Fundamental Aspects of and Failure Modes in High-Temperature Composites

Christos C. Chamis and Carol A. Ginty
Lewis Research Center
Cleveland, Ohio

Prepared for the
35th International SAMPE Symposium and Exhibition
Anaheim, California, April 2–5, 1990
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Christos C. Chamis and Carol A. Glinty
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

Fundamental aspects of and attendant failure mechanisms for high-temperature composites are summarized. These include: (1) in-situ matrix behavior, (2) load transfer, (3) limits on matrix ductility to survive a given number of cyclic loadings, (4) fundamental parameters which govern thermal stresses, (4) vibration stresses and (5) impact resistance. The resulting guidelines are presented in terms of simple equations which are suitable for the preliminary assessment of the merits of a particular high-temperature composite in a specific application.

INTRODUCTION

NASA is currently involved with several programs such as the National Aerospace Plane and the High Speed Civil Transport which will challenge the current state of technology in both materials and structures. To meet the aggressive goals set forth in these programs, high-temperature materials, including metal matrix composites (MMC) and ceramic matrix composites (CMC), are being investigated. The high-temperature nonlinear behavior of these classes of materials is very complex with limited observed characteristics (experimental data) to base a design upon.

As a result, an attempt has been made to identify the fundamental aspects and variables that will affect the high-temperature behavior of these materials. Of primary influence to the composite response is the behavior of the constituents and their interactions with each other. In particular, attention is given to the thermal properties - coefficient of thermal expansion (CTE), thermal conductivity (K), and heat capacity (C) - as well as the mechanical properties: modulus of elasticity (E), shear modulus (G), Poisson's ratio (ν), and strength (S). In addition, other factors such as density (ρ) and fiber volume ratio (FVR) also play a role in the behavior of these materials. The picture is further complicated in that these properties are directional, are changing continuously with temperature, stress, and time, and are dependent upon the fabrication process.

Therefore, the task of identifying the fundamental characteristics and failure modes in high-temperature composites is accomplished by applying fiber composite principles, suitable math models, and acceptable approximate analysis methods to discuss the effects of parameters such as fiber shapes, tensile strength, and matrix ductility. Critical issues are fracture toughness, impact energy, cyclic loads, and thermal stresses. In summary, it is hoped that the simple equations presented will constitute a set of guidelines to conduct a preliminary assessment of the merits of a particular high-temperature composite for a given application. For convenience of reference, the equations are presented in chart form with appropriate schematics. The notation used in the
equations is not uniform, but it is evident from the schematic and the context of each chart.

SIMPLIFIED COMPOSITE MICROMECHANICS

Application of the simple composite mechanics concepts (refs. 1 and 2) leads to the observation that a matrix has a negligible effect on composite longitudinal tensile strength and that fiber fracture is the dominant fracture mode. However, the matrix may control the longitudinal compressive strength, especially at high temperatures. In the high-temperature case the compressive strength will be significantly less than the tensile. The governing equations and respective schematic are summarized in figure 1. Note the equation for the modulus is also included in the summary. The matrix contribution will also be negligible when the matrix is strained to respond nonlinearly. Combinations of temperature and nonlinear effects will degrade the longitudinal compressive strength substantially.

FIBER SHAPES

Elementary considerations of fiber/matrix load transfer lead to the conclusion that circular cross-section fibers require the shortest length to develop the full stress in the fiber. However, in the case of an incomplete interfacial bond, irregular shapes can be selected that can develop the full stress in the fiber within the same length as circular fibers under complete bond. The governing equations and respective schematics are summarized in figure 2. As will be described in a later section, the length of the fiber to transfer the load is also application dependent. For example, composites for impact resistance benefit from longer lengths while static tensile load applications benefit from shorter lengths.

STATISTICAL-LONGITUDINAL TENSILE STRENGTH

The critical length \( l_{cr} \) is an important parameter in evaluating the load transfer at the interface and, thereby, incorporating the statistical variables that influence longitudinal tensile strength (ref. 3). Application of elementary shear-lag theory explicitly relates \( l_{cr} \) to constituent material properties and their respective ratios in the composite. The governing equations and a representative schematic are shown in figure 3. The parameter \( \phi \) is a ratio of the stress transferred in the fiber compared to the fully developed stress. It is given by \( \phi = \sigma_{fI}/k_f S_f T \) and at fracture \( \phi = \sigma_{fI}/S_f T \). Ideally this ratio should be almost 1.0. The most significant parameter in the \( l_{cr} \) equation is \( G_m \), which is the shear modulus at the interface usually taken as that of the matrix or coating. In cases where there is a lack of interfacial bond, \( G_m \approx 0 \), \( l_{cr} \) is infinite. For this case the longitudinal composite modulus \( (E_{lI}, \text{fig. 1}) \) is equal to that of the matrix with holes. For any composite (polymer, metal, or ceramic matrix), if the longitudinal composite modulus is approximately equal to that predicted by the rule of mixtures, then complete load transfer takes place at the interface. This indicates that \( G_m \neq 0 \), and \( l_{cr} \) is relatively small. One way to verify this is to leach out the matrix of fractured specimens, measure broken fiber lengths and compare them to \( l_{cr} \). If the broken lengths are substantially larger than \( l_{cr} \), then the interface bond is poor and vice versa.
PLY MICROSTRESSES - STRESSES IN THE CONSTITUENTS

The fabrication process induces residual stresses in the constituents (ply microstresses). These can be estimated from the explicit equations summarized in figure 4 (ref. 4). Note that these microstresses: (1) can be in tension or compression, (2) depend on relative thermal expansion differences, (3) depend on the temperature change, and (4) depend on the local constituent moduli. These equations can be used to perform parametric studies and identify fiber and/or matrix thermal expansion coefficients for minimum residual stress or for assured durability at service operating conditions. One approach is illustrated in the next section where it is used to estimate the in-situ matrix ductility (strain-to-fracture) required for the composite to survive thermal fatigue without matrix cracking.

The microstress equations previously described can be used to estimate the "in-situ matrix ductility" for the matrix strain to withstand a given ΔT. Suitable equations are summarized in figure 5. This strain value is about 3 percent for the MMC-P100/Cu which is processed at about 1366 K (2000 °F). Also an estimate on the fiber CTE can be obtained. For the same composite (P100/Cu)

\[ \alpha_{f\|} \approx -1.62 \times 10^{-6} \text{ mm/mm-K} \left( -0.9 \times 10^{-6} \text{ in./in.-°F} \right) \]

or greater. By selecting ranges for \( \varepsilon_m \), comparable ranges for \( \alpha_{f\|} \) can be determined. Combinations of ranges for \( \alpha_m \) and \( \alpha_{f\|} \) can also be determined for selected \( \varepsilon_m \) values. These combinations of ranges provide guidance for material research directions. A rule of thumb is to select matrices with an in-situ fracture strain which is greater than 1.5 times the residual strain due to processing.

LOCAL (MICRO) FRACTURE TOUGHNESS

The local fracture toughness can be determined and the significant parameters identified using elementary composite mechanics with fracture mechanics concepts. The procedure is summarized in figure 6. These lead to an equation for the local strain energy release rate (G) as shown at the bottom of the figure. The significant variables in this equation are: (1) the fiber tensile strength \( S_{fT} \) and (2) the displacement \( u \). The equation indicates that the local fracture toughness is mainly due to the local elongation \( u \) of the fiber prior to fracture. This finding is in variance with the traditional belief that fiber pull-out is the most significant event. However, the fiber recession in the matrix absorbs/dissipates the energy released as individual fibers fracture.

The local fracture toughness, defined previously, can be expressed in terms of fiber parameters \( (d_f, S_{fT}) \) and interfacial bond shear strength \( (\tau) \). The equations and a numerical example summarized in figure 7 show that the energy of a single fiber breaking is quite large \( (103,327 \text{ J/m}^2 \left( 590 \text{ in.-lb/in.}^2 \right) ) \). A tough composite will sustain a relatively large number of isolated single-fiber local fractures prior to its fracture.
IMPACT: ENERGY TO FRACTURE

Elementary considerations lead to relationships to assess impact resistance and to identify dominant constituent material parameters. Since composites have fibers which are much stronger than the matrix, the matrix condition at impact is insignificant, especially at high temperatures. A word of caution: The above comments do not apply to structures designed to contain impact. The equations summarized in figure 8 include the three common combinations that bracket the three different types of composite systems: metals, ceramics, and whiskers. It is worth noting that the metal matrix composites at high temperatures behave similarly to polymer matrix composites.

CYCLIC LOADS (FATIGUE): SIGNIFICANT PARAMETERS

The significant variables influencing cyclic-load resistance are readily identified by applying mechanical vibration principles to simple structural components. Governing equations and respective schematics are summarized in figure 9. The magnitude of the cyclic stress is reduced (fatigue life increased) by decreasing the material density (ρ) and/or increasing the modulus (E). Both of these are readily obtainable with composites. Trade-off studies can then be performed to select the most suitable combination (ρ/E) for specific applications.

THERMALLY STRESSED STRUCTURES - SIGNIFICANT VARIABLES

The significant variables that influence thermal stress in a structure are identified by subjecting a panel to a uniform flux and performing a heat transfer analysis. The significant variables are observed in the resulting equation for stress in figure 10. They are modulus (E), thermal expansion coefficient (α), and thermal heat conductivity (K). Composites provide the flexibility to tailor these parameters in order to minimize thermal stresses for specific structural applications.

It is worth noting that increasing the modulus increases the mechanical vibrations fatigue life while the opposite is true for thermal fatigue. It is these competing requirements on material properties that make it appropriate, and even necessary, to consider use of formal structural tailoring methods (ref. 5) in order to select the optimum combination of material properties for a specific application.

STRUCTURAL BEHAVIOR/RESPONSE

The complex behavior of metal matrix composites at high temperatures is comprehensively evaluated using specialty purpose computer codes. Metal Matrix Composite Analyzer (METCAN) is such a computer code under development at the NASA Lewis Research Center (ref. 6). METCAN simulates the nonlinear behavior of high-temperature metal matrix composites (HT-MMC) from fabrication to operating conditions using only room temperature values for the constituent material properties while allowing for the development and growth of an interphase. METCAN is structured to be a user-friendly, portable, stand-alone computer code. It can be used to simulate laminate behavior and/or as a pre- and post-processor to general purpose structural analysis codes with anisotropic
material capabilities. The schematic in figure 11 depicts the computational simulation capability in METCAN.

The in-situ material behavior of the constituents in METCAN is modeled by using a multifactor interaction equation described in figure 12. This multifactor equation is selected to pass through a final and a reference point, subscripts F and O. The nonlinear behavior between these two points is simulated by the exponent. Final and reference values are material characteristics which are generally available, while the exponent is selected from appropriate experiments.

Typical results obtained by METCAN are summarized in table I. The results are for three different fiber volume ratios at room temperature. Comparable results are readily obtained at other temperatures and/or any other condition represented in the material model in figure 12. The results in table I illustrate how METCAN can be used to computationally characterize HT-MMC. Another application of METCAN is to identify the factors that influence composite transverse strength as is described below.

FACTORS AFFECTING GRAPHITE/COPPER METAL MATRIX COMPOSITES

TRANSVERSE STRENGTH BOUNDS

The in-situ matrix properties are more than likely to be different than those of the bulk material. The multitude of possible combinations of factors, influencing in-situ properties, have dramatic effects on composite properties (ref. 7). As can be seen in figure 13, the transverse strength can be anywhere between 14 and 152 MPa (2 and 22 ksi). The low value is indicative of a poorly made composite with no interfacial bond, while the high value represents the most optimistic strength property. Obviously, composites with low-transverse tensile strength have substantial room for improvement. Parametric studies to assess these kinds of effects and identify their respective dominant factors can be routinely performed using METCAN.

SUMMARY

Fundamental concepts and simple equations are summarized to describe the aspects and failure modes in high-temperature metal matrix composites (HT-MMC). These equations are explicit and are used to identify the dominant factors (variables) that influence the behavior of high-temperature materials.

The simple equations are in explicit form and are for: (1) strength; (2) fiber shapes; (3) load transfer, limits on matrix ductility (strain-to-fracture) to survive a given number of cyclic loadings; (4) parameters that govern thermal stresses, vibration stresses, and impact resistance; and (5) in-situ matrix behavior. These equations can be used to perform parametric studies, guide experiments, guide constituent materials research/selection and assess fabrication processes for specific applications. In addition, a computer code is briefly described which includes the integrated and interaction effects of all these factors and which can be used to computationally simulate the history of high-temperature MMC's from consolidation to specified service loading conditions. Many of the factors that influence HT-MM
in specific structural applications are generally competing and would be most effectively evaluated using structural tailoring methods.

REFERENCES


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<th>0.65</th>
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<td>-0.018x10⁻⁶ (-0.01)</td>
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<td>17.3x10⁻⁶ (9.6)</td>
<td>16.9x10⁻⁶ (9.4)</td>
<td>16.4x10⁻⁶ (9.1)</td>
</tr>
<tr>
<td>α₃₃₃, mm/mm-K (μin./in.-°F)</td>
<td>17.3x10⁻⁶ (9.6)</td>
<td>16.9x10⁻⁶ (9.4)</td>
<td>16.4x10⁻⁶ (9.1)</td>
</tr>
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<td>K₁₁₁, W/m-K (Btu-in./°F-hr-in.²)</td>
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<td>12.6 (7.3)</td>
<td>9.3 (5.4)</td>
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<td>C, kcal/kg-K (Btu/1b)</td>
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<td>0.42 (0.1)</td>
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<tr>
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<td>0.30 (0.30)</td>
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<td>v₁₂₃, mm/mm (in./in.)</td>
<td>0.27 (0.27)</td>
<td>0.25 (0.25)</td>
<td>0.24 (0.24)</td>
</tr>
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</table>
APPLY LOAD ALONG THE 1-DIRECTION
THIS INDUCES UNIFORM DISPLACEMENTS
COMPATIBILITY:
\[ E_{m11} = E_{111} = E_{111} \] (1)

FORCE EQUILIBRIUM:
\[ A_{11}^0 a_{11} = A_{11}^0 a_{11} + A_{m11}^0 a_{11} \] (2)

STRESS/STRAIN RELATIONSHIPS:
\[ \sigma_{111} = E_{111} \varepsilon_{111}; \sigma_{111} = E_{111} \varepsilon_{111}; \sigma_{m11} = E_{m11} \varepsilon_{m11} \] (3)

SUBSTITUTING EQ (3) AND EQ (1) IN EQ (2) YIELDS THE COMPOSITE MODULUS
\[ A_{11}^0 E_{111} = A_{11}^0 E_{111} + A_{m11}^0 E_{m11} \]
\[ E_{111} = k_f E_{111} + k_m E_{m11} \] (RULE OF MIXTURES)

COMPOSITE STRENGTH
\[ S_{111,c} = S_{111,c} \left( k_f + \frac{E_{m11}}{E_{111}} k_m \right) - FIBER CONTROLLED \]
\[ = S_{m11,c} \left( k_m + \frac{E_{111}}{E_{m11}} k_f \right) - MATRIX CONTROLLED \]

OBSERVATION: ASSUMING \( E_{111} \gg E_{m11} \) AT FRACTURE
- THE MATRIX HAS NEGLIGIBLE EFFECT ON COMPOSITE STRENGTH
- THE FIBER HAS SIGNIFICANT EFFECT ON COMPOSITE STRENGTH

FIGURE 1. - MICROMECHANICS CONCEPTS FOR LONGITUDINAL STRENGTH.

CYLINDER SMALLEST CIRCUMFERENCE FOR GIVEN AREA
\[ \frac{C}{A} \rightarrow CIRCULAR = \frac{4}{\pi} \]
\[ S_{111} = S_{111} \left( \frac{h_f + h_m E_{m11}}{E_{111}} \right) \]
\[ A \Delta x = C(x) dx \]
\[ A S_{11} = C \Delta x \]
\[ t_{cr} = \frac{A S_{11}}{C} \]
\[ t_{cr} < \text{CIRCULAR} = \frac{d_f}{n} \left( \frac{S_{111}}{T_{111}} \right) \]

THEREFORE:
IRREGULAR-SHAPE FIBERS HAVE SMALLER \( t_{cr} \) THAN CIRCULAR-SHAPE FIBERS

FIGURE 2. - EQUATIONS AND SCHEMATICS FOR ASSESSING FIBER SHAPE.

\[
S_{T1T} = k_{f}S_{T}^{2}\left(S_{T}f_{c}\right)^{-1/2}
\]

- \( k_{f} \): Fiber volume ratio
- \( S_{T} \): Fiber weakest link parameters
- \( f_{c} \): Mean fiber strength

**Fiber Ineffective (Critical) Length**

\[
\left( \frac{d_{c}}{d_{f}} \right) = 1.15 \left[ \frac{1 - k_{f}^{1/2}}{k_{f}^{1/2}} \left( \frac{E_{11}}{G_{m}} \right) \right]^{1/2}
\]

- \( d_{f} \): Fiber diameter
- \( E_{11} \): Fiber longitudinal modulus
- \( G_{m} \): Matrix shear modulus

**DUE TO TEMPERATURE \( \Delta T_d \)**

\[
\begin{align*}
\sigma_{m11} &= \left( \sigma_{f11} - \sigma_{m} \right) \Delta T_d E_{m} \\
\sigma_{f11} &= \left( \sigma_{f11} - \sigma_{f} \right) \Delta T_d E_{11} \\
\sigma_{m22} &= \left( \sigma_{f22} - \sigma_{m} \right) \Delta T_d E_{m} \\
\sigma_{f22} &= \left[ \sqrt{E_{f}} \left( 1 - \sqrt{E_{f}} \right) \sigma_{f} \right] E_{11} \\
\sigma_{m33} &= \left( \sigma_{m} \right) E_{m}
\end{align*}
\]

- \( E_{11} \): Fiber longitudinal modulus
- \( E_{m} \): Matrix modulus

**Equations for Polymer Matrix Composites**

**Figure 3.** Summary of equations for assessing longitudinal statistical strength.

**Figure 4.** Micromechanics equations for residual constituent stresses.
\[
\sigma_{\text{m}11} = (\sigma_{E11} - \sigma_{\text{m}11}) \Delta T \text{m}
\]

\[
\frac{\sigma_{\text{m}11}}{\varepsilon_{\text{m}}} = \varepsilon_{\text{m}11} = \left(\frac{\sigma_{E11}}{\varepsilon_{E11}} - \frac{\sigma_{\text{m}}}{\varepsilon_{\text{m}}}\right)
\]

\[
\Delta T = T_m \text{ (MATRIX MELTING OR CONSOLIDATION TEMPERATURE, WHICHEVER IS SMALLER)}
\]

\[
\sigma_{\text{m}} = 2\sigma_{\text{m}} \text{ (TO ACCOUNT FOR TEMPERATURE DEPENDENCE)}
\]

\[
\varepsilon_{\text{m}} = \frac{1}{2}\varepsilon_{\text{m}} \text{ (DITTO)}
\]

\[
\varepsilon_{\text{m}} > \frac{\varepsilon_{\text{m}11}}{4}
\]

\[
\varepsilon_{\text{m}} = 3\sigma_{\text{m}} \text{ (FOR THERMAL CYCLING INCLUDING 1.5 SAFETY FACTOR)}
\]

**ESTIMATE ON FIBER CTE**

\[
\sigma_{\text{m}11} \leq \sigma_{\text{m}} \left(\frac{k_f \varepsilon_{11} + k_m \varepsilon_{\text{m}}}{k_f \varepsilon_{11} + k_m \varepsilon_{\text{m}}\Delta T}\right)
\]

**OBSERVATIONS:**

- The in situ matrix fracture strain must be greater than 1.5 times the residual strain.
- An estimate on the CTE for the fiber can be established.

**FIGURE 5. - EQUATIONS TO ESTIMATE IN SITU MATRIX DUCTILITY FOR THERMAL FATIGUE.**

\[
\begin{align*}
\text{LOCAL ENERGY} & \quad U = \frac{1}{2} A_f S_{11} \varepsilon_{11} \\
\text{MICRO SERR} & \quad G = \frac{U}{2A_f + \frac{1}{2} \pi u c} = \frac{1}{2} \frac{A_f S_{11} \varepsilon_{11}}{2A_f + \pi u c} \\
S_{11} & \quad S_{11} = S_{11} \left(\frac{k_f + k_m}{E_{11}}\right) \varepsilon_{11} \\
\sigma_{\text{m}} & \quad \sigma_{\text{m}} = \sigma_c \frac{k_m}{k_m} - S_{11} \left(\frac{k_f + k_m}{E_{11}}\right) \\
S_{\text{m}} & \quad S_{\text{m}} = S_{11} \left(\frac{k_f}{k_m} + \frac{E_{11}}{E_f}\right) \text{ (MATRIX WILL NOT FRACTURE)} \\
S_{\text{m}} & \quad S_{\text{m}} = S_{11} \left(\frac{k_f}{k_m} + \frac{E_{11}}{E_f}\right) \text{ (MATRIX WILL FAIL)} \\
\text{THEN:} & \quad G = \frac{U}{2A_f + \pi u c + 4A_m} = \frac{1}{2} \left[\frac{A_f S_{11} \varepsilon_{11}}{2A_f + \pi u c + 4A_m}\right] = \frac{1}{2} \left[S_{11} \varepsilon_{11} \frac{1}{2A_f + \pi u c}\right] \\
\text{THEREFORE:} & \text{ THE ENERGY RELEASED IS NOT ENOUGH TO FRACTURE THE ADJACENT FIBERS WHEN ISOLATED FIBERS FRACTURE PREMATURELY.}
\end{align*}
\]

**FIGURE 6. - EQUATIONS TO ESTIMATE MICRO FRACTURE TOUGHNESS.**
WHEN THE FIBER FRACTURES: \( u = E_{cr} \)

\[
G = \frac{1}{2} A_{SfT} \left\{ \frac{S_{IT}}{t} \right\}^2 \left( \frac{1}{2} + \frac{4 A_{f}}{4 A_{f} + 4 A_{d} \frac{S_{IT}}{t}} \right) = \frac{1}{2} A_{SfT} \left( \frac{S_{IT}}{t} \right) d_{f} \frac{S_{IT}}{t}
\]

\[
G = \frac{1}{2} \frac{d_{f} S_{IT}^2}{2 T + 4 S_{IT}} = \frac{1}{4} \frac{d_{f} S_{IT}^2}{S_{IT} \left( 1 + 2 \left( T/S_{IT} \right) \right)}
\]

\[
G = \frac{d_{f} S_{IT}}{4(1 + 2 \left( T/S_{IT} \right))}
\]

OBSERVATIONS:

FOR INCREASED LOCAL FRACTURE TOUGHNESS IN THE ORDER OF SIGNIFICANT GAIN:

1. INCREASE \( S_{IT} \) (FIBER TENSILE STRENGTH)
2. INCREASE \( d_{f} \) (FIBER DIAMETER)
3. DECREASE \( T \) (INTERFACIAL BOND STRENGTH)

A NUMERICAL EXAMPLE:

\( S_{IT} = 500 \text{ ksi}; d_{f} = 0.005 \text{ in}; T = 15 \text{ ksi} \)

\[
G = \frac{0.005 \text{ in} \times 500 \text{ ksi}}{4(1 + 2(15/500))} = \frac{590 \text{ lb-in.}}{103327 \text{ J/m}^2}
\]

FIGURE 7. - SUMMARY OF EQUATIONS TO ESTIMATE THE STRAIN ENERGY FOR SINGLE FIBER Fracture.

1. \( S_{IT} \gg S_{mt} \) \( E_{f11} \approx E_{ml1} \): \( U = \frac{k_{f} S_{IT}^2}{2E_{f11}} \) (METAL MATRIX COMPOSITES AT HIGH TEMPERATURES)
2. \( S_{IT} \gg S_{mt} \) \( E_{f11} \approx E_{ml1} \): \( U = \frac{S_{IT}^2 \left[ k_{f} + k_{m} \left( E_{ml1}/E_{f11} \right) \right] \left[ k_{f} + k_{m} \left( E_{f11}/E_{ml1} \right) \right]}{2 \left[ E_{f11} \left( E_{mt1} + k_{m} E_{f11} \right) \right] \left[ E_{ml1} + k_{f} E_{f11} \right]} \) (CERAMIC MATRIX COMPOSITES)
3. \( S_{IT} < S_{mt} \) \( E_{f11} \gg E_{ml1} \): \( U = \frac{k_{f} S_{IT}^2}{2E_{f11}} \) (WHISKER REINFORCED COMPOSITES)

FIGURE 8. - SUMMARY OF EQUATIONS TO ESTIMATE IMPACT RESISTANCE IN HIGH TEMPERATURE COMPOSITES.
FOR AXIAL STRESS: \( \sigma = \frac{F \sin \omega t}{1 - \left( \frac{\omega t}{\omega_0} \right)^2} \)

FOR BENDING STRESS: \( \sigma = \frac{F \sin \omega t}{1 - \left( \frac{\omega t}{\omega_0} \right)^2} \)

OBSERVATIONS FOR FATIGUE: FOR A GIVEN FORCING FUNCTION AND GEOMETRY
- FATIGUE IS REDUCED BY:
1. DECREASE IN DENSITY
2. INCREASE IN MODULUS

FIGURE 9. - SUMMARY OF EQUATIONS TO IDENTIFY SIGNIFICANT PARAMETERS FOR CYCLIC LOADS.

\[ \sigma = \frac{k \Delta T}{A} \]
\[ \epsilon = \sigma \Delta T; \sigma = \frac{E \epsilon}{\epsilon_0} = E \sigma \Delta T \]
\[ \Delta T = \frac{Q}{kA} \]

\( \sigma = \frac{E \sigma_0}{kA}; E \left( \frac{1}{A} \right) (Q/k) \)

THERMAL DRIVING FORCE
GEOMETRY CONFIGURATION FACTOR
MATERIAL INSTANTANEOUS MODULUS

OBSERVATION: TO MINIMIZE THERMAL STRESSES FOR A GIVEN HEAT FLUX:
1. DECREASE MODULUS (E)
2. DECREASE THICKNESS (t)
3. DECREASE THERMAL EXPANSION COEFFICIENT (\( \sigma \))
4. INCREASE SURFACE AREA (A)
5. INCREASE THERMAL HEAT CONDUCTIVITY
6. INCREASE STRESS RUPTURE

FOR A FIXED GEOMETRY:
DECREASE \( \sigma \) AND \( \epsilon \), AND INCREASE \( k \)

FIGURE 10. - SIGNIFICANT VARIABLES FOR THERMAL STRESSES.
FIGURE 11. - COMPUTATIONAL SIMULATION CAPABILITY IN METCAN.

\[
\frac{M_p}{M_0} = \left[\begin{array}{c}
\frac{T_F - T}{T_F - T_0} \\
\frac{S_F - S}{S_F - S_0} \\
\frac{\sigma_F - \sigma}{\sigma_F - \sigma_0} \\
\frac{\tau_F - \tau}{\tau_F - \tau_0}
\end{array}\right] \left[\begin{array}{c}
0 \\
0 \\
0 \\
\frac{R_F - R}{R_F - R_0}
\end{array}\right] \ldots
\]

\ldots

\left[\begin{array}{c}
\frac{M_{PF} - M}{M_{PF} - M_0} \\
\frac{N_{PF} - N}{N_{PF} - N_0} \\
\frac{S_{PF} - S}{S_{PF} - S_0} \\
\frac{\tau_{PF} - \tau}{\tau_{PF} - \tau_0}
\end{array}\right] \left[\begin{array}{c}
0 \\
0 \\
\frac{R_{PF} - R}{R_{PF} - R_0}
\end{array}\right] \ldots
\]

VARIABLES:
P = PROPERTY
T = TEMPERATURE
S = STRENGTH
\sigma = STRESS
R = REACTION
N = CYCLES
T = TIME

SUBSCRIPTS:
F = FINAL/CHARACTERISTIC PROPERTY
0 = REFERENCE
M = MECHANICAL
T = THERMAL

FIGURE 12. - MULTI-FACTOR INTERACTION MODEL FOR IN SITU CONSTITUENT MATERIALS BEHAVIOR.
FIGURE 13. - COMPUTATIONALLY SIMULATED EFFECTS OF IN SITU MATRIX BEHAVIOR ON THE TRANSVERSE TENSILE STRENGTH OF A GRAPHITE/COPPER COMPOSITE AT 0.5 FIBER VOLUME RATIO.
Fundamental aspects of and attendant failure mechanisms for high-temperature composites are summarized. These include: (1) in-situ matrix behavior, (2) load transfer, (3) limits on matrix ductility to survive a given number of cyclic loadings, (4) fundamental parameters which govern thermal stresses, (5) vibration stresses and (6) impact resistance. The resulting guidelines are presented in terms of simple equations which are suitable for the preliminary assessment of the merits of a particular high-temperature composite in a specific application.