SYNTHETIC APERTURE
INTERFEROMETRIC RADIOMETER
(SAIR)

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The aperture size requirements of imaging microwave radiometers in geosynchronous orbit rule out filled aperture antenna systems below 10 GHz. In the region 10-30 GHz, filled apertures are only marginally practical. The size requirements in turn aggravate the problems with a mechanically steered antenna beam. Both the aperture size and steering problems are resolved with a Synthetic Aperture Interferometric Radiometer (SAIR).

SAIR imaging is based on a technically and analytically mature discipline, synthetic aperture radio astronomy (refs. 1 and 2). A two-dimensional array of antenna elements can be thinned by more than 99.9%. The remaining elements are cross-correlated and the output is recorded digitally. All antenna beams are then generated simultaneously in software.

THINNED APERTURE RADIOMETERS

- CORRELATION INTERFEROMETRIC IMAGING

- STRONG HISTORICAL PRECEDENT IN RADIO ASTRONOMY

- > 99.9% SAVINGS IN ANTENNA SIZE/WEIGHT

- NO MECHANICAL SCANNING
The cross-correlation of a pair of antenna elements in the SAIR thinned array produces a measurement referred to as the visibility function by radio astronomers. The visibility function is the two-dimensional Fourier transform of the brightness temperature distribution lying within the mainbeam of the individual antenna elements. The relative positions of the pair of elements determine which sample of the visibility function is measured. After enough different samples are made, the measurements are inverse Fourier transformed in software to reconstruct the brightness temperature distribution. The sinusoidal weightings on each sample in the software reconstruction are exactly analogous to the phase shifter settings on a conventional electronically steered phased array. One fundamental difference between a SAIR imager and a phased array, however, is the inability of the phased array to be thinned without raising its sidelobe levels.

\[
V \left( \frac{d_x}{\lambda}, \frac{d_y}{\lambda} \right) = \int_{-\infty}^{\infty} T_B(x, y) \phi \left( \frac{d_x}{\lambda} x, \frac{d_y}{\lambda} y \right) dx dy
\]
Samples are made of the visibility function at a wide variety of different relative antenna element spacings. There must be close and distant spacings, as well as a uniform distribution in-between. Also, the spacings must vary evenly in two perpendicular directions which lie in the plane of the thinned array. There are many configurations in which to construct the thinned array. A ring of elements - pseudo randomly thinned - provides an even distribution of spacings instantaneously (ref. 3). A "Y" configuration is presently used by the Very Large Array radio telescope in Socorro, NM. It relies on the Earth's rotation to produce much of the spacing. A rotating thinned line of elements would generate the proper distribution of spacings after 180° of rotation. Linear thinning algorithms have been studied for SAIR Earth remote sensing (ref. 4).

POSITION CORRELATION
(VARY D_x, y)

EXAMPLES OF SAMPLING LATTICE:

RING

• EXCELLENT "SNAPSHOT" COVERAGE

"Y"

• VLA CONFIGURATION

• USES EARTH ROTATION

ROTATING LINEAR

• MINIMUM REDUNDANCY THINNED ARRAY HERITAGE

• ORBIT STABILIZED ROTATION
The spatial resolution of a SAIR imager can be derived from standard Fourier transform theory. The visibility function is sampled over a region, the extent of which is determined by the maximum spacing between antenna elements in the thinned array. This effectively truncates the visibility function sample space. The truncation produces a smoothing, or "lowpass filtering" of the brightness temperature distribution reconstructed in software. The resulting resolution of the image is the same as would have been measured by a filled aperture, for example a very large reflector, with a diameter equal to the maximum spacing between antenna elements in the SAIR array.

**CORRELATION INTERFEROMETRY SPATIAL RESOLUTION**

- \[ T_B(\lambda, y) = \varphi^{-2} \left\{ \sqrt{\left( \frac{D_x}{\lambda}, \frac{D_y}{\lambda} \right)} \right\} \]

- **V SAMPLE-SPACE HAS FINITE EXTENT**
  \[ \Rightarrow T_B \text{ IMAGE IS "LOWPASS FILTERED"} \]

- **IMAGE RESOLUTION** = \( \frac{1}{V\text{-SPACE MAXIMUM}} \)

  \[ = \frac{\lambda_{\text{min}} h}{D_{x, y_{\text{max}}}} \]
Examples of specific SAIR array configurations illustrate the large savings in size and weight possible relative to conventional filled array imagers. A SAIR imager capable of 100 km resolution from geosynchronous orbit at 10 GHz would require 20 array elements, each approximately 8 cm in diameter, deployed pseudo-randomly in a rotating line 10.7 m long. A phased array with the same image resolution would require 795 times as many elements. A 70 element SAIR imager, with 10 km resolution at 10 GHz, has the imaging performance of a filled array with 22,743 times as many elements. This extreme reduction in required aperture is, to a certain extent, offset by the increased complexity of the cross-correlation circuitry necessary in a SAIR system. Every possible pair of antennas is cross-correlated. For the 70 element array, this implies 2,415 cross-correlators. The streamlining, parallel processing, and, possibly, digitizing of this step is a key area for further development.

### SPATIAL RESOLUTION EXAMPLES AT 10 GHz

<table>
<thead>
<tr>
<th>RESOLUTION (km)</th>
<th>100</th>
<th>50</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_x, y_{max} ) (m)</td>
<td>10.7</td>
<td>21.5</td>
<td>107.5</td>
</tr>
<tr>
<td># ELEMENTS IN FILLED ARRAY</td>
<td>15.9 K</td>
<td>63.7 K</td>
<td>1,592 K</td>
</tr>
<tr>
<td># ELEMENTS IN ROTATING THINNED LINEAR ARRAY</td>
<td>20</td>
<td>30</td>
<td>70</td>
</tr>
<tr>
<td>% FILLED APERTURE</td>
<td>0.13%</td>
<td>0.05%</td>
<td>0.005%</td>
</tr>
</tbody>
</table>
Measurements of the visibility function are made over a region of its sample space. This corresponds to a range of spacings between different array elements. The required distribution of spacings over this range can be specified using Nyquist sampling theory. The field of view (FOV) imaged determines the incremental steps allowed between successive spacings. In the case of a filled phased array imaging the full $2\pi$ steradian half space, this implies the standard half wavelength spacing between elements. From a geosynchronous platform, the FOV is significantly reduced and the increment between element-pair spacings increases to $2.84$ wavelengths. The aliased responses generated by an image reconstruction from Nyquist samples are analogous to antenna grating lobes. These grating lobes are positioned off the limb of the Earth by the Nyquist condition and can be further suppressed, if necessary, by the individual array element patterns.

**CORRELATION INTERFEROMETER SAMPLING THEOREM**

- $V\left(\frac{D_x}{\lambda}, \frac{D_y}{\lambda}\right) = \varphi^2(T_B(x,y))$

- $T_B$ SPATIAL EXTENT IS FINITE
  $\Rightarrow$ 2-d NYQUIST SAMPLING OF V

**EXAMPLE: GEOSTATIONARY PLATFORM**

$\theta = 8.7^\circ$

$h = 35928$ km

$\text{FOV} = 12610$ km

**SAMPLING THEOREM:**

$\Delta \left(\frac{D_x,y}{\lambda}\right) = \frac{h}{\text{FOV}} = 2.84$
Incomplete sampling of the visibility function by a SAIR array configuration results in a loss of beam efficiency in the synthetic antenna pattern. A particular example illustrates the sampling requirements of a SAIR array. Twelve antenna elements are arranged in a circular ring, either uniformly spaced or thinned according to an optimal thinning algorithm (ref. 3). The diameter of the ring is varied and, at each diameter, the beam efficiency of the corresponding synthetic antenna pattern is computed. The uniform distribution degrades as soon as the spacing between elements along the circumference of the ring significantly exceeds the Nyquist criteria. The performance of the thinned ring degrades when its diameter exceeds twelve times the Nyquist criteria. This degradation amounts to a rise in sidelobes near the mainbeam. The spatial resolution of the array is also plotted. A thinned SAIR ring with twelve elements can provide 500 km resolution with 90% beam efficiency. Increasing the resolution to 5 km should raise the number of elements needed to approximately 120 (ref. 5).

RING INTERFEROMETER
BEAM EFFICIENCY

- ASSUMING 12 ANTENNA ELEMENT ARRAY

- OPTIMAL THINNING (T.J. CORNWELL, IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATIONS, Vol. 36, No. 8, AUGUST 1988, pp. 1165-1167)

- BEAM EFFICIENCY ASSUMES UNIFORM APERTURE TAPER

![Graph showing beam efficiency and resolution vs. ring diameter]
The radiometric noise floor of a SAIR image has been studied in detail (ref. 4). The noise floor of the individual measurements of the visibility function is simply that of a conventional correlation radiometer. The two-dimensional Fourier transform of these measurements can be viewed as a weighted summation of $N^2$ random variables, where $N^2$ is the number of measurements or the number of independent pixels in the brightness temperature image. The weights are the sinusoidal kernels of the transform and the resulting image has a noise floor which is approximately $N$ times that of the individual measurements. This accumulation of noise is significantly offset by the fact that all pixels are imaged simultaneously, thus increasing the available integration time over that of a scanning imager. The SAIR system temperature can also be significantly reduced by using small array elements with antenna patterns much broader than the Earth disc. This technique must be traded off, however, against grating lobe suppression by the element patterns.

**CORRELATION INTERFEROMETRY IMAGE SENSITIVITY**

- **INDIVIDUAL MEASUREMENT NOISE FLOOR**
  \[ \Delta V = \frac{T_{sys}}{\sqrt{B\tau}} \]
  \[ \text{WHERE } T_{sys} = T_{Rx} + \frac{\langle T_B \rangle}{2\pi} \]

- **RECONSTRUCTED IMAGE NOISE FLOOR**
  \[ \Delta T = N \Delta V \]
  \[ \text{WHERE } N^2 = \text{# OF PIXELS IN IMAGE} \]

**NOTE:**
- $T_{sys} = T_{Rx} + \langle T_B \rangle = T_{Rx} + 200 \text{ K FOR FILLED APERTURE (PENCIL BEAM)}$
  IMAGERS VERSUS $T_{Rx} + 3 \text{ K FOR CORRELATION INTERFEROMETER}$
- $T_{Rx}$ IS MAJOR DESIGN DRIVER HERE
The noise floor of a brightness temperature image has been computed for three different radiometer systems: a mechanically scanning filled aperture, a SAIR ring array, and a SAIR rotating linear array. Radiometer operating parameters typical on a geostationary platform are assumed. The filled aperture system must sequentially sample the pixels in the image and its integration time per pixel is reduced accordingly. The linear SAIR system must rotate 180° to adequately sample the visibility function and its dwell time per sample is reduced. The SAIR ring array is assumed to have the double advantage of longest integration time and lower system temperature than the filled aperture imager. If this latter advantage is eliminated, then the filled aperture and ring array will have comparable noise floors. For 10 km image resolution, only the rotating linear SAIR array has an unacceptably high noise floor (8.1 K). The ring array, or perhaps some other two dimensional thinned configuration, becomes a performance necessity at this level of resolution.

**CORRELATION INTERFEROMETRY SENSITIVITY EXAMPLE**

- **TOTAL INTEGRATION TIME** \( T = 1 \text{ hr} \)
- **PRE-DETECTION BANDWIDTH** \( B = 100 \text{ MHz} \)
- **RECEIVER NOISE** \( T_{Rx} = 150 \text{ K} \)
- **# OF PIXELS IN IMAGE** \( N^2 \)

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>126</td>
<td>252</td>
<td>1261</td>
</tr>
</tbody>
</table>

- **INDIVIDUAL MEASUREMENT INTEGRATION TIME** \( \tau \)
- **RECONSTRUCTED IMAGE NOISE FLOOR** \( \Delta T \):

<table>
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<tr>
<th>RESOLUTION (km)</th>
<th>100</th>
<th>50</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMAGER</td>
<td>( \tau )</td>
<td>0.08</td>
<td>0.17</td>
</tr>
<tr>
<td>MECHANICAL SCANNING PENCIL BEAM</td>
<td>( \frac{T}{N^2} )</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>RING</td>
<td>( T )</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>ROTATING LINEAR</td>
<td>( \frac{T}{N/2} )</td>
<td>0.26</td>
<td>0.72</td>
</tr>
</tbody>
</table>
REFERENCES


