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THE UNIVERSITY OF ALABAMA

BACKGROUND STUDIES IN GAS IONIZING X-RAY DETECTORS

Prepared by: Hudson B. Eldridge
Academic Rank: Associate Professor
University and Department: University of Central Arkansas Physics Department

NASA/MSFC:
Laboratory: Space Sciences
Division: Astrophysics
Branch: X-Ray Astronomy

MSFC Colleague: Martin C. Weisskopf
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ABSTRACT

The background response of a gas ionizing proportional X-Ray detector is estimated by solving the one dimensional photon transport equation for two regions using Monte Carlo techniques. The solution was effected using the SSL VAX 780 and the CRAY XMP computers at Marshall Space Flight Center. The isotropic photon energy spectrum encompassing the range from 1 to 1000 KeV incident onto the first region, the shield, is taken so as to represent the measured spectrum at an altitude of 3 mb over Palestine, Texas. The differential energy spectrum deposited in the gas region, xenon, over the range of 0 to 100 KeV is written to an output file. In addition, the photon flux emerging from the shield region, tin, over the range of 1 to 1000 KeV is also tabulated and written to a separate file. Published tabular cross sections for photoelectric, elastic and inelastic Compton scattering as well as the total absorption coefficient are used. Histories of each incident photon as well as secondary photons from Compton and photoelectric interactions are followed until the photon either is absorbed or exits from the regions under consideration. A study is made of the effect of shielding thickness upon the energy spectrum deposited in the xenon region for this background spectrum incident upon the tin shield.
ACKNOWLEDGMENTS

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I would like to thank Dr. Gerald R. Karr, whose attention to all details of the program's administration prior to, during, and after, made my visit to Marshall Space Flight Center an enjoyable, intellectually stimulating, and professionally rewarding experience. I am grateful to the National Aeronautics and Space Administration and the American Society for Engineering Education for their support of the Summer Faculty Fellowship Program.
INTRODUCTION

X-ray sources that are of interest to astrophysical research appear very faint when observed from the vicinity of earth. This quite low signal intensity puts severe constraints onto the detectors, especially imaging types, to be used in planned orbiting observatories. In the interest of increasing the signal to noise in future detector design by reducing the background, a study was undertaken of the background sensitivity of a possible type of detector being considered for the AXAF project scheduled for the near future. There are advantages to using a position sensitive proportional detector and considerable development of this type of counter has transpired. As a first step in the study of the background to be expected in such a detector from the ambient photon flux, a one dimensional geometry calculation of the transport through two regions was made.

The mathematical description of the transport of so called "Hard X-rays" through matter yields integral equations that become unwieldy in even the simplest of geometrical arrangements. Instead of an analytic solution, it is frequently adequate to perform a stochastic or Monte Carlo type calculation. In the Monte Carlo calculation, individual photon histories are catalogued as they transit the various media characterized by its cross sections. The result of such a calculation could be either the energy spectrum of photons at a surface in the medium or the energy deposited due to the transport of photons into a volume of the medium.

Central to a Monte Carlo calculation is a sequence of random (or at least pseudo-random) numbers. The sequence used in this calculation was generated by the FORTRAN routine RAN(J) on the SSL VAX 780 or RANF() on the CRAY. In order to test just how random this sequence was, 10 million random numbers were generated and sorted into 100 intervals evenly distributed from 0 to 1. The number in each bin, divided by the 10 million, could be interpreted as the probability of occurrence of a number within each interval. Such a distribution is shown in Figure 1 and it does indeed appear quite uniform. Figure 2, however, shows the deviation from the average of each bin which is a more severe test of randomness and, while the deviations are quite small, some of the "Pseudo" of the randomness of the sequence may be seen.
The incident x-ray spectrum was taken as:

\[
dI/dE = 26(p/3)^{0.2}E^{-1.97} \text{ photons cm}^{-2}\text{s}^{-1}\text{KeV}^{-1}. \quad (1)
\]

An expression which reasonably describes the energy spectrum measured at balloon altitudes over Palastine, Texas (Dean and Dipper 1981). The zenith angle of the photons incident upon the horizontal plane (the tin sheet) was considered isotropic. At a given pressure, the above expression may be considered, when properly normalized over the interval \([0,1]\), the probability density in the variable \(E\). The prescription for selecting the energies at random so that as the number of selections approach infinity the energies will be distributed as the above expression yields:

\[
1-r = (E_0/E)^{0.97} \quad (2)
\]

Where \(r\) is a member of a random sequence over the interval \([0,1]\) which, in practice, is a pseudo-random sequence generated by a computer. \(E_0\) is a cutoff energy which is required for normalization because the expression does not remain finite as \(E\) approaches zero. Also since \(1-r\) is itself just a random sequence it can be replaced by \(r\). The expression may be solved for \(E\) in terms of \(r\) and then the energies \((E)\) calculated from the resulting formula will be distributed according to Equation (1). Inverting Equation (2) yields:

\[
E = (1.0/r)^{1.031} \quad (3)
\]

To verify the validity of the Monte Carlo scheme a program was written to pick one million energies over the interval 1 to 1000 KeV using Equation (3) and the RAN(J) random number generator on the SSL VAX 780 computer. These energies were sorted into 10 KeV intervals to simulate the original spectrum, Equation (1).
Since the same distribution can be calculated analytically by just integrating Equation (1) over each of the 10 KeV intervals, this was done and TABLE 2 shows the comparison of the one million history Monte Carlo simulation with the analytic calculation.

**TABLE 2 - COMPARISON OF MONTE CARLO (A MILLION HISTORIES) WITH THE ANALYTIC CALCULATION OF THE INCIDENT ENERGY SPECTRUM**

<table>
<thead>
<tr>
<th>Interval</th>
<th>Monte Carlo</th>
<th>Analytic</th>
</tr>
</thead>
<tbody>
<tr>
<td>00.00 -&gt; 10.99</td>
<td>23.4613</td>
<td>24.1833</td>
</tr>
<tr>
<td>10.99 -&gt; 20.98</td>
<td>1.8475</td>
<td>1.2211</td>
</tr>
<tr>
<td>20.98 -&gt; 30.97</td>
<td>0.4256</td>
<td>0.4404</td>
</tr>
<tr>
<td>30.97 -&gt; 40.96</td>
<td>0.2210</td>
<td>0.2278</td>
</tr>
<tr>
<td>40.96 -&gt; 50.95</td>
<td>0.1344</td>
<td>0.1396</td>
</tr>
<tr>
<td>50.95 -&gt; 60.94</td>
<td>0.0912</td>
<td>0.0944</td>
</tr>
<tr>
<td>60.94 -&gt; 70.93</td>
<td>0.0668</td>
<td>0.0681</td>
</tr>
<tr>
<td>.... ....</td>
<td>0.0003068</td>
<td>0.0003648</td>
</tr>
<tr>
<td>.... ....</td>
<td>0.0003536</td>
<td>0.0003572</td>
</tr>
<tr>
<td>.... ....</td>
<td>0.0002990</td>
<td>0.0003499</td>
</tr>
<tr>
<td>.... ....</td>
<td>0.0003510</td>
<td>0.0003428</td>
</tr>
<tr>
<td>.... ....</td>
<td>0.0003198</td>
<td>0.0003359</td>
</tr>
<tr>
<td>930.07-&gt;940.06</td>
<td>0.0003276</td>
<td>0.0003292</td>
</tr>
<tr>
<td>940.06-&gt;950.05</td>
<td>0.0003276</td>
<td>0.0003227</td>
</tr>
</tbody>
</table>

In the interest of brevity, the entire spectrum from 1 to 1000 KeV is not presented, only the beginning and the end. As may be noted by examining TABLE 2, 1 million histories are not adequate to represent the desired energy spectrum where the derivative of the curve is large (low energies) and where the magnitude of the flux is very small (high energies). Figure 3 shows a plot of the Monte Carlo simulation of the incident spectrum at the higher energies which is to be compared with Figure 4, the analytic calculation of the incident spectrum, integration of Equation (1) over each interval. Here again the inadequacy of a million histories is evident. Consequently, all subsequent Monte Carlo calculations made using this representation of the incident spectrum were made by compiling 10 million or more photon histories.
A photon of given energy while traversing a medium, in contrast to a charged particle, does not have a well defined range or distance it travels before undergoing any one of a number of possible interactions with the medium. The number of photons, \( dN \), that have an interaction within an infinitesimal distance \( dx \) is given by (Jenkins, Gould, and Gedcke, 1981):

\[
dN = s \, N \, dx
\]  

Where \( s \) is the total cross section in inverse centimeters, \( x \) being in centimeters, and is a function of the photon's energy. \( N \) is the number of photons present at the position \( x \). \( s \) is the sum of the cross sections for the photoelectric, Compton elastic, and Compton inelastic interactions. The energies being considered in this work are below the threshold for pair production. Equation (4) may be rearranged as:

\[
dN/N = s \, dx
\]  

This differential equation has the following solution:

\[
N/N_0 = e^{-sx}
\]  

\( N_0 \) is the flux at \( x = 0 \) and the ratio \( N/N_0 \) is the probability that a photon travels the distance \( x \) before it undergoes an interaction and it is properly normalized for all positive \( x \). This probability function may be solved for \( x \) using a random sequence \( r \) as has been done before:

\[
x = -(1.0/s) \text{ logarithm}(r)
\]  

Where the natural logarithms are to be used.

A FORTRAN program was written which computed a million values of \( x \) using Equation (7), with \( s \) taken as 1, using the function RAN(J) to generate a random sequence. These values were
sorted into 10 equal intervals over the range of 0 to 1 and normalized for comparison with the analytic evaluation of Equation (6) over the same intervals. This comparison is shown in TABLE 2.

TABLE 2 - EVALUATION OF THE FREE PATH DISTRIBUTION
ONE MILLION HISTORIES

<table>
<thead>
<tr>
<th>Interval</th>
<th>Monte Carlo</th>
<th>Analytic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 -&gt; .1</td>
<td>.094876</td>
<td>.0951626</td>
</tr>
<tr>
<td>.1 -&gt; .2</td>
<td>.086219</td>
<td>.0861066</td>
</tr>
<tr>
<td>.2 -&gt; .3</td>
<td>.077935</td>
<td>.0779126</td>
</tr>
<tr>
<td>.3 -&gt; .4</td>
<td>.070229</td>
<td>.0704982</td>
</tr>
<tr>
<td>.4 -&gt; .5</td>
<td>.063695</td>
<td>.0637894</td>
</tr>
<tr>
<td>.5 -&gt; .6</td>
<td>.057681</td>
<td>.0577191</td>
</tr>
<tr>
<td>.6 -&gt; .7</td>
<td>.052426</td>
<td>.0522263</td>
</tr>
<tr>
<td>.7 -&gt; .8</td>
<td>.047123</td>
<td>.0472643</td>
</tr>
<tr>
<td>.8 -&gt; .9</td>
<td>.042736</td>
<td>.0427513</td>
</tr>
<tr>
<td>.9 -&gt; 1.</td>
<td>.038735</td>
<td>.0386902</td>
</tr>
</tbody>
</table>

It may be noted that the Monte Carlo calculation, compiling 1 million histories, differs from the analytic calculation by less than one percent over the entire range of the table.

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THE CROSS SECTIONS

Values of the photoelectric, Compton elastic, Compton inelastic, and total cross sections as functions of the photon energy, were taken from the literature (Veigele 1973), multiplied by the density (Weast 1987), and entered into sequential data files for the SSL VAX 780 and the CRAY computers. There is a separate file for each element. The file structure, not all records, is shown below in TABLE 3. The fields in the first record are number of records in the table, and then the energies of the first four absorption edges. Each subsequent record contains five fields also: energy, total, photoelectric, Compton elastic, and Compton inelastic. The cross section values are in inverse centimeters. Logarithmic interpolation in both energy and cross section is made for energies between the tabulated values and, if the energy at which the cross section is desired falls exactly on an absorption edge, the greater value of the cross section is used. A separate program was written to check the accuracy of the cross section tables' transcription to the files from the journals.

TABLE 3 - STRUCTURE OF XENON CROSS SECTION FILE

37, 0.782, 5.104, 5.453, 34.56
1.0, 60.593, 60.593, 0.4607, 0.0002431
1.145, 43.317, 43.317, 0.04531, 0.0003054
1.145, 47.573, 47.573, 0.64531, 0.0003054
1.5, 24.262, 24.262, 0.04356, 0.0004682
2.0, 11.843, 11.816, 0.4056, 0.0007035
3.0, 4.332, 4.306, 0.03530, 0.001162
4.0, 2.128, 2.098, 0.03054, 0.001560
4.782, 1.372, 1.344, 0.02754, 0.001848
4.782, 40.312, 40.06, 0.02754, 0.001848
5.0, 3.555, 3.530, 0.02679, 0.001923
5.104, 3.380, 3.355, 0.02654, 0.001955
5.104, 4.707, 4.682, 0.02654, 0.001955
5.453, 3.931, 3.906, 0.2529, 0.002065
5.453, 4.732, 4.707, 0.2529, 0.002065
6.0, 3.656, 3.630, 0.2371, 0.002228
8.0, 1.680, 1.660, 0.1878, 0.002754
DETECTOR SIMULATION

The straightforward calculation of the energy deposited in the xenon by the incident photon spectrum discussed above after it had passed through a layer of tin involves the solution of an integral equation wherein the integrand is not a simple function (Jenkins, Gould, and Gedcke, 1981). What is frequently done, instead of analytically solving the equation representing the response of the device, which of course would be generally preferred, is to compromise by performing a "Monte Carlo" or stochastic simulation of the process (Kalos and Whitlock, 1986), (Rubinstein, 1981), and (Hammersley and Handscomb, 1967). The first step in the simulation consists of dividing the physical process into a number of sequential events whose outcomes are governed by known probability distribution functions. The outcome of these events are then determined by properly choosing from these probability distribution functions. The individual histories of the objects under study, in this case x-ray photons, are then compiled. These historical compilations are the desired solution which, in this case is, the differential energy spectrum deposited in the xenon layer by a photon spectrum after it first passes through a layer of tin.

For the particular case under study, the sequence of events in the history of a photon starts by having its energy selected according to Equation (3). Next the direction of the photon, polar angle only, is selected so as to reflect the isotropic flux onto a horizontal plane. This distribution, after considering the solid angle and the projection onto the horizontal plane factors, one has:

\[ p(x) = 2x, \quad (8) \]

where \( x \) is the cosine of the polar angle. Equation (8) may be integrated to give:

\[ r = x^2 \quad (9) \]

Where \( r \) is again a member of a random sequence. Equation (9) may be easily solved for the cosine of the polar angle, \( x \), and that is what will be required in the program.

Once an energy and a polar angle for the photon is selected, it starts its migration through the tin region by having the tin
cross sections written into the active array. The photon history in the tin will be discussed first because, although, the historical sequence is the same in tin as it is in xenon, different historical facts are compiled in the two cases. The free path of this photon, the distance to where it interacts with the medium, is then computed from Equation (7) using the total cross section, \( s \), in this case for tin, and its z coordinate is updated. The z coordinate, serves as a tag to determine which medium the photon is in at any time, is checked against the cumulative thicknesses of the media and, depending upon which medium it is in, the type of interaction is selected. The selection consists of picking a random number and comparing its magnitude with the cumulative fraction of the total cross section that the various types of reactions cross sections are at that energy. To consider a particular example, let’s suppose that the total cross section is 100 while, at that same energy, the photoelectric, Compton elastic, and Compton inelastic are 80, 15, and 5 respectively. Now, if the random number selected is .54321, the interaction would be photoelectric; while, if the random number selected was .96752, the type of interaction would be Compton inelastic.

If the type of interaction turns out to be Compton elastic, a photon with the same energy is started again from this z and the polar angle is chosen uniformly over the interval 0 to \( \pi \). If, however, the interaction was a Compton inelastic interaction, a new photon of reduced energy \( E_1 \) is started at that z. The energy of the emergent photon from this reaction site is given by (Jenkins, Gould, and Gedcke, 1981):

\[
E_1 = \frac{E}{1.0 + \left(\frac{E}{511}\right)\left(1.0 - \cos(x)\right)}
\]

(10)

Its polar angle is also chosen uniformly over the interval 0 to \( \pi \). By choosing the polar angle uniformly here one deviates from reality somewhat for the higher energy photons. The Klein-Nashina formula (Jenkins, Gould, and Gedcke, 1981) should be used for the higher energy photons, which is not uniform in scattering angle but peaks in the forward direction.

Photoelectric interactions give a much wider choice of possibilities. If the photon energy is below the L-edge, it is considered absorbed in the xenon and the history ends in the tin. For energies above the L-edge but below the K-edge, only an L-vacancy could be excited in an atom when the photon was absorbed photoelectrically. The probability that an L-vacancy was formed was calculated from the "jump factors" (Jenkins, Gould,
and Gedcke, 1981) (Conde and dos Santos 1985) (McMaster, Kerr, Mallet, and Hubbell 1969) at the photon energy by comparing its magnitude with a member of the random sequence. This excited ion could then either decay radiatively, the emission of a fluorescent L photon, or it could decay non-radiatively, i.e. Auger processes. Which happened was determined by comparison of a random number and the fluorescent yield $w_{KL}$ (Bambynek, Crasemann, Fink, Frund, Swift, Price, and Rao 1972). A photon whose energy was above the K-edge could create either a K- or L-vacancy in the atom that absorbed it. Which it created was determined by comparing the magnitude of a member of a random sequence and the jump factors for the K and L edges in the same fashion that was done in selecting the type of interaction from the cross sections as was described above. If the random number's magnitude was greater than the combined probability of creating a K- added to the probability of creating an L-vacancy, the photon was considered absorbed (it really excited a higher level but we are not considering them). If a K-vacancy was formed, the probability that it decayed by radiative emission, a fluorescent K photon, or non-radiative (Auger process) was taken as the fluorescent yield $w_K$ (Bambynek, Crasemann, Fink, Frund, Swift, Price, and Rao 1972). It was then decided if the K photon emitted was a K-alpha or a K-beta using the yield ratios as the relative probability (Johnson and White 1985). In case it was a K-alpha, an excited ion was left at that position with an L-vacancy which would have to be dealt with after the K-alpha's history terminated.

In the tin region, a photon's history terminated when it was absorbed or escaped from the surface of the tin opposite the xenon i.e. was reflected back in the incident direction so as to make $z$ less than zero. The energy spectrum of the photons that leave the tin into the xenon is compiled and serves as one of the output files.

The history of the photons that enter the xenon are continued in the same fashion as they were in the tin except they are restarted at the surface of the xenon with the same energy and direction they had when they left the tin. Their paths and interactions are now governed by the xenon cross sections and the absorbed energy spectrum is compiled for them.
RESULTS AND CONCLUSION

The present version of the program provides two output files: ENGDEPXE.DAT and ENGEXTIN.DAT. ENGDEPXE.DAT is the energy deposited in the xenon region over the range of 0 to 100 KeV in 1 KeV intervals. ENGEXTIN.DAT is the photon energy spectrum that penetrates through the tin shield and enters the xenon region. This spectrum covers the energy range of 1 to 1000 KeV in 100 10 KeV intervals. Figure 5 is a plot of the incident spectrum, Equation (3), superimposed over the spectrum exiting a 3/32 inch tin shield. It may be seen that, while the tin shield is very effective in attenuating the background flux at the lower energies, the shield is effectively transparent to the higher energy photons.

Figure 6 is a plot of the spectrum emerging from the tin shield and entering the xenon region except the vertical scale is expanded to emphasise the lower energy part. It may be noted that, except for the fluorescent tin peaks, there are no photons below 60 KeV which penetrate the shield. There is only one fluorescent K peak shown on the plot because the "resolution" of 10 KeV per channel does not permit separation of the K-alpha and K-beta peaks. There appears, on close inspection, even a very small L fluorescent peak in the first channel.

Figure 7 is a plot of the spectrum of the energy deposited in the xenon region (30.0 cm @ 1 atmosphere) for the same shielding as was present in Figure 3. Here both the K-alpha and K-beta fluorescent peaks of tin are resolved and a slight background is present due to higher energy photons that Compton scatter in either the shield or the xenon itself.

Figure 8 is a plot of the background spectrum deposited in the xenon with the tin fluorescence "turned off" for several thicknesses of the tin shield. It may be noted that for 0.1 cm of tin that there is a considerable background. Increasing the thickness of tin to 0.238125 (=3/32 inch) reduces this background considerably but that additional shielding does not appear to reduce the background significantly.
REFERENCES


APPENDIX I

FORTRAN PROGRAM TO SIMULATE AN X-RAY DETECTOR
ON SSL-VAX FILE: [ELDRIDGE]PHOTRAN2.FOR
ON CRAY FILE: EADS::HHBE219.PROGRAMS.FORTRAN(TRAN2)

PROGRAM PHOTRAN2
*COMPUTES THE ENERGY DISTRIBUTION DEPOSITED IN XENON AFTER
* TRANSPORT THROUGH A TIN PLATE SHIELD.
* USES SUBROUTINES; INCPho, XSEC, MFP, XENON
* INPUT FROM KEYBOARD; TINTH, XENONTH, PRESSURE, N(number of histories)
  REAL*4 EP(50), SIGTOT(50), SIGPHO(50), SIGCCO(50), SIGCIN(50),
  2 EB(101), E, SIGT, SIGP, SIGC, SICI, X, INTENSITY(101),
  2 XSECT(2,5,50), EA(101), PHO(101)
  INTEGER*4 J
  COMMON J, EP, SIGTOT, SIGPHO, SIGCCO, SIGCIN, NXC, EB, INTENSITY,
  2 XSECT, TINTH, XENONTH, PRESSURE
  J=987654321  ! SEED THE RANDOM NUMBER GENERATOR
* READ IN FROM THE CONSOLE THE NUMBER OF PHOTONS THAT ARE TO BE DONE
  PRINT *, 'TYPE IN THICKNESS OF THE TIN IN CENTIMETERS(X.XXX)'
  READ (*,2) TINTH
  PRINT *, 'TYPE IN THICKNESS OF THE XENON IN CENTIMETERS(X.XXX)'
  READ (*,2) XENONTH
  PRINT *, 'TYPE IN THE XENON PRESSURE IN ATMOSPHERES(X.XXX)'
  READ(*,2) PRESSURE
  PRINT *, 'TYPE IN THE NUMBER OF PHOTON HISTORIES TO BE DONE(XXX)'
  READ (*,1) N
  1 FORMAT (I8)
  2 FORMAT (F12.6)
* READ IN THE CROSS SECTION DATA FROM FILES: TINXSECT.DAT, XENXSECT.DAT
  OPEN (UNIT=1, FILE='[ELDRIDGE]TINXSECT.DAT', STATUS='OLD')
  OPEN (UNIT=2, FILE='[ELDRIDGE]XENXSECT.DAT', STATUS='OLD')
  DO 50 L=I,2 XXSECT(L,M,N) L=THE ELEMENT; M=TYPE XSECT; N=ENERGY
    READ (L,1000) XSECT(L,1,1), XSECT(L,2,1), XSECT(L,3,1),
    2 XSECT(L,4,1),
    2 XSECT(L,5,1)
  NXC = INT(XSECT(L,1,1))
  DO 10 I=2,NXC  !READ IN THIS ELEMENT'S CROSS SECTIONS
    READ (L,1000) (XSECT(L,M,I), M=1,5)
  10 CONTINUE
  50 CONTINUE
  1000 FORMAT (5F13.6)
  CLOSE (UNIT=1, STATUS='KEEP')
  CLOSE (UNIT=2, STATUS='KEEP')
* SET UP THE ENERGY BINS FOR THE OUTPUT SPECTRUM

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EB(1) = 0.0
DO 20 I = 2, 101
   EB(I) = EB(I-1) + 1.0
20  CONTINUE
EA(1) = 0.0
DO I = 2, 101
   EA(I) = EA(I-1) + 10.0
ENDDO
DO 300 I = 1, N
   CALL INCPSHO(E)
      IF (E) 100, 100, 400
* IF(E .LT. 100.0) GOTO 100
   CTHETA = SQRT(RAN(J))
   SET CROSS SECTION TABLES FOR TIN
   NXC = INT(XSECT(1,1,1))
   DO IN=2,NXC
      EP(IN-1)=XSECT(1,1,IN)
      SIGTOT(IN-1)=XSECT(1,2,IN)
      SIGPHO(IN-1)=XSECT(1,3,IN)
      SIGCOCO(IN-1)=XSECT(1,4,IN)
      SIGCIN(IN-1)=XSECT(1,5,IN)
   END DO
   NXC=NXC-1
500  CALL XSEC(E, SIGT, SIGP, SIGC, SIGI)
   CALL MFPH(SIGT, PATHLEN)
   Z = Z + PATHLEN*CTHETA
   IF(Z .GT. TINTH .AND. Z .LE. (TINTH+XENONTH)) THEN
      DO III = 2, 101
         IF(E.GE.EA(III-1).AND.E.LT.EA(III)) PHO(III-1)=PHO(III-1)+1.0
      ENDDO
      CALL XENON(E, Z, CTHETA)
      GOTO 1100
   ELSE IF (Z .LE. 0.0 .OR. Z .GT. (TINTH+XENONTH)) THEN
      GOTO 1100
   ELSE
      CONTINUE
   END IF
* PHOTON IS STILL INSIDE THE PLATE; SELECT THE TYPE OF INTERACTION
* SET CROSS SECTION TABLES FOR TIN
   NXC = INT(XSECT(1,1,1))
   DO IN=2,NXC
      EP(IN-1)=XSECT(1,1,IN)
      SIGTOT(IN-1)=XSECT(1,2,IN)
      SIGPHO(IN-1)=XSECT(1,3,IN)
      SIGCOCO(IN-1)=XSECT(1,4,IN)
      SIGCIN(IN-1)=XSECT(1,5,IN)
END DO
NXC=NXC-1
200   X = RAN(J)
    Y = SIGP/SIGT
    W = SIGC/SIGT
    V = SIGI/SIGT
    IF (0.0 .LE. X .AND. X .LT. Y) GOTO 600
    IF (Y .LE. X .AND. X .LT. (Y+W)) GOTO 700
* PICK ANGLE (NOT -- KLEIN-NASHINA)!COMPTON INELASTIC
800   THETA = 3.1415927*RAN(J)
    CTHETA= COS(THETA)
* COMPUTE NEW ENERGY DEPENDS ON ANGLE
  E = E/(1.0+(E/511)*(1.0-CTHETA))
GOTO 500
* PHOTOLECTRIC INTERACTION
600   IF(E .LT. 3.929) GOTO 1100
    IF(E .LT. 29.2) GOTO 704
* BOTH K AND L VACANCIES ARE ENERGETICALLY POSSIBLE
    X = RAN(J)
    IF(X .LT. 0.84556) GOTO 701
    IF(X .LT. 0.949529) GOTO 702
* GOT NEITHER K NOR L VACANCY THIS TIME; START NEW PHOTON
GOTO 1100
* GOT AN L VACANCY; DID IT DECAY BY RADIATIVE TRANSITION?
702   X = RAN(J)
    IF(X .GT. 0.064) GOTO 1100
* YES IT WAS A RADIATIVE TRANSITION; ENERGY OF 3.44 KEV
    E = 3.44
    GOTO 700
* GOT A K VACANCY; DID IT DECAY BY RADIATIVE TRANSITION?
701   X = RAN(J)
    IF(X .GT. 0.859) GOTO 1100
* YES, IT WAS A RADIATIVE TRANSITION; K-ALPHA OR K-BETA??
    X=RAN(J)
    IF(X .LE. 0.78) THEN
      E = 25.2   ! IT WAS A K-ALPHA
      KL = KL + 1 ! CREATES AN L VACANCY
      ELSE
      E = 28.5
      END IF
    GOTO 700
* ONLY AN L VACANCY WAS ENERGETICALLY POSSIBLE
704   X = RAN(J)
    IF(X .GT. 0.6732) GOTO 1100
* GOT AN L VACANCY; DID IT DECAY BY RADIATIVE TRANSITION?
GOTO 702
IX-14
* ALSO COME TO HERE FOR ELASTIC COMPTON EVENTS

700 THETA = 3.1415927*RAN(J)
CTHETA = COS(THETA)
GOTO 500

* PHOTON TRAVERSED THE PLATE; START NEXT PHOTON AT THE SURFACE

* FIRST, HOWEVER, ARE THERE ANY L VACANCIES LEFT??

1100 IF(KL .NE. 0) THEN
    KL = KL - 1 ! FINISH OFF THE L-VACANCIES CREATED EARLIER
GOTO 702
END IF

* GO BACK AND START ANOTHER PHOTON

100 Z = 0.0
300 CONTINUE

* UNLESS; OF COURSE, FINISHED; WRITE THE SPECTRUM TO A FILE

OPEN (UNIT=1, FILE='[ELDRIDGE]ENGDEPXE.DAT', STATUS='OLD')
OPEN (UNIT=2, FILE='[ELDRIDGE]ENGEXTIN.DAT', STATUS='OLD')
DO 40 L = 1, 100
  NORMALIZE INTENSITY; FACTOR OF 26.0 FROM ENERGY DISTRIBUTION
  INTENSITY(L) = 26.0*INTENSITY(L)/FLOAT(N)
  PHO(L) = 26.0*PHO(L)/FLOAT(N)
  WRITE (1,900) EB(L), INTENSITY(L)
  WRITE (2,900) EA(L), PHO(L)
40 CONTINUE
CLOSE (UNIT=1, STATUS='KEEP')
CLOSE (UNIT=2, STATUS='KEEP')

* THEN, AT LAST, STOP!!!!!
STOP
END

SUBROUTINE INCPHO(E)

* SELECTS ENERGY OF PHOTON FROM BACKGROUND DIST.
* RETURNS A VALID ENERGY FOR A PHOTON IF SUCCESSFUL; HISTORY BEGINS
* RETURNS 0 FOR PHOTON ENERGY IF NOT SUCCESSFUL; END OF HISTORY

REAL*4 EP(50), SIGTOT(50), SIGPHO(50), SIGCCO(50), SIGCIN(50),
      2     E, SIGT, SIGP, SIGC, SICI, X
INTEGER*4 J
COMMON J, EP, SIGTOT, SIGPHO, SIGCCO, SIGCIN, NXC

* RANDOM ENERGY UNIFORM 0 TO 1000 KEV
X=RAN(J)
IF(X .EQ. 0.0) X=RAN(J)
E=1.0/X**1.031

* DISREGARD PHOTONS WITH: 1 KEV > ENERGY > 1000 KEV
IF(E .LT. 1.0 .OR. E .GT. 1000.0) THEN
    E = 0.0

IX-15
RETURN
* SUCCESS; SO ACCEPT THIS ENERGY; DO PHOTON HISTORY
ELSE
RETURN
END IF
END

SUBROUTINE MFP(SIGT, PATHLEN)
* PICKS THE PHOTON PATH LENGTH FROM THE PROPER DISTRIBUTION
* WHEN FURNISHED THE TOTAL CROSS SECTION
REAL*4 EP(50), SIGTOT(50), SIGPHO(50), SIGCCO(50), SIGCIN(50),
2 E, SIGT, SIGP, SIGC, SICI, X
INTEGER*4 J
COMMON J, EP, SIGTOT, SIGPHO, SIGCCO, SIGCIN, NXC
X = RAN(J)
PATHLEN = -(1.0/SIGT)*ALOG(X)
RETURN
END

SUBROUTINE XSEC(E, SIGT, SIGP, SIGC, SIGI)
* INTERPOLATES TABLES OF CROSS SECTIONS VERSUS ENERGY
* WHEN FURNISHED WITH AN ENERGY FROM 1 TO 1000 KeV
* TOTAL, PHOTOELECTRIC, COMPTON ELASTIC, COMPTON INELASTIC ALL RETURNED
REAL*4 EP(50), SIGTOT(50), SIGPHO(50), SIGCCO(50), SIGCIN(50),
2 E, SIGT, SIGP, SIGC, SICI, X
INTEGER*4 J
COMMON J, EP, SIGTOT, SIGPHO, SIGCCO, SIGCIN, NXC
IF (E .LT. EP(1) .OR. E .GT. EP(NXC)) THEN
* IS ENERGY OUT OF RANGE ?
GOTO 500
* RETURN TO CALLING ROUTINE
ELSE
* INTERPOLATE TABLES TO GET X-SECTIONS FOR THAT ENERGY
* CASE THE ENERGY IS EXACTLY THE FIRST POINT IN THE TABLE
IF(E .EQ. EP(1)) THEN
SIGT=SIGTOT(1)
SIGP=SIGPHO(1)
SIGC=SIGCCO(1)
SIGI=SIGCIN(1)
GOTO 500
END IF
* CASE IF THE ENERGY FALLS EXACTLY ON ANY POINT IN THE TABLE
DO 10 K=2,NXC
  IF (E .EQ. EP(K) .AND. EP(K) .EQ. EP(K+1)) THEN
    SIGT=SIGTOT(K+1) !AT A POINT; AN EDGE
    SIGP=SIGPHO(K+1)
    SIGC=SIGCCO(K+1)
    SIGI=SIGCIN(K+1)
    GOTO 500
  ELSE IF (E .EQ. EP(K)) THEN !AT A POINT IN TABLE
    SIGT=SIGTOT(K)
    SIGP=SIGPHO(K)
    SIGC=SIGCCO(K)
    SIGI=SIGCIN(K)
    GOTO 500
  ELSE
    * OTHERWISE INTERPOLATE TO GET CROSS-SECTIONS AT THAT ENERGY
    IF (E .GT. EP(K-I) .AND. E .LT. EP(K)) THEN
      X=LOG(E/EP(K-I))/LOG(EP(K)/EP(K-I))
      SIGT=EXP(LOG(SIGTOT(K)/SIGTOT(K-I))*X+LOG(SIGTOT(K-I)))
      SIGP=EXP(LOG(SIGPHO(K)/SIGPHO(K-I))*X+LOG(SIGPHO(K-I)))
      SIGC=EXP(LOG(SIGCCO(K)/SIGCCO(K-I))*X+LOG(SIGCCO(K-I)))
      SIGI=EXP(LOG(SIGCIN(K)/SIGCIN(K-I))*X+LOG(SIGCIN(K-I)))
    END IF
    END IF
  END IF
  I0 CONTINUE
END IF
500 RETURN
END

* SUBROUTINE XENON(E, Z, CTHETA)
* REAL*4 EP(50), SIGTOT(50), SIGPHO(50), SIGCCO(50), SIGCIN(50),
  2 EB(101), E, SIGT, SIGP, SIGC, SICI, X, INTENSITY(101),
  2 XSECT(2,5,50), ETOT
* INTEGER*4 J
COMMON J, EP, SIGTOT, SIGPHO, SIGCCO, SIGCIN, NXC, EB, INTENSITY,
  2 XSECT, TINTH, XENONTH, PRESSURE
* SET CROSS SECTION TABLES FOR XENON
NXC = INT(XSECT(2,1,1))
DO IN=2,NXC
  EP(IN-1)=XSECT(2,1,IN)
  SIGTOT(IN-1)=XSECT(2,2,IN)*PRESSURE
  SIGPHO(IN-1)=XSECT(2,3,IN)*PRESSURE
  SIGCCO(IN-1)=XSECT(2,4,IN)*PRESSURE
  SIGCIN(IN-1)=XSECT(2,5,IN)*PRESSURE
END DO
NXC = NXC - 1
Z = TINTH
ETOT = 0.0
500 CALL XSEC(E, SIGT, SIGP, SIGC, SIGI)
    CALL MFP(SIGT, PATHLEN)
    Z = Z + PATHLEN * CTHETA
    IF (Z .LE. TINTH .OR. Z .GT. (TINTH + XENOTH)) GOTO 1100
200   X = RAN(J) ! IN XENON; PICK TYPE INTERACTION
    Y = SIGP/SIGT
    W = SIGC/SIGT
    V = SIGI/SIGT
    IF (0.0 .LE. X .AND. X .LT. Y) GOTO 600 ! PHOTOELECTRIC
    IF (Y .LE. X .AND. X .LT. (Y+W)) GOTO 700 ! COMPTON ELASTIC
800   CTHETA = COS(THETA) ! ISOTROPIC (NOT VERY GOOD)
    EG = E/(1.0+(E/511)*(1.0-CTHETA)) ! NEW ENERGY DEPENDS ON ANGLE
    EE = E-EG ! THE ELECTRON ENERGY (ABSORBED)
    ETOT = ETOT + EE ! TALLY THE DEPOSITED ENERGY
    E = EG ! THE NEW ENERGY OF THE PHOTON
    GOTO 500 ! START A PHOTON FROM THIS POSITION AT NEW ENERGY

* PHOTOELECTRIC INTERACTION
600   IF(E .LT. 4.781) THEN ! ENERGY TOO LOW TO EXCITE L-VACANCY
    ETOT = ETOT + E ! THIS PHOTON ABSORBED; HIGHER LEVEL EXCITATION
    GOTO 1100 ! ANY MORE EXCITED ATOMS ???
END IF
    IF(E .LT. 34.582) GOTO 704 ! L-VACANCY ENERGETICALLY POSSIBLE
* BOTH K AND L VACANCIES ARE ENERGETICALLY POSSIBLE
    X = RAN(J)
    IF(X .LT. 0.8355) GOTO 701 ! GOT A K-VACANCY
    IF(X .LT. 0.949529) GOTO 702 ! GOT AN L-VACANCY
* GOT NEITHER K NOR L VACANCY THIS TIME; START NEW PHOTON
* THE PHOTON WAS COMPLETELY ABSORBED
    ETOT = ETOT + E
    GOTO 1100
702   X = RAN(J) ! GOT AN L-VACANCY; DID IT DECAY BY RADIATIVE TRANSITION?
    IF(X .GT. 0.1) THEN
    ETOT = ETOT + E ! NO; COMPLETELY ABSORBED
    GOTO 1100
END IF
    ETOT = ETOT + E - 4.781 ! YES; RADIATIVE
    E = 4.781 ! ENERGY EMITTED PHOTON
    GOTO 700 ! START 4.781 KEV PHOTON OUT FROM HERE
* GOT A K VACANCY; DID IT DECAY BY RADIATIVE TRANSITION?
701   X = RAN(J)
    IF(X .LT. 0.88) THEN
ETOT = ETOT + E ! NOT RADIATIVE; COMPLETELY ABSORBED HERE
GOTO 1100 ! ANY MORE EXCITED IONS???
ELSE
* YES, IT WAS A RADIATIVE TRANSITION; WAS IT A K-ALPHA OR K-BETA??
  X = RAN(J)
  IF(X .LE. 0.7667) THEN
    ETOT = ETOT + E - 34.582 ! THIS MUCH ENERGY WAS DEPOSITED
    E = 29.801 ! WHEN A K-ALPHA PHOTON WAS EMITTED
    KL = KL + 1 ! ALSO AN L-VACANCY PRODUCED
  ELSE
    ETOT = ETOT + E - 33.644 ! THIS MUCH ENERGY WAS DEPOSITED
    E = 33.644 ! WHEN A K-BETA PHOTON WAS EMITTED
  END IF
END IF
GOTO 700
704 X = RAN(J) ! ONLY AN L-VACANCY WAS ENERGETICALLY POSSIBLE
  IF(X .GT. 0.6527) THEN
    ETOT = ETOT + E ! NO L-VACANCY, HIGHER LEVELS??
    GOTO 1100 ! ANY MORE EXCITED IONS??
  END IF
GOTO 702 ! L-VACANCY WAS PRODUCED; BUT WAS IT RADIATIVE??
* ALSO COME TO HERE FOR ELASTIC COMPTON EVENTS
700 THETA = 3.1415927*RAN(J)
  CTHETA = COS(THETA)
GOTO 500
* PHOTON TRAVERSED THE XENON REGION; START NEXT PHOTON
* BUT FIRST; ARE THERE ANY L VACANCIES LEFT IN THE XENON??
1100 IF(KL .NE. 0) THEN
    E = 4.781
    KL = KL - 1
    GOTO 702
END IF
CALL BINIT(ETOT)
RETURN
END

SUBROUTINE BINIT(E)
REAL*4 EP(50), SIGTOT(50), SIGPHO(50), SIGCCO(50), SIGCIN(50),
  2 EB(101), E, SIGT, SIGP, SIGC, SICI, X, INTENSITY(101),
  2 XSECT(2,5,50), ZL(500)
INTEGER*4 J
COMMON J, EP, SIGTOT, SIGPHO, SIGCCO, SIGCIN, NXC, EB, INTENSITY
* SELECT THE PROPER BIN FOR THIS PHOTON
DO 30 K = 2, 101
IF (E .GT. EB(K-1) .AND. E .LE. EB(K)) THEN
    INTENSITY(K-1) = INTENSITY(K-1) + 1.0
ELSE
    CONTINUE
END IF
30 CONTINUE
RETURN
END
Figure 2
MONTE CARLO CALCULATION OF INCIDENT FLUX

Figure 3
ANALYTIC CALCULATION OF INCIDENT FLUX
INCIDENT AND EMERGENT SPECTRA

Figure 5
ENERGY FLUX LEAVING THE 0.3 CM TIN SHIELD

Figure 6