EVOLUTION OF MIDPLATE HOTSPOT SWELLS: NUMERICAL SOLUTIONS

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ABSTRACT

We simulate numerically the evolution of midplate hotspot swells on an oceanic plate moving over a hot upwelling mantle plume. The plume supplies a Gaussian-shaped thermal perturbation and thermally-induced dynamic support. The lithosphere is treated as a thermal boundary layer with a strongly temperature-dependent viscosity. We consider the two fundamental mechanisms of transferring heat, thermal conduction and convection, during the interaction of the lithosphere with the mantle plume. The transient heat transfer equations, with boundary conditions varying in both time and space, are solved in cylindrical coordinates using the finite difference ADI method on a 100 X 100 grid. The topography, geoid anomaly, and heat flow anomaly of the Hawaiian swell and the Bermuda rise are used to constrain the models. Our results confirm the conclusion of previous work that the Hawaiian swell cannot be explained by conductive heating alone, even if extremely high thermal perturbation is allowed. On the other hand, the model of convective thinning predicts successfully the topography, geoid anomaly and the heat flow anomaly around the Hawaiian islands, as well as the changes in the topography and anomalous heat flow along the Hawaiian volcanic chain. The model constrains the Hawaiian plume to have a half-wavelength of about 500 km, a center heat flux 5-6 times the background value, and a convective current velocity 3-9 times that of the background convective current. Comparatively, the mantle plume for the Bermuda rise is much weaker. Conductive heating is probably the dominant mechanism for the evolution of the Bermuda rise.
INTRODUCTION

Midplate hotspots are usually surrounded by broad shallow sea floor which departs from the topography predicted by the standard cooling model. These anomalous regions are known as hotspot swells. Examples include the Hawaiian swell, the Bermuda rise and the Cape Verde rise. Most profiles of hotspot swells are approximately of Gaussian shape, with peak amplitudes 1-2 km higher above background and wavelengths of 500-2000 kilometers. Hotspot swells are typically associated with anomalous heat flow (8-10 mW/m²) and positive geoid anomalies of 8-8 m at the crests [Crough, 1978, Haxby and Turcotte, 1978, Detrick et al., 1981, Von Herzen et al., 1982, Detrick et al., 1986]. The study of hotspot swells is important in terms of understanding the evolution of oceanic lithosphere, as hotspot swells comprise up to 30-50% of the total oceanic lithosphere [Crough, 1983], and the formation of hotspot swells is probably the second most important cause of vertical movement of oceanic plates, behind only subsidence from the spreading centers. Another less appreciated but very important reason of studying hotspot swells lies in their genetic relationship to mantle plumes. It is generally believed that hotspot swells are the product of lithospheric interaction with mantle plumes. Observations on hotspot swells, such as heat flow anomaly, topography and geoid anomaly, can be regarded as "filtered" (by the lithosphere) information about the mantle plume beneath. Modelling of the processes of the formation of hotspot swells may help us to put constraints on the structure of the mantle plumes and shed light on the dynamics of the mantle.

Among the various models for the formation of hotspot swells, dynamic support [Morgan, 1971, McKenzie et al., 1980, Parsons and Daly, 1983; Watts et al., 1986] and base reheating [Detrick and Crough, 1978, Menard and McNutt, 1982] are the most popular ones. Most models are based on observations of the Hawaiian swell. Recent investigations along the Hawaiian volcanic chain show that the heat flow changes systematically, from near normal at Hawaii to about 25% higher than the background value at Midway [Von Herzen et al., 1982, Detrick et al., 1981]. This delay in appearance of the heat flow anomaly, as well as the good match of the
subsidence of the swell to the prediction of simple cooling models, strongly supports the models of thermal origin. Geoid anomalies, which indicate isostatic compensation in the lower part of the lithosphere, also favor such models [Crough, 1978, Watts, 1980, Crough and Jarrard, 1981, McNutt and Shure, 1986]. The difficulty with these models has been the mechanism of heating. Although thermal conduction is successful in explaining the evolution of the Hawaiian swell after heating, it is almost impossible to produce more than 1000 meter uplift in a few million years as is observed in the Hawaiian swell by purely conductive heating. In most thermal models the problem of uplifting rate is either totally ignored or bypassed [Detrick and Crough, 1978, Detrick et al., 1986]. Our understanding of the structure of the mantle plume underneath the plate is even more rudimentary.

In this work we present the results of numerical simulation of the evolution of midplate hotspot swells on an oceanic plate moving over a hot mantle plume. The transient heat transfer equations, with the lower boundary conditions varying in both time and space, are solved in cylindrical coordinates. The Hawaiian swell and the Bermuda rise are used as examples because of the well-documented data on heat flow measurements, geoid anomaly and topography [Von Herzen et al., 1982, Detrick et al., 1986]. The questions we attempt to answer in this work are: 1) what are the mechanisms of heat transfer in the formation of hotspot swells? and 2) what can we learn about the physical structure of the mantle plumes, such as their temperature and velocity fields? In the following sections we will first present the numerical scheme. We will then discuss the formation of the Hawaiian swell and the Bermuda rise separately. We will show that convective thinning of the lithosphere may play an important role in the Hawaiian case, while the formation of the Bermuda rise is more likely dominated by thermal conduction due to relative weakness of the mantle plume.
NUMERICAL MODELS

Figure 1 shows the geometry of the problem considered in this work. An oceanic plate moves with constant speed over an axisymmetric upwelling mantle plume, which introduces a Gaussian shaped thermal perturbation to the base of the plate. In cylindrical coordinates, the transient heat transfer equation is

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (k r \frac{\partial T}{\partial r}) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (k \frac{\partial T}{\partial \phi}) + \frac{\partial}{\partial z} (k \frac{\partial T}{\partial z}) + \frac{\partial H}{\partial t} \quad (1)$$

where $T = T(r,z,\phi,t)$ is the temperature field, and $\rho$, $C_p$, and $k$ are the density, heat capacity and heat conductivity of the lithosphere, respectively. $H$, the rate of the heat production within the lithosphere, is negligible in oceanic plates for the short time scales we are considering. Assuming constant conductivity and an axisymmetric heat transfer of the lithosphere, equation (1) reduces to

$$\frac{\partial T}{\partial t} = K \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \quad (2)$$

This is an approximation, because within a moving plate the heat transfer is not exactly axisymmetric even with an axisymmetric plume. Here $K = k/\rho C_p$ is the thermal diffusivity. Fig. 1B shows a section of the plate cut perpendicular to its direction of motion. Due to the symmetry of the problem, only half of the section needs to be studied. It is assumed that initially the thermal perturbation is zero and the plate is at thermal equilibrium. Since both the Hawaiian swell and the Bermuda rise are older than 80 million years, this is a reasonable assumption. Boundary conditions are shown in Fig. 1b. The temperature at the Earth's surface is set at zero for all time. The vertical boundary at $r = 0$ (i.e., the axis of the swell) is adiabatic due to the symmetry of the problem. The right boundary is also adiabatic since it is far enough from the heat source that the lateral thermal gradient there proves to be negligible. The lower boundary condition, which varies in both time and space as the plate rides over the
plume, is where most of the complexity arises. In this work we treat the lithosphere as a thermal boundary layer where viscosity is strongly temperature dependent. We then consider two fundamental mechanisms of transferring heat into the lithosphere. In the first case heat is transferred into the plate mainly by conduction. The initial lithospheric material is neither removed nor significantly stretched. Therefore "thinning of lithosphere" occurs only in the sense of uplifting the 1350°C isotherm which marks the initial plate-asthenosphere boundary. In other words the heat flux at the lower boundary is only a function of \( r \) and \( t \): \( q = q(r, t, z=L) \). We will refer to this case as the model of pure conduction. The second case assumes a strong mantle plume. The plume heats the plate and is able to remove the lithospheric material as soon as its temperature reaches the mean temperature of the plume. The lithosphere is therefore mechanically thinned and the lower boundary heat flux is a function of \( r, t, z \): \( q = q(r, t, z) \). We will refer to this case as the model of convective thinning.

Equation (2) with various boundary conditions is solved using the finite difference alternating direction implicit (ADI) method. The grid size used in this work is 100 X 100. The ADI method is unconditionally stable with respect to time step-length. Convergence of the numerical scheme was demonstrated by attainment of equilibrium in a few iterations after a small thermal perturbation. The models are constrained by the observed topography, heat flow and geoid anomaly, which are all related to the thermal structure of the lithosphere and the mantle plume underneath. At the long wavelength involved, we can safely ignore flexural effects in the lithosphere. Under the assumption of isostasy, the topographic uplift can be expressed by

\[
h = \frac{\rho_m}{\rho_m - \rho_w} \int_0^{z_e} \alpha \Delta T(z, r, t) \, dz
\]  

(3)

where \( \rho_m \) and \( \rho_w \) are the density of the mantle and the sea water, respectively, \( \Delta T \) is the thermal perturbation, \( \alpha \) is the volumetric thermal expansion coefficient, and \( z_e \) is the depth of isostatic compensation, i.e., above \( z_e \) each column has the same amount of mass.
Theoretically, \( z_c \) should be at the depth of the plume source region, which we do not know. In previous reheating models \( z_c \) is usually taken as the initial depth of the lithosphere [Mareschal, 1981, McNutt, 1986]. In dynamic support models, the dominant supporting force is the thermally induced buoyancy force and the formula for calculating the topography is similar to equation (3), except that \( z_c \) is taken to be 700 km, the assumed depth of upper mantle convection [Parsons and Daly, 1983, McKenzie et al., 1980]. The key point to notice here is that the topography, geoid anomaly and anomalous heat flow result mainly from the temperature perturbation, as one can see from equation (3), as well as equations (4) and (5) in the following. Because most of the temperature perturbation is concentrated at the lower part of the lithosphere, (or more precisely, near the base of the conductive lid: see Parsons and Daly, [1983] and Courtney and White, [1986]), the difference between the models of dynamic support and lithospheric heating is probably minor in terms of accounting the thermal contributions to the surface topography, geoid anomaly and anomalous heat flow. In this study we take \( z_c \) at 90 km, the depth of the initial base of the lithosphere. The reason of choosing such a initial lithospheric thickness is to keep other parameters, such as the conductivity, base temperature and the background heat flux, in a self-consistent system. Other reasonable choices may alter some details of this work, but is not likely to change the major conclusions.

The isostatic geoid anomaly is given by

\[
N = - \frac{2\pi G}{g} \int_0^{z_c} z \Delta \rho(r,z,t) \, dz \quad [\text{Turcotte and Schubert, 1982}]
\]

\[
= - \frac{2\pi G}{g} \alpha_1 \int_0^{z_c} z \Delta T(r,z,t) \, dz \quad (4)
\]

where \( G \) is Newton's constant and \( g \) is the gravitational acceleration, \( \rho_1 \) is the reference density of the lithosphere (i.e., without thermal perturbation). The surface heat flux is calculated as
q (r,t,z=0) = k \frac{\Delta T(r,t)}{\Delta z} \tag{5}

where \( \Delta T(r,t) \) and \( \Delta z \) are the temperature and distance between the top two rows of the numerical grid.

In the following two sections we will discuss the formation of the Hawaiian swell and the Bermuda rise separately. Physical parameters used in these models are summarized in Table 1.

**HAWAIIAN SWELL**

The Hawaiian swell is one of the best studied midplate swells. The broad area of shallow ocean floor surrounding the Hawaiian islands was first reported by Deitz and Menard [1953]. The Hawaiian-Emperor seamount chain, which extends 3500 km across the central Pacific Ocean, was suggested to be the result of the oceanic plate moving over a melting point or mantle plume [Wilson, 1963, Morgan, 1972]. Most swell models were developed in an attempt to explain the formation of the Hawaiian swell [Walcott, 1970, Watts, 1976, Detrick and Crough, 1978]. The delayed rise in the heat flow, which is near normal (~52±2 mW/m²) around Hawaii but increases to about 25% higher than background at Midway, as well as the downstream subsidence of the swell [Detrick et al., 1981, Von Herzen et al., 1982], leaves little doubt that the swell is of a thermal origin. The difficulty with the thermal models has been the rapidity with which the topographic expression of the swell establishes itself. The distance of the southwest front of the Hawaiian swell to the swell crest near Oahu is about 600 kilometers. With the Pacific plate moving at 99 mm/y, the ocean floor is required to uplift more than 1000 meters in 6-7 million years. Such a uplift rate is too high for conductive base heating to explain, since the thermal conductivity of the lithosphere is very low. Just increasing the strength of mantle plume is unlikely to solve the problem. In one trial solution to the conductive heating problem we put a strong heat source beneath a stationary plate which
produces an initial perturbing heat flux 20 times higher than the background, yet less than 500 meter uplift is predicted by the conductive heating model after 6 million years of heating. The reason for this is that high heat flux across the asthenosphere-plate boundary is very difficult to sustain with time, as one can see from Fig. 2. The perturbing heat flux dropped to about half of its initial value in only 0.1 million year and down to only about 200 mW/m² after one million year. Furthermore, such a high thermal gradient would require the temperature of the lower lithosphere to be as high as 2000°C or more. It is unlikely that the lower lithosphere can sustain such high temperature without both melting and flowing.

In our models the time of beginning of heating is taken when the plate first meets the edge (defined as \( \tau \)) of the Gaussian shaped heat source. It should take a few million years to produce a swell obvious in seafloor topography, about 200 meters. Therefore the heating time in our model should be a few million years longer than that measured from the edge of swells. This, however, can not ease the difficulty associated with the pure conductive heating model. Some kind of convective heating in the lower lithosphere seems necessary.

The problem of uplift rate can be easily solved using the model of convective thinning. Some of the results are presented in Figure 3. Although the topography, geoid anomaly and heat flux around the Hawaiian islands may be explained by various models, the successful models should also explain the changes of topography and heat flow anomaly downstream along the Hawaiian volcanic chain [Crough, 1983]. Fig. 3 shows that the model of convective thinning not only explains the rapid uplift, but also matches well the observed changes of topography and heat flow anomaly after the plate moves off the hotspot. The half wavelength of the heat source used here is 500 kilometers, with the maximum heat flux 5 times higher than the background value. The mean temperature of the plume is taken as 1550°C. Curve A is based on half space cooling, i.e., heat only diffuses to the earth's surface after the plate moves off the plume. Curve B considers the fact that the mantle plume which removes the lower lithospheric material and fills in the space is hotter than the surrounding mantle. Thus
after the plate moves off the plume, heat diffuses not only to the earth's surface, but also to the surrounding mantle. Both curves match the observation well, and Curve B fits slightly better. (The two peaks of the observed topography are in the regions disturbed by seamounts and fracture zones. Hence the data there may not be representative.) It was noticed during the numerical experiments that the modeled heat flow anomalies are always lower than those observed at the young part of the swell, but higher than those observed at the old part of the swell, as one can see in Figure 3. The modeled topography has the same problem though it is less obvious. Similar problems are seen in previous thermal models \[\text{[von Herzen et al., 1982]}\].

The best explanation for this is probably the neglect of magmatic activity in these models. Certain amounts of intrusive sills and dikes are undoubtedly associated with the formation of hotspot swells. This would increase the heat input at the young part of the swell and explain the higher heat flow and topography than the predicted values. As a consequence of this, the heat is brought to the surface more quickly and results in lower heat flow and topography than predicted for the old part of the swell. Figure 4 shows the evolution of the thermal structure of the lithosphere. Profiles are perpendicular to the propagation direction. The heat flux across the lower boundary of the plate at various times is shown at the bottom. Heat is transferred into the lithosphere by the convective current of the mantle plume, which sweeps away the lithospheric material as soon as its temperature reaches the mean temperature of the plume. At time = 9 m.y., more than half of the predicted surface heat flow anomaly, topography and geoid anomaly are contributed by the replacement by the mantle plume. The lithosphere is continuously thinned until the plate moves off the plume. The swell is then dominated by conductive cooling. The perturbing heat in the plume and asthenospheric material that have filled in the lower lithosphere diffuses to the earth's surface as well as to the surrounding mantle, as shown by the negative heat flux in Fig. 4c. The extra heat supplied by the plume will eventually diffuse away and a new thermal equilibrium will be reached. Notice that the thermal gradient is concentrated near the lithosphere-plume boundary during the
interaction. The elastic thickness, usually defined by isotherms between 450°C and 650°C [Watts, 1978, Turcotte and Schubert, 1982, p. 340], is hardly reduced during the formation of the swell.

The geoid anomaly of the Hawaiian swell is +6-8 m near the crest and correlates with topography [Crough, 1978]. The modeled value fits well to the Hawaiian data (Fig. 5). The curvature of the geoid anomaly-topography correlation reflects the fact that the lithosphere is thinned more at the center of the swell than at its margins. Compensation depth at the center predicted by this model is 60-70 km, which agrees with previous works [Crough, 1978, McNutt, 1986]. Figure 6 shows the contour maps of topography, geoid anomaly and heat flow anomaly of the Hawaiian swell predicted by this model. This may be used as a reference for further field investigations. The predicted residual topography is quite comparable to the Hawaiian swell.

The size of the heat source is mainly constrained by the wavelength of the hotspot swell. The mean temperature of the plume and the heat flux it supplies are largely constrained by the topography, surface heat flow anomaly and geoid anomaly. What else can we learn about the plume? One important parameter for studying the dynamics of the plume is its velocity, which may be estimated from this model. Heat transfer across the boundary of a convective fluid and a solid satisfies Newton's law of cooling:

\[ q = h (T_p - T_s) \] (6)

In our case \( q \) is the heat flux across the lower boundary of the lithosphere, \( T_p \) is the mean temperature of the plume current adjacent to the lithospheric base and \( T_s \) is the temperature at the base of the lithosphere, \( h \) is the mean convective heat transfer coefficient, which is a function of the velocity of the plume current relative to the plate. In equation (5) \( q \) and \( \Delta T = (T_p - T_s) \) are constrained in this swell model. The plume velocity may then be estimated from \( h \). There may be two ways to do so, depending on how we approximate the geometry of
the heat transfer problem. If we treat the plume-plate heat transfer as that between a convective current flowing adjacent to a flat plate, \( h \) is related to the plume velocity through two nondimensional numbers: the Nusselt number, \( \text{Nu} = hL/k_p \), and the Reynolds number, \( \text{Re} = VL/\nu \). \( k_p \) is the conductivity of the plume, \( L \) is the characteristic length, \( V \) is the horizontal velocity of the plume relative to the plate, and \( \nu \) is the viscosity of the plume. In this case the Nusselt number can be written as [Incropera and Dewitt, 1985, pp. 293]

\[
\text{Nu} = 0.664 \text{Re}^{1/2} \text{Pr}^{1/3}
\]

where \( \text{Pr} = \frac{K}{\nu} \) is the Prandtl Number. Rewriting \( \text{Re} \) and \( \text{Nu} \) in terms of \( V \) and \( h \), we have

\[
h = \left[ \frac{k_p}{L} 0.664 \left( \frac{V}{\nu} \right)^{1/2} \text{Pr}^{1/3} \right] \sqrt{V}
\]

Referring to (5), we see that the strength of the mantle plume is proportional to the square root of its velocity relative to the plate. Alternatively we may treat the plume-plate heat transfer as that between a free convective fluid and a cold upper plate. In this case the Nusselt number can be written as [Incropera and Dewitt, 1985, p. 395]

\[
\text{Nu} = 0.54 \text{Re}^{1/4}
\]

where \( \text{Ra} \) is the Rayleigh number: \( \text{Ra} = \frac{\alpha g \Delta T L^3}{K \nu} \). We can define a reference velocity \( V_c \), \( V_c = (g\alpha \Delta T L)^{1/2} \), for the free convective fluid. \( (V_c \) is the maximum velocity that may be expected for free convection. It neglects viscous effects and assumes that the input thermal energy is transformed completely to kinetic energy. (See Jaluria, [1980]. Notice that \( V_c \) is vertical here.) Rewriting equation (9) in terms of \( V_c \), we have

\[
h = \left[ 0.54 \left( \frac{L}{\nu} \frac{1}{\sqrt{K}} \right)^{1/4} \right] \sqrt{V_c}
\]

It is very similar to equation (8) and notice that we have once again the square-root relationship between the strength of the mantle plumes and their velocities.

If we know the parameters in the bracket of (8), the radial velocity of the plume can be
calculated from (8) and (5). Or we may estimate the horizontal velocity of the plume adjacent to the plate-base relative to the background convection, by assuming similar physical properties for the plume and the surrounding mantle and comparing heat fluxes introduced to the base of the plate. In this case we have

\[
\frac{V_p}{V_m} = \left[ \frac{-q_m(T_m - T_p)}{q_m(T_p - T_s)} \right] \quad \text{(11)}
\]

where \(V_p, q_p\) and \(V_m, q_m\) are the velocity and heat flux of the plume and background current, respectively. \(T_p\) and \(T_m\) are the mean temperatures of the plume current and the background convective current, respectively. If we assume \(T_m\) is about 50-100 °C higher than the temperature at the lithospheric base, the velocity of the Hawaiian plume is estimated to be 3-10 times higher than that of the background convection near the center of the plume. We should mention that our estimation is based on the heat transfer between a plume current flowing adjacent to a steady plate. Moving of the plate would complicate the flow pattern. However, since the plume current flows radially at the base of the plate, the general effect of the motion of the plate would be increasing the heat transfer on the upstream part of the plume (where the plume current flows in the direction opposite to the moving plate), and does the opposite on the downstream side of the plume. The total effect is probably minor.

BERMUDA RISE

The Bermuda rise is a broad topographic swell in the North Atlantic. It is about 900 km long and 500 km wide, approximately delineated by the 5000 m depth contour \([\text{Detrick et al., 1986}]\). The characteristics of the Bermuda rise include residual topography of 800-1000 meters at the peak, associated with a geoid anomaly of 6-8 m and a heat flow anomaly of 8-10 mW/m². These features are very similar to those of the swells surrounding recent volcanic islands, such as Hawaii, and may indicate a similar origin as that of Hawaiian swell, despite the lack of recent volcanic activity \([\text{Crough, 1983, Detrick et al., 1986}]\). To model the formation of the Bermuda rise is, however, more difficult than the Hawaiian case. Firstly we do not know
the timing of the development of the swell. Secondly there is no clear volcanic chain to show the topography and heat flow changes with time. Most of the history of the Bermuda rise is deduced from studies of the sediments. The major constraints include [Tucholke and Mountain, 1979, Reynolds and Aumento, 1974]: 1) Initial uplift of the swell in middle Eocene (45-50 Ma); 2) Volcanism during late-middle Eocene to early Oligocene (33-43 Ma); 3) Submergence of the volcanic island started in late Oligocene (>25 Ma); 4) Total subsidence since then is no more than 100-200 meters. Geoid anomaly and heat flow data were compiled by Detrick et al. [1986].

Although the poor quality of the Bermuda data does not allow us to model the detailed process of its evolution, it is possible to put some constraints on the major mechanism and the heat source responsible for its formation. It is obvious from the topography and heat flow anomaly of the Bermuda rise that the heat source under the lithosphere at Bermuda must be much weaker than that of the Hawaiian swell. The plate moves only 15mm per year here compared with 99mm/y at Hawaii. The same heat source as that of the Hawaiian swell would cause enormously high heat flow and uplift here. Detrick et al. [1986] have suggested that the strength of the heat source is linearly proportional to the plate velocity, and that the different developing histories of the Hawaiian swell and the Bermuda rise are mainly attributed by the different plate velocities. We have shown in the last section that 1) the strength of the heat source is proportional to the square root of the relative velocity between the plume current and the plate, and 2) since the mantle plume adjacent to the plate base flows radially, movement of the plate would increase heat flux upstream and decrease it downstream. The total effect is probably minor. Besides, it is unlikely that the difference of the developing histories of these two swells can be explained solely by the relative strength of the two plumes without invoking further considerations of the difference in the heat transfer mechanisms. To prove this, we have tried one extreme case, i.e., to assume that the effect of plate motion is only to increase the heat flux across the lower boundary of the plate. Under this assumption,
the Bermuda plume should be $2.6 (=\sqrt{99/15})$ times weaker than that of Hawaiian swell. The results produced by the convective thinning model are presented in Figure 7 (curve A). The squares in Fig. 7 represent the constraints on the history of the Bermuda rise [Tucholke and Mountain, 1979, Reynolds and Aumento, 1974]. The lower limits of these squares are largely arbitrary, but the upper limits are well constrained. Since the present uplift is 800-1000 meters, and since no more than 200 meter subsidence has occurred, the maximum uplift of Bermuda rise was likely no more than 1300 meters. It is clear from Figure 7, curve A, that both the predicted uplift and the subsidence are too large. The heat flow anomaly predicted by this model is even worse. It is about 40 mW/m², 4 times of the observed value! A simple reduction of the strength of the heat source cannot solve the problem. The fact that less than 100-200 m of subsidence occurred in the last 25 million years indicates that the plate was still heated for most of that period, otherwise more than 500 meter subsidence would be predicted by the half-space cooling model, in which case a heat flow anomaly higher than 20 mW/m² would be predicted by the convective thinning model, even when the mean temperature of the plume current is set as low as that of the surrounding mantle.

The Bermuda data clearly indicate a slow heating process with a thick conductive lid. It is interesting to see that the model of pure conduction, which failed to explain the Hawaiian swell, may play a dominant role in the formation of the Bermuda rise. Curve B in Figure 7 is one result of conductive heating. The heat source introduces heat flux about two times higher than the background value, and the half wavelength is 350 km. The predicted uplift satisfies the constraints quite well. The predicted geoid anomaly is $\sqrt{6.5}$ meters at the peak, in the observed range of 6-8 meters. The heat flow anomaly predicted by this model is about 14 mW/m², slightly higher than the 8-10 mW/m² observed value. The compensation depth predicted by this model is 55-70 km. Figure 8 compares the modeled and observed heat flow anomaly patterns across the Bermuda rise. The predicted maps of the residual topography, heat flow anomaly and geoid anomaly for a slow moving plate are shown in Figure 9. The contours
are more circular than those of a fast moving plate (see Figure 6). This is due to a greater contrast of the heating history for the different parts of the fast moving plate. Mantle plumes swept by the moving plate would also cause similar phenomena, but this problem is more complicated and beyond the scope of this work. (For a simple kinematic model for this problem, see Sleep, [1987]).

**DISCUSSION**

We have shown that both conductive heating and convective thinning play important roles in the evolution of hotspot swells. The mechanism of heat transfer at the base of lithosphere depends on the strength of the mantle plume. The Hawaiian swell is probably produced by a strong mantle plume with a flow rate 3-9 times higher than the background convection. This plume heats and removes the material of the lower lithosphere, producing a rapid uplift of more than 1000 meters in 6-10 million years. The plume under the Bermuda rise is undoubtedly weaker than that of the Hawaiian swell. Thermal conduction is the dominant mechanism of transferring heat in this case. This, of cause, does not exclude the possibility of some small degree of convective thinning. A summary of the mantle plume parameters constrained in this work are presented in Table 2.

If the different strength of the mantle plumes indeed reflect the intrinsic properties of various plumes instead of some superficial results of plate velocities, then this information may bear important implications for mantle dynamics. There are good reasons to believe that mantle plumes may come from different depths, have various sources regions and, probably, have different histories. Studies of geochemistry and isotopic chemistry have revealed a large range of variations among hotspot-related basalts [e.g., White, 1985, Kurz et al., 1982]. Through combined studies of chemical and physical parameters of mantle plumes, we may be able to shed some light on the formation and the source region of various mantle plumes through inverse modeling.
One of the questions that may arise in connection with this work is whether the plume current can effectively remove the lithospheric material. Emerman and Turcotte's [1983] model of stagnation flow suggests that the rate of convective thinning should be too low to account for the rapid uplift of the Hawaiian swell. Yet models of thermal thinning indicate that mantle plume can supply enough heat to thin the lithosphere effectively [Spohn and Schubert, 1982]. Models in which viscosity depends on both temperature and pressure would allow more heat to be transported than does a purely temperature-dependent rheology [Fleitout and Yuen, 1984], and the second-order, small-scale convection that may be induced by the mantle plume will significantly increase the rate of convective thinning of the lithosphere [Yuen and Fleitout, 1985]. Despite the debate which may continue until a better knowledge of the physical properties of the lower lithosphere and upper mantle is available, the Hawaiian data clearly indicate that some kind of convective heating must be responsible for its formation. One alternative mechanism is magmatic injection [Withjack, 1979]. The rate and magnitude of the magma injections are not clear, but they are not likely to be significant. Otherwise widespread volcanism would be expected, instead of the narrow limitation of volcanism to the center of volcanic islands. The thermal stress field induced by base heating does not favor the development of fractures which are necessary for the extensive magmatic intrusion. Nonetheless, a certain amount of magmatic intrusion is certainly associated with the formation of hotspot swells, and this mechanism may improve the fit of our predictions to the observations. Other mechanisms, such as flexural deflection, are probably only of minor importance. The maximum elastic fore-bulge due to the loading of volcanic island is less than 500 meters. Heating by the mantle plumes does not significantly affect the elastic thickness of the plate during the formation of the hotspot swells.

CONCLUSIONS

Our numerical models have confirmed that midplate hotspot swells are mainly produced by
the thermal interaction between the lithosphere and the mantle plumes. Conclusions we can draw from this work include:

1) Both thermal conduction and thermal convection may play important roles in the evolution of hotspot swells. The mechanisms of heat transfer at the base of the lithosphere depend on the strength of the mantle plumes. A strong plume may thin the lithosphere convectively and cause rapid uplift, while in the case of a weak mantle plume thermal conduction will dominate the heat transfer.

2) The Hawaiian swell is likely caused by the interaction of a fast moving plate with a strong mantle plume. Convective thinning of the lithosphere is an important mechanism for its formation. As the plate moves off the plume, the space created by removing the lithospheric material is filled with hot plume and asthenospheric material. The heat then diffuses away through conduction, causing the observed pattern of subsidence and heat flow anomaly along the Hawaiian volcanic chain. The perturbing heat flux is 5-6 times higher than the the background value near the center of the plume. The radial velocity of the plume adjacent to the base of the plate is 3-9 times greater than that of the background convection. The compensation depth is 60-70 km.

3) The Bermuda rise is probably the result of a slow moving plate heated by a relatively weak mantle plume. The plume is too weak to remove lithospheric material effectively. Conduction is the dominant mechanism of transferring heat into the plate. The radial velocity of the plume is about 2 times higher than that of the background convection. The maximum perturbing heat flux is 2-4 times that of the background value. The compensation depth is similar to that of the Hawaiian swell.
REFERENCES


Walcott, R. I., Flexure of the lithosphere at Hawaii, Tectonophysics, 9, 435-446, 1970.


FIGURE CAPTIONS

Figure 1: a) Geometry of the problem studied in this work, in which an oceanic plate moves over a upwelling mantle plume. b) A section of the lithosphere cut perpendicular to its direction of motion. The transient heat transfer in this domain satisfies equation (2). $Z$ is taken positive downwards. $r = 0$ is the axis of the swell. Also shown here are the boundary conditions.

Figure 2: Decrease of heat flux across the base of a conductive lithosphere with time after an initial thermal perturbation. The distance $s$ is taken from the center of the plume.

Figure 3: Comparison of the predicted (solid lines) and observed evolution of the residual topography and heat flow anomaly along the crest of the Hawaiian volcanic chain. Data are adopted from Crough [1978] and Von Herzen et al. [1982]. The time is set at zero when the plate first meets the edge (defined as $2a$) of the mantle plume. Curve A is based on half space cooling, while curve B also considers the heat diffusion to the surrounding mantle.

Figure 4: Evolution of the thermal structure of the lithosphere as the oceanic plate moves over the Hawaiian mantle plume. Heat flux across the base of the conductive lid is shown at the bottom. Profiles are perpendicular to the propagation direction. Time is defined as in Fig. 3. a) Time = 3 my, the early stage of the swell formation. b) Time = 9 my, when the plate is about to move off the plume. c) Time = 11 my, plate has moved off the plume. Perturbing heat diffuses to the earth's surface as well as to the surrounding mantle. d) Time = 20 my, heat gradually diffuses to the upper part of the lithosphere, and significant surface heat flow anomalies are expected.

Figure 5: Comparison of the predicted (solid line) and the observed correlation of the geoid anomaly and the uplift of the Hawaiian swell. Data are adopted from Crough [1978].

Figure 6: Maps of the predicted residual topography (a), anomalous heat flow (b) and the geoid anomaly (c) of the Hawaiian swell. Time is defined as in Fig. 3.
Figure 7: Predicted uplift history of the Bermuda Rise. Curve A is produced by the model of convective thinning. Curve B is the result of the conductive heating model. The squares present the constraints deduced from the studies of the sediments [Reynolds and Aumento, 1974; Tucholke and Mountain, 1979]: 1. initial uplift in middle Eocene; 2. volcanism in late Eocene to early Oligocene; 3. the volcanic island started to submerge in late Oligocene. The solid bar represents the present residual topography of the Bermuda Rise. See the text for further discussion.

Figure 8: Comparison of the predicted (solid lines) and the observed anomalous heat flow across the Bermuda Rise. The curve A corresponds to a mantle plume of maximum perturbing heat flux of 90 mW/m² and half-wavelength 400 km. The heat source for curve B has a maximum perturbing heat flux of 110 mW/m², half-wavelength 350 km. Distance is taken from the center of the Bermuda Rise. Data are adopted from Detrick et al. [1986].

Figure 9: Maps of the predicted residual topography (a), heat flow anomaly (b) and the geoid anomaly (c) of the Bermuda Rise. Time is defined as in Fig. 3.
Table 1. Parameters used in our models

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definitions</th>
<th>Values</th>
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<tr>
<td>L</td>
<td>Initial lithospheric thickness</td>
<td>90 km</td>
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<tr>
<td>d</td>
<td>Length of the plate section solved for in equation (2). See Fig. 1b</td>
<td>700 km</td>
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<tr>
<td>k</td>
<td>Thermal conductivity</td>
<td>3.3 W/m°K</td>
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<td>ρₔ</td>
<td>Density of sea water</td>
<td>1050 kg/m³</td>
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<tr>
<td>ρₘ</td>
<td>Mantle density</td>
<td>3300 kg/m³</td>
</tr>
<tr>
<td>ρ₁</td>
<td>Reference density of lithosphere</td>
<td>3300 kg/m³</td>
</tr>
<tr>
<td>α</td>
<td>Volumetric thermal expansion coefficient</td>
<td>$3.5 \times 10^{-6}/°K$</td>
</tr>
<tr>
<td>K</td>
<td>Thermal diffusivity</td>
<td>$8 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$</td>
</tr>
</tbody>
</table>
Table 2. Summary of the mantle plume properties constrained in this work

<table>
<thead>
<tr>
<th>Properties</th>
<th>Hawaiian</th>
<th>Bermuda</th>
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<tbody>
<tr>
<td>Anomalous heat flux</td>
<td>250–300 mW/m²</td>
<td>90–120 mW/m²</td>
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<td>Half wavelength</td>
<td>450–550 km</td>
<td>300–350 km</td>
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<td>Mean temperature of plume (Tp)</td>
<td>~1550–1600°C</td>
<td>~1450–1500°C</td>
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<tr>
<td>Radial velocity of plume relative to background convection ($V_p/V_m$)</td>
<td>3–10</td>
<td>2–4</td>
</tr>
<tr>
<td>Convective thinning?</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

* heat flow and velocity values are those at the center of the swells.
Fig 1.

\[
\tau(r, t, z=0) = 0
\]

\[
\frac{\partial T}{\partial r}igr|_{r=0} = 0
\]

\[
\frac{\partial T}{\partial r}igr|_{r=p} = 0
\]

\[
\frac{\partial T}{\partial \tau}igr|_{\tau=0} = 0
\]

\[
\frac{\partial T}{\partial \tau}igr|_{\tau=p} = 0
\]

\[
\Theta = \alpha \nabla^2 T
\]

\[
q_b = q_b(r, t, z)
\]
L.B. HEAT FLUX VS. TIME

DISTANCE (KM)
Fig. 3.
Time = 3 M.Y.
Fig. 4. b>

Depth (km)

Distance From Swell Axis (km)

L.B. Heat Flux (mW/m²)

Time = 9 M.Y.
Time = 11 M.Y.
Fig. 4. d

**Graph 1:**
- **Y-axis:** Depth (Km)
- **X-axis:** Distance (km)
- Lines indicate depth at various points.

**Graph 2:**
- **Y-axis:** L.B. Heat Flux (mW/m²/m)
- **X-axis:** Distance (km)
- Trend line showing heat flux variation.

**Text:**
- Time = 20. M.Y.
Fig. y

(a)

(b)

(c)

Distance from axis (km)

Time (x 0.1 My.)