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Regional Quasigeoid Determination in Northern Germany And Comparison With GPS

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1. Abstract and Introduction

For the northern part of the Federal Republic of Germany, new quasigeoid solutions have been computed by least squares collocation and FFT techniques using point and mean gravity data, a digital terrain model, and a global geopotential model. As severe accuracy limitations for precise regional quasigeoid determination come from global model uncertainties, different geopotential models have been investigated by combining them with gravimetric data and comparing the quasigeoid heights with GPS and leveling. Optimum results have been obtained by a global model tailored to gravity data in Europe. Collocation and FFT results based on this model agree well. The comparison with GPS and leveling yields r.m.s. discrepancies of ±2 cm over approx. 400 km range.

2. Computation Method

Height anomalies have been determined for the northern part of the Federal Republic of Germany using least squares collocation and FFT techniques. The predicted height anomalies are obtained by

$$\zeta = \zeta_1 + \zeta_2 + \zeta_3 , \tag{1}$$

where ζ_1 is the influence of the spherical harmonic model, ζ_2 is the contribution from a residual terrain model (RTM), and ζ_3 is the contribution from terrestrial gravity field observations. The spherical harmonic model is used as a reference field and yields the major part of the quasigeoid, the terms ζ_2 and ζ_3 being typically less than 0.5 ... 1.0 m.

After subtracting the effect of a global geopotential model and a residual terrain model from all observations, the contribution of terrestrial gravity field observations (ζ_3) has been computed by least squares collocation and integral formulas. The main drawback of collocation, being the solution of a normal equation system with as many unknowns as the number of observations, may be overcome to a certain extent with modern vector computers (see *Denker* and *Wenzel* 1987). On the other hand, the use of integral formulas evaluated by FFT with gridded data is also possible on a mini computer, and thus making this technique very attractive from the computational point of view. The spectral computation of the disturbing potential and its functionals by FFT is based on flat-earth approximations. Thus, Stokes' integral formula may be written as a two-dimensional convolution in the form

$$\zeta_3 = \frac{1}{2\pi\gamma} s * \Delta g' , \qquad s = (x^2 + y^2)^{-\frac{1}{2}} , \tag{2}$$

where $\Delta g'$ are the reduced gravity anomalies. The convolution of the kernel function s with the data $\Delta g'$ is most easily done in the frequency domain. Using the analytical transform of s, formula (4) can be written as

$$\zeta_3 = \frac{1}{2\pi\gamma} \mathbf{F}^{-1} \left\{ \frac{2\pi}{\omega} \widetilde{\Delta g}'(u,v) \right\} , \qquad \omega = \sqrt{u^2 + v^2} , \tag{3}$$

where $\mathbf{F}^{-1}$ denotes the inverse Fourier transform, (u, v) are the frequencies and $\widetilde{\Delta g}'$ is the Fourier transform of $\Delta g'$.

3. Data Collection and Evaluation

For the determination of the long wavelength part of the earth's gravity field, the geopotential models GPM2 complete to degree and order 200 (*Wenzel* 1985), OSU86F complete to degree and order 360 (*Rapp* and *Cruz* 1986), and IFE87E2 complete to degree and order 360 (*Bašić* 1988) have been considered. The third model IFE87E2 is based on GPM2 and has been tailored to available $12' \times 20'$ mean gravity anomalies in Europe (*Bašić* 1988). The computation procedure consists of a spherical harmonic analysis of the non-global distributed differences between start model and terrestrial data, and the obtained potential difference coefficients are then added to the original coefficients resulting in an improved geopotential model fitted to the regional gravity field data.

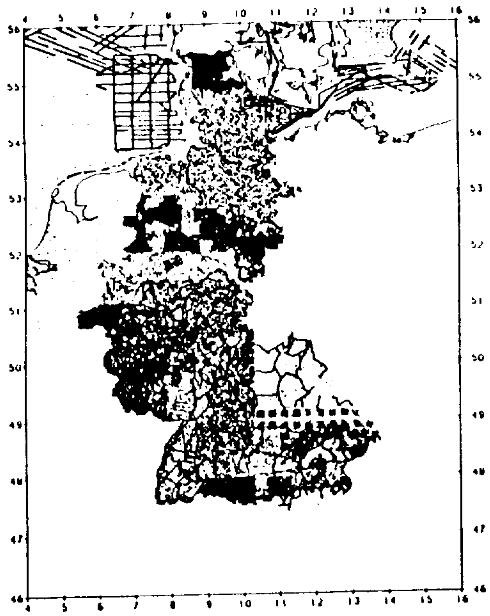

Figure 1: Distribution of Point Gravity Data

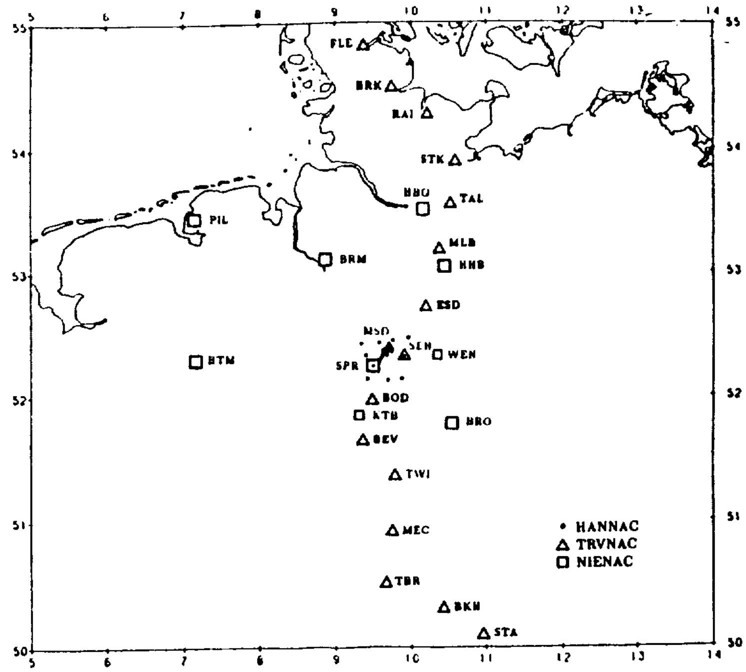

Figure 2: Distribution of GPS Stations

For the computation of terrain reductions, 30″ × 50″ mean elevations are available for the area of the Federal Republic of Germany.

Point gravity data have been extracted from the standard data base PFA, existing at IFE. Since the collected point values are located mainly in the Federal Republic of Germany (see fig. 1), additionally 6′ × 10′ and 12′ × 20′ mean gravity anomalies have been extracted from the IFE gravity data base for the evaluation of the outer zone.

All data sets have been checked carefully for gross errors by applying different procedures, for details see *Denker* and *Wenzel* (1987) and *Denker* (1988).

In addition, GPS results from three different campaigns (HANNAC, NIENAC, TRVNAC) observed with TI 4100 dual frequency receivers are available for northern Germany (see fig. 2). The internal error estimates for ellipsoidal height differences from GPS do not exceed 3 cm. The GPS coordinates are refering to the WGS84 reference system, which can be assumed to coincide with the gravimetric reference frame. The GPS stations have been connected to the national leveling network by spirit leveling to a nearby benchmark for the TRVNAC campaign and partly by the trigonometric method for the other campaigns.

4. Practical Results

In order to study the impact of different geopotential fields on gravimetric quasigeoid determinations, the three models GPM2, OSU86F and IFE87E2 have been tested by comparing the quasigeoid heights derived from GPS and leveling with values computed from these three models as well as from combination solutions with gravimetric and topographic data. For this task, the FFT method was used because of the high speed of this algorithm and the generally good agreement with corresponding collocation solutions.

For the FFT computations, the RTM-reduced gravity data were gridded in a 1.′0 × 1.′5 grid for the area 47.°5 – 57.°5 N and 3° – 15° E using point data and 6′ × 10′ mean values in areas with no point data available. The gravimetric solutions were performed in one step for the whole area of North Germany yielding a 576 × 480 grid for FFT. Due to periodicity effects of FFT, a cosine tapered window was used for the outer 10 grid points.

The topography was taken into account by a residual terrain model (RTM) reduction using a 6′ × 10′ moving average filter for the construction of the reference topography, for details see *Denker* and *Wenzel* (1987).The maximum values of the obtained RTM-effects are approx. 30 mgal for gravity anomalies, 3″ for vertical deflections resp. 8 cm for height anomalies.

Fig. 3 shows a comparison of quasigeoid heights derived from GPS and leveling for stations of the

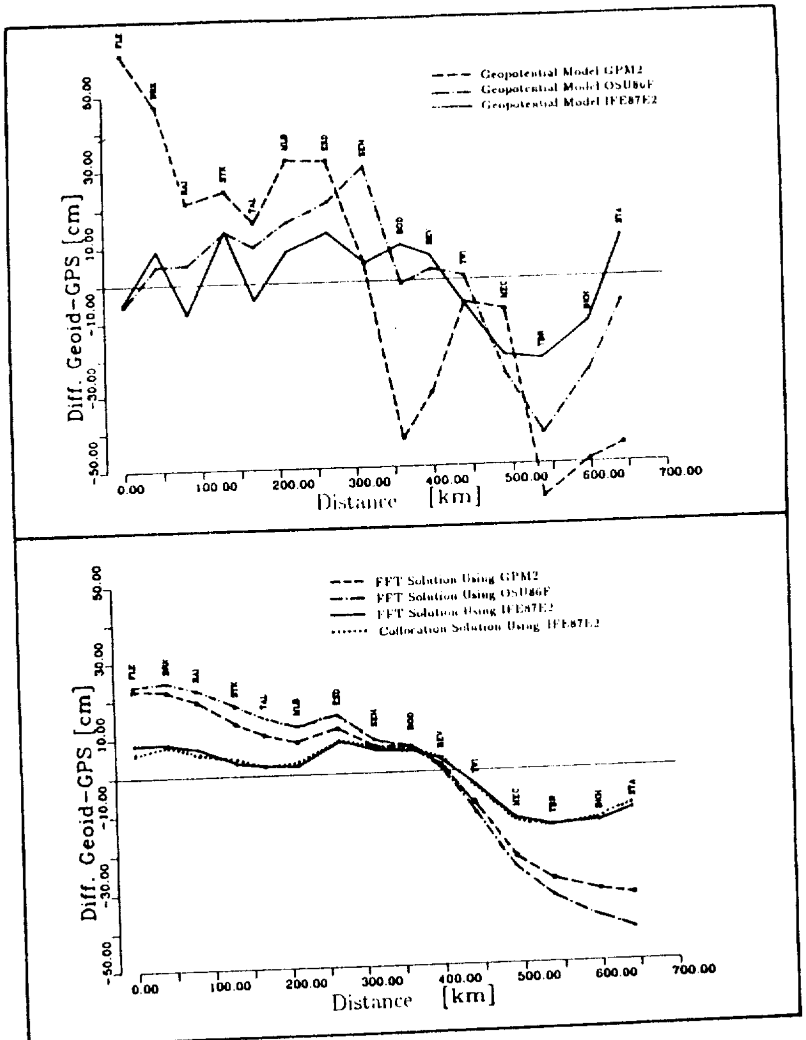

Figure 3: Comparison of GPS and Leveling With Quasigeoid Heights From Geopotential Models and From Gravimetric Combination Solutions for Stations of the GPS Traverse (TRVNAC)

Station		ζ	Differences (Bias Fit)		Differences (Bias And Tilt Fit)	
No.	Name	(Coll.)	Coll.	FFT	Coll.	FFT
1	SPR	44.228	−0.062	−0.052	−0.004	−0.001
2	WEN	43.323	−0.001	−0.010	0.074	0.054
3	BRO	46.031	−0.205	−0.156	−0.052	−0.023
4	HHB	41.414	0.046	0.038	0.033	0.021
5	BRM	41.202	0.103	0.098	0.028	0.032
6	HBG	40.729	0.036	0.036	−0.047	−0.041
7	PIL	40.971	0.157	0.138	−0.028	−0.021
8	BTM	44.224	0.059	0.043	0.027	0.022
9	KTB	45.767	−0.132	−0.134	−0.030	−0.043
R.M.S.			±0.108	±0.093	±0.040	±0.032

Table 3: Comparison of Gravimetric Height Anomalies With GPS and Leveling for the NIENAC Campaign (Units are m)

Station		ζ	Differences (Bias Fit)	
No.	Name	(Coll.)	Coll.	FFT
1	FLE	40.827	0.063	0.085
2	BRK	40.428	0.082	0.086
3	RAI	40.236	0.058	0.073
4	STK	40.154	0.045	0.033
5	TAL	40.594	0.025	0.026
6	MLB	41.126	0.035	0.023
7	ESD	42.235	0.089	0.084
8	SEH	43.518	0.065	0.058
9	BOD	45.152	0.062	0.054
10	BEV	46.246	0.036	0.033
11	TWI	46.753	−0.043	−0.036
12	MEC	47.738	−0.136	−0.126
13	TBR	48.324	−0.149	−0.146
14	BKH	47.652	−0.134	−0.139
15	STA	47.182	−0.098	−0.109
R.M.S.			±0.084	±0.084
R.M.S.*			±0.020	±0.024
* Only Stations FLE–BEV				

Table 1: Comparison of Gravimetric Height Anomalies With GPS and Leveling for the TRVNAC Campaign (Units are m)

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Station		ζ	Differences (Bias Fit)	
No.	Name	(Coll.)	Coll.	FFT
1	MSD	43.403	−0.003	−0.004
2	LIN	43.489	0.008	0.008
3	VEL	43.464	0.031	0.031
4	BEN	43.559	−0.006	−0.013
5	RON	43.657	−0.030	−0.042
6	MHL	43.589	−0.015	−0.018
7	LVA	43.444	0.002	0.005
8	GEH	43.833	−0.041	−0.044
9	SPR	44.228	0.014	0.028
10	ALT	43.384	−0.010	−0.023
11	MEY	43.281	0.026	0.031
12	LHG	43.135	0.007	0.014
13	SLG	42.992	0.031	0.039
14	BAN	43.824	−0.030	−0.031
15	SEH	43.518	0.018	0.018
16	HAS	44.633	−0.005	−0.002
17	WIT	44.386	0.007	0.005
18	SOR	44.214	−0.005	−0.002
R.M.S.			±0.020	±0.024

Table 2: Comparison of Gravimetric Height Anomalies With GPS and Leveling for the HANNAC Campaign (Units are m)

GPS traverse (TRVNAC) with values computed from three different geopotential models (upper part) as well as values computed by FFT on the basis of these models, gravity data and RTM-contributions (lower part). As expected, the solutions based on model IFE87E2 tailored to gravity data in Europe yields the best agreement with GPS and leveling. Using this model, the contribution of terrestrial gravity data takes a maximum value of about 50 cm. The other two models have larger long to medium wavelength errors, which are essentially presurved in the combination solutions. This problem might be overcome with a larger data collection area, but then the advantages of high-degree geopotential models are lost. However, the computation of tailored models in connection with a small cap size of local gravity field data will in many cases be less expensive than the use of existing geopotential models with a large cap size.

In addition, collocation solutions were computed using the tailored model IFE87E2. Due to the large amounts of data, the computations were blocked in $1° \times 2°$ areas using a larger data collection area (see *Denker* 1988). Altogether, 11 blocks were computed to provide coverage of northern Germany. As compared to the FFT method, the computation time necessary for all 11 collocation blocks is approx. two orders of magnitude larger. The covariance functions required for the computations were assumed the same for all blocks; the model covariance function selected on the basis of empirical data has a gravity anomaly variance of 100 $mgal^2$ and a correlation length of about 20 km. One unsolved problem in this context is to fit different partial solutions together. In practical tests it was found, that discrepancies between adjacent blocks are mainly dependent on the size of the data collection area, on the quality of the used geopotential model and on the degree variances contained in the covariance model for the low degrees. However, if the used covariance function contains long wavelength components, collocation performs an estimate of corresponding field structures for each block being one of the main reasons for the occuring discrepancies at the block boundaries. In order to keep the discrepancies between adjacent blocks below 1 cm, it was finally decided to assume the reference field to be errorless up to degree 72. Through this assumption, changes in the predicted height anomalies are of long wavelength nature resulting in biases up to 2 cm and tilts < 2 cm/100 km. Formal error estimates from collocation (without the assumption of an errorless reference field up to a certain degree) are in the order of 1 cm/50 km and 2 cm/100 km for height anomalies.

The FFT and the collocation solution have been evaluated by comparing the predicted height anomalies with GPS and leveling from different campaigns, the discrepancies found are listed in tables 1–3 (after subtraction of a common bias for each campaign). As can be seen, the collocation and the FFT results agree well within a few cm. For the TRVNAC campaign, both solutions show jumps of approx. 10 cm between stations BEV/TWI and TWI/MEC (see fig. 3). There are indications that these discrepancies are caused by the GPS data processing, but this remains to be clarified in the future. If only the northern part of the traverse with a length of approx. 400 km is considered, the r.m.s. discrepancy amounts to ± 2.0 cm.

For the HANNAC campaign with maximum interstation distances of about 50 km, the r.m.s. discrepancies are approx. ± 2.0 cm for the FFT and the collocation solution.

The third GPS data set available in North Germany is the NIENAC campaign with maximum interstation distances of 300 km. The r.m.s. difference is about ± 10 cm, but a detailed analysis shows a slope about an axis with an azimuth of approx. 45°, which is probably due to the GPS solution as a similar behaviour is not visible for the GPS traverse (TRVNAC). After taking additional tilts in northern and eastern direction into account, the r.m.s. discrepancy reduces to about ± 4 cm.

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