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POTENTIAL THEORY OF RADIATION

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ABSTRACT

This study aims to develop a new theoretical method by which the structure of a radiation field can be predicted by a radiation potential theory, similar to a classical potential theory. The introduction of a scalar potential is justified on the grounds that the spectral intensity vector is irrotational. The vector is also solenoidal in the limits of a radiation field in complete radiative equilibrium or in a vacuum. This method provides an exact, elliptic type equation that will upgrade the accuracy and the efficiency of the current CFD programs required for the prediction of radiation and flow fields.

A number of interesting results emerge from the present study. First, a steady state radiation field exhibits an "optically modulated inverse square law" distribution character. Secondly, the unsteady radiation field is structured with two conjugate scalar potentials. Each is governed by a Klein-Gordon equation with a frictional force and a restoring force. This steady potential field structure and the propagation of radiation potentials are consistent with the well known results of classical electromagnetic theory. The study also recommends the extension of the radiation potential theory for spray combustion and hypersonic flow.

1. INTRODUCTION

Thermal radiation plays an important role in the broad area of engineering and scientific applications involving high temperature flow processes. In high performance rocket engines[1], industrial boilers and furnaces, as well as in many astrophysical flow fields[2], the radiative transfer frequently makes a dominant contribution to the energy redistribution, mass, and the momentum transfer in the participating media.

Recently, an assessment of the impact of radiation in rocket engine performance was conducted by this writer[3]. It was concluded that the radiative heat loss and the enhancement of the burning rate of droplets by radiation can significantly affect the overall performance of low-intermediate enthalpy engines. The results of this preliminary study were brought to the attention of the research community of liquid propulsion technology. Subsequently, the JANNAF workshop[4] on "Radiation effect on flow characteristic in combustion chambers" was conducted to indentify the problem areas that could have potential impacts on rocket engine hardware design, performance, life cycles thermal material fatigue, and reliability as well as the radiative modulation on spray flow field behavior and combustion instability.

The present study is the continuation of the radiation research which aims to promote a state-of-the-art understanding of the phenomena and to upgrade the predictive capability as well as the accuracy of Computational Fluid Dynamics (CFD) codes with radiative heat transfer.

The specific goals of this study are (1) to establish a theoretical link between two principal theories of radiation, electromagnetic (EM) theory vs. radiative transfer (RT) theory, and (2) to develop a unified theoretical methodology that can be incorporated in a computational alogrithm and a grid structure similar to those of the participating fluid.

The first problem treated is a fundamental issue of establishing a unified view of radiation by examining the structures of the radiation field predicted by two rival theories; the EM and RT models.

The most remarkable findings that emerged from this study are (1) the radiation field intensity vector is irrotational and therefore the field vector is derivable from a scalar potential, (2) a steady state field structure predicted by RT theory obeys an elliptic type equation in much the same manner as an electro- magnetic static field and (3) a non-steady radiative field is governed by a wave equation which has the same general features of an electro-magnetic field characterized by the Maxwell equation.

The second problem in this study is the establishment of an exact radiation potential equation for a radiation modulated Computational Fluid Dynamics program. This study is further motivated by the fact that the radiation equations currently adopted in the majority of the Computational Fluid Dynamics (CFD) approaches are approximate equations[5]; e.g., Two-Flux model, Six-Flux model, discrete-ordinate or zonal method. The accuracy of these equations in general is not compatible with that of the conservation laws and associated submodels which are more sophisticated in the process description and mathematical characterization; e.g.,$k - \varepsilon$ model, combustion-turbulence interaction and spray-droplet models[6].

Since the equation governing the radiation potential in the present theory is of an elliptic type, a numerical algorithm and grid net work similar to those of the Navier-Stokes equations, or turbulent flow conservation laws can be adopted for the simultaneous prediction of the radiation and flow field. These advantages facilitate computational efficiency and accuracy.

The potential theory serves to provide a new physical perception and an useful methodology toward the understanding of radiation processes in high temperature flow phenomena. It also provides a gauge for the comparative assessment of the validity and limitation of the classical electromagnetic theory and the phenomenological radiative transfer approach.

This writer recommends that the steady state potential theory be implemented for the prediction of the combustion flow field in a selected liquid rocket combustion chamber, such as TRW's variable thrust engine, to test the viability of the present method. It is also suggested that this theory be extended to treat the interaction of radiation with droplets and particles in a spray combustion environment.

Additionally, spontaneous or non-steady radiation processes in combustion environments may be examined to assess the radiation induced ignition, vaporization and combustion in advanced propulsion systems.

2. BASIC PROPERTIES OF SPECTRAL INTENSITY VECTOR AND RADIATION POTENTIAL

The radiative transfer equation[7] (RTE) which governs the spectral intensity is given by

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \nabla \cdot \bar{s} I_\nu = \beta_\nu (S_\nu - I_\nu) \tag{1}$$

where c is the speed of light in a vacuum, I_ν is the spectral intensity of radiation in the frequency interval ν and $\nu + d\nu$, β is the volumetric extinction, which is the sum of the absorption coefficient K_ν and scattering coefficient σ_ν, $\bar{s}$ is the unit propagation vector and S_ν is the sum of emitted and in-scattered radiation given by

$$\begin{aligned} S_\nu =& (\eta_\nu/\beta_\nu) + (\sigma_\nu/4\pi\beta_\nu) \times \\ &\times \iint \chi_v(\bar{s}' \to \bar{s}, \quad \nu' \to v) I_{\nu'}(\bar{s}') d\Omega' dv' \end{aligned} \tag{2}$$

in which η_ν is the emission coefficient, χ_ν is the scattering function and Ω is a solid angle.

2.1 Irrotationality of Spectral Intensity Vector at Steady State

The spectral intensity vector is defined as the vector which has the magnitude equal to the spectral intensity I_ν and the direction coincides with that of the propagation of a ray of light. The spectral intensity vector at a steady state can be shown to be "irrotational" by the following mathematical procedure.

By a simple quadrature the solution of Eq.(1). is given by

$$I_\nu = I_o \exp\left(-\int_{\bar{r}_o}^{\bar{r}} \beta_\nu \bar{s}' \cdot d\bar{r}'\right) + \int \beta_\nu S_\nu \exp\left(-\int_{\bar{r}'}^{\bar{r}} \beta_\nu \bar{s}'' \cdot d\bar{r}''\right) d\bar{s}' \cdot d\bar{r}' \qquad (3)$$

Note that Eqn (3) is an intergral equation if S_ν contains the scattering term which is represented by the third term on the right hand side of Eq.(2). In order to show the irrotationality of the spectral intensity vector, the curl of $I_\nu \bar{S}$ must vanish.

$$\begin{aligned}\nabla \times (I_\nu \bar{s}) = \Bigg\{ & I_{\nu o}(-\beta_\nu \bar{s}) \exp(\left(-\int_{\bar{r}_o}^{\bar{r}} \beta_\nu \bar{s}' \cdot d\bar{r}'\right) \\ & - \beta_\nu \bar{s} \int_{r_o}^{\bar{r}} \beta_\nu S_\nu \exp\left(-\int_{r_o}^{\bar{r}'} \beta_\nu \bar{s}'' \cdot d\bar{r}''\right) \bar{s}' \cdot d\bar{r}' \\ & + \beta_\nu S_\nu \bar{s} \Bigg\} \times \bar{s} = -(\beta_\nu I_\nu - \beta_\nu S_\nu)\bar{s} \times \bar{s} = 0\end{aligned} \qquad (4)$$

The irrotationality of a spectral intensity vector allows one to introduce a "radiation potential ϕ_ν" from which the spectral intensity vector can be calculated as follows

$$\nabla \phi_\nu = I_\nu \bar{s} \qquad (5)$$

Additionally, a ray of light propagates in the direction normal to an equipotential surface, as shown in the following.

Since $\bar{s}$ is a unit vector, the magnitude of the spectral density vector I_ν is given by

$$\sqrt{\nabla\phi_\nu \cdot \nabla\phi_\nu} = I_\nu \tag{6}$$

Thus from Eq.(5), the unit propagation vector $\bar{s}$ is

$$\bar{s} = \frac{\nabla\phi_\nu}{I_\nu} = \frac{\nabla\phi_\nu}{\sqrt{\nabla\phi_\nu \cdot \nabla\phi_\nu}} = \bar{n} \tag{7}$$

Hence the propagation vector coincides with the unit vector, $\bar{n}$, normal to an equipotential surface.

By substituting (5) into (1) one finds that the radiation potential ϕ_ν obeys the following "Radiation Potential Equation" (RPE)

$$\nabla^2\phi_\nu + \beta_\nu\bar{s} \cdot \nabla\phi_\nu = \beta_\nu S_\nu \tag{8}$$

The structure of a radiation field and spectral intensity vector can be predicted by the solution of Eq.(8). A general solution of Eq.(8) will be discussed in section 3.

2.2 Solenoidal Characteristics of Spectral Intensity Vector for Two Limiting Cases

An observation of Eq.(1) suggests that when the right hand side of the equation vanishes, the spectral intensity vector is solenoidal or divergent free, providing that the radiation is independent of time. This special case corresponds to (1)a complete radiative equilibrium, and (2) non-participating media i.e. $\beta_\nu = 0$ including a vacuum space.

In the special cases listed above, the radiation potential ϕ_ν satisfies the Laplace equation

$$\nabla^2\phi_\nu = 0$$

Thus the determination of the distribution of the spectral intensity vector for these limited cases is equivalent to the boundary value problem of a "Laplace" potential field. Principle tools and the results of classical potential theories (such as the method of image, conformal mapping, as well as Green functions for Dirichlet and Neumann boundary value problems) can be used with or without modification for the prediction of the flow coupled or decoupled radiation field problems.

3. POTENTIAL THEORY OF STEADY STATE RADIATION

The determination of a radiation intensity in a domain bounded internally or externally by a physical surface is reduced to an appropriate boundary value problem of RPE, Eq.(8).

A general solution of Eq.(8), "optically modulated inverse square law", is discussed in the following section.

3.1 Optically Modulated Inverse Square Law

In order to determine the radiation field one has to predict I_ν simultaneously with the conservation laws of a flow field because the source term S_ν appearing on the right hand side of Eq.(8) depends , in general, on the properties of a gas flow field and radiation.

In the following analysis, however, the S_ν will be treated as a source term so that the formal solution of Eq.(8) can be expressed in an internal form which contains S_ν in an intergrand.

· The general solution is first expressed by the method of potential splitting as follows

$$\nabla \phi_\nu = \exp\left\{ - \int_{r_o}^{\bar{r}} \beta_\nu \bar{s}' \cdot d\bar{r}' \right\} \nabla \psi_\nu \tag{9}$$

Subsequently by substituting (9) into (8) one obtains

$$\nabla^2 \psi_\nu = \exp\left\{ \int_{\bar{r}_o}^{\bar{r}} \beta_\nu \bar{s}' \cdot d\bar{r}' \right\} \beta_\nu S_\nu \tag{10}$$

Thus a particular solution of Eq.(10) is what may be termed as an "optically retarded inverse square law potential" given by

$$\psi_\nu = \iiint \frac{\beta_\nu S_\nu(\bar{r}') \exp(\int_{\bar{r}_o}^{\bar{r}'} \beta_\nu \bar{s}'' \cdot d\bar{r}'')}{|\bar{r} - \bar{r}'|} dV' \tag{11}$$

The spectral intensity vector is obtained by substituting Eq.(11) into (9) as follows

$$I_\nu \bar{s} = \nabla \psi_\nu \doteq - \iiint \frac{\beta_\nu S_\nu(\bar{r}')}{|\bar{r} - \bar{r}'|^2} \exp\left\{ - \int_{\bar{r}'}^{\bar{r}} \beta_\nu \bar{s}'' \cdot d\bar{r}'' \right\} dV' \tag{12}$$

Observation of Eq.(2) suggests that the intensity decay pattern has two primary characteristics; inverse square and exponential decay in source strength due to an opacity factor. This decay pattern is one of the basic characteristics of the radiation heat transfer and will be referred to as "optically modulated inverse square law".

3.2 Steady State Radiation Potential Solution

A general solution satisfying a prescribed potential and its normal derivative at a physical boundary shown in Fig.1 can be constructed from the solution of an associated potential ψ_ν. Since ψ_ν satisfies the Poisson equation, the general solution is given by the following classical expression

$$\begin{aligned}\psi_\nu(\bar{r}) = &\iiint \frac{\beta_\nu S_\nu(\bar{r}') \exp(\int_{\bar{r}_o}^{\bar{r}'} \beta_\nu \bar{s}'' \cdot d\bar{r}'')}{|\bar{r} - \bar{r}'|} dV' \\ &+ \iint \psi_\nu(\bar{r}_o) \frac{\partial}{\partial n} \frac{1}{|\bar{r} - \bar{r}_\gamma|} d\sum\nolimits_o - \iint \frac{\partial \psi(\bar{r}_o)}{\partial n} \frac{1}{|\bar{r} - \bar{r}_o|} d\sum\nolimits_o\end{aligned} \tag{13}$$

where $\psi_\nu(\bar{r}_0)$ and $\frac{\partial \psi(\bar{r}_0)}{\partial n}$ is the value of the potential and its normal derivative at $\bar{r}_0$, located on the boundary. The numerical values of $\frac{\partial \psi_0}{\partial n}$ are related with those of $\frac{\partial \phi_\nu}{\partial n}$ as follows

$$\frac{\partial \psi_\nu}{\partial n}(\bar{r}_o) = \lim_{\bar{r} \to r_o} \bar{n} \cdot \nabla \psi_\nu = \lim_{\bar{r} \to \bar{r}_o} \bar{n} \cdot \exp\left(\int_{\bar{r}_o}^{\bar{r}} \beta_\nu \bar{s}' \cdot d\bar{r}' \right) \nabla \phi_\nu$$

Thus the numerical value of $\frac{\partial \psi_\nu}{\partial n}$ at $r = r_0$ is equal to that of $\frac{\partial \psi_\nu}{\partial n}$; i.e.

$$\frac{\partial \psi_\nu(\bar{r}_o)}{\partial n} = \frac{\partial \phi_\nu(\bar{r}_o)}{\partial n}$$

The numerical values of $\psi_\nu(\bar{r}_0)$ must be calculated from the prescribed value of $\phi_\nu(\bar{r}_0)$ by integrating Eq.(9) as follows

$$\psi_\nu(\bar{r}) = \psi_\nu(\bar{r}_{o,o}) + \int_{\bar{r}_{o,o}}^{\bar{r}} \exp(\int \beta_\nu \bar{s}' \cdot d\bar{r}') d\psi_\nu \tag{14}$$

where $\bar{r}_{0,0}$ is a reference point on the wall.

An additional step is required to determine the value of $\psi_\nu(\bar{r}_0)$ in terms of $\phi_\nu(\bar{r}_0)$. Firstly, Eq.(14) will be specialized to the boundary by replacing $\bar{r}$ by $\bar{r}_0$, i.e.

$$\psi_\nu(\bar{r}_o) = \psi_\nu(\bar{r}_{o,o}) + \int_{\bar{r}_{o,o}}^{\bar{r}_o} \exp(\lambda) d\phi_\nu \tag{15}$$

where

$$\lambda = \int_{\bar{r}_{o,o}}^{\bar{r}_o} \beta_\nu \bar{s}' \cdot d\bar{r}'_o \tag{16}$$

Since the distribution of a potential is prescribed as function ϕ_ν i.e.

$$\phi_\nu = \phi_\nu(\lambda)$$

Finally, by subsituting (17) into (16) one obtains

$$\psi_\nu(\bar{r}_o) = \psi_\nu(\bar{r}_{o,o}) + \int_{\bar{r}_{o,o}}^{\bar{r}_o} e^\lambda f(\lambda) d\lambda \tag{18}$$

where

$$f(\lambda) = \frac{d\phi_\nu}{d\lambda}_{r=r_o}$$

The spectral intensity vector is derived from Eqs.(9) and (13) as follows

$$\begin{aligned} I_\nu \bar{s} = \nabla\psi_\nu = & -\iiint \frac{\beta_\nu s_\nu(\bar{r}')}{|\bar{r} - \bar{r}|^2} \exp\left(\int_{\bar{r}_o}^{\bar{r}'} \beta_\nu \bar{s}'' \cdot d\bar{r}''\right) \bar{s}' dV' \\ & + \iint \frac{\partial\psi_\nu/\partial n}{|\bar{r} - \bar{r}_o|^2} \exp\left(\int_{\bar{r}'_o}^{\bar{r}'} \beta_\nu \bar{s}'' \cdot d\bar{r}'\right) \bar{s}_o \, d\sum_0 \\ & - \iint \psi_\nu(\bar{r}_o) \frac{\partial}{\partial n}\left\{\frac{\bar{s}_o}{|r - \bar{r}_o|^2}\right\} d\sum_0 \end{aligned} \tag{19}$$

where

$$S'_i = \frac{x_i - x'_i}{\sqrt{\sum(x_j - x'_j)^2}} \tag{20}$$

$$S_{ai} = \frac{x_i - x_{oi}}{\sqrt{\sum(x_j - x'_j)^2}} \tag{21}$$

and $\psi_\nu(\bar{r}_0)$ is given by Eq.(18).

In general, the method of Green functions can be used to predict the solution corresponding to the boundary value problem with a prescribed normal derivative of the radiation potential. For instance, the solution with a prescribed inhomogeneous Neumann boundary condition is given by

$$\begin{aligned} I_\nu \bar{s} = \nabla\phi_\nu = & \iiint \nabla G(\bar{r}/\bar{r}') \beta_\nu S_\nu(\bar{r}') \exp\left(-\int_{\bar{r}_o}^{\bar{r}} \beta_\nu \bar{s}'' \cdot d\bar{r}''\right) dV \\ & + \iint \nabla G_\nu(\bar{r}/\bar{r}_o) \frac{\partial\psi^*_\nu(\bar{r}_o)}{\partial n} \exp\left(-\int_{\bar{r}_o}^{\bar{r}} \beta_\nu \bar{s}'' \cdot d\bar{r}''\right) d\sum\nolimits_o \end{aligned} \tag{22}$$

where

$$\frac{\partial\psi^*_\nu(\bar{r}_o)}{\partial n} = \frac{\partial\psi_\nu(\bar{r}_o)}{\partial n} - \iiint \nabla G(\bar{r}_o/\bar{r}')\beta_\nu S_\nu(\bar{r}') dV \tag{23}$$

The Green function G satisfies the following equation

$$\nabla^2 G(\bar{r}/\bar{r}') = -4\pi\,\delta(\bar{r} - \bar{r}') \tag{24}$$

and homogeneous Dirichlet condition i.e. $G(\bar{r}_0/\bar{r}') = 0$

3.3 Structural Equivalence Between the Radiation Potential Theory and the Electro-magneto Static Theory

The fact that the radiative potential equation Eq.(8a) and (8b) is an elliptic type suggests the structural similarity between the radiation field predicted by the Radiation Potential Theory, and the electro-magnetic static field which is governed by the Laplace or the Poisson equation in electromagnetic theory. However, it must be pointed out that the radiation potential ϕ_ν and spectral intensity vector $I_\nu \bar{s}$ are not the same physical quantities as the electro-magnetic static potential and electro-magnetic intensity. Hence, complete equivalence between the two theories will require the knowledge of the functional relationship between those of the electromagnetic properties and those of the radiation field. This issue will be discussed further in section 4.

4. POTENTIAL THEORY OF NON-STEADY RADIATION

Unsteady radiation phenomena involve the propagation and interaction of light with matter in a participating fluid. The objective of this section is to develop a hyperbolic type equation that governs what may be termed a "conjugate spectral intensity". The proposed formulation of a "Radiation Wave Equation" exhibits another interesting insight into the structural equivalence between the radiative transfer and electromagnetic theories.

4.1 Conjugate Spectral Intensities

The radiative transfer equation Eq.(1) does not remain invariant under the inverse transformation of the propagation vector $\bar{s}$. Thus, it is appropriate to define two independent spectral intensities; I_ν^+ and I_ν^- which satisfy the following pair of equations.

$$\frac{1}{C}\frac{\partial I_\nu^\pm}{\partial t} \pm \bar{S}\cdot\nabla I_\nu^\pm = \beta_\nu(S_\nu - I_\nu^\pm) \tag{25}$$

The equation governing I_ν^+ is the radiative transfer equation with the propagation vector being equal to $\bar{s}$, whereas the conjugate equation governing I_ν^- has a propagation vector of $-\bar{s}$.

Following a mathematical proof similar to that which was presented in section 2.1, one can show that the conjugate spectral intensity vectors are irrotational and therefore derivable from two conjugate radiation potentials i.e.

$$I_\nu^\pm \bar{s} = \nabla\phi_\nu^\pm \tag{26}$$

Substitution of Eq.(26) into (25) gives

$$\nabla\cdot\bar{L}^\pm\phi_\nu^\pm = \beta_\nu S_\nu \tag{27}$$

where $\bar{L}^\pm$ are first order vectorial operators

$$\bar{L}^\pm = \frac{\bar{s}}{c}\frac{\partial}{\partial t} \pm \nabla + \beta_\nu\bar{s} \tag{28}$$

Two vectors $\bar{L}^\pm\phi_\nu^\pm$ can be decomposed into two components as follows

$$\bar{L}^\pm\phi_\nu^\pm = \nabla\psi^\pm + \nabla\times\bar{W}^\pm \tag{29}$$

Where $\psi^\pm$ and $\bar{W}^\pm$ are scalar and vector potentials respectively. Two conjugate scalar potentials $\psi_\nu^\pm$ are governed by the following Poisson equation

$$\nabla^2\psi_\nu^\pm = \beta_\nu S_\nu \tag{30}$$

whereas the vectorial potential functions obey the following homogeneous vectorial equation with repeated curl operators.

$$\nabla \times \nabla \times \bar{W}^{\pm} = 0 \tag{31}$$

The absence of the source term in Eq.(31) is demonstrated, initially, by applying a curl operator in Eq.(29). The result is

$$\nabla \times \bar{L}^{\pm}\phi_{\nu}^{\pm} = \nabla \times \nabla \times \bar{W}^{\pm} = \nabla \times (\frac{\bar{s}}{c}\frac{\partial}{\partial t} \pm \nabla + \beta_n u \bar{s})\phi_{\nu}^{\pm} \tag{32}$$

Secondly, the irrotationality of $\bar{L}^{\pm}\phi_{\nu}^{\pm}$ is proved by showing that the first and third terms appearing on the right hand side of Eq.(32) vanish. The proof is shown in the following

$$\nabla \times (\bar{s}\phi_{\nu}^{\pm}) = \nabla \times (\bar{s}\int^{\bar{r}} I_{\nu}^{\pm}\bar{s}' \cdot d\bar{r}') = \bar{s} \times I_{\nu}^{\pm}\bar{s} \equiv 0 \tag{33}$$

In order to establish the structural equivalence between the radiative transfer and electromagnetic theory, one has to demonstrate the similarity in the type of the partial differential operations for the conjugate radiation potentials and the electromagnetic potentials. This is discussed in the next subsection.

4.2 Conjugate Radiative Wave Equations: Klein-Gorden Equations

According to the irrotational characteristcis of $\bar{L}^{\pm}\phi_{\nu}^{\pm}$, one expresses

$$\bar{L}^{\pm}\phi_{\nu}^{\pm} = \nabla\psi_{\nu}^{\pm} \tag{34}$$

By applying the vectorial conjugate operator to Eq.(34), one obtains

$$\bar{L}^{\mp} \cdot \bar{L}^{\pm}\phi_{\nu}^{\pm} = \bar{L}^{\mp} \cdot \nabla\psi_{\nu}^{\pm} \tag{35}$$

The above equations, (35), are rewritten into two components explicitly as follows

$$\frac{1}{c^2}\frac{\partial^2\phi_\nu^+}{\partial t^2} - \nabla^2\phi_\nu^+ + 2\frac{\beta_r}{c}\frac{\partial\phi_\nu^+}{\partial t} + (\beta_\nu^2 - \bar{s}\cdot\nabla\beta_\nu)\phi_\nu^+ = (\frac{\bar{s}}{c}\frac{\partial}{\partial t} + \beta_\nu\bar{s})\cdot\nabla\psi_\nu^+ - \beta_\nu S_\nu \quad (36)$$

$$\frac{1}{c^2}\frac{\partial^2\phi_\nu^-}{\partial t^2} - \nabla^2\phi_\nu^- + 2\frac{\beta_\nu}{c}\frac{\partial\phi_\nu^-}{\partial t} + (\beta_\nu^2 + \bar{s}\cdot\nabla\beta_\nu)\phi_\nu^- = (\frac{\bar{s}}{c}\frac{\partial}{\partial t} + \beta_\nu\bar{s})\cdot\nabla\psi_\nu^- + \beta_\nu S_\nu \quad (37)$$

where $\psi_\nu^\pm$ are given by

$$\psi_\nu^\pm = \iiint \frac{\beta_\nu S_\nu(\bar{r}')}{|\bar{r}-\bar{r}'|}dV' + \iint \psi_\nu^\pm(\bar{r}_0)\frac{\partial}{\partial n}\frac{1}{|\bar{r}-\bar{r}_0|}d\bar{Z}_o$$
$$-\iint \frac{\partial\psi_\nu^\pm(\bar{r}_o)}{\partial n}\frac{1}{|\bar{r}-\bar{r}_o|}d\bar{Z}_o \quad (38)$$

in which $\psi_\nu^\pm(\bar{r}_0)$ and $\frac{\partial\psi_\nu^\pm(\bar{r}_0)}{\partial n}$ are the potentials and their normal derivatives on the boundary surface respectively.

The pair of the wave equations (36) and (37) are Klein-Gordon Equations with a friction force represented by the term involving the first order derivative with respect to time. The difference between two conjugate wave equations (36) and (37) is the sign of the second restoring term $\beta_\nu^2 \mp \bar{s}\cdot\nabla\beta_\nu$ and the source terms appear on the, $\pm\beta_\nu s_\nu$, present on the right hand side of Eqs.(36) and (37). The structural similarity between the two theories; RT and EM is the key feature of the present theory. Presently, however, the functional interrelation between the properties of the two fields has not being indentified. Thus the complete similarity between two fields can not be fully established. It may be mentioned, nevertheless, that there is an indirect method for the interrelation of the energy-stress tensors of an electromagnetic field and that of a photon gas field. Such photon gas-electromagnetic equivalence[8] has been established in the frame work of the relativistically covariant theory of photon gas conservation laws derived from the conservation of proper photon density. This latter equation corresponds to the covariant radiative transfer equation in a four-dimensional space.

Observation of Eqs.(36) and (37) reveals the following basic features. First, two equations are reduced to a conventional wave equation when β_ν vanishes. This is consistent with the well known result of the propogation of an electromagnetic wave in a vacuum. Secondly, in an optically thin medium, i.e., $\beta_\nu << 1$ restoring forces for two conjugate waves are different from each other depending on the sign of the gradient of β_ν in the direction of the propagation. For example, the coefficient of a restoring force is positive in the direction of the increasing β_ν. However in an optically thick media $\beta_\nu >> 1$, the restoring force coefficient is positive for two conjugate waves. Thirdly, an opacity retards the wave propagation. Finally, the radiation potential ϕ_ν^+ is degenerated by the sink $\beta_\nu S_\nu$ whereas it is generated by a source term $\beta_\nu S_\nu$ for the ϕ_ν^- wave. Furthermore both $\phi_\nu^\pm$ waves are partially generated by the source term proportional to the temporal variation of the gradient of an associated potential $\phi_\nu^\pm$ and the product of the opacity with the latter quantity.

5. CONCLUSION

The potential theory of radiation presented in this paper provides a new physical perception of the structure of a radiation field through the identification of the basic characteristics of the irrotationality of a spectral intensity vector. Scalar potential representation of a radiation field allows a unique interpretation of the laws of radiative decay, effects of the boundary values on the field variables for the case of steady state, propagation, retardation, as well as the generation of a radiation wave for unsteady radiation. This preliminary theoretical development is focused ,by design, upon a narrowly scoped mathematical problem, and on the identification of the basic commonality between radiative transfer and electromagnetic theories. The comparative study yields a confirmation of a structural similarity between two fields: radiation and electromagnetic . It is projected, without proof, that the ultimate equivalence theory will serve to determine the validity limit of the existing phenomenogical theory of radiative transfer. The results of such a comparative study will serve to guide the formulation of a new radiative transfer theory.

In the mean time, the steady state potential equation seems to be a viable equation for the prediction of radiation coupled problems because of its accuracy and elliptic type of equation. This formulation has not presently been applied in modern CFD calculations. However, in view of an increasing interest in radiation phenomena in high performance liquid rocket engines and hypersonic flows, the present method is recommended for further advancement toward the numerical prediction of the radiation effects on a gas phase flow as well as for spray combustion processes. This potential method is also expected to provide a unique mathematical tool for the prediction of the radiation fields of various geometrical objects. Such options should be explored to enhance the radiation modelling activity.

Finally, the unsteady radiation phenomena is a largely unexplored area. The effects of highly pulsed radiation on propellent heating, vaporization and ignition have a practical significance on the engine start-up and instability of liquid rocket engines.

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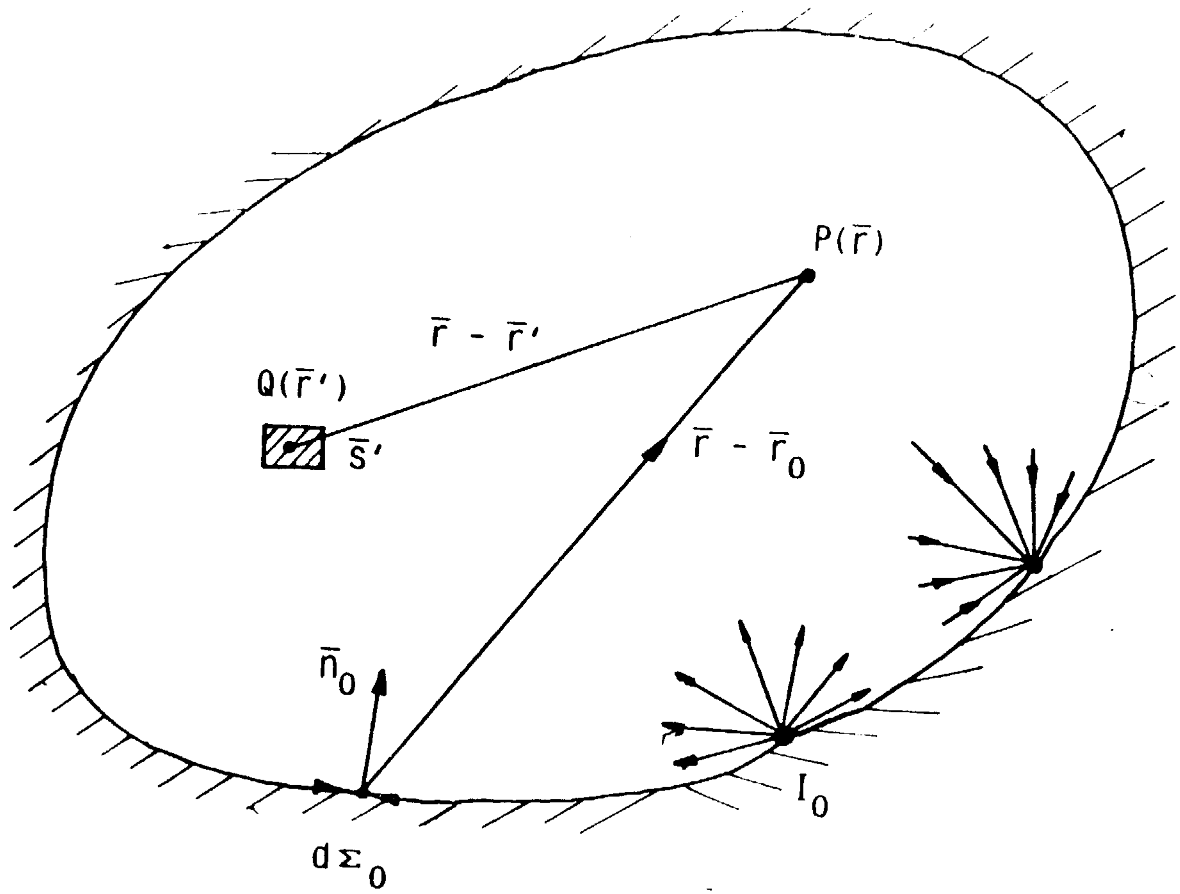

Fig. 1 Schematic of the radiation field bounded by a boundary surface