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New Atmospheric Turbulence Model for Shuttle Applications


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New Atmospheric Turbulence Model for Shuttle Applications

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PREFACE

NASA/ Marshall Space Flight Center (MSFC) has developed a new atmospheric turbulence model which is more realistic and less conservative when applied in space shuttle reentry simulations involving engineering calculations of reaction control system fuel expenditures. Both Georgia Institute of Technology and BDM Corporation were contracted to update the required turbulence velocity sigmas and length scales, and to apply them in a white noise filter technique to arrive at a more realistic engineering turbulence model. This model is also envisioned to be useful in other type spacecraft and aircraft simulation studies. This project was funded by the NASA-Johnson Space Center Space Shuttle Office.
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It has been determined at NASA [1] that the currently used atmospheric turbulence (wind) model [2,3] for space shuttle reentry simulation is overly conservative. Use of this model in shuttle reaction engine fuel usage calculations assumes severe turbulence all the way from reentry to landing. Johnson Space Center (JSC) sets reaction control system redlines based on these fuel predictions. However, in reality, the orbiter generally returns from space with approximately 270 kg (600 lb) of extra, unused fuel aboard creating an unneeded weight excess. Turbulence in the real atmosphere is patchy or intermittent with quiescent zones present. Therefore, the Environmental Analysis Branch of Marshall Space Flight Center’s (MSFC) Earth Science and Applications Division developed a more realistic engineering-oriented turbulence model that can now be used in shuttle simulation work to select a more rational fuel reserve redline value, along with other potential atmospheric turbulence applications.

This modeling task was accomplished in two parts. First, Dr. C.G. Justus of the Georgia Institute of Technology updated the statistical turbulence data base by a literature search to arrive at better estimates of anisotropic horizontal and vertical turbulence velocity standard deviations (\(\sigma_u\) and \(\sigma_w\)), and length scale parameters (L_x and L_z), from near-surface to 200-km altitude. The y-component (v) was not explicitly calculated, but is generated identically to the z-component. These model statistics are available in the form of a program subroutine to evaluate turbulence u’s and L’s as a function of altitude. This task is fully documented as part II of this report, and was taken completely from the Georgia Tech final report [4] and reprinted in its entirety here. The results from this task have also been presented at a recent technical conference [5].

The second part of this task was done by Dr. C.W. Campbell and M.K. Doubleday of BDM Corporation who applied and modeled the new turbulence statistics of Justus [4] in a procedure which inputs Gaussian white noise into a low-pass linear filter to output the simulated turbulence in a Gaussian time series. The transfer function of the filter was selected to produce a desired von Karman spectrum, with a more realistic probability distribution. In Campbell’s study, for longitudinal spectra, transfer function approximations to the von Karman transfer function, up to fifth order, were derived versus differing sampling rates. The corresponding transverse transfer function is one order higher. The resulting longitudinal and transverse equations used can be directly coded into the shuttle reentry simulation, or into any other type of vehicle flight simulation procedure. The only inputs to the equations are the appropriate turbulence length scale and relative wind velocity turbulent intensity and the sampling rate. Campbell’s work is documented in the BDM final report [6] which is also presented in this report as part III. This work has also been presented at technical conferences [7,8]. Larry McWhorter at NASA/JSC is currently implementing this new turbulence model in his shuttle reentry fuel simulation work.
REFERENCES


II. UPDATED TURBULENCE STATISTICS AND SUBROUTINE

A. BACKGROUND

As evidenced by the information in Table 1, from a review of turbulence models by Moorhouse and Heffley (1986), there are a wide variety of techniques which have been used in turbulence simulation. The purpose of this study was to develop a turbulence model for use in space shuttle reentry simulations, which will be simple to use, computationally fast, and consistent with techniques of Monte Carlo modeling and digital filtering, being developed by other researchers for NASA (Campbell and Fichtl, 1985; Campbell, 1986).

Previous NASA methods for simulation of turbulence for aerospace vehicle flight simulations include the turbulence simulation technique of Fichtl (1977) and the turbulence criteria and model presented in Section 2.4 of Turner and Hill (1982). The Fichtl approach was used in Appendix 10.10 of “Natural Environment Design Requirements for the Space Shuttle” (NASA, 1975), and formed the basis for the shuttle simulation turbulence tapes of Tatom and Smith (1982). The Turner and Hill approach has also been adopted and recommended as the turbulence design criteria for other NASA projects (e.g., Adelfang, 1987).

The turbulence model proposed here is based on these techniques, with updates and modifications. The method of Turner and Hill is based on a model probability distribution p(σ) given by:

\[
p(\sigma) = \sqrt{\frac{2}{\pi}} \left( \frac{P_1}{b_1} \right) \exp \left[ -\frac{\sigma^2}{2b_1^2} \right] + \sqrt{\frac{2}{\pi}} \left( \frac{P_2}{b_2} \right) \exp \left[ -\frac{\sigma^2}{2b_2^2} \right],
\]  

(1)

(their equation 2.38), where \(b_1\) and \(b_2\) are the standard deviations of rms gust velocity in nonstorm and storm turbulence, and \(P_1\) and \(P_2\) are the fractions of flight time or distance flown in nonstorm and storm turbulence. Equation (1) assumes a fraction \(P_0\) for flight time or distance in smooth air, such that

\[
P_0 + P_1 + P_2 = 1 .
\]

(2)

It should be noted that, for consistency with equation (2), equation (1) should have an added term \(P_0 \delta(\sigma)\), where \(\delta\) is the Dirac delta function.
B. BASIS OF THE REVISED MODEL

There are several changes in the form of the model developed. The half-Gaussian distribution of equation (1) has the unrealistic feature that the most probable value (the mode) of $\sigma$ in the $p(\sigma)$ distribution is $\sigma = 0$. The alternate form of distribution suggested is the Rayleigh, which has a more probable value (mode) closer to the mean (expected) value of the distribution. The Rayleigh distribution is given by

$$p(\sigma) = \left(\frac{\sigma}{b}\right) \exp \left\{ -\left(\frac{\sigma}{b}\right)^2 \right\},$$

(3)

which has a mean value $\sigma = (\pi/2)^{1/2}b$, a mode of $b$, and a standard deviation of $[2 - \pi/2]^{1/2}b$.

For the cumulative probability $p(\sigma \geq \sigma_1)$, the Rayleigh distribution produces a Gaussian distribution

$$p(\sigma \geq \sigma_1) = \exp\left\{ -\left(\frac{\sigma_1}{b}\right)^2 \right\}.$$

(4)

For implementation in the Monte Carlo series simulation (Campbell and Fichtl, 1985; Campbell, 1986), the values of $\sigma$ can be evaluated from Gaussian-distributed components, $\sigma_i$ and $\sigma_j$ (not to be confused with spatial vector components) by the relation $\sigma = (\sigma_i^2 + \sigma_j^2)^{1/2}$, where $\sigma_i$ and $\sigma_j$ are each selected from a Gaussian (normal) distribution, that is

$$p(\sigma_i) = \left(2\pi b^2\right)^{-1/2} \exp\left\{ -\left(\frac{\sigma_i}{b}\right)^2 \right\},$$

(5)

and similarly for $p(\sigma_j)$.

Equation (3) is used separately to evaluate the probability distribution for no turbulence (with $\sigma = 0$ and probability $P_0$), moderate turbulence (with $\sigma$ selected from the distribution with $b = b_1$ and probability $P_1$), and severe turbulence (with $\sigma$ selected from the distribution with $b = b_2$ and probability $P_2$). Instead of using these probability distributions additively, as in equation (1), each of these distributions is used separately, according to which severity level of turbulence is being encountered (none, moderate, or severe). A turbulence severity parameter $\alpha$ (with value selected from a uniform random distribution between 0 and 1) is used to determine the severity level of the turbulence: there is no turbulence if $\alpha < P_0$; the turbulence is severe if $\alpha \geq 1 - P_2$, and moderate otherwise. Minimum vertical depths for layers of moderate and severe turbulence are also specified. Thus, once the series simulation enters a zone with moderate or severe turbulence, it must remain at this severity level until at least the specified minimum depth (or the specified minimum horizontal extent) has been traversed, before it can return to a lower severity level.
The effects of vertical correlations between \( \sigma \) values at one altitude with those at adjacent altitudes are also included. This feature incorporates the fact that rms turbulence gust magnitudes \( \sigma \) must vary more-or-less continuously along the trajectory [not discontinuously as if selected by independent calls upon the probability distribution of equation (3)]. The turbulence \( \sigma \) value changes abruptly, however, when transitioning from one severity layer to another (with only a one-step, linear interpolation smoothing being applied each time a new intensity layer is encountered). No correlations are assumed between sigmas in two layers which are spatially separated by a layer of lower turbulence magnitude. Thus, each time a layer of higher than current turbulence severity is encountered, the random number generator sequence for the sigma selections is reinitialized.

A major portion of the model development project has been a literature survey to develop revised parameter values for the data on turbulence intensities (mean \( \sigma \) values), scales, and probabilities of intensity levels. In addition, new parameter values were required for the vertical scale of the \( \sigma \) interlevel correlation, and the minimum vertical sizes for moderate and severe turbulence layers. Anisotropic horizontal and vertical values are provided for the turbulence intensity and scale parameters.

C. MODEL DESCRIPTION

The literature survey for updating the turbulence parameter values consisted of a search of the Scientific and Technical Aerospace Abstracts, the International Aerospace Abstracts, and the Meteorological and Geoastrophysical Abstracts for the period 1970 to present. Parameter values resulting from this literature review have been incorporated into the turbulence simulation subroutine.

Information on horizontal and vertical scales of turbulence (integral scale in the velocity correlation function) were averaged from data in McCloskey et al. (1971), U.S. Department of Defense (1975), Fichtl (1977), Hasty (1977), Justus et al. (1980), Turner and Hill (1982), Frost et al. (1985), Murrow (1986), and Reid and Vincent (1987). Resulting average values for these horizontal and vertical scales are shown in Figure 1 as a function of altitude from the surface to 200 km.

Values of the sigma components \( \sigma_i \) and \( \sigma_j \) to be selected from the Rayleigh distribution by equation (4) are assumed to be correlated over horizontal and vertical separations with scale values (integral scale) which are related to the horizontal and vertical scales of the turbulent velocity, through a ratio which was assumed to vary from 10.0 at the surface to 5.0 at 20 km and higher. For example, with a horizontal turbulence velocity scale of 0.52 km at the surface, the horizontal sigma scale is assumed to be 5.2 km. Assumptions such as these were required because no observational data were found to provide direct estimates of the sigma scales. These ratio values (5–10) were assumed since turbulent velocity statistics cannot accurately be determined from measurements unless the turbulence is stationary (relatively constant sigma) over at least 5–10 velocity scale values; hence a presumption that the sigma scales relative to the velocity scales are of at least this magnitude range (with larger ratio values anticipated to occur at lower altitudes).
For data on the average value $\bar{\sigma}_u$ (the average longitudinal turbulence intensity), data were averaged over information taken from Pershikov (1969), McCloskey et al. (1971), Ryan et al. (1971), Fichtl (1977), Kao et al. (1977), Waco (1978), Justus et al. (1980), Vinnichenko et al. (1980), Moorhouse and Woodcock (1982), Murrow et al. (1982), Turner and Hill (1982), Hocking (1983), Frost et al. (1985), Hill (1986), Murrow (1986), and Andrews et al. (1987). For the Fichtl (1977) values of $\bar{\sigma}_u$, a weight value of 0.6 was used, since these values were taken to represent extreme values rather than average values.

Data on the average value $\bar{\sigma}_w$ (or on the ratio of $\bar{\sigma}_w/\bar{\sigma}_u$) were taken from McCloskey et al. (1971), Ryan et al. (1971), U.S. Department of Defense (1975), Fichtl (1977), Waco (1978), Vinnichenko et al. (1980), Moorhouse and Woodcock (1982), and Turner and Hill (1982). All of these data were interpreted in terms of an average ratio for $\bar{\sigma}_w/\bar{\sigma}_u$, which was then multiplied by the average $\bar{\sigma}_u$, determined above, to arrive at the final value for the average $\bar{\sigma}_w$.

Values of the ratio ($\gamma = \bar{\sigma}_2/\bar{\sigma}_1$) of the magnitude of severe turbulence ($\bar{\sigma}_2$), relative to the magnitude of moderate turbulence ($\bar{\sigma}_1$), were averaged from data in Kao et al. (1977), Moorhouse and Woodcock (1982), and Turner and Hill (1982). No values for the $\gamma$ intensity ratio were found for altitudes above 18 km; therefore a value of $\bar{\sigma}_2$ equal to 1.5 times $\bar{\sigma}_1$ was assumed at 30 km and higher, with a smooth transition from the value of $\gamma = 1.84$ at 18 km to $\gamma = 1.5$ at 30 km.

Values of the total probability $P = P_1 + P_2$ for encountering turbulence (either moderate or severe) were taken and averaged from McCloskey et al. (1971), Wilson et al. (1971), Ehrenberger (1975), Waco (1976), Hasty (1977), Zimmerman and Murphy (1977), Turner and Hill (1982), and Ehrenberger (1987). Values for $\rho = P_2/P_1$, the ratio of the probability of encountering severe turbulence to that for encountering moderate turbulence, were obtained and averaged from McCloskey et al. (1971), Waco (1976), and Turner and Hill (1982). No values of $\rho$ were found in the literature for altitudes above 30 km (where $\rho = 0.1$). A steady decrease to a value of $\rho = 0.01$ was assumed at 120 km, with $\rho = 0.01$ between 120 and 200 km. From values of the total probability $P$, and the ratio $\rho$, separate values of $P_1$ and $P_2$ were obtained from

$$P_1 = P/(1 + \rho) \quad \text{and} \quad P_2 = \rho P/(1 + \rho) \quad (6)$$

From values of $\rho$ and the severity magnitude ratio $\gamma$, separate values for average sigma for moderate turbulence ($\bar{\sigma}_1$) and for severe turbulence ($\bar{\sigma}_2$) can be obtained from the overall average sigma $\bar{\sigma}$ for both horizontal and vertical components ($u$ and $w$) via

$$\bar{\sigma}_{1u} = \frac{(1 + \rho)\bar{\sigma}_u}{(1 + \rho \gamma)} \quad \text{and} \quad \bar{\sigma}_{2u} = \frac{\gamma(1 + \rho)\bar{\sigma}_u}{(1 + \rho \gamma)} \quad (7)$$

$$\bar{\sigma}_{1w} = \frac{(1 + \rho)\bar{\sigma}_w}{(1 + \rho \gamma)} \quad \text{and} \quad \bar{\sigma}_{2w} = \frac{\gamma(1 + \rho)\bar{\sigma}_w}{(1 + \rho \gamma)} \quad (8)$$
Values for the resulting averages $\bar{\sigma}_{2u}$ and $\bar{\sigma}_{2w}$ as a function of altitude for severe turbulence are shown in Figure 2. The profiles of $\bar{\sigma}_{1u}$ and $\bar{\sigma}_{1w}$ values for moderate turbulence are shown in Figure 3. Values of the average sigmas for composite turbulence, $\bar{\sigma}_c$, defined to be probability-weighted values of $\bar{\sigma}_1$ and $\bar{\sigma}_2$, are defined by

$$\bar{\sigma}_c = P_0 \cdot (0) + P_1 \bar{\sigma}_1 + P_2 \bar{\sigma}_2$$  \hspace{1cm} (9)$$

for both $u$ and $w$ components [where the normalization $P_0 + P_1 + P_2 = 1$ has been used, from equation (2)]. The resulting composite turbulence $\bar{\sigma}_c$ values are shown in Figure 4, as functions of height, for both horizontal and vertical components.

Up to about 20 km, the magnitudes (Figures 2–4) and scales (Figure 1) for the turbulence model are taken to represent atmospheric turbulence as conventionally defined. Above this height, the magnitudes and scales are taken to represent either gravity waves (which in high-speed flight may produce the same dynamical effects on the vehicle as does turbulence), or the turbulence which may result from gravity wave breaking or gravity wave dissipation at critical levels.

For the minimum vertical extent of turbulence layers, data values from Ehrenberger (1975), Röttger (1980), Barat (1982), Tanaka and Yamanaka (1984), Yamanaka and Tanaka (1984), Cot and Barat (1986), and Ehrenberger (1987) were averaged. No data on vertical layer thickness were available above the 60–90 km layer, so an extrapolated increase rate in the vertical layer size above this altitude range was assumed.

Horizontal minimum extent of turbulence layers was derived from average data values taken from Ehrenberger (1975), Murrow et al. (1982), Hasty (1977), and Waco (1978). No data were available above 25 km, so an extrapolated height increase for the horizontal layer size was assumed above this height. No clear data were available on the relative sizes for severe turbulence relative to the size for moderate turbulence layers. Therefore the minimum vertical and horizontal sizes for severe turbulence layers were taken to be 1/2 that for the moderate layers.

The attached program listing shows the model implementation as subroutine TURBSIG, which is to be used for simulating turbulence intensity values ($\sigma$'s) at a given altitude $z$ (0 ≤ $z$ ≤ 200 km). The subroutine returns values for $\sigma_u$ and $\sigma_w$, the horizontal and vertical sigma components (in m/s), and $L_x$ and $L_z$, the horizontal and vertical integral scale values (in km).

Several model options (specified by the input parameter MODEL) are available: MODEL = 0 for the complete model with all specified parameters (probabilities $P_0$, $P_1$, $P_2$, average sigma values, $\bar{\sigma}_{1u}$, $\bar{\sigma}_{2u}$, $\bar{\sigma}_{1w}$, $\bar{\sigma}_{2w}$, scales $L_x$, $L_z$, etc.) with the local, spatially dependent, sigma values selected from the appropriate Rayleigh distribution for moderate and severe turbulence intensity levels; MODEL = 1 for moderate turbulence throughout the whole run, with sigma values selected from the Rayleigh distribution which has $\bar{\sigma}_1$; MODEL = 2 for severe turbulence throughout the whole run with sigma values selected from the Rayleigh distribution which has $\bar{\sigma}_2$; MODEL = 3 for composite turbulence (probability-weighted average of severe, moderate, and non-turbulent) throughout the whole run, with sigmas selected from the Rayleigh distribution which
has $\sigma = \bar{\sigma_c}$; MODEL = 4 for no turbulence throughout the whole run ($\sigma = 0$); MODEL = 5 for moderate turbulence with $\sigma = \bar{\sigma_1}$ (no Rayleigh distribution) throughout the whole run; MODEL = 6 for severe turbulence with $\sigma = \bar{\sigma_2}$ throughout the whole run (no Rayleigh distribution); and MODEL = 7 for composite turbulence with $\sigma = \bar{\sigma_c}$ throughout the whole run (no Rayleigh distribution).

The main program TESTSIG is designed to be used for producing test output of the TURBSIG subroutine, and illustrates how TURBSIG is to be used in the reentry trajectory simulation programs. The user must select a starting random number (any odd positive integer) for the random number generator (RAND), and a value of MODEL must be selected. The TURBSIG subroutine must also be called once with displacement values $dx = 0$ and $dz = 0$, in order to initialize all of its variables. On subsequent calls, the trajectory program must pass to the subroutine the displacement values ($dx$ and $dz$, in km) of current position from previous position. Only the magnitudes of the displacements are of importance, so positive or negative values may be used.
REFERENCES


Figure 1. Horizontal and vertical scales (km) for the turbulence model as a function of height.
Figure 3. Horizontal and vertical magnitudes for moderate turbulence as a function of height.
Composite Turbulence Magnitude

Figure 4. Horizontal and vertical magnitudes for composite turbulence (probability-weighted severe, moderate, and no turbulence cases) as a function of height.
## Table 1. A Survey of Atmospheric Disturbance Models (Moorhouse and Heffley, 1986)

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<th>Model</th>
<th>Key Features</th>
<th>Sources</th>
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<td>Dryden turbulence</td>
<td>A convenient spectral form based on an exponential autocorrelation function for the axial component</td>
<td>Dryden (1961)</td>
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<tr>
<td>von Karman turbulence</td>
<td>A spectral form for which the correlation function includes a finite micro-scale, thus the relative proportion of spectral power at high frequencies exceeds that of the Dryden.</td>
<td>von Karman (1961), Houbolt (1973)</td>
</tr>
<tr>
<td>Ornstein-Uhlenbeck turbulence</td>
<td>A spectral form with first-order longitudinal and transverse components</td>
<td>Gaonkar (1980)</td>
</tr>
<tr>
<td>Etkin one-dimensional turbulence</td>
<td>The local turbulent velocity field is approximated by a truncated Taylor series which yields uniform and gradient components. High frequency spectral components eliminated on the basis of aircraft size. Based on Dryden form, but gradient spectra are non-realizable unless simplified.</td>
<td>Etkin (1961), Etkin (1959a, b), Etkin (1972)</td>
</tr>
<tr>
<td>power spectra</td>
<td>A discrete gust waveform</td>
<td>Moorhouse and Woodcock, (1982)</td>
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<td>Versine gust</td>
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<tr>
<td>Lappe low-altitude turbulence model</td>
<td>Experimentally-obtained data of vertical gust spectra, mean wind speed, and lapse rate were used to develop a low-level turbulence model. The turbulence spectra are presented for different types of terrain, height, and meteorological conditions.</td>
<td>Lappe (1966)</td>
</tr>
<tr>
<td>Multiple point source turbulence</td>
<td>A two-dimensional gust field generated from two or more noise sources having prescribed correlation functions and located spanwise or lengthwise on the vehicle.</td>
<td>Etkin (1980), Holley and Bryson (1975), Skelton (1968)</td>
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<td>Holley-Bryson random turbulence shaping filters</td>
<td>A matrix differential equation formulation of uniform and gradient components including aircraft size effects. Filter equation coefficients determined from least squares fit to multipoint-source-derived correlation functions.</td>
<td>Holley and Bryson (1975)</td>
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<td>University of Washington non-Gaussian atmospheric turbulence model</td>
<td>Non-Gaussian model using modified Bessel functions to simulate the patchy characteristics of real-world turbulence. Spectral properties are Dryden and include gust gradients.</td>
<td>Reeves et al. (1974), Reeves (1969)</td>
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<td>Delft University of Technology non-Gaussian structure of the simulated turbulent environment</td>
<td>Non-Gaussian model similar in form to the University of Washington model, but uses the Hilbert transformation to model intermittency as well as patchiness. Includes University of Washington model features extended to approximate transverse turbulence velocities and gradients.</td>
<td>van de Moesdijk (1978)</td>
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<td>The Netherlands National Aerospace Laboratory model of non-Gaussian turbulence</td>
<td>Similar to the Royal Aeronautical Establishment model, but extended to include patchiness and gust gradient components and transverse velocities.</td>
<td>Jansen (1977a, 1977b)</td>
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<td>University of Virginia turbulence model</td>
<td>Models patchiness by randomizing gust variance and integral scale of basic Dryden turbulence.</td>
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<td>MIL Standard turbulence model</td>
<td>First order difference equation implementation of turbulence filters based on 8785 Dryden turbulence and refitted rolling gust intensity.</td>
<td>Hoh et al. (1982)</td>
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<td>Indian Institute of Science non-stationary turbulence model</td>
<td>Non-stationary turbulence is obtained over finite time windows by modulating a Gaussian process with either a deterministic or random process. The result is patchy-like turbulence, similar to the University of Washington model, except the time-varying statistics of turbulence are presented for the deterministic modulating functions.</td>
<td>Gaonkar (1980)</td>
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<td>FAA wind shear models</td>
<td>Three-dimensional wind profiles for several weather system types including fronts, thunderstorms, and boundary layer. The profiles are available in table form.</td>
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<tr>
<td>STI wind shear model</td>
<td>Time and space domain models of mean wind and wind shear (ramp wave forms) are combined with MIL-F-8785C Dryden turbulence to obtain the total atmospheric disturbance. The magnitudes of the mean wind and wind shear are evaluated in terms of the aircraft’s acceleration capabilities.</td>
<td>Hoh and Jewell (1976), Hefley and Jewell (1978)</td>
</tr>
<tr>
<td>Sinclair frontal wind shear model</td>
<td>A generic model of frontal surface wind shear derived from a reduced-order of Navier-Stokes equations. Relatively simple to use and can match the overall characteristics of measured wind shears.</td>
<td>Jewell et al. (1979), Sinclair and West (1978)</td>
</tr>
</tbody>
</table>
### TABLE 1. (Continued)

<table>
<thead>
<tr>
<th>Model</th>
<th>Key Features</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIL-F-8785B atmospheric disturbance model</td>
<td>Intensities and scale lengths are functions of altitude and use either Dryden or von Karman spectral forms or a one minus cosine gust. Also spectral descriptions of rotary gusts.</td>
<td>U.S. Dept. of Defense (1969), Chalk et al. (1969)</td>
</tr>
<tr>
<td>MIL-F-8785C atmospheric disturbance model</td>
<td>Same as 8785B with the addition of a logarithmic planetary boundary layer, a vector shear and a Naval carrier airwake model.</td>
<td>U.S. Dept. of Defense (1980)</td>
</tr>
<tr>
<td>ESDU atmospheric turbulence</td>
<td>Rather general, but contains comprehensive descriptive data for turbulence intensity, spectra, and probability density.</td>
<td>Anonymous (1974, 1975)</td>
</tr>
<tr>
<td>Boeing atmospheric disturbance model turbulence</td>
<td>A comprehensive model of atmospheric disturbances that includes mean wind, wind shear, and random turbulence. Turbulence is Gaussian and uses linear filters that closely approximate the von Karman spectral form. Mean wind and turbulence intensity are functions of meteorological parameters.</td>
<td>Barr et al. (1974)</td>
</tr>
<tr>
<td>Wasicko carrier airwake model</td>
<td>Includes mean wind profile, effect of ship motion, and turbulence.</td>
<td>Durand (1967)</td>
</tr>
<tr>
<td>Naval ship airwake model</td>
<td>Includes free air turbulence filters plus steady, periodic, and random components of airwake which are functions of space and time.</td>
<td>U.S. Dept. Defense (1980), Nave (1978)</td>
</tr>
<tr>
<td>Vought airwake model for DD-963 class ships</td>
<td>Combined random and deterministic wind components for free air and ship airwake regions. Based on wind tunnel flow measurements.</td>
<td>Fortenbaugh (1978)</td>
</tr>
</tbody>
</table>
TABLE 1. (Concluded)

<table>
<thead>
<tr>
<th>Model</th>
<th>Key Features</th>
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</tr>
</thead>
<tbody>
<tr>
<td>STI wake vortex encounter model</td>
<td>A two-dimensional model of the flow field due to the wake vortex of an aircraft is presented. The parameters of the flow-field model are weight, size, and speed of the vortex-generating aircraft, and distance and orientation of the vortex-encountering aircraft. Strip theory is used to model the aerodynamics of the vortex-encountering aircraft.</td>
<td>Johnson and Teper (1974)</td>
</tr>
<tr>
<td>Campbell and Sandborne wind shear and turbulence model</td>
<td>Spatial model based on joint airport weather studies (JAWS) microburst data. Permits calculation of aerodynamic loads over body of aircraft.</td>
<td>Campbell and Sandborne (1985)</td>
</tr>
<tr>
<td>Zhu and Etkin microburst model</td>
<td>Generic spatial model of microburst velocity components based on potential flow singularity distribution involving only three adjustable parameters.</td>
<td>Zhu and Etkin (1985)</td>
</tr>
</tbody>
</table>
C..... PROGRAM - TESTSIG to test the TURBSIG subroutine
implicit double precision (a-h,o-z)
Open(21,File='sigmau',Status='New')
Open(22,File='sigmaw',Status='New')
Open(23,File='scaleu',Status='New')
Open(24,File='scalew',Status='New')
Open(25,File='severity',Status='New')
Open(26,File='model',Status='New')
1  Write(*,5)
5 Format(' Enter starting random number (odd integer): ') Read(*,n1)
   if(n1.le.0)goto 99
   if(MOD(n1,2).ne.1)goto 1
   z = rand(n1)
   Write(*,10)
10 Format(' Enter model number 0-7: ') Read(*,*)model
   Write(S,15) Format(' Enter starting height: ') Read(S,Z)
   Call turbsig(model,z,O.,O.,sigmau,sigmaw,xlwind,zlwind,isev)
15 Format(' Enter displacements DX, DZ in km: ') Read(*,*)dx,dz
   x = 0.
   xmax = 99999.
   if(abs(dz).le.0.)then
      Write(*,23) Format(' Enter maximum x value to simulate: ') Read(*,*)xmax
23   endif
25 Call turbsig(model,z,dx,dz,sigmau,sigmaw,xlwind,zlwind,isev)
20 Format(' Enter displacements DX, DZ in km: ') Read(*,*)dx,dz
   x = 0.
   xmax = 99999.
   if(abs(dz).le.0.)then
      Write(*,23) Format(' Enter maximum x value to simulate: ') Read(*,*)xmax
23   endif
25 Call turbsig(model,z,dx,dz,sigmau,sigmaw,xlwind,zlwind,isev)
30 Format(5f10.2,i3) 40 Format(2f10.2) 50 Format(f10.2,i4)
   Z = Z - abs(dz)
   x = x + abs(dx)
   if (Z.lt.0.0.or.x.gt.xmax)goto 99
   goto 25
SUBROUTINE TURBSIG(MODEL,Z,DX,DZ,SIGMAU,SIGMAW,XLWIND,ZLWIND, & ISEV)

C
C Simulation of turbulent wind standard deviation, sigma, selected
C from a Rayleigh distribution, with parameters which are a
C function of height Z.
C Three turbulence intensities are simulated: a non-turbulent
C background, moderate turbulence, and severe turbulence.
C Frequencies of occurrence and minimum persistence of
C layers are specified for moderate and severe intensities.
C Correlation lengths for the turbulent wind field and for
C the sigmas for the wind field are also specified
C independently.
C The characteristics of the Rayleigh distribution
C for standard deviation (sigma) are determined by the expected
C value (average value) of the sigma distribution.
C
C Input subroutine arguments are:
C MODEL – 0 for complete model with all specified parameters;
C sigmas selected from Rayleigh distribution
C 1 for moderate turbulence throughout the whole run;
C sigmas selected from Rayleigh distribution
C 2 for severe turbulence throughout the whole run;
C sigmas selected from Rayleigh distribution
C 3 for composite turbulence (average of severe,
C moderate and non turbulence) throughout the
C whole run; sigmas selected from Rayleigh
C distribution
C 4 for no turbulence throughout the whole run
C 5 for moderate turbulence of average sigma
C throughout the whole run
C 6 for severe turbulence of average sigma
C throughout the whole run
C 7 for composite turbulence of average sigma
C throughout the whole run
C Z - Current altitude in km
C DX - Horizontal displacement since last position, in km
C DZ - Vertical displacement since last position, in km.
C Note - To initialize values call TURBSIG with both
C DX = 0 and DZ = 0
C
C Output subroutine arguments are:
C SIGMAU - Current turbulence standard deviation for the
C horizontal wind components, in m/s
C SIGMAW - Current turbulence standard deviation for the
C vertical wind component, in m/s
C XLWIND - Current horizontal scale for turbulent wind, in km
C ZLWIND - Current vertical scale for turbulent wind, in km
C ISEV - Severity parameter (1 = non turbulent,
C 2 = moderate turbulence, 3 = severe turbulence,
C or 0 = composite turbulence: weighted average
C of categories 1-3)
C

implicit double precision (a-h,o-z)
C
The following static variables have values which should
C remain unchanged from one call of the subroutine to another.
C Some compilers require declaration of such static variables;
C others do not.
C Static sigmu,sigmyu,sigmxw,sigmyw,sigsxu,sigsyu,
C & sigsxw,sigsyw,xsev,psigu,psigw,nsev
double precision LSIGX(32),LSIGZ(32),LWINDX(32),LWINDZ(32),
& IMODZ(32),LSEVZ(32),IMODX(32),LSEVX(32),maxz(3),maxx(3)
dimension hgt(3),SIGMXBAR(32),SIGSXBAR(32),
& SIGMZBAR(32),SIGSZBAR(32),PM(32),PS(32),sigu(3),sigw(3)
data pi,one,two/3.14159265359d0,1.0d0,2.0d0/
data AFAC,BFAC/19.51615854016301d0,1.00041693941~45578d~/
C
Heights for turbulence model parameters, in km
& 35.,40.,45.,50.,55.,60.,65.,70.,75.,80.,85.,90.,100.,
& 110.,120.,140.,160.,180.,200./
C
Mean Value for moderate turbulence sigmas, m/s (horizontal)
Data SIGMXBAR/1.25,1.65,1.65,2.04,2.13,2.15,2.23,2.47,
& 2.62,2.44,2.21,2.26,2.71,3.73,4.59,5.26,6.22,7.27,8.7,
& 10.1,11.3,15.9,19.2,22.6,27.3,33.2,35.6,42.3,44.3,48.2,
& 48.9,49.5/
C
Mean Value for moderate turbulence sigmas, m/s (vertical)
Data SIGMZBAR/98.,1.36,1.43,1.68,1.69,1.69,1.73,1.79,
& 1.91,2.10,2.07,1.99,2.09,2.39,2.58,2.87,3.25,4.21,4.40,
& 4.42,4.05,5.04,6.3,8.3,10.3,11.8,11.4,10.7,10.8,11.7,
& 11.8,12.0/
C
Mean Value for severe turbulence sigmas, m/s (horizontal)
Data SIGMXBAR/3.06,3.90,4.35,6.24,7.16,7.59,7.72,7.89,
& 6.93,5.00,4.07,3.85,4.34,5.60,6.89,7.89,9.33,10.90,13.06,
& 15.1,16.9,23.8,28.7,33.8,40.9,49.8,53.3,63.4,66.4,47.2,2,
& 73.3,74.2/
C
Mean Value for severe turbulence sigmas, m/s (vertical)
Data SIGSZBAR/2.41,3.21,3.78,5.13,5.69,5.98,6.00,5.71,
& 5.05,4.31,3.81,3.38,3.43,3.59,3.87,4.30,4.88,6.31,6.60,
& 6.63,6.0,7.5,9.5,12.4,15.4,17.7,17.1,16.0,16.1,17.6,
& 17.8,18.1/
C
Horizontal scale for turbulence sigmas, in km
Data LSIGX/5.2,8.3,8.6,9.4,8.8,8.3,9.2,12.6,18.,21.,28.,
& 510.,555.,605.,660.,765.,1000.,1160.,1350.,1500.,1500./
C
Vertical scale for turbulence sigmas, in km
Data LSIGZ/3.2,6.2,7.7,8.5,8.4,8.1,8.3,11.,14.,16.,18.,
& 52.,56.,64.,79.,88.,100.,111.,122./
C
Horizontal scale for turbulence winds, in km
Data LWINDX/0.520,0.832,0.902,1.04,1.04,1.04,1.23,1.80,
& 2.82, 3.40, 5.00, 8.64, 12.0, 28.6, 35.4, 42.6, 50.1, 57.9, 66.0,
& 74.4, 83.2, 92.3, 102., 111., 121., 132., 153., 200., 232., 270.,
& 300., 300. /

c... Vertical scale for turbulence winds, in km
Data LWINDZ/0.323, 0.624, 0.831, 1.01, 0.98, 1.10, 1.54,
& 2.12, 2.60, 3.34, 4.41, 6.56, 8.88, 8.33, 6.2, 5.3, 6.0, 6.8,
& 7.5, 8.2, 9.0, 9.7, 10.4, 11.2, 12.7, 15.8, 17.6, 20.0, 22.2, 24.3/

c... Probability for encountering moderate turbulence
Data PM/.867, .199, .079, .0738, .0650, .0704, .0677, .0502, .0368,
& .0337, .0277, .0180, .0185, .0249, .0318, .0366, .0455,
& .0682, .0917, .1620, .2336, .3066, .3810, .5769, .7767, .9804,
& .9901, .9901, .9901, .9901 /

c... Probability for encountering severe turbulence
Data PS/.010, .025, .0111, .0063, .0049, .0043, .0034,
& .0027, .0024, .0020, .0015, .0018, .0025, .0032, .0039,
& .0045, .0068, .0083, .0130, .0164, .0194, .0205, .0231, .0233,
& .0196, .0099, .0099, .0099, .0099, .0099 /

c... Minimum vertical size for moderate turbulence layer, km
Data LMODZ/0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.4, 0.50, 0.56, 0.72, 0.88, 1.04, 1.20, 1.36, 1.48, 1.64,
& 1.80, 1.96, 2.12, 2.28, 2.44, 2.60, 3.44, 4.08, 4.72, 5.36, 6.0/

c... Minimum vertical size for severe turbulence layer, km
Data LSEVZ/0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.32, 0.32, 0.32, 0.32, 0.32, 0.32, 0.32, 0.32, 0.32, 0.32, 0.32, 0.32,
& 0.32, 0.25, 0.20, 0.28, 0.36, 0.44, 0.52, 0.60, 0.68, 0.74, 0.82, 0.90, 0.98, 1.06, 1.14, 1.22, 1.40, 1.72, 2.04, 2.36, 2.68, 3.0 /

c... Minimum horizontal size for moderate turbulence layer, km
Data LMODX/91., 74., 60., 59., 66., 69., 66., 62., 54., 38., 27.,
& 121., 132., 153., 200., 232., 270., 300., 300. /

c... Minimum horizontal size for severe turbulence layer, km
Data LSEVX/46., 37., 30., 30., 3.3., 33., 33., 31., 27., 19., 14.,
& 61., 66., 77., 100., 116., 135., 150., 150. /

if(model.gt.7.or.model.lt.0) stop ' Invalid Model Number! ' 

c... Non-turbulent case
if(model.eq.4) then
   SIGMAU = 0.
   SIGMAW = 0.
   XLWIND = 99999.
   ZLWIND = 99999.
   ISEV = 1
   return
endif
sqr2pi = dsqrt(two/pi)

c... Find height index for interpolation
j = 0
do 10 i = 1,32
   if(Z.lt.hgt(33-i))goto 10
   j = 33-i
   goto 20
10 continue
20 if(j.lt.1) j = 1
   if(j.gt.31) j = 31
Interpolate parameters on height
\[ \text{delz} = (z - \text{hgt}(j))/(\text{hgt}(j+1) - \text{hgt}(j)) \]

Average sigmas for moderate turbulence
\[ \begin{align*}
\text{smxb} &= \text{sigmxb}(j) + \text{delz}*(\text{sigmxb}(j+1) - \text{sigmxb}(j)) \\
\text{smzb} &= \text{sigmzb}(j) + \text{delz}*(\text{sigmzb}(j+1) - \text{sigmzb}(j))
\end{align*} \]

Average sigmas for severe turbulence
\[ \begin{align*}
\text{ssxb} &= \text{sigssxb}(j) + \text{delz}*(\text{sigssxb}(j+1) - \text{sigssxb}(j)) \\
\text{sszb} &= \text{sigsszb}(j) + \text{delz}*(\text{sigsszb}(j+1) - \text{sigsszb}(j))
\end{align*} \]

Scales for sigmas
\[ \begin{align*}
\text{XLSIG} &= \text{lsigx}(j) + \text{delz}*(\text{lsigx}(j+1) - \text{lsigx}(j)) \\
\text{ZLSIG} &= \text{lsigz}(j) + \text{delz}*(\text{lsigz}(j+1) - \text{lsigz}(j))
\end{align*} \]

Scales for turbulent winds
\[ \begin{align*}
\text{XLWIND} &= \text{lwindx}(j) + \text{delz}*(\text{lwindx}(j+1) - \text{lwindx}(j)) \\
\text{ZLWIND} &= \text{lwindz}(j) + \text{delz}*(\text{lwindz}(j+1) - \text{lwindz}(j))
\end{align*} \]

Moderate turbulence case
\[ \text{if(mmodel.eq.5) then} \]
\[ \begin{align*}
\text{SIGMAU} &= \text{smxb} \\
\text{SIGMAW} &= \text{smzb} \\
\text{ISEV} &= 2 \\
\text{return}
\end{align*} \]
\[ \text{endif} \]

Severe turbulence case
\[ \text{if(mmodel.eq.6) then} \]
\[ \begin{align*}
\text{SIGMAU} &= \text{ssxb} \\
\text{SIGMAW} &= \text{sszb} \\
\text{ISEV} &= 3 \\
\text{return}
\end{align*} \]
\[ \text{endif} \]

Interpolate probabilities for encountering turbulence
\[ \begin{align*}
\text{pmz} &= \text{PM}(j) + \text{delz}*(\text{PM}(j+1) - \text{PM}(j)) \\
\text{psz} &= \text{PS}(j) + \text{delz}*(\text{PS}(j+1) - \text{PS}(j))
\end{align*} \]

Composite turbulence case
\[ \text{if(mmodel.eq.7) then} \]
\[ \begin{align*}
\text{SIGMAU} &= \text{pmz}*(\text{smxb}) + \text{psz}*(\text{ssxb}) \\
\text{SIGMAW} &= \text{pmz}*(\text{smzb}) + \text{psz}*(\text{sszb}) \\
\text{ISEV} &= 0 \\
\text{return}
\end{align*} \]
\[ \text{endif} \]

Minimum thickness for moderate and severe turbulence layers
\[ \begin{align*}
\text{maxz}(2) &= \text{LMODZ}(j) + \text{delz}*(\text{LMODZ}(j+1) - \text{LMODZ}(j)) \\
\text{maxz}(3) &= \text{LSEVZ}(j) + \text{delz}*(\text{LSEVZ}(j+1) - \text{LSEVZ}(j)) \\
\text{maxx}(2) &= \text{LMODX}(j) + \text{delz}*(\text{LMODX}(j+1) - \text{LMODX}(j)) \\
\text{maxx}(3) &= \text{LSEVX}(j) + \text{delz}*(\text{LSEVX}(j+1) - \text{LSEVX}(j))
\end{align*} \]

Rayleigh distribution standard deviations from mean values
\[ \begin{align*}
\text{smx} &= \sqrt{2}\pi*\text{smxb} \\
\text{smz} &= \sqrt{2}\pi*\text{smzb} \\
\text{ssx} &= \sqrt{2}\pi*\text{ssxb} \\
\text{ssz} &= \sqrt{2}\pi*\text{sszb}
\end{align*} \]

all = probability of non-turbulent severity category
\[ \text{all} = \text{one} - (\text{pmz} + \text{psz}) \]

als = probability of less than severe category
\[ \text{als} = \text{one} - \text{psz} \]

Use dx = 0 and dz = 0 to initialize values
if(abs(dx).le.0.0.and.abs(dz).le.0.0)then
  c........ Randomize initial sigma values
  sigmxu = rand(0)*smx
  sigmyu = rand(0)*smx
  sigmxw = rand(0)*smz
  sigmyw = rand(0)*smz
  sigsxu = rand(0)*ssx
  sigsyu = rand(0)*ssx
  sigsxw = rand(0)*ssz
  sigsyw = rand(0)*ssz
  c... Calculate sigma values
  sigu(1) = 0.
  sigw(1) = 0.
  sigu(2) = dsqrt(sigmuxu**2 + sigmyu**2)
  sigu(3) = dsqrt(sigsxu**2 + sigsyu**2)
  sigw(2) = dsqrt(sigmwxw**2 + sigmyw**2)
  sigw(3) = dsqrt(sigsxw**2 + sigsyw**2)
  c... Moderate or severe turbulence cases
  if(model.eq.1.or.model.eq.2)then
    SIGMAU = sigu(model+1)
    SIGMAW = sigw(model+1)
    ISEV = model+1
    return
  endif
  c... Composite turbulence case
  if(model.eq.3)then
    SIGMAU = pmx*sigu(2) + psx*sigu(3)
    SIGMAW = pmx*sigw(2) + psx*sigw(3)
    ISEV = 0
    return
  endif
  c... Select sigma values according to severity level (alpha)
  alpha = rand(0)
  if(alpha.gt.all.and.alpha.lt.als)jsev = 2
  if(alpha.le.all)jsev = 1
  if(alpha.ge.als)jsev = 3
  SIGMAU = sigu(jsev)
  SIGMAW = sigw(jsev)
  c......... Store turbulence parameter values for next cycle
  psigu = sigmau
  psigw = sigmaw
  ISEV = jsev
  xsev = 0.
  nsev = 1
  return
  endif
  c... Correlations for sigmas
  delx = Dsqrt((DX/XLSIG)**2 + (DZ/ZLSIG)**2)
  if(delx.lt.0.05d0) then
    rhosig = one - AFAC*delx**2
  else
    rhosig = Dexp(-BFAC*delx)
  endif
  betasig = dsqrt(one - rhosig**2)
III. DIGITAL FILTER TECHNIQUES AND TURBULENCE MODELING

A. BACKGROUND

The currently used turbulence model for the space shuttle reentry simulation is overly conservative in that severe turbulence is assumed from reentry altitude to the Earth's surface. In the real atmosphere, turbulence is intermittent with large quiescent zones. From reentry to 10 km altitude, shuttle control is by reaction engines. Johnson Space Center (JSC) sets reaction control system redlines based on the predictions of shuttle reentry simulations. Because of the overly conservative model, the orbiter usually lands with about 270 kg (600 lb) of extra fuel. A realistic turbulence simulation model was developed to help in more rational selection of redlines. The model development was performed in two parts: a stochastic turbulence intensity model and revision of the turbulence simulation equations. This paper documents the second part of the development. Part one was performed by another contractor. The two parts are easily integrated into a single subroutine; a recommended subroutine structure is presented.

B. INTRODUCTION

The current turbulence model for the space shuttle reentry simulation assumes severe turbulence from reentry to the Earth's surface. This model is overly conservative because in the real atmosphere, turbulence occurs in patches embedded in large quiescent zones. Reaction engines provide control of the orbiter from reentry to about 10 km altitude. Fuel budgets for the orbiter are based on the reentry simulations with the conservative turbulence model. As a result, the orbiter lands with excess fuel. Figure 1 shows the fuel budgets and usage for a typical flight. The orbiter takes off with about 2,270 kg (5,000 lb) of fuel. About 45 kg (100 lb) of fuel is used for step-away from the external tank after separation. The amount of fuel used in on-orbit maneuvers varies and is greatest for rendezvous missions. Currently, on-orbit operations cease when fuel levels drop to the 770-kg (1,700-lb) redline. Typically, the shuttle lands with 270 kg (600 lb) of extra fuel. For some missions, this redline limits operations. For example, the Hubble Space Telescope (HST) mission needs at least 180 kg (400 lb) of additional fuel to put the HST into a higher orbit than is normally available from the shuttle.

A more realistic turbulence model will permit more rational selection of reaction control fuel redlines. The task of revising the turbulence model was performed in two parts: development of a stochastic turbulent intensity model and revision of the turbulence simulation difference equations. The first part was done by Justus [1]. The second part is the subject of this study. Part one will be discussed briefly, and part two in detail. Most derivations for part two are presented in the appendix. All difference equations and associated parameters are presented in a form appropriate for coding into flight simulation models. Each equation was tested and verified for accuracy and stability.
Turbulence is normally simulated as is shown in Figure 2. Gaussian white noise is input to the low pass filter. The output simulated turbulence is also Gaussian with the desired spectrum, e.g., von Karman. The transfer function of the filter is selected so that the desired output spectrum is obtained. If the filter is linear and the probability distribution of the input noise is Gaussian, then the output time series will also have a Gaussian probability distribution. Atmospheric turbulence is often not Gaussian. The linear filter output can be randomly modulated to obtain a more realistic output probability distribution. This study deals with generation of the Gaussian turbulence rather than with nonlinear modulation of the Gaussian time series. These refinements can be added if deemed necessary.

The most consistently observed characteristic of atmospheric turbulence is the power spectral density fall-off with frequency to the $-5/3$ power. Only occasionally will different fall-offs be observed in turbulence spectra. The $-5/3$ fall-off is consistent with Kolmogorov's local isotropy hypothesis and with von Karman's spectrum. The difficulty for simulators is the irrational form of the von Karman spectrum. In theory, an irrational spectrum gives rise to either an infinite order differential equation or a finite, noninteger order differential equation. Investigators have defined derivatives of noninteger order and Tatom [2] has used solutions of noninteger order differential equations to simulate irrational processes. The computational efficiency of these solutions has not yet been demonstrated.

Frequently, the rational Dryden spectrum, which falls off as frequency to the $-2$ power, is used to simulate turbulence. Frost and Wang [3] have shown quantitative differences between simulations of fixed stick aircraft landings flying in Dryden and in von Karman turbulence. The differences are in the standard deviation of the touch-down point and in the standard deviation of aircraft sink rate. The von Karman spectrum gave lower values of these standard deviations than the Dryden model. The Dryden spectrum is, in fact, a crude approximation to the von Karman spectrum. The Frost and Wang study provides justification for looking at better approximations.

Campbell [4] approximated the von Karman spectrum with a higher order rational approximation. The discretized Campbell model was shown to have instabilities over some ranges of sampling rates. In theory, the Campbell equations were stable over all sampling rates. The instability apparently arose from finite precision computer arithmetic. This hypothesis was supported by the fact that using extended precision arithmetic delayed the onset of instability.

Mantey [5] observed a similar phenomenon and pointed out that if the difference equations were cast in modal form, the sensitivity of the equations to finite precision arithmetic instabilities was minimized. Consequently, a revision of the Campbell formulation was performed that incorporated the Mantey approach. Casting the equations in modal form permitted the analytical determination of the dependence of output turbulence standard deviation on time step. Campbell used a least squares fit of the observed standard deviation-time step curve in his original paper.

In this study, a class of transfer function approximations to the von Karman transfer function was obtained. For longitudinal spectra, approximations up to fifth order were derived. For the
The dimensionless transfer functions and corresponding spectra are presented in Figure 3. The dimensionless transfer functions can be converted to dimensional form by replacing $s$ by $aLs/V$ where $L$ is the appropriate turbulent length scale, $V$ is the velocity of the vehicle relative to the wind, and $"a"$ is von Karman’s constant (1.339). From the figure, the fifth order approximation falls close to the von Karman spectrum. To better understand the closeness of the approximation, five simulations were run with the same input Gaussian white noise source data. The fifth order approximation was plotted versus the first through the fourth order approximation time series. These graphs are shown on the right in Figure 3. If the agreement was perfect, these curves would fall along a straight 45-degree line. The greater the scatter about the 45-degree line, the worse the agreement. The figure shows that the agreement becomes better as the order of the approximation increases. Similar plots were done for different sampling rates. These graphs are not shown, but the agreement becomes worse for a given approximation as the sampling rate increases.

Simulations were performed at several dimensionless sampling rates, with each of the approximations and their corresponding transverse equations, to test for instabilities. These approximations were formulated using the Advanced Continuous Simulation Language (ACSL) in single precision arithmetic. The results, presented in Tables 1 through 3, show that all simulations are stable except for the highest order and the lowest sampling rates. Second and fourth order Runge-Kutta and Euler integration were used in the simulations. Since divergence occurred for exceptionally long sampling intervals, use of even the fifth order approximation should be feasible for shuttle reentry simulations. In any case, the third order simulation was always stable and will be presented in detail.

The difference equations for the third order approximation are presented in Figure 4. The equations for both the longitudinal and transverse equations are presented. Longitudinal equations will be applied for the along wind, $x$-component of turbulence. The transverse equations will be used for both the $y$- and $z$-components. The equations, as shown, can be directly coded into the shuttle reentry simulation or into flight simulations for any other vehicle. Required inputs to the equations are the appropriate turbulence length scale, the appropriate relative wind velocity, turbulent intensity, and the sampling rate. The derivation of the equations is presented in the appendix. The discretization of the continuous equations was ideal in that output discrete points appear as points from the ideally sampled continuous signal. The ideal discretization is easy because of the modal form of the equations. The simulation equations currently being used at JSC do not use this ideal sampling because of the requirement for speed in a real-time simulation.
D. INTEGRATION OF THE TURBULENCE MODEL

Justus developed a model for turbulent intensities that realistically selects turbulent intensities from a Rayleigh distribution. He provided a subroutine that offered eight options for turbulent intensities. These options range from zero turbulence to stochastic selection of moderate or severe intensities to severe turbulence at all altitudes. For the stochastic case, intensities are selected from Rayleigh distributions. Between adjacent altitude bands, the intensities are correlated. The Justus model provides all the inputs required by the turbulence equations except for the sampling rate. His model has a number of required inputs not currently used in the turbulence simulation subroutine used by JSC. The inputs required by the Justus model are shown in Figure 5. Justus developed a subroutine that calculates turbulence parameters for the x- and z-components of turbulence. The y-component is not explicitly calculated but is generated identically to the z-component. The y-component must have its own random number generator but otherwise can use coding analogous to that for the z-component. Inputs required for Justus' subroutine and not currently supplied by the JSC model are the changes in position of the vehicle from the last time step and the previous values of turbulent intensity. All of these new inputs are readily available from the current JSC model. The outputs from the model are the turbulence length scales and standard deviations required by the simulation equations. The Justus submodule and the turbulence update equations described in the previous section are essentially independent except for the Justus model outputs (length scales and standard deviations). As a result, the two efforts are easily integrated. The flow chart shown in Figure 6 describes the integration of the two elements into a revised subroutine for the JSC simulation. The turbulence equation options available are the original first order equations and the third order equations presented here.

E. RECOMMENDED INVESTIGATIONS

A higher order, stable approximation to the von Karman spectrum and corresponding difference equations were presented in this report. The third order approximation permits simulations much closer to real turbulence than simulations based on the rational Dryden spectrum. The remaining issue is the relative importance of the higher order approximation to shuttle reentry simulation. Simulations based on the third order approximation require much more computation than first order simulations. Frost and Wang [3] give some support to the idea that better approximations are required. Running a number of simulations with the current shuttle reentry simulation using both the current turbulence model and the model with the revised turbulence equations will resolve the importance of higher order turbulence models.

The measure of importance of the higher order simulation will be the relative variability of fuel usage for the two turbulence equations. The mean fuel weight after landing will likely be about the same for both sets of simulation equations. The standard deviation about the mean will be different for the two models. The variation of fuel usage about the mean for the same mission affects the on-orbit redline selection. Each set of equations can be used to generate a probability distribution of fuel remaining at landing. If the standard deviations of the probability distributions are significantly different, use of the results from the higher order simulations are recommended for redline selection.
REFERENCES


Figure 1. On-orbit maneuvering system (OMS) fuel budget.

Figure 2. Gaussian turbulence simulation.
Figure 3. Performance of higher order rational turbulence models.
DIFERENCE EQUATION FORM

\[ x_{n+1} = Gx_n + Hu_n \]
\[ y_{n+1} = Cx_n \]

WHERE
- \( x_n \) = STATE VECTOR
- \( y_n \) = SIMULATED TURBULENT VELOCITY
- \( u_n \) = GAUSSIAN WHITE NOISE INPUT
- \( G \) = COEFFICIENT MATRIX DEFINED BELOW
- \( H \) = COLUMN VECTOR DEFINED BELOW
- \( C \) = ROW VECTOR DEFINED BELOW

INPUTS

- \( \sigma_u \) = LONGITUDINAL TURBULENCE STANDARD DEVIATION
- \( L_u \) = LONGITUDINAL TURBULENT LENGTH SCALE
- \( V \) = VEHICLE VELOCITY RELATIVE TO THE WIND
- \( \tau \) = SAMPLING INTERVAL

PARAMETERS AND CONSTANTS

- \( \beta = 1.339 L_u/V \)
- \( \lambda_1 = -1.02025059165 \)
- \( \lambda_2 = -1.6766157076134 \)
- \( \lambda_3 = -8.1031337007321 \)
- \( E_i = \exp(\lambda_i\tau/\beta) \)
- \( c_i = c_{c_i} + \frac{c_{c_2}}{\lambda_i} + \frac{c_{c_3}}{\lambda_i^2} \)
- \( \sigma_y = \left[ \sum_{i=1}^{3} \frac{c_i^2 h_i^2 (E_i-1)^2}{\lambda_i^2 (1-E_i^2)} + 2 \sum_{i=1}^{3} \sum_{j=1}^{2} \frac{c_i c_j h_i h_j (E_i-1)(E_j-1)}{\lambda_i \lambda_j (1-E_i E_j)} \right]^{1/2} \)

MATRIX AND VECTOR DEFINITION

- \( G = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{bmatrix} \)
- \( H = \frac{\sigma_u \beta}{\sigma_y} \begin{bmatrix} h_{c_1}(E_1-1)/\lambda_1 \\ h_{c_2}(E_2-1)/\lambda_2 \\ h_{c_3}(E_3-1)/\lambda_3 \end{bmatrix} \)
- \( C = \frac{1}{\beta} [c_1, c_2, c_3] \)

Figure 4a. Equations for the third order longitudinal turbulence model.
DIFFERENCE EQUATION FORM

\[ X_{n+1} = GX_n + Hu_n \]
\[ Y_{n+1} = CX_n \]

WHERE
\[ x_n = \text{STATE} \]
\[ y_n = \text{SIMULATED TURBULENT VELOCITY} \]
\[ u_n = \text{GAUSSIAN WHITE NOISE INPUT} \]
\[ G = \text{COEFFICIENT MATRIX DEFINED BELOW} \]
\[ H = \text{COLUMN VECTOR DEFINED BELOW} \]
\[ C = \text{ROW VECTOR DEFINED BELOW} \]

INPUTS
\[ \sigma_w = \text{TRANSVERSE TURBULENCE STANDARD DEVIATION} \]
\[ L_w = \text{TRANSVERSE TURBULENT LENGTH SCALE} \]
\[ V = \text{VEHICLE VELOCITY RELATIVE TO THE WIND} \]
\[ \tau = \text{SAMPLING INTERVAL} \]

PARAMETERS AND CONSTANTS

\[ \beta = 1.339 \frac{V}{L_w} \]
\[ \lambda_1, \lambda_2, \text{AND } \lambda_3 \text{ ARE SAME} \]
\[ \lambda_4 = -1. \]
\[ c_i = c_{c_1} + \frac{c_{c_2}}{\lambda_i} + \frac{c_{c_3}}{\lambda_i^2} + \frac{c_{c_4}}{\lambda_i^3} \]
\[ \sigma_y = \left[ \sum_{i=1}^{4} \frac{c_i^2}{\lambda_i^2} \right]^{1/2} + \sum_{i=1}^{4} \sum_{j=1}^{4} \frac{c_i c_j h_{c_i} h_{c_j}}{\lambda_i \lambda_j} \frac{(E_i-1)(E_j-1)}{(1-E_i E_j)} \]

MATRIX AND VECTOR DEFINITION

\[ G = \begin{bmatrix} E_1 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_3 & 0 \\ 0 & 0 & 0 & E_4 \end{bmatrix} \]
\[ H = \frac{\sigma_w \beta}{\sigma_y} \begin{bmatrix} h_{c_1}(E_1-1)/\lambda_1 \\ h_{c_2}(E_2-1)/\lambda_2 \\ h_{c_3}(E_3-1)/\lambda_3 \\ h_{c_4}(E_4-1)/\lambda_4 \end{bmatrix} \]
\[ C = \frac{1}{\beta} \begin{bmatrix} c_{c_1}, c_{c_2}, c_{c_3}, c_{c_4} \end{bmatrix} \]

Figure 4b. Equations for the third order transverse turbulence model.
### JUSTUS MODEL INPUTS

- **SELECTED OPTION**
  - 0: COMPLETE MODEL WITH \( \sigma \) SELECTED FROM RAYLEIGH DISTRIBUTION
  - 1: MODERATE TURBULENCE WITH \( \sigma \) SELECTED FROM RAYLEIGH DISTRIBUTION
  - 2: SEVERE TURBULENCE, \( \sigma \) SELECTED FROM RAYLEIGH DISTRIBUTION
  - 3: COMPOSITE TURBULENCE (AVERAGE OF SEVERE, MODERATE AND NON TURBULENCE).
  - 4: NO TURBULENCE
  - 5: MODERATE TURBULENCE, \( \sigma = \sigma_M \)
  - 6: SEVERE TURBULENCE, \( \sigma = \sigma_S \)
  - 7: COMPOSITE TURBULENCE, \( \sigma = (\sigma_S + \sigma_M)/2 \)

- **CURRENT ALTITUDE**
- **DISPLACEMENTS SINCE LAST TIME STEP (DX, DZ)**
- \( \sigma_U, \sigma_W \) FROM PREVIOUS TIME STEP

---

**Figure 5.** Characteristics of the Justus [1] turbulence model.
Figure 6. Recommended turbulence simulation subroutine structure.
### TABLE 1. STABILITY OF EULER INTEGRATION TURBULENCE SIMULATION

<table>
<thead>
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### TABLE 2. STABILITY OF SECOND ORDER RUNGE KUTTA SIMULATIONS

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TABLE 3. STABILITY OF FOURTH ORDER RUNGE KUTTA TURBULENCE SIMULATIONS

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**TRANSVERSE COMPONENT**

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APPENDIX

DERIVATION OF SIMULATION EQUATIONS FOR HIGHER ORDER APPROXIMATIONS

The turbulence simulation equations for the third order approximation in normal and state-space form are:

\[ y''' + k_1 y'' + k_2 y' + k_3 y = r_1 u'' + r_2 u' + r_3 u \]  \hspace{1cm} (1)

\[
\begin{bmatrix}
  x_1' \\
  x_2' \\
  x_3'
\end{bmatrix}
= 
\begin{bmatrix}
  -k_1 & -k_2 & -k_3 \\
  1 & 0 & 0 \\
  0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix}
+ 
\begin{bmatrix}
  1 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  u
\end{bmatrix}
\hspace{1cm} (2)
\]

\[ y = [r_1, r_2, r_3] \begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix} \hspace{1cm} (3) \]

These equations have distinct, real eigenvalues and consequently can be transformed to modal form by using the matrix whose columns are eigenvectors of the coefficient matrix. For a matrix in the above form, the appropriate transformation is given by:

\[ T = \begin{bmatrix}
  1 & 1 & 1 \\
  1/\lambda_1 & 1/\lambda_2 & 1/\lambda_3 \\
  1/\lambda_1^2 & 1/\lambda_2^2 & 1/\lambda_3^2
\end{bmatrix} \hspace{1cm} (4)\]

where \( \lambda_i \) = eigenvalues.

The resulting equations are in the form \( x = Ax + Bu \) where \( A \) is the following diagonal matrix.
The matrix $B$ is given by $B = T^{-1}U$ where $U = [1 0 0]^T$. The superscript $T$ indicates the column vector transpose of the row vector $[1 0 0]$. The eigenvalues in equation (5) are in dimensionless form. To convert them to dimensional form, the eigenvalues must be divided by $1.339 \frac{L}{V}$, $L$ is the turbulent length scale and $V$ is the vehicle velocity relative to the wind. To perform a simulation, the above equations must be discretized. The discretization chosen is in the following form.

$$x_{n+1} = Gx_n + Hf u_n,$$

where

$f$ = a factor to ensure the correct standard deviation of $y$

$G = e^{At}$

$H = A^{-1}(e^{At} - I)B$

$\tau$ = sampling interval, and

$I = 3x3$ identity matrix.

Once the difference equation is obtained, the variance of the output $y$ is required. For equations in the $x = Gx + Hu$ form, the covariance of the state vector is given by the expected value of $x^T x = P$. $P$ is given by the solution to the Lyapunov equation

$$P = GP^T + HH^T.$$

The unknown quantity in this equation is a matrix. Because $G$ is a diagonal matrix, $P$ can be determined easily.

$$P_{ij} = \frac{h_i h_j}{1 - e^{\lambda_i \tau / \beta} e^{\lambda_j \tau / \beta}}.$$
where \( h_i \) is the ith component of \( H \).

Once \( P \) is known, the covariance of \( y \) can be determined from:

\[
\sigma_y^2 = CPC^T .
\]

The preceding equation is expanded in Figure 4. The above approach can be applied for the equations of any order. The general equation is:

\[
\sigma_y^2 = \sum_{i=1}^{n} \frac{c_i^2 h_i^2 (E_i - 1)^2}{\lambda_i^2 (1 - E_i^2)} + 2 \sum_{j=i+1}^{n} \sum_{i=1}^{n-1} \frac{c_i c_j h_i h_j (E_i - 1)(E_j - 1)}{\lambda_i \lambda_j (1 - E_i E_j)}
\]

All parameters in the above equation are presented in Figure 4.

Derivations in this appendix were specifically for the third order equations; the higher order equations are in identical form. Only the eigenvalues and the constants are different for different equations.
**Abstract**

This report presents an updated NASA atmospheric turbulence model, from 0 to 200 km altitude, which was developed to be more realistic and less conservative when applied to space shuttle reentry engineering simulation studies involving control system fuel expenditures. The prior model used extreme turbulence ($3\sigma$) for all altitudes, whereas in reality severe turbulence is patchy within quiescent atmospheric zones. The updated turbulence model presented in this report is designed to be more realistic. The prior turbulence statistics ($\sigma$ and $L$) have been updated and have been modeled accordingly.

**Key Words (Suggested by Author(s))**

- Atmospheric turbulence
- Turbulence model
- Winds, surface, aloft
- Space shuttle simulations

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