ABSTRACT

The Doppler tracking method is currently the only technique available for broadband gravitational wave searches in the $10^{-4}$ to $10^{-1}$ Hz "low-frequency" (LF) band. In this paper I give a brief review of the Doppler method, a discussion of the main noise sources, and a review of experience with current spacecraft and the prospects for sensitivity improvements in an advanced Doppler tracking experiment.

I. RESPONSE OF DOPPLER LINK TO GRAVITATIONAL WAVES

The Doppler link between the earth and a distant spacecraft (thought of here as two free test masses separated by distance L) measures their relative dimensionless velocity $\Delta v/c = \Delta f/f_o = y$ as a function of time, where $\Delta f$ is the perturbation in the Doppler frequency and $f_o$ is the nominal radio frequency of the link. An incident gravitational wave of strain amplitude $h$ causes small perturbations in the tracking record. These perturbations are of order $h$ in $\Delta f/f_o$ and are replicated three times in the Doppler data (Estabrook and Wahlquist 1975). The sum of the Doppler perturbations of the three pulses is zero; pulses with duration longer than $-\Delta f/f_o$ produce overlapping responses in the tracking record and the net response cancels to first order. The system has a passband to gravitational excitation: the low-frequency band edge is set by pulse cancellation to $-c/L$, while thermal noises limit the high-frequency response to $\sim 1/30$ sec.

II. NOISE SOURCES

The main noise sources in spacecraft gravitational wave experiments are briefly summarized in this section. Schematic spectra of these sources are plotted in Figure 1. Spectra of actual data are given, e.g., in Armstrong, Woo, and Estabrook (1979), Hellings et al. (1981), Anderson et al. (1984), Anderson and Mashhoon (1985), and Armstrong, Estabrook, and Wahlquist (1987). Transfer functions of the noises to the observable have been summarized by Armstrong (1988), along with signal processing techniques to exploit the differences between signal and noise signatures.

At frequencies higher than $1/30$ sec, thermal noise, mainly from finite signal-to-noise ratio on the downlink, dominates. This noise has a power spectrum of fractional frequency $\Delta f/f_o$ going as (Fourier frequency)$^2$. At lower frequencies, propagation noise and instrumental instability are important. Propagation noise results from radiowave phase scintillations imposed by irregularities in the media between earth and spacecraft (troposphere, ionosphere, solar wind). Charged particle scintillations (ionosphere and solar wind) dominate current generation (S-band radio link $- f_o = 2.3$ GHz) experiments (Wahlquist et al. 1977; Woo and Armstrong 1979). Plasma scintillation reaches a broad minimum in the antisolar direction to $\Delta f/f_o = (3 \times 10^{-15})$ (8.4 GHz/link radio frequency)$^2$. Plasma scintillation data have a "red" spectrum: $S_y \sim \text{(frequency)}^{-0.7}$.
Water vapor fluctuations dominate tropospheric scintillation at microwave frequencies (Hogg et al. 1981; Resch et al. 1984; Treuhaft (this volume)), although fluctuations in the "dry component" (Shannon et al. 1979) may be important in future experiments. The index of refraction of tropospheric irregularities is independent of radio frequency (at microwave wavelengths), so that their level in $\Delta f/f_0$ is also independent of radio frequency. At the high elevation angles relevant to a gravity wave track, the effect, although highly variable, is typically $S_y \sim 10^{-25}$ (f/0.001 Hz)$^{-0.4}$ Hz$^{-1}$ (Armstrong and Sramek 1982).

A fundamental low-frequency noise is instrumental instability (including clock noise), signal distribution instability, transmitter and receiver instability, mechanical stability of the antenna, spacecraft transponder stability, etc. Because the Doppler method is a "one-armed interferometer," frequency stability in the Doppler link is fundamental to achieving good sensitivity. The ground system aspects are discussed by Kursinski (this volume), and should enter at $\sim 5 \times 10^{-15}$ for Galileo-era experiments.

Nongravitational forces (examples are spacecraft buffeting, leaking thrusters, irregularities in the spacecraft spin rate for Doppler measurements using circularly polarized signals) are noise sources. In the Galileo-era the most important of these can be calibrated and removed with engineering telemetry to a level less than the propagation and instrumental noise levels.

### III. CURRENT SENSITIVITY AND FUTURE PROSPECTS

Current generation long-duration experiments are limited by plasma scintillation noises to $1\sigma$ sensitivities $\sim 5 \times 10^{-14}$ for bursts, $\sim 1.5 \times 10^{-14}$ for broadband searches for sinusoids, and $S_y \sim 10^{-23}$ Hz$^{-1}$ for the spectral density of a background. Using selected short-duration data sets with very low plasma noise (but still apparently plasma-limited, as evidenced by dual-frequency downlink data) sensitivities can be much better; see, e.g., Figures 1 and 2 of Hellings et al. (1981). These levels of sensitivity can be compared with wave amplitudes at the earth from plausible sources (Thorne 1987; Wahlquist 1987; and Wahlquist (this volume)).

In the Galileo-era, X-band uplink will reduce plasma noise to parts in $10^{15}$ for bursts. For X-band uplink experiments, plasma noise, uncalibrated tropospheric scintillation noise, and station stability enter at comparable levels. Galileo will have lower noise levels and smaller resolution bandwidths than S-band experiments, allowing $\langle 1\sigma \rangle$ sensitivity to sinusoids of $\sim 3 \times 10^{-16}$.

Increasing the radio frequency to, say, K-band ($\sim 32$ GHz) uplink or using multifrequency links to isolate the plasma noise, gravitational wave observations can provide very high immunity to plasma noise and very sensitive gravitational wave experiments. If flight-qualified precision timekeeping becomes practical, then the possibility of onboard extraction of one- and two-way Doppler, separately, offers improved ways to discriminate gravity wave and noise signatures — see, e.g., Vessot (this volume). To fully exploit the plasma noise immunity of a K-band link would require improved timekeeping on the ground and precision tropospheric monitoring; at these levels there may also be important impacts on the quality of the spacecraft transponder. Calibration of both the wet and dry troposphere to yield residuals smaller than than $\sim 5\%$ of the total would also be required to reduce residual...
tropospheric noise to a level comparable with the plasma noise. Instrumental stability at $10^{-16}$ would also be required. Such a system ($f_0 \approx 32$ GHz, precision tropospheric monitor, high instrumental stability, high SNR radio links) could, for long tracking arcs, have $(1\sigma)$ sensitivity at $3 \times 10^{-17}$ for sinusoids.

IV. CONCLUSION

Spacecraft Doppler experiments in the Galileo-era will have substantial sensitivity improvements over the current-generation (S-band uplink) prototypes. With improvements such as higher radio frequency links, high instrumental stability on the ground and in the spacecraft, very high signal-to-noise ratio radio links, and precision tropospheric monitoring, sensitivities $3 \times 10^{-17}$ for sinusoidal waves appear possible. Improvements to sensitivities significantly better than this are, I think, impractical for observations with one station on the earth and using only the two-way Doppler observable. The difficult problems of (1) tropospheric monitoring (wet and dry components) at these levels, (2) frequency standard stability, and (3) low-level systematic errors in reliably removing the "known" motion of the station at these levels, will play roles. Sensitivity improvement in the LF band, significantly better than the levels discussed here, will likely require moving all the test masses into space and using interferometric techniques.

ACKNOWLEDGMENTS

I thank F. B. Estabrook and H. D. Wahlquist for valuable discussions on all aspects of the Doppler gravitational wave method. This work was performed at the Jet Propulsion Laboratory, under contract with NASA.

REFERENCES

Kursinski, E. R. "High-Stability Radio Links" (this volume).
Treuhaft, R. "Tropospheric Monitoring Technology" (this volume).
Wahlquist, H. D. "Detecting Gravity Waves from Binary Black Holes" (this volume).
FIG. 1.—Spectra of fractional frequency fluctuations, $S_y(f)$, versus Fourier frequency, for the main noise sources in the LF band. Current generation experiments have S-band radio links and are plasma noise limited. Galileo-class experiments (X-band up- and downlinks) will have substantially reduced plasma noise; these will be limited by some combination of the X-band plasma noise, unmonitored troposphere, and station stability. Advanced experiments involving, say, K-band (32 GHz) radio links, $-10^{-16}$ station stability, very high signal-to-noise ratio radio links, and precision tropospheric monitoring, could reach sensitivities $-3 \times 10^{-17}$ for sinusoidal waves.
DISCUSSION

SHAPIRO: Has the problem of isolation of the transmitter from the receiver (for the X-band uplink and downlink) been solved or is it intended to use two antennas at each site, one for the transmitter and the other for the receiver?

ARMSTRONG: Galileo-era X-band gravitational wave experiments will be supported by the DSN’s new 34-meter high-efficiency antennas, with both transmission and reception at the same antenna. An X-band uplink/downlink capability similar to that of the 34-meter antennas is also being planned for the 70-meter network, which could then be used with the 34-meter antennas for simultaneous two-spacecraft coincidence experiments in the 1990’s.

SCHUMAKER: How well correlated are the plasma induced phase fluctuations at S- and X-band frequencies—i.e., how immune to plasma noise is a dual frequency microwave system? Can this be translated into an equivalent single higher-frequency (e.g., an optical frequency) for which the $1/f^2$ plasma noise would be as small?

ARMSTRONG: The leading contributor of plasma noise at opposition is the solar wind, which is to a good approximation collisionless and only very weakly magnetized. From cold plasma theory, refractive index squared is given by $n^2 = [1-(f_p/f)^2]$, where $f_p$ is the plasma frequency (~30kHz for the near-earth solar wind). Since the plasma frequency is so small compared with the radio frequency, the $1/f^2$ leading term is essentially “exact”. Subtle complications (e.g., geometric optics paths at the two frequencies not quite the same, imperfect plasma correlation because of different Fresnel zone sizes, collisional and magnetized plasma effects) are potential problems, but should enter at sensitivity levels well below those discussed in this paper.