Optimum Element Density Studies
for Finite-Element Thermal Analysis
of Hypersonic Aircraft Structures

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ABSTRACT

Different finite-element models previously set up for thermal analysis of the space shuttle orbiter structure were discussed and their shortcomings were identified. Element density criteria were established for the finite-element thermal modelings of space shuttle orbiter-type large, hypersonic aircraft structures. These criteria were based on rigorous studies on solution accuracies using different finite-element models having different element densities set up for one cell of the orbiter wing. Also, a method for optimization of the transient thermal analysis computer central processing unit (CPU) time was discussed. Based on the newly established element density criteria, the orbiter wing midspan segment was modeled for the examination of thermal analysis solution accuracies and the extent of computation CPU time requirements. The results showed that the distributions of the structural temperatures and the thermal stresses obtained from this wing segment model were satisfactory and the computation CPU time was at the acceptable level. The studies offered the hope that modeling the large, hypersonic aircraft structures using high-density elements for transient thermal analysis was possible if a CPU optimization technique was used.

INTRODUCTION

During the early stage of space shuttle development, the heat transfer analysis of the complex orbiter structure was carried out by using the so-called "plug" method. Only selected small, local regions of the orbiter were modeled with three-dimensional finite-difference (or lumped parameters) plug thermal models (fig. 1). Structural temperatures calculated from the plug models were then interpolated to obtain structural temperatures in the unmodeled regions. Approximately 90 percent of the temperature inputs used in the early days of orbiter thermal stress analysis was obtained by this type of interpolation. The procedure was tedious, laborious, and expensive. For structures with steep temperature gradients near the heat sinks (for example, the orbiter wing spar and rib caps), the structural temperature distribution obtained from the interpolation method could be erroneous and could cause inaccurate thermal stress predictions.

After the development of finite-element heat transfer analysis computer codes such as the structural performance and resizing (SPAR) finite-element thermal analysis computer program (ref. 1), it became possible to model larger regions of the orbiter structure including the whole wing. The use of the finite-element method instead of the conventional lumped-parameter (or finite-difference) method enables the use of the same thermal model as a structural model for thermal stress calculations by simply removing elements set up for the thermal protection system (TPS) (not a major mechanical load-carrying structural component). In the past several years, Ko and others (refs. 2 to 9) conducted extensive heat transfer and thermal stress analyses of the space shuttle orbiter using a series of finite-element models set up for three wing segments, one fuselage cross section, and the whole wing. These thermal models were used to calculate orbiter structural temperatures which were correlated with the actual flight-measured data during the initial orbiter tests of the space shuttle Columbia (refs. 2 to 8). The earlier thermal models set up for the orbiter structure were by no means perfect and had shortcomings in the light of element density, element size distributions, and extent of the region modeled.

In the finite-element heat transfer and thermal stress analysis of small structural components, the element density may be freely increased at will to obtain highly accurate solutions without the worry that the limit of the computer memory core space might be reached or exceeded. However, in the finite-element thermal modeling of large, hypersonic aircraft structures (such as the space shuttle and space plane), the use of high-density finite elements could increase the number of radiation view factors tremendously, and could require prohibitive computer time and/or computer core space requirements in the transient heat transfer analysis using these radiation view factors. In the wing box-type structures, each time the number of radiation elements is doubled, the number of radiation view factors would be nearly quadrupled. Thus, in the finite-element heat transfer analysis of large aerospace structures, the highest desirable element density is governed by the time requirement and/or by the memory capacity of the computer used. The corresponding structural model for thermal stress analysis using the same element density as
the thermal model has far less nodal points because the non-load-carrying heat shields are removed. It is, therefore, not the model to be used to set up the criteria for maximum allowable element density. Preliminary studies in this area were carried out by Ko and others (refs. 10 to 12) to investigate solution accuracies obtained from different finite-element models having different element densities set up for one wing cell of the orbiter. These studies formed the foundation of the state of the art in finite-element modelings of large, hypersonic aircraft structures.

In this report, several past finite-element models set up for the orbiter structure are reviewed and their shortcomings are identified. The criteria for the element density and element size distribution required for finite-element modeling of larger orbiter wing regions are discussed in detail. Finally, the report shows how to optimize the SPAR transient heat transfer analysis computation CPU time when the high-density elements are used.

NOMENCLATURE

\[ C \]
- capacitance matrix

\[ C_{21} \]
- two-node forced convection element

\[ C_{41} \]
- four-node forced convection element

\[ \text{CPU} \]
- central processing unit

\[ C_{\text{QUAD2}} \]
- quadrilateral membrane and bending element

\[ C_{\text{CROD}} \]
- two-node tension–compression–torsion element

\[ E_{23} \]
- bar elements

\[ E_{25} \]
- zero-length element for elastically connected, geometrically coincident joints

\[ E_{31} \]
- triangular membrane element

\[ E_{41} \]
- quadrilateral membrane element

\[ E_{44} \]
- quadrilateral shear panel element

\[ F_{ij} \]
- radiation view factor from element \( i \) to element \( j \)

\[ \text{FRSI} \]
- flexible felt reusable surface insulation

\[ H \]
- convection load vector

\[ \text{HRSI} \]
- high temperature reusable surface insulation

\[ i \]
- integer, 1, 2, 3, . . .

\[ \text{JLOCS} \]
- joint locations

\[ j \]
- integer, 1, 2, 3, . . .

\[ K \]
- system matrix \( = K_k + K_r + K_h \)

\[ K_h \]
- convection matrix

\[ K_k \]
- conduction matrix

\[ K_r \]
- radiation matrix

\[ K_{21} \]
- two-node line conduction element

\[ K_{31} \]
- three-node area conduction element

\[ K_{41} \]
- four-node area conduction element

\[ K_{61} \]
- six-node volume conduction element

\[ K_{81} \]
- eight-node volume conduction element
KTIME: control command in SPAR computer program to specify the time interval at which the system matrix $K$ is to be factored.

LRSI: low temperature reusable surface insulation.

NASTRAN: NASA structural analysis.

$Q$: source load vector.

$R$: radiation load vector.

R21: two-node area radiation element.

R31: three-node area radiation element.

R41: four-node area radiation element.

RCONV: control command in SPAR computer program to set the convergence criterion for the radiation load vector computations.

RNITER: control command in SPAR computer program to set the maximum number of iterations permitted during the computations of radiation load vectors.

RTIME: control command in SPAR computer program to specify the time increments at which the radiation load vectors are to be computed.

SIP: strain isolation pad.

SPAR: structural performance and resizing.

STS-5: space transportation system, flight 5.

$T$: absolute temperature.

$T_2$: final time of SPAR transient thermal analysis.

TPS: thermal protection system.

$Y_o$: station in $y$-axis.

$[\dot{\cdot}]$: time derivative.

$\sigma_x$: chordwise stress.

$\sigma_y$: spanwise stress.

$\tau_{xy}$: shear stress.

**REVIEW OF PAST MODELS**

**Wing and Fuselage Cross Sections**

Figures 2 and 3 show two typical past thermal models (WS240, FS877) set up for the orbiter wing midspan and midfuselage cross sections (ref. 5). The corresponding structural models were obtained by removing the TPS elements. In both figures, the sizes of the thermal and the corresponding structural models are compared. Notice that the structural models are always simpler (less nodal points). These thermal models were extensively used to calculate orbiter structural temperatures which were correlated with the actual flight data obtained from initial orbital tests of the space shuttle Columbia (refs. 2 to 8). The WS240 model (fig. 2) has a reasonable number of elements to give satisfactory structural temperature distributions in the chordwise direction, but not in the spanwise direction. Also, the FS877 model (fig. 3), which is two-dimensional, can give good structural temperature distribution in the fuselage circumferential direction, but not in the fuselage axial direction.
Both of these thermal models, when converted to structural models by removing TPS elements, are not capable of yielding accurate thermal stress predictions because of the following reasons. It is well known that the magnitudes of the computed thermal stresses are sensitive to the modeling parameters (such as element density, element size distributions, and extent of the region modeled) (refs. 11 and 12). For the case of biaxial stress fields which occur in orbiter-type structures (especially wing and fuselage skins), the manner in which the boundary conditions are applied can greatly affect the magnitudes of calculated thermal stresses in the proximity of the tracted boundaries or free edges. Because the thermal stresses attenuate to zero at free edges, the structural models must have dimensions sufficiently large enough to be beyond the influence of the edge effects. Both models mentioned earlier lack the capacity to fulfill those structural modeling requirements.

Whole Wing

In order to eliminate the modeling shortcomings that occurred in the WS240 models (thermal and structural), the whole wing of the orbiter was modeled (refs. 8 and 9). Figure 4 shows the past thermal model, WING, set up for the orbiter whole wing. In the figure the sizes of both the thermal model and the structural model (TPS landing gear and wheel well door elements removed) are compared. In this case the number of nodes for the structural model is approximately 10 percent of that of the thermal model.

Because of the fear of encountering and dealing with a tremendous number of radiation view factors, the element sizes used in the WING model were relatively coarse. The WING thermal model required 48,034 radiation view factor computations and can only give “roof”-shaped structural temperature distributions. Therefore, the thermal stresses calculated based on this type of temperature distribution may not give accurate predictions. Thus, finer elements are required for the whole wing model, for which the optimum manageable element density has to be determined first.

ELEMENT DENSITY

Because of the need of finding the optimum element density and element size distribution for modeling orbiter whole wing-type structures, Ko and others (refs. 11 and 12) set up five different finite-element models (A, B, C, D, and E) with different element densities for one orbiter wing cell (located at midspan bay 3 (fig. 5)), and examined the effect of element density on the finite-element solution accuracies. These studies laid the foundation, or State of the art, for choosing optimum element density in finite-element modeling of whole orbiter wing-type structures.

Figure 6 shows the wing’s lower skin temperature distributions predicted from the five thermal models. Except for model A, which had the same skin element density as that of the WING model (fig. 4), models B, C, D, and E gave very close values of the structural temperatures at the center region of the wing’s lower skin.

Figure 7 shows how the number of radiation view factors $F_{ij}$ increases as the number of radiation elements $R_{41}$ (or element density) increases. For this particular type of wing cell structure, doubling the number of radiation elements $R_{41}$ would cause the number of radiation view factors $F_{ij}$ to nearly quadruple.

Model E gives smooth structural temperature distribution (fig. 6), and its element density level can be considered an attractive element density to use in modeling the whole orbiter wing-type structures. However, the SPAR transient heat transfer analysis using model E required extremely long computer CPU time because of the computations of the time-dependent radiation exchange vector using the 93,868 radiation view factors $F_{ij}$. Thus, the use of model E element density level in modeling the whole orbiter wing-type structures could be beyond the current computer capabilities (memory core space and computation speed).

The structural temperature distribution given by model E has a flat region near the wing skin central region in the chordwise direction. In this region, slightly coarser elements, as in the element density level of model D, could be used to reduce CPU time without sacrificing the solution accuracy. As shown in the Central Processing Unit.
Time Optimization section of this report, the SPAR CPU time for model D is 21 to 27 percent of that for model E. The logical criteria for setting the element density and size distribution for modeling large, complex aerospace structures could be somewhere between the element density levels of models D and E (for example, eight elements in chordwise direction and six elements in spanwise direction, with finer elements located near the heat sinks).

CENTRAL PROCESSING UNIT TIME OPTIMIZATION

In the finite-element transient heat transfer analysis using the SPAR program, the governing matrix equation used is of the form (ref. 1)

\[(K_k + K_r + K_h)T + C\dot{T} = Q + R + H\]  

(1)

where

- \(K_k\) is the conduction matrix,
- \(K_r\) is the radiation matrix,
- \(K_h\) is the convection matrix,
- \(T\) is the absolute temperature,
- \(C\) is the capacitance matrix,
- \(Q\) is the source load vector,
- \(R\) is the radiation load vector,
- \(H\) is the convection load vector, and
- \(\dot{\cdot}\) denotes time derivative.

In the computations using equation (1), there are several key control parameters which could be adjusted to reduce the SPAR CPU time. However, at the same time the solution accuracy may be affected by adjusting those control parameters. Therefore, it is vital to find out the optimum values of those control parameters which will drastically reduce the SPAR CPU time and yet will hardly affect the solution accuracy. Those control parameters are:

- **KTIME** = control command to specify the time interval at which the system matrix \(K(= K_k + K_r + K_h)\) is to be factored (see eq. (1)).
- **RTIME** = control command to specify the time increments at which the radiation load vectors \((R)\) are to be computed (see eq. (1)).
- **RCONV** = control command to set the convergence criterion for the radiation load vector computations.
- **RNITER** = control command to set the maximum number of iterations permitted during the computation of radiation load vectors \((R)\).

Figure 8 shows the SPAR transient thermal analysis (up to 3000 sec from reentry) computation CPU time (using the ELXSI 6400 computer) plotted as a function of the number of joint locations of thermal models for two sets of KTIME/RTIME values. By increasing both the values of KTIME and RTIME from 2 sec to 25 sec, the SPAR CPU time could be drastically reduced, and the solutions remain practically unchanged. The peak skin temperatures and actual SPAR CPU time associated with the different thermal models are tabulated in table 1. Notice that the use of model E, which required a SPAR CPU time several times longer than that for model D, did not show any advantage in solution accuracy.
Table 1. SPAR computation CPU time and peak skin temperatures associated with different wing cell thermal models.

<table>
<thead>
<tr>
<th>Model</th>
<th>SPAR CPU time, min</th>
<th>Peak skin temperature, °F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>A</td>
<td>19.38</td>
<td>5.48</td>
</tr>
<tr>
<td>B</td>
<td>102.13</td>
<td>23.18</td>
</tr>
<tr>
<td>C</td>
<td>177.87</td>
<td>35.88</td>
</tr>
<tr>
<td>D</td>
<td>301.02</td>
<td>56.45</td>
</tr>
<tr>
<td>E</td>
<td>1428.08</td>
<td>204.28</td>
</tr>
</tbody>
</table>

For conducting a more thorough study of the effects of KTIME, RTIME, and other SPAR control parameters previously mentioned, and for finding the optimum values of these parameters, the thermal model D (fig. 5) was used because it required much less SPAR CPU time than model E.

Table 2. SPAR computation CPU times and peak skin temperatures based on thermal model D.

<table>
<thead>
<tr>
<th>KTIME, sec</th>
<th>RTIME, sec</th>
<th>RCONV</th>
<th>RNITER</th>
<th>SPAR CPU time, min</th>
<th>Peak skin temperature, °F</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>12</td>
<td>301.02</td>
<td>117.26</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>0.0001</td>
<td>12</td>
<td>231.08</td>
<td>117.36</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>0.0005</td>
<td>12</td>
<td>219.95</td>
<td>117.44</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>0.0001</td>
<td>12</td>
<td>56.45</td>
<td>117.30</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>0.0005</td>
<td>5</td>
<td>53.98</td>
<td>116.78</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>0.0005</td>
<td>12</td>
<td>50.10</td>
<td>117.15</td>
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<tr>
<td>50</td>
<td>10</td>
<td>0.0005</td>
<td>12</td>
<td>47.02</td>
<td>116.87</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>0.0005</td>
<td>12</td>
<td>45.48</td>
<td>117.07</td>
</tr>
</tbody>
</table>

Table 2 summarizes the results of this investigation. Figure 9 shows the data of table 2 plotted in the KTIME/RTIME space. Notice that by increasing KTIME from 2 sec to 25 sec (holding RTIME = 2 sec), the SPAR CPU time could be reduced from 301.02 min (point A) down to 219.95 min (point B). This is an approximately 27-percent reduction in SPAR CPU time. Further increase in the value of KTIME offered very little gain in saving SPAR CPU time. Next, by increasing RTIME from 2 sec to 25 sec (keeping KTIME = 25 sec), the SPAR CPU time could be further reduced from 219.95 min (point B) down to 56.45 min (point C). This gives another 54-percent reduction (based on point A) in the SPAR CPU time without affecting the solution accuracy. Further increase in the value of
RTIME beyond point C did not save much SPAR CPU time. At point C the SPAR CPU time is only 19 percent of that at point A, and the solution error is only 0.04 °F (compared to that at point A). Additional increase in both values of KTIME and RTIME to 50 sec (point D) had very little improvement in CPU time, and the solution error started to show up (0.19 °F error). Thus, point C could be chosen as the optimum point for selecting the values of KTIME and RTIME. The reductions of the values of RCONV and RNITER from 0.0001 and 12, respectively, had little effect on the improvement of the SPAR CPU time (table 2).

**MIDSPAN MODEL**

**Element Density**

Based on the knowledge gained from the studies of element density requirements and the SPAR CPU time optimization described earlier, the element density criteria is established for the finite-element modelings of the orbiter wing-type structures. Before modeling the orbiter whole wing with high-density elements, it is wise to first model one segment of the orbiter wing with high-density elements and explore the CPU time requirements in the SPAR transient thermal analysis. The CPU time found for the wing segment model may then be extrapolated to estimate the CPU time requirement for the whole wing model having the same element density as the wing segment model. The said wing segment has four bays and is located at the midspan bounded by two adjacent wing ribs located at \( Y_o - 226 \) and \( Y_o - 254 \), respectively.

The thermal model MIDSPAN for this wing segment is shown in figure 10. In the light of structural temperature distributions obtained from models D and E (fig. 6), the lower and upper skins of each bay of the wing segment were modeled with eight and six different-sized elements, respectively, in the chordwise and spanwise directions with finer elements used near the heat sinks (spar and rib caps). The MIDSPAN SPAR thermal model has 4064 joint locations, 1149 radiation elements R41, and 137,328 radiation view factors \( F_{ij} \). The corresponding NASTRAN (ref. 13) structural model has 722 grid points (18 percent of the thermal model). The sizes of the MIDSPAN thermal and structural models are compared in table 3.

Table 3. Summary of thermal and structural models for an orbiter wing midspan segment.

<table>
<thead>
<tr>
<th>Item</th>
<th>Thermal</th>
<th>Structural</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes</td>
<td>4,046</td>
<td>722</td>
</tr>
<tr>
<td>Number of elements</td>
<td>363 K21, 1,634 K41, 6 K61, 2,730 K81, 1,149 R41, 764 C41</td>
<td>674 CQUAD2, 303 CROD, 764 C41</td>
</tr>
<tr>
<td>Number of ( F_{ij} )</td>
<td>137,328</td>
<td>-- --</td>
</tr>
<tr>
<td>Total CPU time, min(^a)</td>
<td>464</td>
<td>13</td>
</tr>
<tr>
<td>( F_{ij} ) computation CPU time, min</td>
<td>38</td>
<td>-- --</td>
</tr>
</tbody>
</table>

\(^a\)ELXSI 6400 computer  
KTIME = RTIME = 25 sec  
RCONV = 0.0001, RNITER = 12  
\( T_2 = 3000 \) sec
Central Processing Unit Time

By using the previously established optimum values of $K_{\text{TIME}} = R_{\text{TIME}} = 25$ sec, transient heat transfer analysis was performed on the MIDSPAN thermal model. This thermal model required 464 min computation CPU time (table 3) for the thermal analysis duration of 3000 sec. The corresponding structural model took 13 min computation CPU time. It took only 38 min to compute the vast number (137,328) of radiation view factors. It must be emphasized that once the radiation view factors are computed, they are stored in the computer file, and no more computations are required in subsequent computer runs of the same thermal model. Therefore, in the transient thermal analysis most of the computation CPU time is used in computing time-dependent structural temperatures, not in computing radiation view factors, even though their number is enormous. This finding eliminated the fear of encountering a huge number of view factors in the thermal modelings of large, complex aerospace structures using finer elements.

Structural Temperatures

The calculations of the structural temperatures in the MIDSPAN thermal model were based on the STS-5 surface heating rates shown in figure 11. The distributions of skin temperatures in time sequence are shown in figure 12. The most severe (highest values and gradients) temperature distributions occurred at 1700 sec from reentry, and gradually the severity tapered off as the time increased. Judging from the temperature distribution surfaces for each bay, the use of an 8 by 6 system of element density gave acceptable smoothness of the skin temperature distributions. Therefore, an 8 by 6 system of element density could be considered as the element density criteria for modeling such types of aerospace structures.

Thermal Stresses

Based on the structural temperature distributions at 1700 sec, the thermal stresses $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ in the wing skins were calculated and are shown in figures 13 to 15, respectively. Again the distributions of all three stresses exhibited acceptable smoothness, emphasizing that 8 by 6 system element density criteria were satisfactory. Notice that $\sigma_x$, whose direction is transverse to the hat stringers of the wing skins, has a higher magnitude than that of $\sigma_y$, and is therefore more critical.

Thermal Deformations

Figure 16 shows the undeformed and deformed shapes of the MIDSPAN structural model. The lower and upper skins of each bay bulged outwardly, with the maximum displacement of 0.08552 in. occurring near the midregion bay 1's upper skin. This amount of deformation is way below the thickness (0.16 in.) of the SIP. A small region in bay 1's upper skin, near the spar cap separating bays 1 and 2, dented slightly inward. By observing figure 16, the MIDSPAN structural model gave an acceptable detailed deformation field.

CONCLUSIONS

The element density level required in the finite-element thermal analysis of orbiter wing-type structures was investigated, and the optimization of transient thermal analysis computation central processing unit (CPU) time was discussed. The newly established element density criteria for each wing cell were eight elements in the chordwise direction and six elements in the spanwise direction. The resulting distributions of structural temperatures, thermal stresses, and thermal displacements were found to be satisfactory by using this level of element density in the thermal modeling of an orbiter wing segment.
The computation CPU time required for calculating the large number of radiation view factors comprised a small fraction of the total CPU time required for the computation of time-dependent structural temperatures. The study of the wing segment model, using the computer CPU time optimization technique, shows that the estimated computation CPU time required for the thermal analysis of the whole orbiter wing-type structures (with an 8 by 6 element density system) may be within the acceptable range.

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*Dryden Flight Research Facility*

*National Aeronautics and Space Administration*

*Edwards, California, August 22, 1988*
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Figure 1. Locations of local three-dimensional plug thermal models for space shuttle orbiter.
Figure 2. SPAR thermal model WS240 setup for orbiter wing midspan cross section. TPS elements removed to convert to structural model (ref. 5).
<table>
<thead>
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<tr>
<td>JLOC</td>
<td>605</td>
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<tr>
<td>K21 element</td>
<td>250</td>
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<td>6</td>
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<td>R21 element</td>
<td>124</td>
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<tr>
<td>C21 element</td>
<td>96</td>
</tr>
<tr>
<td>$F_{ij}$</td>
<td>1564</td>
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<tr>
<td>GRID point</td>
<td>62</td>
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<tr>
<td>CQUAD2 element</td>
<td>89</td>
</tr>
<tr>
<td>CROD element</td>
<td>9</td>
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</table>

Figure 3. SPAR thermal model FS877 setup for orbiter midfuselage cross section. TPS and cargo bay door elements removed to convert to structural model (ref. 5).
Figure 4. SPAR thermal model WING setup for orbiter wing. TPS, wheel well door, and landing gear removed to convert to structural model (refs. 8 and 9).
Figure 5. SPAR thermal models A, B, C, D, E setups for orbiter wing midspan bay 3. The K81 elements for TPS and SIP not shown; TPS and SIP removed to convert to structural models.
Figure 6. Distributions of orbiter wing lower skin temperatures predicted from thermal models A, B, C, D, E; time = 1,700 sec, STS-5.
Figure 7. Number of radiation view factors as a function of number of radiation elements, orbiter wing midspan bay 3.
Figure 8. SPAR computation CPU time as a function of number of joint locations, $T_2 = 3000$ sec.
Figure 9. Three-dimensional plots of SPAR CPU time in KTIME–RTIME space.
Figure 10. SPAR thermal model MIDSPAN setup for orbiter wing midspan segment; TPS removed to convert to structural model.
Figure 11. Surface heating rates at $|Y_o|240$, STS-5.
Figure 12. Orbiter wing skin temperature distributions.

Time = 500 sec
Figure 12. Continued.
Figure 12. Continued.
Upper skin

MIDSPAN

Lower skin

Time = 1700 sec

Figure 12. Continued.
Figure 12. Continued.
Figure 12. Continued.
Figure 12. Concluded.
Figure 13. Distributions of chordwise stress $\sigma_x$ in orbiter wing skins; time = 1700 sec, STS-5.
Figure 14. Distributions of spanwise stress $\sigma_y$ in orbiter wing skins; time = 1700 sec, STS-5.
Figure 15. Distributions of shear stress $\tau_{xy}$ in orbiter wing skins; time = 1700 sec, STS-5.
Maximum vertical displacement, 0.08552 in.

Deformed

Undeformed

Figure 16. Deformed shape of orbiter wing midspan segment; time = 1700 sec, STS-5.
Different finite-element models previously set up for thermal analysis of the space shuttle orbiter structure were discussed and their shortcomings were identified. Element density criteria were established for the finite-element thermal modelings of space shuttle orbiter-type large, hypersonic aircraft structures. These criteria were based on rigorous studies on solution accuracies using different finite-element models having different element densities set up for one cell of the orbiter wing. Also, a method for optimization of the transient thermal analysis computer central processing unit (CPU) time was discussed. Based on the newly established element density criteria, the orbiter wing midspan segment was modeled for the examination of thermal analysis solution accuracies and the extent of computation CPU time requirements. The results showed that the distributions of the structural temperatures and the thermal stresses obtained from this wing segment model were satisfactory and the computation CPU time was at the acceptable level. The studies offered the hope that modeling the large, hypersonic aircraft structures using high-density elements for transient thermal analysis was possible if a CPU optimization technique was used.