical thickness τ₆s is assumed to vary according to:

\[
τ₆s (Z_o) = τ₆s (0) \exp \left( \frac{-Z_o}{g} \right)
\]

where \(Z_o\) is the height of the surface above sea level in kilometers. In order to avoid a need for a new radiative transfer computation for each height of a surface, the computations are performed for \(Z_o = 0.4\) km, and the look-up table is adjusted each time the user specifies a different height. The algorithm adjusts the look-up table to account for the different molecular optical thickness by adjusting the wavelength of the radiation. Substitution of the relationship between \(λ\) and τ₆s (eq. 18) yields
\[ \lambda(Z_0) = \lambda(0.4) \exp \left[ \frac{(Z_0 - 0.4)}{36} \right] \]  

(20)

For example, for \( Z_0 = 0 \) km, \( \lambda \) will increase by 1.1\%, whereas for \( Z_0 = 2 \) km, \( \lambda \) will increase by 4.3\%. The error introduced by this method is in the effective scattering phase function, but this change is negligible relative to the general uncertainty in the scattering phase function due to the uncertainty in the aerosol size distribution.

The relation between upward radiance and surface reflectance for each spectral band is computed for four aerosol optical thicknesses, \( \tau_a = 0.0, 0.25, 0.50, \) and 1.00, except for TM bands 5 and 7 where only the first two values are used. These values are selected to cover the range of aerosol optical thicknesses which could be expected for most remote sensing applications.

3.6 Measurement Altitudes

The radiative transfer computations are tabulated at three measurement altitudes above the ground: 0.45 km, 4.5 km, and 80 km. The algorithm will use these altitudes to interpolate to the input measurement altitude. Since 80.0 kilometers is above the atmosphere, corrections to satellite measurements are made with the 80 km tables.

For altitudes less than 0.45 km, it is assumed that the aerosols and the absorbing gases are well mixed and it is possible to linearly interpolate between the atmospheric optical properties (\( L_o \) and \( T \)) at \( Z = 0 \) km and at \( Z = 0.45 \) km above the ground:

\[
L_o (Z) = L_o (Z = 0.45) \left( \frac{Z_{0.45}}{Z} \right)
\]

(21a)

\[
T (Z) = \frac{(T(0.45) Z + (0.45-Z)) / 0.45}{Z_{0.45}}
\]

(21b)

For altitudes above 4.5 km, interpolations are performed between values of \( L_o \) and \( T \) at 4.5 km and 80 km. Although the atmospheric model contains aerosols between these altitudes, the interpolations are based on the assumption that \( L_o \) and \( T \) depend essentially on molecular scattering between 4.5 and 80 km. The interpolations are performed linearly as a function of the molecular scattering coefficient \( \sigma_{gs}(Z) \):

\[
\sigma_{gs} (Z) = \sigma_{gs} (0) \exp \left( -\frac{Z}{9} \right)
\]

(22)

Hence the transmission and \( L_o \) become
Between 0.45 km and 4.5 km the interpolation is uncertain due to the inhomogeneity of the aerosol layer. Since the linear interpolation used below 0.45 km is usually suitable to extrapolate for heights below 1 km, and the aerosol concentration decreases rapidly above 3 km so that the interpolation used above 4.5 km is appropriate for extrapolation to heights between 3 and 4.5 km, the algorithm extrapolates from these two regions to the desired height \( h \) \((0.45 \text{ km} < Z < 4.5 \text{ km})\), and chooses the result that shows a minimal atmospheric effect. Thus, the lower values of \( L_0 \) and higher values of \( T \) are chosen for the solution.

4.0 Look-up Tables

The look-up tables used by the correction algorithm contain the normalized radiances and fluxes computed by the radiative transfer routines. The values generated are stored to be used by the main program. Eight look-up tables are produced, one table for each of the 2 AVHRR bands of 0.639 and 0.845 μm and one for each of the 6 TM bands of 0.486, 0.587, 0.663, 0.837, 1.663 and 2.189 μm. Each table is arranged for 3 heights of 0.45, 4.5 and 80.0 kilometers. Each table contains values for 9 solar zenith angles \( \theta_0 \) \((10, 20, 30, 40, 50, 60, 66, 72, \text{ and } 78^\circ)\), 13 observation zenith angles \( \theta \) \((0^\circ \text{ to } 78^\circ, \text{ every } 6^\circ)\), 19 observation azimuth angles \( \phi \) \((0^\circ \text{ to } 180^\circ, \text{ every } 10^\circ, \text{ plus } 5^\circ \text{ and } 175^\circ)\), and 4 aerosol optical thicknesses \( \tau_a \) \((0.0, 0.25, 0.50 \text{ and } 1.0)\) for all wavelengths, except for 1.663 and 2.189 μm where only first two optical thicknesses of 0.0 and 0.25 are used.

5.0 Computational Procedure

5.1 General

Input data to the correction program consists of the date, time, measurement wavelength, aerosol and gaseous absorption optical thicknesses at the measurement wavelength \( \lambda \), solar zenith angle \( \theta_0 \), observation scan angle \( \theta' \), observation azimuth angle \( \phi \), height of the surface above sea level \( Z_o \), measurement height \( Z^m \), and the measured spectral radiance in absolute \( L^m \) or reflectance \( L^{m'} \) units (option 1), or surface reflectance \( \rho \) (option 2). The wavelength, solar and observation angles, and aerosol optical thickness

\[
T(Z) = (T(80.0) - T(4.5)) \cdot \frac{\exp(-4.5/9) - \exp(-Z/9)}{\exp(-4.5/9) - \exp(80/9)}
\]

\[
L_0(Z) = (L_0(80.0) - L_0(4.5)) \cdot \frac{\exp(-4.5/9) - \exp(-Z/9)}{\exp(-4.5/9) - \exp(80/9)}
\]
data must be in the ranges discussed above as no extrapolation is performed. If the selected wavelength or altitude does not match the values used to construct the look-up table, the algorithm interpolates on wavelength and altitude as described above.

If option 1 is selected, the program computes the surface reflectance \( \rho \), total spectral irradiance on the surface \( F_g \), and total spectral radiance of the ground \( L_g \) in the direction of observation using eq. 2, 3, 4 and 5. The total spectral irradiance \( F_g \) is computed in Watts/m\(^2\)/\( \mu \)m and total upward spectral radiance \( L_g \) is computed in Watts/m\(^2\)/\( \mu \)m/sr. If option 2 is selected, the program computes the absolute and normalized radiances \( L_m \) and \( L_m' \), total spectral irradiance on surface \( F_g \) and the total upward spectral radiance \( L_g \) in Watts/m\(^2\)/\( \mu \)m/sr.

The program for making atmospheric corrections consists of a main program called FIFEWAV. The computations are performed in single precision and seven subroutines are called at different stages by the main program. The listings of the main program and subroutines are given in section 9. The statement numbers for the program and subroutines are given in parentheses. Table 3 lists the variables appearing in the main program and associated subroutines and their equivalent in the text. The asterisk superscript indicates interpolated values. The subroutines called by the main program FIFEWAV are listed below:

Subroutine \textbf{READIN} (6350 - 6580): This subroutine reads the input data from unit number 5 and writes it on unit number 6. The purpose of this subroutine is to perform a check on the input data set.

Subroutine \textbf{FINDW} (6620 - 6860): This subroutine checks to see if \( \lambda^m \) falls in the range of wavelengths for which look-up tables are available and picks two wavelengths between which \( \lambda^m \) falls. The subroutine prints an error message and returns the control to the main program if \( \lambda^m \) does not fall in the range of wavelengths look-up table provides. The main program processes a new data point.

Subroutine \textbf{INTSFX} (6900 - 7100): This subroutine computes values of the sun-earth distance \( R \) for 365 days of the year. These values are returned to the main program, which chooses the value of \( R \) for the day the measurements were made.

Subroutine \textbf{INTGHT} (7140 - 8530): This subroutine computes interpolated values of \( L_o \) and \( T/\pi \) for the measured height \( MHGHT \) (eq. 21a, 21b, 23 and 24). The transmission \( T \) is divided by \( \pi \) to account for the \( \pi \) appearing in eq. (1).

25
Table 3. Variables appearing in FIFEWAV program and associated subroutines, and their equivalent in the text.

<table>
<thead>
<tr>
<th>VARIABLES IN PROGRAM</th>
<th>VARIABLES IN TEXT</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSRAD</td>
<td>$L_g$</td>
</tr>
<tr>
<td>AIRR</td>
<td>$F_g^*$</td>
</tr>
<tr>
<td>AMUO</td>
<td>$\cos(\theta_0^m)$</td>
</tr>
<tr>
<td>ANGLE, PHI</td>
<td>$\varphi$</td>
</tr>
<tr>
<td>ARAD</td>
<td>$L_g^*$</td>
</tr>
<tr>
<td>FDOWN</td>
<td>$F_d$</td>
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<td>$F_o^*$</td>
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<td>FINT</td>
<td>$L_o^*$</td>
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<td>FOIRR</td>
<td>$F_o$</td>
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<tr>
<td>FT</td>
<td>$T^*/\pi$</td>
</tr>
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<td>INT</td>
<td>$L_o$</td>
</tr>
<tr>
<td>IRRID</td>
<td>$F_g$</td>
</tr>
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<td>$Z^m$</td>
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</tr>
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<td>MTHETO</td>
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</tr>
<tr>
<td>MWAV</td>
<td>$\lambda^m$</td>
</tr>
<tr>
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<td>$F_{d^*}$</td>
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<td>OPTH</td>
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</tr>
<tr>
<td>PIT</td>
<td>$T/\pi$</td>
</tr>
<tr>
<td>R, RR</td>
<td>$R$</td>
</tr>
<tr>
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<td>$\rho$</td>
</tr>
<tr>
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</tr>
<tr>
<td>SBAR</td>
<td>$s$</td>
</tr>
<tr>
<td>THE</td>
<td>$\theta$</td>
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<tr>
<td>THET0</td>
<td>$\theta_0$</td>
</tr>
<tr>
<td>WAV</td>
<td>$\lambda$</td>
</tr>
</tbody>
</table>
Subroutine INTERP (8570 - 9020): The purpose of this subroutine is to return to the main program the interpolated values of $L_0^*$, $F_d^*$, $T^*/\pi$, and $s^*$. This is a general purpose interpolation routine and is called at different stages by the main program. This subroutine does not allow any extrapolation and an error message is printed when the values are out of bounds. The asterisk indicates in most cases values interpolated from the look-up table.

Subroutine INTEXP (9040 - 9840): This subroutine sends exponentially interpolated values of $L_0^*$, $F_d^*$, $T^*/\pi$ and $s^*$ for the measured optical thickness $MTAU$. It calls a systems subroutine ZXGSN residing in IMSL math package.

Subroutine ZXGSN (9520): This subroutine is called by subroutine INTEXP, it computes the minimum of function on certain interval which is used for exponential interpolation.

PROCEDURE:

Subroutine READIN is called (490) to read the input data from unit number 5 and write it on unit number 6 as well as to perform a check on the input data. The main program reads the first two data cards from unit number 5 and writes the labels for output on unit number 6 and 56 (690 - 840). The program reads from unit 20 the solar spectral irradiances, which are used to compute the solar flux for the measured wavelength $MWAV$ (870 - 950). The program enters into a loop over the number (INUM) of data points to be processed (1020). The program reads the values of time, $\lambda^m$, $\tau_a^m$, $\theta_0^m$, $\theta^m$, $\phi^m$, $Z^m$, $L^m$ or $L^m'$, $\taugL$, $\tau gH$, and $Z_0$ if NOPT is 1, and values of time, $\lambda^m$, $\tau_a^m$, $\theta_0^m$, $\theta^m$, $\phi^m$, $Z^m$, $\rho^m$, $\tau gL$, $\tau gH$ and $Z_0$ if NOPT is 2 (1190 - 1230). If values of absorptions $\tau gL$ and $\tau gH$ are 0.0, default values are computed by interpolating linearly from the values of $TAUGL(\lambda)$ and $TAUGH(\lambda)$ (330 - 340) for the measured wavelength $\lambda^m$ (1300 - 1600).

Subroutine FINDW is called to select the look-up table for the two wavelengths between which $\lambda^m$ lies by assigning the proper file number $NFILE1$ and $NFILE2$ (1700 - 1740). A new data point is read if subroutine FINDW does not find the input wavelength lying in the range of wavelengths for which look-up table is provided (0.486 - 2.2 $\mu$m).

The ratio of earth-sun distance ($R$, eq. 8) is computed for 365 days of the year by subroutine INTSFX. The program (1870 - 2020) changes the month and day to the
Julian day if MOPT is 1. The corrected values of incident solar flux $F'_0$ are computed by using eq. 8 (2150).

The observation scan angle of satellite or aircraft is changed to observation zenith angle at the ground by using eq. 9 (2250).

Statements (2610 - 3040) read the look-up table chosen for the two wavelengths on each side of the measured wave length $\lambda^m$. The parameters $\theta, \phi, \lambda, \tau_a, \theta_o, s, F'_d, L'_o,$ and $T/\pi$ are read for four optical thicknesses, except for wavelengths of 1.663 $\mu$m and 2.189 $\mu$m, for which look-up tables are available for only two optical thicknesses of 0.0 and 0.25. The subroutine INTHGH (3120) is called to compute the interpolated values of $L'_o$ and $T/\pi$ for the measured height MHGHT (eq. 21a, 21b, 23 and 24). The statements (3170 - 3180) adjust the look-up table for the surface height SHGHT and then interpolate values of $s, F'_d, L'_o$ and $T/\pi$ (3230 - 3410) for the measured wavelength $\lambda^m$ (see documentation sec. 3.1). The excess or deficit gaseous absorption in the upper and lower atmosphere is computed (3450 - 3500) using $\tau_{gL}$ and $\tau_{gH}$ whose values are supplied as input parameters; or if values read are zero, default values computed earlier in program (1300 - 1600) are used. New values of $s, L'_o, F'_d,$ and $T/\pi$ are computed (3540 - 3780) after adjusting for the excess and deficit of gaseous absorption (eq. 17a, 17b, 17c, and 17d).

The next step is to compute the interpolated values of $L'_o, F'_d,$ and $T/\pi$ for the measured height and geometry ($\theta_o^m, \theta^m,$ and $\phi^m$). The mesh of the tables is small enough to allow accurate linear interpolations. The interpolations are made for each of the four aerosol optical thicknesses: 0.00, 0.25, 0.50, and 1.00. $L'^*_{o}$ is interpolated from the data set $L'_o$ for $\theta_o^m, \phi^m,$ and $\theta^m$; $F'^*_{d}$ is interpolated from the data set $F'_d$ for $\theta_o^m$; and $T'^*/\pi$ is interpolated from the data set $T/\pi$ for $\theta_m$. To make these interpolations, subroutine INTERP (4050 - 4140) is called to compute the interpolated values of both $L'_o$ and $F'_d$ for $\theta_o^m$. The part of program between 4270 to 4430 calls subroutine INTERP to compute the interpolated values of $L'_o$ for $\phi_m$. Subroutine INTERP is called again to compute the interpolated value of $L'_o$ and $T/\pi$ for $\theta_m$ (4680 - 4770). If the measured $\theta_o^m, \phi^m,$ or $\theta^m$ are out of the range of data tabulated for $\theta_o, \phi,$ and $\theta$, subroutine INTERP returns control to the main program after printing an appropriate message. It does not allow any extrapolation. The program will then process a new data point if the error message is printed.

28
Subroutine INTEXP (4910) is called to perform exponential interpolations on four radiation parameters \( L'^o, F'^d, s, \) and \( T'^\pi \) for measured optical thickness \( MTAU(4960-5190) \). Subroutine INTEXP checks the four parameters one by one for linearity and sends the control back to the main program if any of the functions are linear. The main program then calls the subroutine INTERP to perform linear interpolation.

Statement 5270 sends control to statement 5550 to compute the spectral radiance, if NOPT is 2. Statements 5280 to 5380 use the interpolated values \( L'^o, F'^d, \) and \( T'^\pi \) to compute the surface reflectance \( \rho \), total spectral irradiance \( F_g \), and total spectral radiance of the ground \( L_g \) in the direction of observation (eq. 2, 3, and 4). The total spectral irradiance \( F_g \) is computed in Watts/m\(^2\)/\( \mu \)m, and the total upward spectral radiance \( L_g \) is computed in Watts/m\(^2\)/\( \mu \)m/sr. Statements 5440 - 5450 write the output on FORTRAN logical units 6 and 56.

Statements 5540 - 5790 use the interpolated values \( L'^o, F'^d, \) and \( T'^\pi \) to compute the spectral radiance \( L'^m \), total spectral irradiance \( F'_g \), and total spectral radiance of the ground \( L'_g \) in the direction of observation (eq. 2, 3, and 4) if MOPT is 2. The total spectral irradiance \( F'_g \) is computed in Watts/m\(^2\)/\( \mu \)m, and the total upward spectral radiance \( L'_g \) is computed in Watts/m\(^2\)/\( \mu \)m/sr. The output is written on FORTRAN logical units 6 and 56.

5.2 Input

Input data to this program consists of two input files. The first file (FORTRAN logical unit number 5) describes input to a particular case, while the second file (FORTRAN logical unit number 20) reads solar spectral irradiances (Neckel and Labs, 1984) as function of wavelength. The program uses 8 look-up tables which are not to be changed by the user (FORTRAN logical unit numbers 7 - 14). In order to demonstrate the input data, four cases representative of different options are presented. The input cards for all four runs are listed in Table 4.

INPUT FROM UNIT NUMBER 5: (see Table 4)

1. The first card contains the labels to identify the input parameters in step 2 below.

2. The second card contains the options for determining the units for the measured reflectance or radiance, the format for the date of the measured data, and the option either to compute the surface reflectance or radiance.
<table>
<thead>
<tr>
<th>I/OPT</th>
<th>M/OPT</th>
<th>N/OPT</th>
<th>INUM</th>
<th>MON</th>
<th>DAY</th>
<th>YEAR</th>
<th>TIME</th>
<th>ZONE</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>7</td>
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<td>CDT</td>
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<td>MTAU</td>
<td>MTHET0</td>
<td>MTHET</td>
<td>MPHI</td>
<td>MHGHT</td>
<td>MINT</td>
<td>TAUH</td>
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<td>150.00</td>
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<td>00.0001</td>
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(RUN 2)

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<th>YEAR</th>
<th>TIME</th>
<th>ZONE</th>
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</thead>
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<td>MPHI</td>
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<td>0.00</td>
<td>0.00</td>
<td>80.00</td>
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<td>0.00</td>
<td>4.50</td>
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<td>0.00</td>
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<td>4.50</td>
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(RUN 4)

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<td>00.0001</td>
</tr>
<tr>
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<td>0.45</td>
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<td>150.00</td>
<td>0.45</td>
<td>00.05000</td>
<td>00.0001</td>
</tr>
</tbody>
</table>
IOPT, MOPT, NOPT (FORMAT: 315)

For IOPT = 1 measured radiance should be in reflectance units - $L_m'$.  
For IOPT = 2 measured radiance should be in absolute radiance units of Watts/m²/sr/μm.  
For MOPT = 1 the day, month and year should be provided.  
For MOPT = 2 the day of year (Julian date) and year should be given.  
For NOPT = 1 surface reflectance (RHOS) is computed.  
For NOPT = 2 radiance (MINT) is computed for the input geometry and height; the measured surface reflectance (MRHO) is input. The variable name MINT is used for programming convenience. MINT is a computed and not measured when NOPT = 2.

Run 1 and run 2 choose the IOPT = 1 option for selecting reflectance units of measured radiance and NOPT = 1 option to compute surface reflectance. MOPT (the option to select the day of year) is set to 1 and 2, respectively. Run 3 chooses IOPT = 2 and MOPT = NOPT = 1. Run 4 selects NOPT to be 2 and IOPT and MOPT to be 1.

3. The third card contains the labels to identify the input parameters in step 4 below.

4. The fourth card contains the information about the number of data points INUM to be processed, the date, and information about the time zone (e.g. Central, Pacific, or Mountain and Daylight or Standard Time). The information about the time zone is not used for any computations. There are two options for this input card depending on the value of MOPT (FORMAT 415, 30A1):

For MOPT = 1: INUM, IMONTH, IDAY, IYEAR, TIME ZONE

e.g.: (1, 12, 25, 1987, CDT) for 1 data point to be processed for December 25, 1987, and the time zone is CDT or Central Daylight Time.

For MOPT = 2: INUM, IDAY, IYEAR, TIME ZONE

e.g.: (1, 359, 1987, CDT)
The fourth row of run 1 reads number of data points, month, day, and year. The fourth row of run 2 reads number of data points, the Julian day and year for which the measurements are made. The radiance is read in reflectance units for both runs 1 and 2.

5. The fifth card contains the labels to identify the input parameters in step 6 below.

6. The sixth card can be repeated for the number of data points INUM. The data points should be for the same date. The following names in parentheses are the variable names used by the program. The sixth card contains values of time in hours and minutes (without punctuation; this input parameter is not used in any computations but is merely for the record) (MTIME), measured wavelength $\lambda^m$ (MWAV) in micrometers, measured aerosol optical thickness $\tau^m_a$ (MTAU) for the measured wavelength $\lambda^m$, solar zenith angle $\theta^m_0$ (MTHET0) in degrees, observation scan angle $\theta^m$ (MTHET) in degrees, observation azimuth angle $\varphi^m$ (MPHI) in degrees, observation height $Z^m$ (MHGHT) in kilometers, and spectral radiance $L^m$ or $L^m$ (MINT) from satellite or aircraft in reflectance or absolute units depending on option (IOPT ) read in the first card, $\tau_{gL}$ (TAUGL), $\tau_{gH}$ (TAUGH), and $Z_0$ (SHGHT) in the following format:

```
MTIME, MWAV, MTAU, MTHET0, MTHET, MPHI, MHGHT, MINT, TAUGH, TAUGL, SHGHT
```

```
FORMAT (I5,1x, F 6.3, F 6.3, 4F7.2,F10.5,3F8.4)
```

Card 6 for run 3 reads the measured radiance in absolute units. Card 6 for run 4 when NOPT is 2 reads the surface reflectance value MRHO in the following format:

```
MTIME, MWAV, MTAU, MTHET0, MTHET, MPHI, MHGHT, MRHO, TAUGH, TAUGL, SHGHT
```

```
FORMAT (I5,1x, F 6.3, F 6.3, 4F7.2,F10.5,3F8.4)
```

**INPUT FROM LOOK-UP TABLE (UNIT NUMBERS 7 - 14):**

The input from the look-up table is read after the execution of the statements given in parentheses. Subroutine FINDW (1610) searches the data base and then reads only the

32
required table for the two wavelengths between which measured wavelength $\lambda^m$ (MWAV) lies. The following are obtained:

1. values of observation zenith angle $\theta$ in degrees (2610, 2650)

2. values of observation azimuth angle $\phi$ in degrees (2610, 2660)

3. values of wavelength $\lambda$ in micrometers, aerosol optical thickness $\tau_a$, reflectance of atmosphere $s$, flux incident on surface $F_d$ (2830 - 2840)

4. a blank line

5. values of atmospheric radiance $L_b'$ in reflectance units (2920 - 2950)

6. values of transmission from surface $T/x$ (3000).

5.3 Output

A successful execution of this FORTRAN program will result in the output described below, which is written to FORTRAN logical unit numbers 6 and 56. Four cases representative of the different options discussed in section 5.2 are shown in Table 5.

OUTPUT TO UNIT NUMBER 6: (see Table 5.)

The input data set is written as it is read from unit number 5. The message 'END OF INPUT DATA SET AS READ FROM UNIT NUMBER 5' marks the end of input data set. The next line of this output contains the information about the date for which measurements are made:

For $\text{MOPT} = 1$, month, day, year, and time zone will be written (Table 5, run 1).

For $\text{MOPT} = 2$, Julian day, year, and time zone will be written (Table 5, run 2).

Each subsequent line gives the measured and derived output for each data point processed. The first seven entries on a line provide information about the following input parameters as read from logical unit number 5: time (Central, Pacific, or Mountain and Daylight or Standard) in hours and minutes, measured wavelength ($\lambda^m$) in micrometers, measured aerosol optical thickness ($\tau_a^m$), solar zenith angle ($\theta_o^m$) in degrees at the time of observation, measured observation zenith angle ($\theta^m$) in degrees, measured observation azimuth angle ($\phi^m$) in degrees, and measured observation height ($Z^m$) in kilometers. The
### Table 5. Examples of Computed Products with Input from Table 4.

(RUN1)

Start of input data as read from unit number 5

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<table>
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**Julian date 209/1987 time zone CDT**

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</tr>
<tr>
<td>1015</td>
<td>0.486</td>
<td>0.25</td>
<td>26.00</td>
<td>18.00</td>
<td>150.00</td>
<td>0.45</td>
<td>29.02</td>
<td>0.0524</td>
<td>1701.</td>
<td>27.07</td>
<td>0.050</td>
</tr>
</tbody>
</table>
eighth and ninth entries depend on IOPT. For IOPT = 1, the measured radiance $L_m^m$ is given in reflectance units and labeled under RELATIVE RADIANCE; the corresponding radiance in absolute units is computed by the program and is labeled under RADIANCE. For IOPT = 2, the measured radiance $L_m^m$ is given in absolute units and is labeled under RADIANCE; the corresponding radiance in reflectance units is computed by the program and is labeled under RELATIVE RADIANCE. Next three surface quantities are computed and given as values for total spectral irradiance $F_g^s$, total spectral radiance $L_g^s$, and surface reflectance $\rho$ (eq.2,3 and 4). The output in Table 5 for run 4 computes the eighth and ninth columns, since NOPT = 2 and the surface reflectance is input (column 12).

OUTPUT TO UNIT NUMBER 56: (see Table 6)

The label for this output shown in table 6 reads INTERPOLATED RADIATION PARAMETERS. The second line gives information about the date and time zone of the input data set. Each subsequent line gives the measured and derived output for each data point processed. The first seven entries on a line provide information about the following input parameters as read from logical unit number 5: time (Central, Pacific, or Mountain and Daylight or Standard) in hours and minutes, measured wavelength ($\lambda_m^m$) in micrometers, measured aerosol optical thickness ($\tau_a^m$), solar zenith angle ($\theta_o^m$) in degrees at the time of observation, measured observation zenith angle ($\theta_m^m$) in degrees, measured observation azimuth angle ($\phi_m^m$) in degrees, and measured observation height ($Z_m^m$) in kilometers. The eighth and ninth entries depend on IOPT and NOPT. For IOPT = 1 and NOPT = 1, the measured radiance $L_m^m$ is given in reflectance units and labeled under RELATIVE RADIANCE; the corresponding radiance in absolute units is computed by the program and is labeled under RADIANCE. For IOPT = 2 and NOPT = 1, the measured radiance $L_m^m$ is given in absolute units and is labeled under RADIANCE; the corresponding radiance in reflectance units is computed by the program and is labeled under RELATIVE RADIANCE. If NOPT = 2 the values of radiance in absolute and reflectance units are computed by the program. Next three quantities are computed and given as values for $L_o^s$, $F_d^s$, $s$, and transmission $T$ (eq. 6).

5.4 Summary of Error Messages

A summary of error messages is given in Table 7. First, examples of input data are given followed by the different error messages that would result. For the sixth row and second column of the upper table, where the input measured wavelength $\lambda_m^m = 0.345$, it
<table>
<thead>
<tr>
<th>DATE</th>
<th>7/ 28/ 1987</th>
<th>TIME ZONE CDT</th>
<th>INTERPOLATED RADIATION PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIME</td>
<td>WAVELENGTH</td>
<td>OPTICAL THICKNESS</td>
<td>ZENITH DEG</td>
</tr>
<tr>
<td>MICROMETERS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1010</td>
<td>0.486</td>
<td>0.25</td>
<td>26.00</td>
</tr>
<tr>
<td>1011</td>
<td>0.486</td>
<td>0.25</td>
<td>26.00</td>
</tr>
<tr>
<td>1012</td>
<td>0.486</td>
<td>0.25</td>
<td>26.00</td>
</tr>
<tr>
<td>1013</td>
<td>0.486</td>
<td>0.25</td>
<td>26.00</td>
</tr>
<tr>
<td>1014</td>
<td>0.486</td>
<td>0.25</td>
<td>26.00</td>
</tr>
<tr>
<td>1015</td>
<td>0.486</td>
<td>0.25</td>
<td>26.00</td>
</tr>
<tr>
<td>TIME</td>
<td>WAVELENGTH</td>
<td>OPTICAL THICKNESS</td>
<td>SOLAR ZENITH</td>
</tr>
<tr>
<td>------</td>
<td>------------</td>
<td>-------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>1010</td>
<td>0.345</td>
<td>0.250</td>
<td>26.00</td>
</tr>
<tr>
<td>1011</td>
<td>0.845</td>
<td>3.000</td>
<td>26.00</td>
</tr>
<tr>
<td>1012</td>
<td>0.486</td>
<td>0.250</td>
<td>79.00</td>
</tr>
<tr>
<td>1013</td>
<td>0.486</td>
<td>0.250</td>
<td>26.00</td>
</tr>
<tr>
<td>1014</td>
<td>0.486</td>
<td>0.250</td>
<td>26.00</td>
</tr>
</tbody>
</table>

**TABLE 7. EXAMPLES OF ERROR MESSAGES FOR VARIOUS ERRORS IN THE INPUT DATA**

**START OF INPUT DATA AS READ FROM UNIT NUMBER 5**

<table>
<thead>
<tr>
<th>IOPT</th>
<th>MOPT</th>
<th>NOPT</th>
<th>INUM</th>
<th>MON</th>
<th>DAY</th>
<th>YEAR</th>
<th>TIME</th>
<th>DATE</th>
<th>ZONE</th>
<th>CDT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>7</td>
<td>28</td>
<td>1987</td>
<td>000</td>
<td>5/28</td>
<td>1987</td>
<td>CDT</td>
</tr>
</tbody>
</table>

**END OF INPUT DATA SET AS READ FROM UNIT NUMBER 5**

**DATE 7/28/1987 TIME ZONE CDT**

**FLIGHT LEVEL**

**GROUND LEVEL**

**MEASURED WAVELENGTH (MWAV) 0.345 IS OUT OF RANGE. THE DATA POINT # 1 IS NOT PROCESSED. ACTUAL RANGE 0.486-2.189 UM.**

**MEASURED OPTICAL THICKNESS (MTAU) 3.000 IS OUT OF RANGE. ACTUAL RANGE 0.0-1.0. THE DATA POINT # 2 IS NOT PROCESSED.**

**MEASURED SOLAR ZENITH ANGLE (MTHET0) 79.0 IS OUT OF RANGE. ACTUAL RANGE IS 10-70 DEGREES. THE DATA POINT # 3 IS NOT PROCESSED.**

**MEASURED OBSERVATION ZENITH ANGLE (MTHETA) 74.4 IS OUT OF RANGE. ACTUAL RANGE IS 0-72 DEGREES. THE DATA POINT # 4 IS NOT PROCESSED.**

**MEASURED OBSERVATION AZIMUTH ANGLE (MPHI) 181.0 IS OUT OF RANGE. ACTUAL RANGE IS 0-180 DEGREES. THE DATA POINT # 5 IS NOT PROCESSED.**
does not lie between the range (0.486 - 2.2 \mu m), for which the look-up table is available (section 4.0). The subroutine FINDW prints an error message '\textit{Measured wavelength 0.345 is out of range. The data point #1 is not processed; Actual range (0.486 - 2.2 \mu m)}. For the following rows (2, 3, 4, and 5) of input data, the measured aerosol optical thickness $\tau_m$, solar zenith angle $\theta_O$, observation zenith angle $\theta_m$, and observation azimuth angle $\phi_m$ are out of the ranges of values provided by the look up table (section 4.0). The subroutine INTERP prints error messages stating the variables which are out of range.

\textbf{6.0 Sensitivity Study}

A careful error analysis would be extensive, because so many measurement and surface parameters are involved. Instead an attempt is made to estimate the maximum errors that might occur during FIFE in estimating the surface reflectance ($\rho$). The errors given here are are not root-mean-square errors. The surface radiance $L_g$ error can be calculated with the reflectance error and eq. 5. The irradiance at the ground ($F_g$ (eq.4) and also $F_d$ (eq. 4 and 5)) is not affected appreciably by the perturbations. The main source of error usually is caused in estimation of the path radiance ($L_o$). The absolute surface reflectance error is insensitive to strength of the reflectance.

The perturbations are placed in three categories: model errors, interpolation errors, and uncertainties in the input data (input errors). The model parameters studied are:

1) uncertainty in the aerosol single scattering phase function
2) uncertainty due to the neglect of polarization in the radiative transfer computations
3) uncertainty in water vapor and ozone absorption
4) uncertainty in aerosol absorption
5) uncertainty in the height distribution of aerosols

Since the algorithm must interpolate parameters on aerosol optical thickness $\tau_a$, solar zenith angle $\theta_O$, observation zenith angle $\theta$, azimuth angle $\phi$, wavelength $\lambda$, and height $Z$, the errors introduced by these interpolations are studied. Finally, because the input data will have some uncertainty associated with them, the effects of errors in the input radiance, input aerosol optical thickness, and input geometry are studied. The studies described above are mostly performed for a wavelength $\lambda = 0.486 \mu m$, since the atmospheric effects generally are the greatest at the shortest wavelength. Unless stated otherwise, the
The atmospheric correction algorithm causes errors because the model does not represent exactly the state of the atmosphere at the time of an observation. The simulated measured remote radiance or surface reflectance is first computed with a radiative transfer model that contains the perturbation. These computations produce a given perturbed radiance \( \Delta L_m \) at the sensor altitude, as well as the other radiation parameters. The perturbed radiance \( \Delta L_m \) serves as an input in the correction algorithm, resulting in a derived perturbed surface reflectance \( \rho + \Delta \rho \).

### 6.1.1 Aerosol Single Scattering Phase Function

The scattering effects of aerosols increase with increasing aerosol optical thickness, which usually is largest at the shortest wavelengths; but compared with molecular scattering, the relative contribution by the aerosols to backward scattering decreases with decreasing wavelength. Therefore, an intermediate wavelength of 0.639 \( \mu \text{m} \) (AVHRR band 1) is used for the phase function sensitivity study.

The phase function sensitivity test is performed using two phase functions which differ appreciably from the one used in the model. As discussed previously, the aerosol single scattering phase function used in the algorithm is computed for two log-normal aerosol size distributions combined, which are chosen to represent accumulation and coarse particle modes. The first phase function used in this sensitivity analysis is chosen from a
Table 8. Case numbers assigned to Sensitivity Tests for the Atmospheric Correction Algorithm

<table>
<thead>
<tr>
<th>KEY</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>Solar zenith angle ($^\circ$)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Observation zenith angle ($^\circ$)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength ($\mu$m)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Surface reflectance for no error</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>Derived surface reflectance for the given error</td>
</tr>
<tr>
<td>$%\rho$</td>
<td>Percentage error in derived surface reflectance</td>
</tr>
<tr>
<td>$\tau_a$</td>
<td>Aerosol optical thickness</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Azimuth angle ($^\circ$)</td>
</tr>
<tr>
<td>$F_g$</td>
<td>Irradiance (Watts/m$^2$-$\mu$m) on a horizontal surface at the ground for no error</td>
</tr>
<tr>
<td>$F_{gd}$</td>
<td>Derived irradiance (Watts/m$^2$-$\mu$m) on a horizontal surface at the ground for the given error</td>
</tr>
<tr>
<td>$%F_g$</td>
<td>Percentage error in the derived irradiance</td>
</tr>
<tr>
<td>$L_d$</td>
<td>Input radiance (reflectance units) for the given error</td>
</tr>
<tr>
<td>$L_g$</td>
<td>Upward radiance (Watts/m$^2$/\mu$m/sr) at the ground for no error</td>
</tr>
<tr>
<td>$%L_g$</td>
<td>Percentage error in the derived radiance at the ground</td>
</tr>
<tr>
<td>$L_{gd}$</td>
<td>Derived radiance (Watts/m$^2$/\mu$m/sr) at the ground for the given error</td>
</tr>
</tbody>
</table>
Table 8 continued

\( I_m = \) Input radiance (reflectance units) at sensor height for no error

\( aL = \) Percentage error in input radiance at sensor height

\( z = \) Altitude (km)

### A1 Model Errors

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
</table>

### Aerosol Single Scattering Phase Function:

1. Single mode, log-normal aerosol size distribution \((n = 1.349 - 0.008i)\), geometric mean mass radius \(r_m = 0.60 \mu m, \sigma = 0.6\)

2. Power law aerosol size distribution \((\delta = 4)\), refractive index \(n = 1.53 - 10^{-2i}\)

### Polarization:

3. Wavelength = 0.486 \(\mu m\) (TM band 1)

4. Wavelength = 0.639 \(\mu m\) (AVHRR band 1)

5. Wavelength = 0.845 \(\mu m\) (AVHRR band 2)

### Water vapor absorption:

6. Water vapor absorption optical thickness \(\tau_g = 0.0486\) (AVHRR band 2)
Table 8 continued

7. Water vapor absorption optical thickness $\tau_g = 0.1235$ (AVHRR band 2)

Ozone absorption:

8. Ozone absorption optical thickness $\tau_g = 0.039$ (TM band 2)

9. Ozone absorption optical thickness $\tau_g = 0.024$ (TM band 2)

Aerosol single scattering albedo:

10. Aerosol single scattering albedo $\omega_o = 0.84$

10a. Aerosol single scattering albedo $\omega_o = 0.77$

Aerosol height distribution:

11. Model 1. 50 km visibility, rural in boundary layer; 50 km visibility spring/summer in upper troposphere

12. Model 2. 23 km visibility, rural in boundary layer; 23 km visibility spring/summer in upper troposphere

13. Model 3. 10 km visibility, rural in boundary layer; 10 km visibility spring/summer in upper troposphere
Table 8 continued

B) Interpolation Errors.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>For input $T_a = 0.375$, algorithm interpolates between $T_a = 0.25$ and $T_a = 0.50$</td>
</tr>
<tr>
<td>15</td>
<td>For input $\Theta_0 = 26^\circ$, algorithm interpolates between $\Theta_0 = 20^\circ$ and $\Theta_0 = 30^\circ$</td>
</tr>
<tr>
<td>16</td>
<td>For input $\Theta = 20^\circ$, algorithm interpolates between $\Theta = 18^\circ$ and $\Theta = 24^\circ$</td>
</tr>
<tr>
<td>17</td>
<td>For input $\phi = 145^\circ$, algorithm interpolates between $\phi = 140^\circ$ and $\phi = 150^\circ$</td>
</tr>
</tbody>
</table>

18-32 Descriptions are given in Table 9.

C) Input Errors.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>Input error in radiance of $5%$</td>
</tr>
<tr>
<td>34</td>
<td>Input error in radiance of $10%$</td>
</tr>
<tr>
<td>35</td>
<td>Input error in aerosol optical thickness of $\Delta T_a = 0.10$ for true $T_a = 0.25$</td>
</tr>
<tr>
<td>36</td>
<td>Input error in aerosol optical thickness of $\Delta T_a = 0.05$ for true $T_a = 0.25$</td>
</tr>
<tr>
<td>37</td>
<td>Input error in aerosol optical thickness of $\Delta T_a = 0.20$ for true $T_a = 1.00$</td>
</tr>
</tbody>
</table>
Table 8 continued

38 Input error in aerosol optical thickness of $\Delta \tau_\lambda = 0.10$ for true $\tau_\lambda = 1.00$

39 Input error in solar zenith angle of $\Delta \theta_0 = 4^\circ$ for true $\theta_0 = 30^\circ$

40 Input error in solar zenith angle of $\Delta \theta_0 = 2^\circ$ for true $\theta_0 = 30^\circ$

41 Input error in observation zenith angle of $\Delta \theta = 2^\circ$ for true $\theta = 0^\circ$

42 Input error in observation zenith angle of $\Delta \theta = 4^\circ$ for true $\theta = 0^\circ$

43 Input error in azimuth angle of $\Delta \phi = 2^\circ$ for true $\phi = 0^\circ$

44 Input error in azimuth angle of $\Delta \phi = 4^\circ$ for true $\phi = 150^\circ$
Table 9. Summary of sensitivity results.

**KEY**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>Surface reflectance for no error</td>
</tr>
<tr>
<td>%ρ</td>
<td>Percentage error in derived surface reflectance</td>
</tr>
<tr>
<td>Lg</td>
<td>Upward radiance (Watts/m²/sr/μm) at the ground for no error</td>
</tr>
<tr>
<td>%L</td>
<td>Percentage error in input radiance at Z = 80 km</td>
</tr>
</tbody>
</table>

**A1 Model errors.**

Geometry 1: \( \theta_o = 30^\circ, \ \phi = 0^\circ, \ \tau_a = 0.25, \ Z = 80 \ km \)

Geometry 2: \( \theta_o = 30^\circ, \ \phi = 18^\circ, \ \tau_a = 0.25, \ Z = 80 \ km \)

<table>
<thead>
<tr>
<th>Case</th>
<th>( \lambda (\mu\text{m}) )</th>
<th>( L_g )</th>
<th>%L</th>
<th>ρ</th>
<th>%ρ</th>
<th>( L_g )</th>
<th>%L</th>
<th>ρ</th>
<th>%ρ</th>
<th>Description of perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.639</td>
<td>0.0741</td>
<td>-9.7</td>
<td>0.05</td>
<td>-18.0</td>
<td>0.0803</td>
<td>11.8</td>
<td>0.05</td>
<td>-24.0</td>
<td>single mode, log-normal aerosol size distribution</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(refractive index ( n = 1.349 - 0.0001 ))</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>geometric mean mass radius ( r_m = 0.60 \ \mu\text{m}, \ \sigma = 0.6 )</td>
</tr>
<tr>
<td>2</td>
<td>0.639</td>
<td>0.0741</td>
<td>15.4</td>
<td>0.05</td>
<td>28.0</td>
<td>0.0803</td>
<td>13.1</td>
<td>0.05</td>
<td>26.0</td>
<td>power law aerosol size distribution (( \nu = 4 )),</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(refractive index ( n = 1.53 - 10^{-8} ))</td>
</tr>
<tr>
<td>3</td>
<td>0.486</td>
<td>0.1122</td>
<td>2.3</td>
<td>0.05</td>
<td>5.9</td>
<td>0.1230</td>
<td>3.1</td>
<td>0.05</td>
<td>10.2</td>
<td>polarization: ( \lambda = 0.486 \ \mu\text{m} ) (TM band 1)</td>
</tr>
<tr>
<td>4</td>
<td>0.639</td>
<td>0.0746</td>
<td>1.0</td>
<td>0.05</td>
<td>2.0</td>
<td>0.0783</td>
<td>1.3</td>
<td>0.05</td>
<td>2.1</td>
<td>polarization: ( \lambda = 0.639 \ \mu\text{m} ) (AVHRR band 1)</td>
</tr>
<tr>
<td>Case</td>
<td>$\lambda$</td>
<td>$L_g$</td>
<td>$L_L$</td>
<td>$\rho$</td>
<td>$%\rho$</td>
<td>$L_g$</td>
<td>$L_L$</td>
<td>$\rho$</td>
<td>$%\rho$</td>
<td>Description of perturbation</td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>--------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>--------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>5</td>
<td>0.845</td>
<td>0.1635</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
<td>0.1594</td>
<td>0.3</td>
<td>0.2</td>
<td>0.5</td>
<td>polarization: $\lambda = 0.845$ $\mu$m (AVHRR band 2)</td>
</tr>
<tr>
<td>6</td>
<td>0.845</td>
<td>0.1635</td>
<td>9.9</td>
<td>0.2</td>
<td>11.5</td>
<td>0.1594</td>
<td>9.9</td>
<td>0.2</td>
<td>11.0</td>
<td>water vapor absorption optical thickness $\tau_H^\lambda = 0.0486$, model $\tau_H^\lambda = 0.0933$</td>
</tr>
<tr>
<td>7</td>
<td>0.845</td>
<td>0.1635</td>
<td>-6.1</td>
<td>0.2</td>
<td>-7.0</td>
<td>0.1594</td>
<td>-6.1</td>
<td>0.2</td>
<td>-7.0</td>
<td>water vapor absorption optical thickness $\tau_H^\lambda = 0.1235$, model $\tau_H^\lambda = 0.0933$</td>
</tr>
<tr>
<td>8</td>
<td>0.587</td>
<td>0.0830</td>
<td>1.9</td>
<td>0.05</td>
<td>4.0</td>
<td>0.0902</td>
<td>2.0</td>
<td>0.05</td>
<td>4.0</td>
<td>ozone absorption optical thickness $\tau_O^\lambda = 0.039$, model $\tau_O^\lambda = 0.032$</td>
</tr>
<tr>
<td>9</td>
<td>0.587</td>
<td>0.0830</td>
<td>-1.3</td>
<td>0.05</td>
<td>-2.0</td>
<td>0.0902</td>
<td>-1.3</td>
<td>0.05</td>
<td>-4.0</td>
<td>ozone absorption optical thickness $\tau_O^\lambda = 0.024$, model $\tau_O^\lambda = 0.032$</td>
</tr>
<tr>
<td>10</td>
<td>0.486</td>
<td>0.1122</td>
<td>-0.3</td>
<td>0.05</td>
<td>0.0</td>
<td>0.1230</td>
<td>-3.4</td>
<td>0.05</td>
<td>-10.0</td>
<td>aerosol single scattering albedo $\omega_o = 0.84$, model $\omega_o = 0.96$</td>
</tr>
</tbody>
</table>

Geometry: $\theta_o = 64^\circ$, $\theta_o = 56^\circ$, $\phi = 15^\circ$, $\omega_o = 0.091$, $2 = 80$ km, $\rho = 0.6$, $\lambda = 0.845$ $\mu$m

10a | 0.845 | 0.342 | -10.5 | 0.60 | -10.7 | reduced aerosol $\omega_o$ to 0.77 |

11 | 0.486 | 0.1122 | 0.0 | 0.05 | 0.0 | 0.1230 | 0.0 | 0.05 | 0.0 | aerosol height distribution (50 km visibility) |

12 | 0.486 | 0.1122 | -0.1 | 0.05 | 0.0 | 0.1230 | -0.1 | 0.0 | -0.0 | aerosol height distribution (23 km visibility) |

13 | 0.486 | 0.1122 | -0.2 | 0.05 | 0.0 | 0.1230 | -0.3 | 0.05 | 0.0 | aerosol height distribution (10 km visibility) |
### Table 9 continued

**B1. Interpolation errors.**

**Geometry:** \( \theta_o = 30^\circ, \ \phi = 180^\circ, \ \phi_o = 150^\circ, \ t_a = 0.25, \ z = 80 \text{ km}, \ \rho = 0.05, \ \lambda = 0.486 \text{ \( \mu \)m}, \ L_g = 0.1230 \)

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<td>For input ( \lambda = 0.500 \text{ ( \mu )m} ), algorithm interpolates between ( \lambda = 0.486 \text{ ( \mu )m} ) and ( \lambda = 0.586 \text{ ( \mu )m} )</td>
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### Table 9 continued

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<td>Errors for large solar and nadir angles. Interpolations are made on $\tau_a$, $\theta_0$, $\theta$, and $\phi$. For $Z = 80$ km, $\lambda = 0.639$ $\mu$m, $\tau_a = 0.35$, $\theta_0 = 74^\circ$, $\theta = 56^\circ$, $\phi = 15^\circ$, $p = 0.05$</td>
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### C1. Input errors.

Geometry: $\theta_0 = 30^\circ$, $\theta = 0^\circ$, $\phi = 0^\circ$, $\tau_a = 0.25$, $Z = 80$ km, $p = 0.05$, $\lambda = 0.486$, $Lg = 0.1122$

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Table 10. Sensitivity Results

A1 Model errors.

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| 2    | 0.639 | 30.0 | 0.0 | 0.0 | 0.25 | 80.00 | 0.0741 | 0.0855 | 15.4 | 0.050 | 0.064 | 28.0 | 1449.7 | 1451.9 | 0.1 | 23.1 | 29.5 | 28.1 |
|      | 4.50 | 0.0637 | 0.0742 | 16.4 | 0.050 | 0.062 | 24.0 | 1449.7 | 1451.6 | 0.1 | 23.1 | 28.8 | 24.7 |
|      | 0.45 | 0.0501 | 0.0537 | 7.2 | 0.050 | 0.054 | 8.0 | 1449.7 | 1450.3 | 0.0 | 23.1 | 25.0 | 8.2 |
| 18.0 | 150.0  | 80.00 | 0.0803 | 0.0909 | 13.1 | 0.050 | 0.063 | 26.0 | 1449.7 | 1451.7 | 0.1 | 23.1 | 29.1 | 26.1 |
|      | 4.50 | 0.0684 | 0.0785 | 14.8 | 0.050 | 0.062 | 24.0 | 1449.7 | 1451.5 | 0.1 | 23.1 | 28.6 | 24.0 |
|      | 0.45 | 0.0517 | 0.0552 | 6.8 | 0.050 | 0.054 | 8.0 | 1449.7 | 1450.3 | 0.0 | 23.1 | 24.9 | 8.0 |
Table 10 continued

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B) Interpolation errors.

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library of phase functions that were computed for 19 models of the accumulation mode. The size distribution is a single log-normal aerosol size distribution with an index of refraction of \( n = 1.349 - 0.008i \), geometric mean mass radius \( r_m = 0.6 \mu m \), and standard deviation of the logarithm of the radius \( \sigma = 0.61 \). This phase function is compared with the phase function used in the algorithm in Figure 5 along. The second phase function used in the sensitivity analysis is chosen assuming a power law aerosol size distribution. The power chosen is \( \nu = 4 \) where

\[
\frac{dn}{d\ln r} \sim r^{-\nu}
\]

and the index of refraction is \( n = 1.53 - 10^{-7}i \). This phase function is also shown in Figure 5. Chayanova and Shifrin (1966) found this power law most closely matched the model number 4 (the surface visibility is 20 km) aerosol phase function measured by Barteneva (1960). These two models are designated as cases 1 and 2 in Tables 8-10. Figure 5 shows that the bimodal log-normal phase function used in the algorithm lies between the two phase functions used in the sensitivity study for scattering angles greater than about 50°. Therefore, these two phase functions are chosen to represent extreme experimental conditions where the true aerosol scattering in the backward direction does not match the scattering assumed in the model.

Both of the phase functions used in the sensitivity analysis produced errors in the derived surface reflectance of approximately ±20-30%. The phase function derived using the single mode log-normal aerosol size distribution produced surface reflectance values too small, while the phase function derived using the power size distribution produced surface reflectance values too large. It should be noted that the two phase functions were chosen to represent the possible extremes in the scattering phase function (for large scattering angles) so that the errors associated with uncertainties in the phase function should usually be smaller.

Large negative reflectance errors (case 1) indicate that the algorithm will return negative reflectances, if the actual reflectance is weak. The large negative percentage reflectance error corresponds to an error magnitude of \( \Delta \rho = -0.012 \). If the simulation surface reflectance was less than +0.012, the algorithm would return a negative reflectance.

The large errors associated with these uncertainties in the aerosol single scattering phase function indicate that the algorithm in its present form is limited to applications where the aerosol scattering phase function closely matches the function used in the algorithm.
Figure 5. Aerosol single scattering phase functions derived from: 1) bimodal log-normal aerosol size distribution used in the unperturbed model (---), 2) single log-normal aerosol size distribution with an index of refraction \( n = 1.349 - 0.008i \), geometric mean mass radius \( r_m = 0.6 \mu m \), standard deviation of the logarithm of the radius \( \sigma = 0.61 \) (---), and 3) power law aerosol size distribution with \( \nu = 4 \) and an index of refraction \( n = 1.53 - 10^{-7}i \) (---).
However, the algorithm could be easily modified for use with other aerosol size distributions by replacing the look-up tables of path radiance, downward flux, transmission, and atmospheric backscattering ratio with values generated for a more appropriate aerosol size distribution. It is not necessary for these values to be generated by the same Dave (1972 a,b,c,d) radiative transfer code used to derive the values used here as long as the values tabulated are defined as in eq. (6-8).

6.1.2 Polarization

The algorithm which performs the atmospheric corrections uses tabulated radiances and fluxes computed with a Dave radiative transfer code. This code is based upon the scalar form of the radiative transfer equation, where the assumption is that the light scattered by the atmosphere and the earth's surface is unpolarized. If the atmospheric optical thickness is small this assumption is satisfactory, since the primary source of the light scattered by the atmosphere is the direct, unpolarized sunlight. The polarization errors are caused by second and higher order scattering of light that is polarized, and increase with optical thickness and decreasing wavelength. Light scattered by the surface, and especially by the atmosphere, is polarized.

Therefore, sensitivity tests are performed to estimate surface reflectance errors caused by neglect of polarization. In these tests, polarization is accounted for by another Dave code, similar in all other aspects to the scalar code. These sensitivity tests are performed for three wavelengths (cases 3,4,5): 0.486 μm (TM band 1), 0.639 μm (AVHRR band 1), and 0.845 μm (AVHRR band 2). Errors in derived surface reflectance associated with the assumption of unpolarized light are less than 3% for λ ≥ 0.639 μm (cases 4 and 5), but increase to approximately 10% for λ = 0.486 μm (TM band 1). Although the results are not shown in the tables, the errors associated with the assumption of unpolarized light generally decrease with decreasing aerosol optical thickness because less multiple scattering occurs.

6.1.3 Water Vapor and Ozone Absorption

Another potential source of error which is examined is the effect of using an incorrect value of the gaseous absorption. Since most of the gaseous absorption in the Landsat TM and NOAA AVHRR visible and near IR bands is contributed by either water vapor or ozone, the sensitivity tests are run at those wavelengths where absorption by these
constituents is the largest. For water vapor this occurs for AVHRR band 2 while for ozone this occurs for TM band 2.

Since the amount of an absorbing gas is variable, the algorithm uses a weighted average of the gaseous absorption values computed for the tropical, mid-latitude summer, and mid-latitude winter models given in the LOWTRAN code. Most of the weight is given to the mid-latitude summer profile (see eq. 16). In the case of AVHRR band 2, the algorithm uses a water vapor gaseous absorption optical thickness of $\tau_g = 0.0933$. The gaseous absorption optical thicknesses computed using the mid-latitude winter and tropical profiles are used for the sensitivity studies. Therefore, two tests are made to estimate the uncertainty due to water vapor absorption; the first test uses the mid-latitude winter value of $\tau_g = 0.0486$ (case 6) while the second test uses the tropical value of $\tau_g = 0.1235$ (case 7).

A change in the total amount of water vapor between the algorithm and mid-latitude winter profiles (case 6) causes errors as large as 11% in the derived surface reflectance for AVHRR band 2. Since the difference in the total water vapor amounts between the algorithm and tropical profiles is smaller than the difference between the algorithm and mid-latitude winter profiles, the magnitude of the error that results from using the tropical profile is less--7% (case 7). These errors can be reduced significantly by applying the water vapor correction given in Section 3.3.

A second set of tests is conducted to determine the sensitivity of the model to changes in ozone absorption. Since TM band 2 has the largest ozone absorption, the ozone absorption sensitivity tests are run at this wavelength. As in the case of water vapor absorption discussed above, the algorithm uses a weighted average of the mid-latitude winter, mid-latitude summer, and tropical profiles computed using the LOWTRAN code to derive a value of ozone absorption optical thickness of $\tau_g = 0.032$ for TM band 2. The first of two sensitivity tests are run using the mid-latitude winter ozone absorption optical thickness $\tau_g = 0.039$ (case 8) while the second test is run using the tropical value of $\tau_g = 0.024$ (case 9). The magnitude of errors in the the derived surface reflectance in TM band 2 caused by variations in mid-latitude winter and tropical profiles to the algorithm ozone amounts is 2-4% (cases 8 and 9).
6.1.4 Aerosol Absorption

The sensitivity of the correction algorithm to aerosol absorption is tested by changing the aerosol single scattering albedo $\omega_0$, which is the ratio of aerosol scattering to extinction. The algorithm uses values of $\omega_0$ derived for the 70%-relative-humidity, rural model of Shettle and Fenn (1979); in this model the aerosol single scattering albedo varies from 0.95 at $\lambda = 0.486 \, \mu m$ to 0.86 at $\lambda = 2.550 \, \mu m$. Values of the single scattering albedo for the visible spectrum, reported by Waggoner et al. (1981), range from $0.54 \leq \omega_0 \leq 0.61$ for urban industrial areas, $0.73 \leq \omega_0 \leq 0.87$ for urban residential areas, and $0.89 \leq \omega_0 \leq 1.00$ for rural areas. These values agree with the values used by Shettle and Fenn. In the sensitivity test (case 10), a value of $\omega_0 = 0.84$ at 0.486 $\mu$m is based upon the average measured values in residential urban areas of Michigan and Missouri (Waggoner et. al., 1981). Since the algorithm is designed to correct radiances measured over rural sites, this should represent a rather extreme (but possible) departure from the usual case.

The derived surface reflectance is somewhat sensitive to changes in aerosol absorption (case 10). This sensitivity appears to depend strongly on geometry, as the resulting errors in the derived reflectance vary between -10 to 0% for the case when the single scattering albedo decreased from 0.95 to 0.84.

Another study of the effect of aerosol absorption error on the derived surface reflectance is given out of sequence (case 10a ), because the unperturbed model is different. The emphasis is on a large solar zenith angle ($\theta_0 = 64^\circ$), long path through the atmosphere ($\theta = 56^\circ$), and rather large aerosol optical thickness ($\tau_a = 0.35$). Interpolations are made on all these variables plus the azimuth. Also, the simulated surface reflectance is high (0.6). The error (-0.11%) is significant. The error would be less with smaller optical thickness, however.

6.1.5 Vertical Distribution of Aerosols

The next sensitivity test deals with errors which result from uncertainty in the vertical distribution of aerosols. The altitude distribution of aerosols used in the algorithm is based on the 'average' distribution described by Braslau and Dave (1973) which is shown in Figure 4. The sensitivity of the results to this distribution is tested by using three different profiles which are constructed using the aerosol models described by Shettle and Fenn (1979). In these models, the atmosphere is divided into four regions: boundary layer (0 - 2 km), upper troposphere (2 - 9 km), stratosphere (9 - 30 km), and upper atmosphere.
(30 - 100 km). All three of the models use the background aerosol models for the stratosphere and upper atmosphere regions; the perturbation models differ in the boundary layer and upper troposphere regions. The first model (case 11) uses the 50-km-visibility, rural model in the boundary layer and the 50-km-visibility, spring/summer model in the upper troposphere. The second model (case 12) uses the 23-km-visibility, rural model in the boundary layer and the 23-km-visibility, spring/summer model in the upper troposphere. The third model (case 13) uses the 10-km-visibility, rural model in the boundary layer and the 23-km-visibility, spring/summer model in the upper troposphere. The vertical distributions of aerosol corresponding to these three models are shown in Figure 4. These profiles are normalized to produce an optical thickness of \( \tau_a = 0.25 \) at 0.550 \( \mu \text{m} \). Since the average visibility in the midwestern U.S. is about 20 km (Husar and Holloway, 1984), a maximum visibility of 50 km and a minimum visibility of 10 km are chosen to represent the extremes used for the construction of the altitude distribution of aerosols.

Uncertainty in the height distribution of aerosols has a negligible effect on the surface reflectance when measured from the top of the atmosphere, since the aerosol distribution is normalized such that the total aerosol optical thickness below the sensor remains the same (cases 11, 12 and 13). However, if a sensor is within the atmosphere, there will be small differences in the optical thickness of aerosols above and below the sensor, depending upon which aerosol distribution is used. The magnitude of the derived reflectance errors is less than 7% at aircraft heights (Table 10).

6.1.6 Bidirectional Reflectance Errors

The final model error concerns the assumption that the surface is Lambertian. Surfaces do not reflect light according to Lambert's Law as is assumed for the current atmospheric correction algorithm. Lee and Kaufman (1986) calculated the error in the derived surface reflectance for a model which also assumed Lambert reflection. Their study utilized actual surface bidirectional reflectances for pasture as measured by Kriebel (1977). The absolute errors in estimates of surface reflectance are a few hundredths when the solar zenith angle is small because the surface is nearly Lambertian. The errors are also small where the surface reflectance is weak. The derived surface reflectance errors become large (about 0.1), however, for moderate haze, when the surface reflectance is both high and strongly anisotropic. Errors in the derived surface reflectance and radiance, but not the irradiance, will not be significant if the surface reflectance is weak; however, the errors can be important for strong bidirectional reflectance.
6.2 Interpolation Errors

The correction algorithm is based on a series of look-up tables relating surface reflectance to measured upward radiance for various aerosol optical thicknesses and geometries. As a result, the algorithm must interpolate to determine the reflectance corresponding to an arbitrary input radiance, aerosol optical thickness, gaseous absorption optical thickness, altitude, and geometry. (No extrapolation is permitted.) The simulations are chosen to show the largest interpolation errors. The interpolation errors are computed by comparing the reflectance errors derived from simulated radiances with those derived with the correction algorithm.

The first interpolation sensitivity test (case 14) is performed using the standard test input parameters described earlier ($\lambda = 0.486 \ \mu m$, $\theta_o = 30^\circ$, $\theta = 0^\circ$, $\phi = 0^\circ$), except with an aerosol optical thickness of $\tau_a = 0.375$, which requires the algorithm to interpolate between $\tau_a = 0.25$ and $\tau_a = 0.50$. Note that the reflectance errors are deviations from the correct value of $\rho = 0.05$ for cases 14-21 and 24-32; the unperturbed reflectance changes only for cases 22 and 23. The next run (case 15) tests the interpolation on solar zenith angle. The solar zenith angle interpolation test is run using the standard test input parameters except with a solar zenith angle of $26^\circ$ which requires the algorithm to interpolate between $\theta_o = 20^\circ$ and $\theta_o = 30^\circ$. Similar tests are run for the observation zenith angle (case 16) and the azimuth angle (case 17). The observation zenith angle interpolation test was run using $\theta = 20^\circ$, which requires the algorithm to interpolate between $\theta = 18^\circ$ and $\theta = 24^\circ$. The azimuth angle interpolation test was run using $\phi = 145^\circ$ so that the algorithm interpolated between $\phi = 140^\circ$ and $\phi = 150^\circ$. Separate interpolations on $\tau_a$, $\theta$, and $\phi$ result in errors less than 1% in the derived surface reflectance, while the interpolation on $\theta_o$ can result in an error as large as 4%.

Errors associated with interpolation on wavelength are shown in cases 18 - 20. The input wavelengths are 0.5, 0.6 and 0.775 $\mu m$. In these cases, the error in derived surface reflectance is at most 6%.

Since the algorithm may also interpolate on gaseous absorption, sensitivity tests are also made to determine the uncertainty involved with this interpolation. In cases 21-23 the algorithm uses the input gaseous absorption value $\tau_g$ computed using the LOWTRAN 6 code and the radiances associated with TM band 4 (0.840 $\mu m$) (which has a narrow bandwidth and relatively small gaseous absorption) to determine the surface reflectance.
seen by the AVHRR band 2 (which has a wide bandwidth and relatively large gaseous absorption). These tests show the algorithm underestimates the surface reflectance by only about 1-2%.

Sensitivity tests are also made to determine the errors associated with adjustments to the look-up tables corresponding to different input surface heights. As discussed in section 3.5, the algorithm makes the adjustment by changing the wavelength slightly (eq. 20). Cases 24 and 25 show that this adjustment results in rather large errors in the derived surface reflectance of -8%.

Errors associated with interpolation on the measurement altitude $Z$ are studied. The radiative transfer computations are tabulated at three altitudes: 0.45 km, 4.5 km, and 80 km. In cases 26 to 28 input observation altitudes of $Z = 0.23$ km, 2.96 km, and 6.5 km are used. The derived surface reflectance errors are as large as 8%.

Finally, errors associated with interpolation for large solar and viewing zenith angles are given. Interpolations are made with respect to $\tau_a$, $\theta_o$, $\theta$, and $\phi$; the surface reflectance $p = 0.05$ (cases 29-32). Case 29 shows that the error is appreciable for a rather large optical thickness. This large error depends on the nonlinear change in radiance as the azimuth increases from $180^\circ$ to $200^\circ$. If the same test is made, except that the azimuth is $165^\circ$, the error reduces to 8% (case 30). When the optical thickness decreases to $\tau_a = 0.1$ (case 31), however, the error has a large negative value. The derived value is $p = 0.034$, compared with the simulation value of $p = 0.050$. Again, if the azimuth is changed from near-forward (case 31) to near-backward (case 32), the magnitude of the error decreases to 6%.

6.3 Input Errors

A third set of sensitivity tests is performed to estimate the errors in the derived surface reflectance resulting from errors or uncertainties in the input data. Each test is performed with the algorithm model, except that an incorrect value of one of the input parameters is used.

The first test introduces a 5% error into the input radiance (case 33). As would be expected, the algorithm is very sensitive to such an error. If the input radiance error increases to 10% (case 34 in Tables 8 and 10), the reflectance error doubles to 28%. The relative errors in the derived surface reflectance can be as large as two to three times the relative errors in the input radiance.
Errors in the input optical thickness occur because of measurement errors, and the optical thickness is not measured at the same time and place of the remote measurement. Tests are made with an error in the input aerosol optical thickness of $\Delta \tau_a = 0.05$ for the case where the correct input is $\tau_a = 0.25$ (case 36), and for $\Delta \tau_a = 0.1$ when $\tau_a = 1.0$ (case 38). The derived surface reflectance errors caused by optical thickness errors are largest when the surface reflectance is weak. Hence, the errors are computed for visible reflectance of visible light from vegetation ($\rho = 0.05$). The resulting surface reflectance error of 0.003 is insensitive to errors of 0.05 for moderate aerosol optical thickness (case 36). For $\tau_a = 1.0$ and larger aerosol optical thickness error of 0.1, the error in the derived surface reflectance increases to $\Delta \rho = 0.014$ (case 38).

The solar zenith angle test is run by introducing an error of $\Delta \theta_o = 2^\circ$ (case 40) when the correct input is $\theta_o = 30^\circ$. Similarly, the sensitivity of the results to errors in the input observation zenith angle is tested by introducing an error of $\Delta \theta = 2^\circ$ (case 41) when the correct input is $\theta = 0^\circ$. Finally, the sensitivity to errors in the input azimuth angle is tested by introducing an error of $\Delta \phi = 2^\circ$ for the case when the correct input is $\phi = 0^\circ$ (case 44). Errors in the input values of solar zenith, observation zenith, and azimuth angles generally result in negligible errors of less than 2% in the derived surface reflectance.

### 7.0 Conclusion

An algorithm is developed to account for atmospheric effects when deriving surface reflectance properties from visible and near-infrared radiances measured by aircraft or satellite over rural areas. The radiance that would be measured for a given surface reflectance can be derived, also. The algorithm uses a tabulated set of radiances computed for various wavelengths, solar and observation angles, and aerosol optical thicknesses. All aerosol parameters have been assumed, except for the aerosol optical thickness, which is an input value. Since the algorithm performs essentially interpolations, it is fast; therefore, it is well suited for reducing observations in many wavelengths. Otherwise, the effect of the atmosphere requires many radiative transfer computations.

Large errors in derived parameters, rather than rms errors are estimated. Among the largest model errors are those caused by uncertainties in the aerosol scattering phase function; in this case surface reflectance and radiance errors reach ±20-30%. Thus, in its current configuration, the algorithm is suitable for only a rural, bimodal log-normal aerosol size distribution. However, the algorithm could be easily modified for use with other
aerosol size distributions by replacing the radiance look-up tables with values generated for a more appropriate aerosol size distribution.

Errors in the derived surface reflectance can be large when either the slant path through the atmosphere of sunlight or light reflected from the ground is long. The uncertainty in the amount of water vapor causes an error of 5% in the reflectance for AVHRR band 2. This error can be reduced significantly by using measurements of the total amount of water vapor at the time of measurement. The mesh for the tabulated radiation parameters is fine enough so that linear interpolation results in small reflectance errors (<4%).

Of the errors in the input parameters required by the correction algorithm, errors in the measured radiance can result in large errors in the derived radiance and reflectance but not irradiance. Absolute measured radiance errors of 5-10% are expected. Therefore, the derived surface radiances and reflectances will deviate by at least 5-10% from the correct values even without any errors in the atmospheric correction algorithm. Large errors in the derived surface reflectance can also result from errors in the input values of aerosol optical thickness. Optical thickness errors should be less than 0.05 so that the corresponding surface reflectance errors are less than 10% for a dark surface (ρ = 0.05).

Acknowledgement: We appreciate Mr. Brian Markham's early efforts to use the tables, and thereby bring to our attention places to improve their accuracy.

8.0 References


Fraser, R.S., Y.J. Kaufman, and R.L. Mahoney, 1984: Satellite measurements of aerosol mass and transport, Atmos. Environ., 18, 2577-2584.


9.0 FORTRAN Listing

The FORTRAN code for the atmospheric correction algorithm is given in the pages which follow. The line numbers are listed to the right of each line.
THIS SOFTWARE WAS DEVELOPED BY
SHANA MATTOO, ARC AND GSFC
C109D, BLD22, TEL # 286-2120

C THIS MAIN PROGRAM IS BASED ON THE ATMOSPHERIC CORRECTION OF UPWARD RADIANCE MEASURED FROM AIRCRAFT OR SATELLITE. IT DERIVES DOWNWARD IRRADIANCE, UPWARD RADIANCE, AND REFLECTANCE ALL FOR THE GROUND LEVEL. THIS PROGRAM ALSO HAS A OPTION TO COMPUTE THE UPWARD RADIANCE IF THE SURFACE REFLECTANCE IS PROVIDED.

C SUBROUTINES USED:
1. READIN
2. FINDW
3. INTSFX
4. INTGH
5. INTERP
6. INTEXP
7. SYSTEM SUBROUTINE IMSL(ZXGSN) (LOOK IN SUBROUTINE INTEXP)

REAL * 4 MWAV, MTAU, MINT, MTHET0, MTHET, MPHI, MHGHT, RHOS, MRHO, SBAR
REAL * 4 HGHT(3), THE(13), INT(9, 4, 13, 19), WAV, OPTH(4), FDOWN(9, 4)
REAL * 4 SBAR(4), PIT(13, 4), THE0(9), NEW(30), YY(1), NEWN(30)
REAL * 4 SBARN(2, 4), PITW(2, 13, 4, 3), FDOWNW(2, 9, 4)
REAL * 4 INTW(2, 9, 4, 13, 19), INTWH(2, 9, 4, 13, 19), PITWH(2, 13, 4)
REAL * 4 NEWINT(4, 13, 19), NFDOWN(4), NEWNEW(4, 13), FINT(4), FT(4)
REAL * 4 F, AIRR, ARAD, PHI(19)
REAL * 4 FOIRR(100), WAVE(100), R(365)
REAL * 4 WAVEN(8)/.486, .587, .639, .663, .837, .845, 1.663, 2.189/
REAL * 4 TAUGH(8)/.0070, .0320, .0247, .0174, .0021, .0152, .0077, .0091/
REAL * 4 TAUGL(8)/.0130, .0142, .0208, .0218, .0628, .1156, .1380, .0930/
CHARACTER * 1 LINE(132)
CHARACTER * 1 LINE2(80)
CHARACTER * 1 LL(30)
INTEGER ANGLE(19)

C DEFINE PI
PI=ARCOS(-1.00)

C CALL SUBROUTINE READIN

C SUBROUTINE READIN READS THE INPUT DATA AND WRITES IT ON UNIT 6, TO PERFORM A CHECK ON INPUT DATA.

C LINE2 READS THE TITLE FOR THE VALUES OF FOLLOWING INPUT QUANTITIES.
C READ THE IOPT, MOPT AND NOPT. IOPT IS OPTION FOR UNITS OF RADIANCE.
C IF IOPT IS 1, UNITS SHOULD BE IN REFLECTANCE UNITS; IF IOPT=2 C THE UNITS OF REFLECTANCE SHOULD BE IN ABSOLUTE UNITS. IF MOPT IS 1 C THE DAY, MONTH AND YEAR FOR WHICH DATA IS MEASURED SHOULD BE
C GIVEN; AND IF MOPT IS 2 THE DAY OF THE YEAR AND THE YEAR SHOULD BE
C GIVEN. NOPT IS OPTION FOR COMPUTING SURFACE REFLECTANCE OR RADIANCE.
C IF NOPT IS 1 SURFACE REFLECTANCE IS COMPUTED AND IF NOPT IS 2 RADIANCE IS COMPUTED.

81
READ(5,1003) LINE2
READ(5,9) IOPT, MOPT, NOPT
C
C LINE2 READS THE TITLE FOR THE VALUES OF FOLLOWING INPUT QUANTITIES:
C INUM THE NUMBER OF DATA POINTS TO BE PROCESSED, MONTH, DAY, YEAR AND
C TIME ZONE FOR WHICH MEASURED DATA ARE PROVIDED.
C
READ(5,1003) LINE2
IF(MOPT .EQ. 1) READ(5,10) INUM, IMONTH, IDAY, IYEAR, LL
IF(MOPT .EQ. 2) READ(5,10) INUM, IDAY, IYEAR, LL
READ(5,1003) LINE2
C
C WRITE THE LABELS FOR THE OUTPUT.
C
WRITE(56,1108)
WRITE(56,1109)
IF(MOPT .EQ. 1) WRITE(6,11) IMONTH, IDAY, IYEAR, LL
IF(MOPT .EQ. 1) WRITE(56,11) IMONTH, IDAY, IYEAR, LL
IF(MOPT .EQ. 2) WRITE(6,21) IDAY, IYEAR, LL
IF(MOPT .EQ. 2) WRITE(56,21) IDAY, IYEAR, LL
WRITE(6,12)
WRITE(56,22)
C
C READ LABS AND NECKEL DATA FOR SOLAR FLUX
C
DO 121 IJ = 1, 4
121 READ(20,1003) LINE
IK=1
DO 122 IJ = 1, 20
READ(20,23) WAVE(IK), FOIRR(IK), WAVE(IK+1), FOIRR(IK+1), WAVE(IK+2),
1   FOIRR(IK+2)
IK=IK+3
122 CONTINUE
IK=IK-1
C
C START OF LOOP FOR THE NUMBER OF THE DATA POINTS TO BE PROCESSED
C LFILE IS USED TO SET THE VALUE FOR THE FILE NUMBER NFILE TO READ
C THE LOOK-UP TABLE FOR CHOSEN WAVELENGTH.
C
LFILE=6
2000 DO 999 NUM=1, INUM
   IF(NUM .GT. 1) REWIND NFILE1
   IF(NUM .GT. 1) REWIND NFILE2
C
C * READ MEASURED PARAMETERS
C
C TIME IN CDT (MTIME IN HOURS AND MINUTES), WAVELENGTH (MWAV IN
C MICROMETERS), OPTICAL THICKNESS (MTAU) FOR MWAV, SOLAR ZENITH ANGLE
C MTHTO IN DEGREES), OBSERVATION SCAN ANGLE (MTHTH IN DEGREES),
C OBSERVATION AZIMUTH ANGLE (MPhi IN DEGREES), OBSERVATION HEIGHT (MHGT)
C IN KILOMETERS), INTENSITY (MINT IN REFLECTANCE OR ABSOLUTE UNITS) IF
C NOPT IS 1; AND SURFACE REFLECTANCE (MRHO) IF NOPT IS 2.
C TAUW (ABSORPTION FOR CARBON DIOXIDE AND OZONE), TAUW (ABSORPTION
C FOR WATER) FOR MEASURED WAVELENGTH, HEIGHT FROM THE SURFACE ABOVE SEA-
C LEVEL (SHGHT IN KILOMETERS). (LOOK DOCUMENTATION PAGE 19)
C
C IF(NOPT.EQ.1)
1 READ(5,13) MTIME, MWAV, MTAU, MTHT0, MTHTH, MPhi, MHGT, MINT, TAUGLW,
1   TAUW, SHGHT
IF (NOPT .EQ. 2)
  1READ (5,13)MTIME, MWAV, MTAU, MTHET0, MTHET, MPH, MHGHT, MRHO, TAUWL, 
     1 TAUWH, SHGHT
C
C IF TAUWL (ABSORPTION FOR WATER) IS 0.0 THE DEFAULT VALUE IS COMPUTED BY INTERPOLATING LINEARLY FOR THE MEASURED WAVELENGTH MWAV. IF MWAV IS OUT OF RANGE NO EXTRAPOLATION IS ALLOWED, THE PROGRAM SENDS CONTROL TO PROCESS NEW DATA POINT.
C
C IF (TAUWL .GT. 0.0) THEN
GO TO 360
ELSE
   CALL INTERP (1,8,MWAV,WAVEN,TAUWL,YY, IJK,NUM,LOPT)
IF (LOPT .EQ. 0) THEN
   WRITE (6,24) MWAV, NUM
GO TO 999
ELSE
   TAUWL = YY (1)
ENDIF
ENDIF

C
C IF TAUGH (ABSORPTION FOR GASES) IS 0.0 THE DEFAULT VALUE IS COMPUTED BY INTERPOLATING LINEARLY FOR THE MEASURED WAVELENGTH MWAV. IF MWAV IS OUT OF RANGE NO EXTRAPOLATION IS ALLOWED, THE PROGRAM SENDS CONTROL TO PROCESS NEW DATA POINT.
C
360 CONTINUE
IF (TAUGH .GT. 0.0) THEN
   GO TO 350
ELSE
   CALL INTERP (1,8,MWAV,WAVEN,TAUGH,YY, IJK,NUM,LOPT)
   IF (LOPT .EQ. 0) THEN
      WRITE (6,25) MWAV, NUM
      GO TO 999
   ELSE
      TAUGH = YY (1)
   ENDIF
ENDIF

350 CONTINUE
C
C******************************************************************************
C* CALL TO SUBROUTINE FINDW *  
C******************************************************************************
C SUBROUTINE FINDW PICKS TWO WAVELENGTHS WAV1 AND WAV2 BETWEEN WHICH MEASURED WAVELENGTH MWAV LIES AND SETS FILE NUMBERS NFILE1 AND NFILE2 TO READ THE REQUIRED DATA SET FOR THE TWO WAVELENGTHS FROM LOOK-UP TABLE. THE MWAV SHOULD BE IN RANGE OF 0.486 - 2.189 UM, NO EXTRAPOLATION IS ALLOWED.
C
CALL FINDW (MWAV, II,NUM, KOPT, IMM, WAV1, WAV2)
IF (KOPT.EQ.0) GO TO 999

NFILE1 = LFILE+II
NFILE2 = LFILE+IMM

C******************************************************************************
C* CALL TO SUBROUTINE INTSFX *  
C******************************************************************************
C SUBROUTINE INTSFX INTERPOLATES TO COMPUTE THE VALUES OF R FOR 365 DAYS. THESE VALUES WILL BE USED TO COMPUTE THE CORRECTED VALUES OF SOLAR FLUX FOR THE DAY MONTH AND YEAR FOR WHICH MEASURED DATA IS GIVEN. R IS SUN-EARTH DISTANCE .

83
CALL INTSFX(R)

CHANGE MONTH AND DAY TO THE JULIAN DATE IF MOPT IS 1.

IF (MOPT.EQ.2) THEN
 IQ=IDAY
 ELSE
 IS=IDAY+1
 IM=IMONTH-1
 IQ=0
 DO 20 IJ=1,IM
 IF (MOD(IYEAR,4).EQ.0) THEN
 IQ=IQ+LEAP(IJ)
 ELSE
 IQ=IQ+NLEAP(IJ)
 ENDIF
 20 CONTINUE
 IQ=IQ+IS
ENDIF

COMPUTE SOLAR FLUX FOR THE MEASURED WAVELENGTH MWAV FROM LABS AND NECKEL DATA BY INTERPOLATION

IJK=0
LNUM=0
CALL INTERP(1,IK,MWAV,WAVE,FOIFTR,YY,IJK,LNUM,LOPT)

COMPUTE CORRECTED SOLAR FLUX FOR THE DAY OF YEAR FOR WHICH MEASUREMENT ARE MADE. (SEE DOCUMENTATION EQ.6)

R(IQ)=1.
FFLUX=YY(1)/(R(IQ)*R(IQ))

COMPUTE THE OBSERVATION ZENITH ANGLE FOR THE SCAN ANGLE OF THE SATELLITE OR AIRCRAFT (SEE DOCUMENTATION EQ.9)
RAD= RADIUS OF EARTH

DEGRAD=ARCOS(-1.)/180.
RADDEG=180./ARCOS(-1.)
RAD=6370.
MTHET=ASIN((1.+(MHGHT/RAD)))*(SIN(MTHET*DEGRAD)))*RADDEG

IF IOPT EQ 1 CONVERT MEASURED RADIANCE TO ABSOLUTE UNITS.
IF IOPT IS 2 CONVERT MEASURED RADIANCE FROM ABSOLUTE UNITS TO REFLECTANCE UNITS.

AMU0=COS(MTHETO*DEGRAD)
IF(IOPT.EQ.1 .AND. NOPT.EQ.1 ) AMINT=(MINT*FFLUX*AMU0)/PI
IF(IOPT.EQ.2 .AND. NOPT.EQ.1 ) AMINT=MINT
IF(IOPT.EQ.2 .AND. NOPT.EQ.1 ) MINT=(MINT*PI)/(FFLUX*AMU0)
C  ***********************************************************************
C * READ FROM LOOK-UP TABLE.                                         *
C  ***********************************************************************
C  ***********************************************************************
C  WAV=WAVELENGTHS (8) (.486, .587, .639, .663, .837, .845, 1.663, 2.189) *
C  * MICROMETERS.                                                 *
C  * LTHET0=NUMBER OF THETA0(SOLAR ZENITH ANGLE)                  *
C  * THETO=(10,20,30,40,50,60,66,72,78 DEGREES)                     *
C  * LTHE=NUMBER OF THE(OBSERVATION ZENITH ANGLE)                  *
C  * THE=(0,6,.....72 DEGREES EVERY 6 DEGREES)                     *
C  * LPHI =NUMBER OF PHI(OBSERVATION AZIMUTH ANGLE)                *
C  * PHI=(0,5,10,20,30,40,60,80,90,100,110,120,130,140,150,160,170, *
C  * 175,180 DEGREES)                                             *
C  * LTAU =NUMBER OF OPTH(OPTICAL THICKNESS)                       *
C  * OPTH=(0.0,.25,.50,1.0)                                        *
C  * LHGT = NUMBER OF HGHT(OBSERVATION HEIGHTS)                    *
C  * HGHT=(80.0,4.5,.45 KM)                                        *
C  ***********************************************************************
C
LTHET0=9
LTHE=13
LPHI=19
LTAU=4
LHGT=3
C READ OBSERVATION ZENITH ANGLE(THE), AND OBSERVATION AZIMUTH ANGLE
C FROM LOOK-UP TABLE. STORE INTEGER ARRAY ANGLE IN REAL ARRAY PHI.
C
READ (NFILE1,1000) (THE(I),I=1,LTHE)
READ (NFILE1,1001) (ANGLE(I),I=1,LPHI)
READ (NFILE2,1000) (THE(I),I=1,LTHE)
READ (NFILE2,1001) (ANGLE(I),I=1,LPHI)
DO 30 IPHI=1,LPHI
  30 PHI(IPHI)=ANGLE(IPHI)
C READ WAVELENGTH(WAV), OPTICAL THICKNESS(OPTH), SOLAR ZENITH ANGLE
C (THE0), REFLECTANCE OF ATMOSPHERE(SBARW), FLUX INCIDENT ON
C SURFACE(FDOWNW) FROM LOOK-UP TABLE.
C
C IF WAVELENGTH IS 1.663 OR 2.189, THE LOOK-UP TABLE IS AVAILABLE ONLY
C FOR 2 OPTICAL THICKNESSES OF 0.0 AND .25.
C
DO 9995 IWAV = 1,2
  IF(IWAV .EQ. 1)NFILE = NFILE1
  IF(IWAV .EQ. 2)NFILE = NFILE2
  IF(NFILE .GT. 12)LTAU=2
  DO 100 ITHETO=1,LTHETO
    DO 101 ITAU=1,LTAU
      DO 102 IHGHT=1,LHGT
        READ (NFILE,1002) WAV, OPTH(ITAU), THETO(ITHETO), SBARW(IWAV, ITAU),
                          1 FDOWNW(IWAV, ITHETO, ITAU)
        102 CONTINUE
    101 CONTINUE
  100 CONTINUE
C
C READ ATMOSPHERIC RADIANCE (INTW) AS FUNCTION OF SOLAR ZENITH ANGLE,
C OPTICAL THICKNESS, HEIGHT, OBSERVATION ZENITH ANGLE, AND OBSERVATION
C AZIMUTH ANGLE FROM LOOK-UP TABLE.
C
DO 103 ITHE=1,LTHE
  READ (NFILE,1004) (INTW(IWAV, ITHE, ITAU, IHGHT,ITHE, IPHI),IPHI=1,10)
  103 CONTINUE
READ TRANSMISSION FROM SURFACE (PITW) AS FUNCTION OF OBSERVATION
C ZENITH ANGLE, OPTICAL THICKNESS, AND HEIGHT.
C
READ (NFILE, 1005) (PITW(IWAV, ITHE, ITAU, IHGHT), ITHE = 1, LTHE)
102 CONTINUE
101 CONTINUE
100 CONTINUE
9995 CONTINUE
C
C **************************************************************
C *CALL TO SUBROUTINE INTHGH*
C **************************************************************
C
CALL INTHGH (LTAU, LTHE, LPHI, LTHETO, INTW, MHGHT, PITW, INTWH, PITWH)

WAV1 = WAV1 * EXP((SHGHT -.4) / 36.)
WAV2 = WAV2 * EXP((SHGHT -.4) / 36.)
C
C INTERPOLATE INTWH, PITWH, SBARW, FDOWNW FOR THE MEASURED WAVELENGTH
C MWAV. (SEE DOCUMENTATION 3.1, PAGE 12 - 15)
C
ALPHA = ( ALOG (MWAV) - ALOG (WAV2) ) / ( ALOG (WAV1) - ALOG (WAV2) )
DO 200 ITAU = 1, LTAU
SBAR (ITAU) = EXP ((ALPHA * ALOG (SBARW (1, ITAU))) +
1 (1. - ALPHA) * ALOG (SBARW (2, ITAU)))
DO 201 ITHE = 1, LTHE
PIT (ITHE, ITAU) = EXP ((ALPHA * ALOG (PITWH (1, ITHE, ITAU))) +
1 (1. - ALPHA) * ALOG (PITWH (2, ITHE, ITAU)))
DO 202 IPHI = 1, LPHI
DO 203 ITHETO = 1, LTHETO
FDOWN (ITHET0, ITAU) = EXP ((ALPHA * ALOG (FDOWNW (1, ITHET0, ITAU))) +
1 (1. - ALPHA) * ALOG (FDOWNW (2, ITHET0, ITAU)))
203 CONTINUE
202 CONTINUE
201 CONTINUE
200 CONTINUE
C
C COMPUTE DELTAH AND DELTAL AS EXCESS OR DEFICIT IN UPPER (OZONE, CARBON DIOXIDE) AND LOWER (WATER) ATMOSPHERE RESPECTIVELY.
C
AVTAGH = EXP ((ALPHA * ALOG (TAUGH (II))) +
1 (1. - ALPHA) * ALOG (TAUGH (IMM)))
DELTAL = TAUGHW - AVTAGH
AVTAGW = EXP ((ALPHA * ALOG (TAUGL (II))) +
1 (1. - ALPHA) * ALOG (TAUGL (IMM)))
DELTAL = TAUGLW - AVTAGL
C
C COMPUTE FDOWN, SBAR, PIT AND INT AFTER ADJUSTING FOR THE EXCESS AND DEFICIT OF GASEOUS ABSORPTION (SEE EQ. 17A, 17B, 17C, 17D)
DO 908 ITHETO=1,LTHETO
AMUTH0 = COS (ITHETO (ITHETO) * DEGRAD)
THDOWN=EXP (-DELTAL/AMUTH0)
TLDOWN=EXP (-DELTAL/AMUTH0)
DO 908 ITAU=1,LTAU
908 FDOWN (ITHETO, ITAU) = FDOWN (ITHETO, ITAU) * THDOWN * TLDOWN
DO 909 ITAU=1,LTAU
909 SBAR (ITAU) = SBAR (ITAU) * EXP (-DELTAL * 2)
DO 910 ITAU=1,LTAU
910 THUP = EXP (-DELTAL/AMUTH)
TLUP = EXP (-DELTAL/AMUTH)
910 PIT (ITHE, ITAU) = PIT (ITHE, ITAU) * THUP * TLUP
DO 912 ITAU=1,LTAU
912 AMUTH = COS (THE (ITHE) * DEGRAD)
THUP = EXP (-DELTAL/AMUTH)
TLUP = EXP (-DELTAL/AMUTH)
912 INT (ITHETO, ITAU, ITHE, IPHI) = INT (ITHETO, ITAU, ITHE, IPHI) * 
1 (EXP (-((DELTAL/2.1 + DELTAL) * (1./AMUTH) + (1./AMUTH0))))
701 CONTINUE
701 DO 240 I=1,30
NEW(I) = 0.0
NEWN(I) = 0.0
240 CONTINUE
C ** INTERPOLATION OF ATMOSPHERIC RADIANCE AND FLUX INCIDENT ON GROUND*
C ** ON MEASURED SOLAR ZENITH ANGLE. **
C ** INT (ITHETO, OPTh, IGMT, THE, PHI) IS INTERPOLATED FOR*
C MEASURED MTHETO TO GET NEW VARIABLE NEWINT (ITAU, THE, PHI).*
C FDOWN (ITHETO, ITAU) IS INTERPOLATED FOR MEASURED MTHETO TO*
C GET NEW VARIABLE NFDOWN (ITAU).*
C
C CALL SUBROUTINE INTERP TO INTERPOLATE INT. C
C IF MTHETO IS OUT OF RANGE OF VALUES OF THETO SUBROUTINE INTERP C
C RETURNS THE VALUE OF LOPT = 0 AND THE NEW DATA POINT WILL BE READ. C
C
CALL INTERP (1, LTHETO, MTHETO, THETO, NEW, YY, IJK, NUM, LOPT)
IF (LOPT.EQ.0) GO TO 999
NEWINT (ITAU, ITHE, IPHI) = YY (1)
IJK = IJK + 1
C
C CALL SUBROUTINE INTERP TO INTERPOLATE FDOWN. C
C IF MTHETO IS OUT OF RANGE OF VALUES OF THETO SUBROUTINE INTERP C
C RETURNS THE VALUE OF LOPT = 0 AND THE NEW DATA POINT WILL BE READ. C
C
CALL INTERP (1, ITHETO, MTHETO, THETO, NEWN, YY, IJK, NUM, LOPT)
IF (LOPT.EQ.0) GO TO 999
NFDOWN (ITAU) = YY(1)

106 CONTINUE
105 CONTINUE
104 CONTINUE

C******************************************************************************
C* INTERPOLATION OF THE ATMOSPHERIC RADIANCE ON MEASURED AZIMUTH
C* ANGLE.
C******************************************************************************
C NEWINT(TAU, THETA, PHI) IS INTERPOLATED FOR MEASURED MPHI
C TO GET NEW VARIABLE NEWNEW(TAU, THETA).
C
DO 241 I = 1, 30
  NEW(I) = 0.0
241 NEWN(1) = 0.0
  DO 108 ITAU = 1, LTAU
  DO 109 ITHE = 1, LTHE
  DO 110 IPHI = 1, LPHI
    NEW(IPHI) = NEWINT(ITAU, ITHE, IPHI)
110 CONTINUE
  IJK = 3

C CALL SUBROUTINE INTERP TO INTERPOLATE ON PHI.
C IF MPHI IS OUT OF RANGE OF VALUES OF PHI SUBROUTINE INTERP
C RETURNS THE VALUE OF LOPT = 0 AND THE NEW DATA POINT WILL BE READ.
C
C CALL INTERP (1, LPHI, MPHI, PHI, NEW, YY, IJK, NUM, LOPT)
IF (LOPT.EQ.0) GO TO 999
  NEWNEW(ITAU, ITHE) = YY(1)
109 CONTINUE
108 CONTINUE

C******************************************************************************
C* INTERPOLATION OF ATMOSPHERIC RADIANCE AND TRANSMISSION/PI FROM
C* SURFACE ON MEASURED OBSERVATION ZENITH ANGLE.
C******************************************************************************
C NEWNEW(OTHET, THE) IS INTERPOLATED FOR MEASURED MTHET
C TO GET NEW VARIABLE FINT(TAU). TRANSMISSION FACTOR PIT IS
C INTERPOLATED TO GET FT(TAU).
C
DO 242 I = 1, 30
  NEW(I) = 0.0
242 NEWN(1) = 0.0
  DO 111 ITAU = 1, LTAU
  DO 112 ITHE = 1, LTHE
    NEW(ITHE) = NEWNEW(ITAU, ITHE)
112 CONTINUE
  IJK = 4

C CALL SUBROUTINE INTERP TO INTERPOLATE RADIANCE ON MTHET.
C IF MTHET IS OUT OF RANGE OF VALUES OF THE SUBROUTINE INTERP
C RETURNS THE VALUE OF LOPT = 0 AND THE NEW DATA POINT WILL BE READ
C
C CALL INTERP (1, ITHET, MTHET, THE, NEW, YY, IJK, NUM, LOPT)
IF (LOPT.EQ.0) GO TO 999
  FINT(ITAU) = YY(1)
  IJK = 5

C
CALL SUBROUTINE INTERP TO INTERPOLATE PIT ON MTHET.
CALL INTERP (1, LTHE, MTHET, THE, NEWN, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
FT (ITAU) = YY (1)
111 CONTINUE

CALL SUBROUTINE INTERP WHICH PERFORMS EXPONENTIAL INTERPOLATION,
IF VALUE OF IGNORE RETURNED FROM SUBROUTINE INTEXP IS 1 LINEAR INTER-
POLATION IS PERFORMED USING SUBROUTINE INTERP.
CALL INTEXP (OPTH, FINT, MTAU, YY, LTAU, IGNORE)
CALL INTERP (1, LTAU, MTAU, OPTH, FINT, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
ENDIF
FINTN = YY (1)
CALL INTEXP (OPTH, NFDOWN, MTAU, YY, LTAU, IGNORE)
IF (IGNORE .EQ. 1) THEN
CALL INTERP (1, LTAU, MTAU, OPTH, NFDOWN, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
ENDIF
FDOWN = YY (1)
CALL INTEXP (OPTH, FT, MTAU, YY, LTAU, IGNORE)
IF (IGNORE .EQ. 1) THEN
CALL INTERP (1, LTAU, MTAU, OPTH, FT, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
ENDIF
FTN = YY (1)
CALL INTEXP (OPTH, SBAR, MTAU, YY, LTAU, IGNORE)
IF (IGNORE .EQ. 1) THEN
CALL INTERP (1, LTAU, MTAU, OPTH, SBAR, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
ENDIF
SBARN = YY (1)

CALL SUBROUTINE INTEXP WHICH PERFORMS EXPONENTIAL INTERPOLATION,
IF VALUE OF IGNORE RETURNED FROM SUBROUTINE INTEXP IS 1 LINEAR INTER-
POLATION IS PERFORMED USING SUBROUTINE INTERP.
CALL INTEXP (OPTH, FINT, MTAU, YY, LTAU, IGNORE)
CALL INTERP (1, LTAU, MTAU, OPTH, FINT, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
ENDIF
FINTN = YY (1)
CALL INTEXP (OPTH, NFDOWN, MTAU, YY, LTAU, IGNORE)
IF (IGNORE .EQ. 1) THEN
CALL INTERP (1, LTAU, MTAU, OPTH, NFDOWN, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
ENDIF
FDOWN = YY (1)
CALL INTEXP (OPTH, FT, MTAU, YY, LTAU, IGNORE)
IF (IGNORE .EQ. 1) THEN
CALL INTERP (1, LTAU, MTAU, OPTH, FT, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
ENDIF
FTN = YY (1)
CALL INTEXP (OPTH, SBAR, MTAU, YY, LTAU, IGNORE)
IF (IGNORE .EQ. 1) THEN
CALL INTERP (1, LTAU, MTAU, OPTH, SBAR, YY, IJK, NUM, LOPT)
IF (LOPT .EQ. 0) GO TO 999
ENDIF
SBARN = YY (1)

COMPUTE SURFACE REFLECTANCE RHO (SEE DOCUMENTATION EQ. 2, 3)

C COMPUTE SURFACE REFLECTANCE RHO = (MINT - FINTN) / (FDOWN * FTN)
C SURFACE REFLECTANCE RHO = F / (1 + (F * SBARN (TAU)))
C IF NOPT IS 2 REFLECTANCE IS COMPUTED.

IF (NOPT .EQ. 2) GO TO 56
F = (MINT - FINTN) / (FDOWN * FTN)
RHOS = F / (1 + (F * SBARN (TAU)))
C **********************************************************
C * COMPUTE IRRADIANCE IN WATTS/METER**2/UM, AND ABSOLUTE RADIANCE *
C * IN WATTS/METER* **2/UM/SR. *
C **********************************************************
C COMPUTE IRRADIANCE AIRR=(FDOWN*FFLUX* MU0)/(1-SBARN.RHOS)
C COMPUTE ABSOLUTE RADIANCE=(RHOS*FDOWN*FFLUX*AMU0)/(PI*(1-SBARN*RHOS))
C .
AIRR=(FDOWN * FFLUX* AMU0)/(1.-SBARN*RHOS)
ARAD=(RHOS*FDOWN*FFLUX* AMU0)/(PI*(1.-SBARN*RHOS))
C **********************************************************

C *WRITE OUTPUT, ALL MEASURED PARAMETERS WHICH ENTER AS INPUT; AND *
C *COMPUTED IRRADIANCE AIRR, ABSOLUTE RADIANCE ARAD, AND SURFACE *
C *REFLECTANCE AT GROUND LEVEL. *
C **********************************************************
WRITE (6,1006) MTIME, MWAV, MTAU, MTHET0, MTHET, MPH1, MHGHT, AMINT, MINT,
1AIRR, ARAD, RHOS
WRITE (56,1107) MTIME, MWAV, MTAU, MTHET0, MTHET, MPH1, MHGHT, AMINT,
1MINT, FINTN, FDOWNN, FTN, SBARN
GO TO 9987
C
C* NOPT IS 2 *
C* COMPUTE REFLECTANCE MINT (SEE DOCUMENTATION (ABSTRACT)) *
C**********************************************************
C COMPUTE MINT=FINTN+(FTN*FDOWNN*RHOS)/(1.-SBARN*RHOS)
C
56 CONTINUE
RHOS=MRHO
MINT=FINTN+(FTN*FDOWNN*RHOS)/
1(1.-SBARN*RHOS))
C**********************************************************
C * COMPUTE IRRADIANCE IN WATTS/METER**2/UM, AND ABSOLUTE RADIANCE *
C * IN WATTS/METER* **2/UM/SR. *
C**********************************************************
C COMPUTE IRRADIANCE AIRR=(FDOWN*FFLUX* MU0)/(1-SBARN.RHOS)
C COMPUTE ABSOLUTE RADIANCE=(RHOS*FDOWN*FFLUX*AMU0)/(PI*(1-SBARN*RHOS))
C .
AIRR=(FDOWN * FFLUX* AMU0)/(1.-SBARN*RHOS)
ARAD=(RHOS*FDOWN*FFLUX* AMU0)/(PI*(1.-SBARN*RHOS))
C**********************************************************

C* WRITE OUTPUT, ALL MEASURED PARAMETERS WHICH ENTER AS INPUT; AND *
C* COMputed IRRADIANCE AIRR, ABSOLUTE RADIANCE ARAD, AND SURFACE *
C* REFLECTANCE AT GROUND LEVEL. *
C**********************************************************
IF(IOPT .EQ.1 .AND. NOPT.EQ.2) AMINT=(MINT*FFLUX*AMU0)/PI
IF(IOPT .EQ.1 .AND. NOPT.EQ.2) AMINT=MINT
IF(IOPT .EQ.2 .AND. NOPT.EQ.2) MINT=(MINT*PI)/(FFLUX*AMU0)
WRITE (6,1006) MTIME, MWAV, MTAU, MTHET0, MTHET, MPH1, MHGHT, AMINT, MINT,
1AIRR, ARAD, RHOS
WRITE (56,1107) MTIME, MWAV, MTAU, MTHET0, MTHET, MPH1, MHGHT, AMINT,
1MINT, FINTN, FDOWNN, FTN, SBARN
9987 CONTINUE
999 CONTINUE
WRITE (6,1007)
STOP
9 FORMAT(315)
10 FORMAT(415,30A1)
11 FORMAT(/,' DATE ',I3,'/','I3','/','I5,' TIME ZONE','30A1)
90
1 'TIME', 2X, 'WAVELENGTH', 2X, 'OPTICAL', 2X, 'SOLAR', 2X, FIP05890
1 'OBSERVATION', 2X, 'OBSERVATION', 2X, 'OBSERVATION', 12X, 'RELATIVE', 1X, FIP05900
1 '|' , 2X, 'DOWNWARD', 5X, 'UPWARD', '/', 18X, 'THICKNESS', 1X, FIP05910
1 'ZENITH', 3X, 'ZENITH', 7X, 'AZIMUTH', FIP05920
1 6X, 'HEIGHT', 5X, 'RADIANCE', 2X, 'RADIANCE', 1X, '|', 3X, FIP05930
14X, 'ANGLE', 8X, 'ANGLE', 23X, 'RADIANCE*PI/F0/0', '10X', 'W/M^2/UM/SR' FIP05950
1 '|' , 67X, '|'
13 FORMAT (I5, 1X, F6.3, F6.3, 4F7.2, F10.5, 3F8.4) FIP05990
21 FORMAT (' ' JU]),N DATE ', I3', '/', 15, 'TIME ZONE', 30A1) FIP06000
22 FORMAT (' TIME', 2X, 'WAVELENGTH', 2X, 'OPTICAL', 2X, 'SOLAR', 2X, FIP06010
1 'OBSERVATION', 2X, 'OBSERVATION', 2X, 'OBSERVATION', 12X, 'RELATIVE', 5X, FIP06020
1 'LSUBO', 2X, 'FSUBD', 18X, 'THICKNESS', 1X, FIP06030
1 'ZENITH', 3X, 'ZENITH', 7X, 'AZIMUTH', FIP06040
1 6X, 'HEIGHT', 5X, 'RADIANCE', 2X, 'RADIANCE', 2X, FIP06050
1 'SUPERPRIME', 1X, 'SUPERPRIME', 2X, 'T', 2X, 'S', 29X, 'ANGLE', FIP06060
14X, 'ANGLE', 8X, 'ANGLE', 28X, 'RADIANCE*PI/F0/0', '/', FIP06070
1 'W/M^2/UM/SR') FIP06090
23 FORMAT (3 (8X, F6.4, F11.4, 2X)) FIP06100
24 FORMAT (' MEASURED WAVELENGTH (MWAV)', 'F6.3', 'IS OUT OF RANGE THE DATA POINT #', I4, 'IS NOT PROCESSED. RANGE IS 0.486-2.189') FIP06110
25 FORMAT (' MEASURED WAVELENGTH (MWAV)', 'F6.3', 'IS OUT OF RANGE THE DATA POINT #', I4, 'IS NOT PROCESSED. RANGE IS 0.486-2.189') FIP06130
1000 FORMAT (6X, 13F6.0) FIP06150
1001 FORMAT (4X, 1916) FIP06160
1002 FORMAT (11X, F7.3, 19X, F5.2, 22X, F5.0, 5X, F7.4, 11X, F6.4) FIP06170
1003 FORMAT (132A1) FIP06180
1004 FORMAT (10E12.4) FIP06190
1005 FORMAT (3X, 13F6.5) FIP06200
1 F7.2, 3X, F8.2, 4X, F8.4, 1X, '|', 3X, F8.0, 3X, F7.1, 5X, F7.3) FIP06220
1008 FORMAT (2F6.3) FIP06240
1009 FORMAT (3X, 13F6.3) FIP06250
1108 FORMAT (45X, 'INTERPOLATED RADIATION PARAMETERS') FIP06270
1109 FORMAT (43X, '-----------------------------------------------') FIP06280
END FIP06290
**SUBROUTINE READIN**

```fortran
CHARACTER*1 LINE(132)
WRITE(6,200)
DO 10 I=1,3
READ(5,100) LINE
WRITE(6,100) LINE
10 CONTINUE
READ(5,101) INUM, (LINE(I),I=6,132)
WRITE(6,101) INUM, (LINE(I),I=6,132)
READ(5,100) LINE
WRITE(6,100) LINE
DO 20 I=1,INUM
READ(5,100) LINE
WRITE(6,100) LINE
20 CONTINUE
WRITE(6,201)
REWIND 5
100 FORMAT(132A1)
101 FORMAT(15,127A1)
200 FORMAT(/, 1X, 'START OF INPUT DATA AS READ FROM UNIT NUMBER 5',/)
201 FORMAT(/, 1X, 'END OF INPUT DATA SET AS READ FROM UNIT NUMBER 5',/)
RETURN
END
```

**SUBROUTINE FINDW**

```fortran
SUBROUTINE FINDW (MWAV, II, INUM, KOPT, IMM, W1, W2)
REAL*4 MWAV, WAV(8) = .486, .587, .639, .663, .837, .845, 1.663, 2.189/
C THIS SUBROUTINE PICKS TWO WAVELENGTHS W1 AND W2 BETWEEN WHICH MEASURED WAVELENGTH MWAV LIES. THE SUBROUTINE PRINTS A ERROR MESSAGE IF THE MWAV DOES NOT FALL IN THE RANGE OF .486 - 2.189 UM. THE CONTROL IS RETURNED TO MAIN PROGRAM WITH VALUE OF KOPT.
II = 7
DO 10 IJ=1,IK
IF(MWAV.LT.WAV(I) .OR. MWAV.GT.WAV(IK+1)) GO TO 20
IF(MWAV.GE.WAV(IJ) .AND. MWAV.LE.WAV(IJ+1)) GO TO 30
10 CONTINUE
GO TO 20
30 IMM = IJ+1
W1 = WAV(IJ)
W2 = WAV(IJ+1)
KOPT = 1
RETURN
20 WRITE(6,2) MWAV, INUM
KOPT = 0
2 FORMAT( 'MEASURED WAVELENGTH(MWAV) ',F6.3,' IS OUT OF RANGE.THE DATA POINT #',I4,' IS NOT PROCESSED. ACTUAL RANGE .486-2.189 UM')
RETURN
END
```
**SUBROUTINE INTSFX**

This subroutine interpolates values of RR for 365 days of year. RR is sun Earth distance.

```plaintext
REAL * 4 RR(365), DAY(365)
REAL * 4 AMON(25) /1.15., 32., 46., 60., 74., 91., 106.,
 1121., 135., 152., 166., 182., 196., 213., 227.,
1 224., 258., 274., 288., 305., 319., 325., 345., 365./
REAL * 4 R(25) /.9832, .9836, .9853, .9878,
1.9909, .9945, .9993, 1.0033, 1.0076,
1.0109, 1.0140, 1.0158, 1.0167, 1.0165, 1.0149, 1.0128,
1.0092, 1.0057, 1.0011, .9972, .9925, .9892, .9860,
1.9843, .9843/ 
DO 100 I = 1, 365
DAY(I) = I
100 CONTINUE
IJK = 0
NUM = 0
CALL INTERP (365, 25, DAY, AMON, R, RR, IJK, NUM,LOPT)
RETURN
END
```

**SUBROUTINE INTHGH**

This subroutine interpolates the values of INTW and PITW for the measured height MHGHT.

```plaintext
REAL * 4 INTW(2,9,4,3,13,19), INTWH(2,9,4,13,19), PITW(2,13,4),
1 PITWH)
REAL * 4 INTWH1(2,9,4,13,19), PITWH1(2,13,4),
1 PITWH2(2,9,4,13,19), PITWH2(2,13,4)
C INTERPOLATES IF MEASURED HEIGHT IS LESS THAN OR EQUAL TO .45KM
C (SEE DOCUMENTATION EQU. 21A, 21B)
C IF (MHGHT .LE. HGHT(3)) THEN
DO 100 IWAV =1,2
DO 101 ITAU = 1, LTAU
DO 102 ITHE = 1, LTHE
PITWH(IWAV, ITHE, ITAU) = (PITW(IWAV, ITHE, ITAU, 3) * MHGHT) +
1 (HGHT(3) - MHGHT) / HGHT(3)
102 CONTINUE
101 CONTINUE
100 CONTINUE
```

93
C INTERPOLATES IF MEASURED HEIGHT LIES BETWEEN .45 AND 4.5 KM
C (SEE DOCUMENTATION 3.6)
C
ELSE
   IF( MHGHT .GT. HGHT(3) .AND. MHGHT .LT. HGHT(2)) THEN
C
C EXTRAPOLATES FOR THE HEIGHT LE. TO .45 KM.
C
DO 200 IWAV = 1,2
DO 201 ITAU = 1,LTAU
DO 202 ITHE = 1,LTHE
   PITWH1(IWAV,ITHE,ITAU) = ((PITW(IWAV,ITHE,ITAU,3) * MHGHT) +
  1 (HGHT(3) - MHGHT)) / HGHT(3)
DO 203 IPHI = 1,LPHI
DO 204 ITHETO = 1,LTHETO
   INTWH1(IWAV,ITHETO,ITAU,ITHE,IPHI) =
   1 (INTW(IWAV,ITHETO,ITAU,ITHE,IPHI) .LE. 0.0)
   1 INTWH1(IWAV,ITHETO,ITAU,ITHE,IPHI) = 1.E-6
204 CONTINUE
203 CONTINUE
202 CONTINUE
201 CONTINUE
200 CONTINUE
C
C EXTRAPOLATES BETWEEN THE HEIGHTS 4.5 KM AND 80.0 KM.
C
Z = (1. - (EXP(-MHGHT/9.)))
Z1= (1. - (EXP(-HGHT(2)/9.)))
Z2= (1. - (EXP(-HGHT(1)/9.)))
DO 300 IWAV = 1,2
DO 301 ITAU = 1,LTAU
DO 302 ITHE = 1,LTHE
   PITWH2(IWAV,ITHE,ITAU) =
   1 PITW(IWAV,ITHE,ITAU,2) +
   1 (((PITW(IWAV,ITHE,ITAU,1) -
   1 PITW(IWAV,ITHE,ITAU,2)) / (Z2 - Z1)) * (Z2 - Z1))
DO 303 IPHI = 1,LPHI
DO 304 ITHETO = 1,LTHETO
   INTWH2(IWAV,ITHETO,ITAU,ITHE,IPHI) =
   1 INTW(IWAV,ITHETO,ITAU,ITHE,IPHI) +
   1 (((INTW(IWAV,ITHETO,ITAU,ITHE,IPHI) -
   1 INTW(IWAV,ITHETO,ITAU,ITHE,IPHI)) / (Z2 - Z1)) * (Z2 - Z1))
304 CONTINUE
303 CONTINUE
302 CONTINUE
301 CONTINUE
300 CONTINUE
C
C CHOSSES THE VALUES THAT SHOW NOMINAL ATMOSPHERIC EFFECT. LOWER VALUES
C OF INTENSITY AND HIGHER VALUES OF PIT ARE PICKED.
DO 400 IWAV = 1, 2
DO 401 ITAU = 1, LTAU
DO 402 ITHE = 1, LTHE
IF (PITWH1(IWAV, ITHE, ITAU) .LE. PITWH2(IWAV, ITHE, ITAU)) GO TO 405
   PITWH(IWAV, ITHE, ITAU) = PITWH1(IWAV, ITHE, ITAU)
DO 403 IPHI = 1, LPHI
DO 404 ITHETO = 1, LTHETO
INTWH(IWAV, ITHETO, ITAU, ITHE, IPHI) =
1 * INTWH1(IWAV, ITHETO, ITAU, ITHE, IPHI)
404 CONTINUE
403 CONTINUE
402 CONTINUE
401 CONTINUE
400 CONTINUE

C C INTERPOLATE FOR THE HEIGHT BETWEEN 4.5 KM. AND 80.0 KM.

C ELSE
   IF ( MHGHT .GE. HGHT(2)) THEN
      Z = (1. - (EXP(-MHGHT/9.)))
      Z1 = (1. - (EXP(-HGHT(2)/9.)))
      Z2 = (1. - (EXP(-HGHT(1)/9.)))
      WRITE(6,10001) Z, Z1, Z2
   DO 600 IWAV = 1, 2
   DO 601 ITAU = 1, LTAU
   DO 602 ITHE = 1, LTHE
   PITWH(IWAV, ITHE, ITAU) =
1 * PITWH(IWAV, ITHE, ITAU, 2) +
1 * PITW(IWAV, ITHE, ITAU, 2)) / (Z2 - Z1)) * (Z - Z1))
   DO 603 IPHI = 1, LPHI
   DO 604 ITHETO = 1, LTHETO
   INTWH(IWAV, ITHETO, ITAU, ITHE, IPHI) =
1 * INTWH(IWAV, ITHETO, ITAU, 2, ITHE, IPHI) +
1 * ((INTW(IWAV, ITHETO, ITAU, 2, ITHE, IPHI) -
1 INTW(IWAV, ITHETO, ITAU, 2, ITHE, IPHI)) / (Z2 - Z1)) * (Z - Z1))
604 CONTINUE
603 CONTINUE
602 CONTINUE
601 CONTINUE
600 CONTINUE
ELSE
   ENDIF
ENDIF
END
SUBROUTINE INTERP(I,M,X1,X,Y,Y1,IJK,INUM,LOPT)
C
C THIS SUBROUTINE IS A GENERAL PURPOSE ROUTINE AND INTERPOLATES
C LINEARLY.VALUE OF Y1 IS INTERPOLATED FOR X1.NO EXTRAPOLATION IS
C ALLOWED.
REAL*4 X1(I),X(M),Y(M),Y1(I)
DO 290 LK = 1,I
XBAR=X1(LK)
LL=M-1
DO 230 IL=1,LL
IF(XBAR .LT.X(I))GO TO 300
IF(XBAR .GT.X(LL+1)) GO TO 300
IF(XBAR .GE.X(IL) .AND.XBAR.LE. X(IL+1)) GO TO 250
GO TO 230
250 PPHI=X(IL)
SPHI=X(IL+1)
P INTEN=Y(IL)
SINTEN=Y(IL+1)
Y1(LK)=PINTEN+((SINTEN-PINTEN)*((XBAR-PPHI)/(SPHI-PPHI))
GO TO 290
230 CONTINUE
290 CONTINUE
LOPT=1
RETURN
C THE PROGRAM DOES NOT ALLOW ANY EXTRAPOLATION .IT WILL PRINT AN ERROR
C MESSAGE .IJK IDENTIFIES THE ARRAY BEING SENT FOR INTERPOLATION.
300 LOPT=0
IF(IJK .EQ.1) WRITE(6,1) XBAR, INUM
IF(IJK .EQ.2) WRITE(6,1) XBAR, INUM
IF(IJK .EQ.3) WRITE(6,1) XBAR, INUM
IF(IJK .EQ.4) WRITE(6,1) XBAR, INUM
1 FORMAT(1X,' MEASURED SOLAR ZENITH ANGLE(MTHETO)',F5.1,' IS OUT OF RANGE.ACTUAL RANGE 10 -78 DEGREES.THE DATA POINT #',14,' IS NOT PROCESSED.')
2 FORMAT(1X,' MEASURED OBSERVATION AZIMUTH ANGLE(MPHI)',F6.1,' IS OUT OF RANGE.ACTUAL RANGE 0 -180 DEGREES.THE DATA POINT #',I4,' IS NOT PROCESSED.')
3 FORMAT(1X,' MEASURED OBSERVATION ZENITH ANGLE(MTHETA)',FS.1,' IS OUT OF RANGE.ACTUAL RANGE 0 -72 DEGREES.THE DATA POINT #',14,' IS NOT PROCESSED.')
4 FORMAT(1X,' MEASURED OPTICAL THICKNESS(MTAU) ','F5.3,' IS OUT OF RANGE . ACTUAL RANGE 0.0-1.0 . THE DATA POINT #',I4,' IS NOT PROCESSED.')
1SED.'
RETURN
END
SUBROUTINE INTEXP (X, Y,MTAU, YY, LTAU, IGNORE)
C
C THIS SUBROUTINE IS A GENERAL PURPOSE ROUTINE AND INTERPOLATES
C EXPONANTIALLY.IGNORE SENDS VALUE OF 0 IF SUBROUTINE INTEXP IS
C USED AND 1 IF IT FINDS THE FUNCTION TO BE LINEAR.
REAL * 4 X(LTAU),Y(LTAU),MTAU,YY(1),TOL,C
COMMON/FF/XS(3),YS(3)
EXTERNAL FMIN
C VALUE OF IGNORE IS SET TO 0 AND IF MEASURED OPTICAL THICKNESS IS LESS
C THEN OR EQUAL TO 0.25 THEN THE FUNCTIONS OF OPTICAL THICKNESSES OF
C 0.0, 0.25, 0.50 ARE USED.
C
    IGNORE=0
    IF(MTAU.LE. 0.25)THEN
        MM=1
        MN=3
    ENDIF
    IS=0
    DO 10 IJ = MM, MN
        HS=IS+1
        XS(IS)=X(IJ)
        YS(IS)=Y(IJ)
    CONTINUE
C
C IF MEASURED OPTICAL THICKNESS IS GREATER THAN 0.25 THEN FUNCTIONS
C OF OPTICAL THICKNESSES OF 0.25, 0.50 AND 1.00 ARE USED.
C
    ELSE
        MM=2
        MN=4
        ENDIF
    IS=0
    DO 10 IJ = MM, MN
        HS=IS+1
        XS(IS)=X(IJ)
        YS(IS)=Y(IJ)
    CONTINUE
C
C SYSTEMS SUBROUTINE IMSL (ZXGSN) IS CALLED TO GET THE MINIMAL OF
C FUNCTION FMIN.
C
A IS LOWER END-POINT OF THE INTERVAL IN WHICH MINIMUM OF FMIN IS TO BE
LOCATED.
C B IS THE UPPER END-POINT OF THE INTERVAL.
C TOL IS LENGTH OF THE FINAL SUBINTERVAL CONTAINING THE MINIMUM.
C C IS OUTPUT , THE APPROXIMATE MINIMUM OF THE FUNCTION FMIN ON THE
ORGINAL INTERVAL(A,B)
C IER RETURNS THE VALUE OF 0 IF NO ERROR IS FOUND .
C
    A = 0.01
    AA = 0.01
    B = 10
    TOL=1.E-4
    CALL ZXGSN(FMIN,A,B,TOL,C,IER)
C
C THE IF STATEMENT BELOW DECIDES IF THE FUNCTION IS LINEAR OR NOT.
C IF THE FUNCTION IS LINEAR THE VALUE OF IGNORE IS SET TO 1 AND
C CONTROL IS SEND BACK TO MAIN PROGRAM.
C
    IF((C-AA) .LT. .001)THEN
        IGNORE=1
        RETURN
    ELSE
C
C IF FUNCTION IS NOT LINEAR ,EXPONANTIAL INTERPOLATION IS USED, AND INTE
C INTERPOLATED VALUE FOR MTAU IS YY(1) IS COMPUTED FOR THE FUNCTION.
C
      BB=(YS(1)-YS(2))/((EXP(-C*XS(1)))-(EXP(-C*XS(2)))
      AA=YS(1)-(BB *(EXP(-C*XS(1))))
      YY(1)=AA+(BB *(EXP(-C*MTAU)))
      ENDIF
    RETURN
    END
FUNCTION FMIN(C)

FUNCTION FMIN IS FUNCTION TO BE MINIMIZED.

REAL C
COMMON/FF/XS(3), YS(3)
P = (YS(1) - YS(2)) / (YS(3) - YS(2))
P = EXP(-C * XS(1)) - EXP(-C * XS(2))
PG = EXP(-C * XS(3)) - EXP(-C * XS(2))
PP = PF/PG
FMIN = (P - PP)**2
RETURN
END
This report describes a simple and fast atmospheric correction algorithm used to correct radiances of scattered sunlight measured by aircraft and/or satellite above a uniform surface. The atmospheric effect, the basic equations, a description of the computational procedure, and a sensitivity study are discussed. The program is designed to take the measured radiances, view and illumination directions, and the aerosol and gaseous absorption optical thicknesses to compute the radiance just above the surface, the irradiance on the surface, and surface reflectance. Alternatively, the program will compute the upward radiance at a specific altitude for a given surface reflectance, view and illumination directions, and aerosol and gaseous absorption optical thicknesses. The algorithm can be applied for any view and illumination directions and any wavelength in the range 0.48 µm to 2.2 µm. The relation between the measured radiance and surface reflectance, which is expressed as a function of atmospheric properties and measurement geometry, is computed using a radiative transfer routine. The results of the computations are tabulated in a look-up table, which forms the basis of the correction algorithm. The algorithm can be used for atmospheric corrections in the presence of a rural aerosol. The sensitivity of the derived surface reflectance to uncertainties in the model and input data is discussed.