HEAVILY LOADED JOINTS FOR ASSEMBLING AEROBRAKE SUPPORT TRUSSES

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Civil Engineering Trusses: Evaluation, Application and Transition to Aerobrakes

Earth Evolution

Apparently, primitive man discovered that by tying diagonals into the corners of his framed huts, the structures could resist lateral loads. Armed with his basic intuition man adopted the resulting triangulation method of connecting wood members. Historically, it is difficult to find absolute proof of the use of trusses to span large spaces. Correspondingly, nature has few examples of the use of this structural form. However, a carved seal survives from the third millennium B.C. discovered at Susa, Persia revealing what appears to be a Warren-type truss supporting a granary. Temple ruins from the Greek Mycenaean period suggest roof spans that could have only been achieved by a trussing system. We know the truss in a form as we conceive it today was used to enclose the great spaces of the Roman basilicas. The Pantheon portico, constructed about 130 A.D., lasted until it was torn down in 1625. This bronze truss was very accurately depicted in the drawings of Palladio prior to its destruction. Medieval times,
spanning from the Roman Empire to the Renaissance, saw the truss form continue in use across Northern Europe in the application of half-timber framing.

The Renaissance, largely through the efforts of Andrea Palladio (1518-1580), saw the truss emerge in both bridge and roof construction. His Palladin truss, still in use today, ushered in the era of the engineered truss—based on present day understanding of statics. The Swiss brothers Goubermann built a 390-foot span timber truss bridge in 1758.

The migration across North America saw a great deal of empirical engineering employed, intuitively, by American engineers accommodating the expansion west across a vast array of rivers and mountain ranges. The first of these bridges was built of wood but some wrought iron (tension members) and cast iron (compression members) were integrated into the proliferating number of wooden truss bridges. Very quickly, however, these trusses became all metal paralleling the rapid development of the iron and steel industry in the USA. The first patent for an all metal truss bridge in the USA was in 1831.
Planar truss applications in civil engineering structures continued to provide alternatives for the designer who was seeking minimum mass through optimization of form. London's 1853 Crystal Palace took full advantage of this rationale in the long span roof and floor iron truss system then employed. The Hall of Machines with 375-foot span trussed arches at the 1889 International Exhibition in Paris was surpassed only by Gustave Eifel's magnificent use of the truss in the tower at this same event bearing his name to this day.

The Twentieth Century witnessed a continued expansion of trusses in civil engineering by capitalizing upon the use of steel. Aircraft and automobile factories required large open spaces therefore calling upon the truss for long span, lightweight, stiff framing systems in the 1930's. A 20th Century civil structure that played a major role in the industrial expansion of America beginning in the 1920's was the electric transmission steel trussed tower. This application exemplified the use of light, economical, easy to transport, pre-manufactured elements which are readily assembled and required minimum maintenance—-not unlike our task at
hand. The skyscraper, too, provided more economical applications starting in the 1960's with the incorporation of vertical trusses into their skeletal frames. The oil industry showed an early need for trussed towers along with today's offshore drilling rigs.

We see that transportation and industry were major driving factors behind the development of the truss in the 20th Century. Moreover, though, we see the move to outer space first through the giant truss supported radio telescopes, the rocket launch towers and the remarkable Vehicle Assembly Building (1967) having a record volume 129 million cubic feet, expandable to 178 million cubic feet, all embraced by trussed vertical bays all capped a horizontal roof truss.

3-D Trusses

Now, literally moving into another dimension with trusses, we encounter tri-dimensional trussing. Therefore, a truss may be on a continuous surface which is not flat and be stressed with forces having both tangential and perpendicular components to this surface. Correspondingly, a truss may have members in more than one plane. An example being the tri-dimensional truss which is
made up of elements located within three planes forming a triangle in section. Space frames are a form of tri-dimensional trusses. They find applications horizontally as roof and floor systems and vertically as walls and columns. Trusses and space frames correspond, respectively, to one way and two way plates. A space frame is a system of two parallel grids connected by diagonal members or an assembly of three dimensional modular units formed by the edges of tethedra (polyhedra).

**Joints**

Joints have always been the most critical element in the manufacture of trusses. When Caesar crossed the Rhine it was on a wood truss bridge held together with bronze hardware made in Spain. More recently, the advent of welding has greatly minimized large gusset plates previously needed for riveting and bolting truss connections. The acceptance of welded construction ushered in the use of tubular elements for trussing which due to their symmetrical sections minimized joint eccentricities and maximized their structural efficiency, especially for compression members. So, without gusset plates and with the efficient symmetry of tubes
eccentricities were practically eliminated. Their development enhanced the evolution in Germany during the 1920's of space frame systems. The popular architectural philosophy of this period was the application of industrial methods to building technology with the consequent development of the first space frame systems.

The MERO joint evolved in this environment and is still in use today along with another currently popular system from Germany, the Octaplatte. Other well known standard systems available are: from North America, the Unistrut and Triodetic; from France, the Unibat; and from Britain, the Nodus and Space Deck. With truss applications, according to the situation, the choice can be a standard, commercially available system or a specifically engineered solution fabricated solely for a specific condition. With this option available we recognize the difficulty in categorizing space frames by type of node for there is an infinite number of possible factors affecting connector types: shapes such as flat, bent or built-up gusset plates; clamps as used in scaffolding; hubs, stars and internally threaded polyhedra or externally threaded node projections; required assembly or a finished manufactured item; strut and tie preassembled elements.
such as solid, tubular or open sections with member connections achieved by bolting, welding, gluing, keying along with other special techniques.

Construction of civil space structures is a vital aspect relating to the design. Foundation conditions play a major factor especially for large roof structures and can rule out the space truss in the case of poor soil conditions. Moreover, though, depending on scale and sequence of erection, temporary support or lifting systems may be elected including large preassembled pieces in the latter case and smaller building blocks for falsework erection. Preassembled space frame elements can be shipped over the road in pieces as large as 60' x 15', a familiar size to the space shuttle.

**Aerobrake**

Thus, in gathering our experience with trusses on earth from civil engineering applications we are brought to what may be termed the leading edge of gravitational space frame technology. There are design considerations which will push loading beyond the one-G environment such as seismic and live load impacts with a
corresponding fractional G environment in the sea. Passing beyond into the zero-G environment casts a different perspective upon the design, construction, performance, maintenance and cost for space trusses. More specifically, however, the aerobrake design will call for related parameters that include assembly in a zero-G area with corresponding performance requirements in a greatly magnified G zone. So, when looking for a data-base for 3-D joints in space frames we can only draw upon experience which exhibits slightly less or a little more than one-G structures to lead us into solutions for 3-D space truss joints whose gravity conditions can range from zero to six G's.

Recognizing the infinite possibility for types of 3-D joints on earth we first conclude that premanufactured strut and tie and joints are necessary because of the narrow, sectionalized, weight sensitive road we have available to us in the belly of the 15-foot diameter, 60-foot long cargo bay carried atop the costly propellent system of the shuttle. Therefore, this eliminates from our consideration the most heavily loaded joint systems now available in the form of engineered, sole solutions which have a developmental cast of less than 10% of the system construction
costs.

The great majority of 3-D truss applications in space will be for zero gravity loading. This includes trusses for platforms, beams, antennas, semi-monocoque shelters and sunshields. The semi-monocoque aerobrake truss being the exception. The enormous transient loadings this truss configuration will be subjected to will again draw our 3-D joint experience far from the mark when compared to earth-bound comparisons.

Industrial experience with 3-D trusses will amortize developmental cost over many specific applications. In some cases, these will number in the thousands. However, the aerobrake developmental costs, due to its unique utilization in space, will be applicable only to a relatively few cases, moreover, these standardized components are so anomalous to sole, zero-G structures that they would not prove economical in comparison, therefore, their inventory would be separate.

The major consideration of this study is to produce a 3-D joint subject to loads exceeding those of the typical earth bound, premanufactured systems. They will contain strut-tie elements which can be joined together in a zero-G environment without tools
or parts assembly by a relatively high loaded 3-D nodal system. All of the data base studies have joints with many parts, having high erection tolerances, requiring intermediate construction phases and using erecting equipment and tools. The data base information shows no evidence of specific length variations in strut-tie elements. Their installation or replacement requires some method of removing or applying external loads to allow for length adjustments.

Utilizing the new composite materials will reduce delivery loads and increase the strength to weight ratios to minimize the delivery cost per volume of materials. Adequate stiffness can be achieved through a wealth of experience from the truss data-base. Also from the data-base we have a firm understanding of methods and significance of focused axial loads through the vortex of the nodal joints.

Construction of the aerobrake would be done either piece by piece from portable component packages through EVA starting at the center and developing the truss outward towards its limiting perimeter. This process could be achieved through EVA but, to reduce this type of activity, larger preassembled blocks could be
developed and delivered from the shuttle's bay with temporarily attached, radio controlled, video monitored thrusters for attachment to the expanding perimeter of the aerobrake.

**Conclusion**

Historically, we have seen how closely tied the development of the truss has been to materials development and technology transfer. Corresponding to the rapid change in the 1830's from wood to all metal trusses is today's transition from metals to composites. This latter development has amazingly reached the point where today the volume of plastic exceeds the volume of metals produced.

Nevertheless, our conclusion is that the aerobrake truss will account for a relatively small fraction of 3-D truss joints and truss elements in space structures due to the wide range of loading conditions it is subjected to, whereas the great preponderance of truss structures to be built in outer space are for the lightly loaded zero-G environment. Correspondingly, we have seen that in civil structures the heavy trusses are invariably custom designed as contrasted to the more lightly loaded roof systems utilizing
premanufactured components available in the market place. When considering the data base available for all civil truss structures we find it to be practically infinite, therefore, we have selectively identified certain premanufactured joints that provide a general readily available data base for lightly loaded 3-D structures in space. Moreover, the heavily loaded 3-D joints, premanufactured for shuttle transport, zero-G assembly and multiple-G transient gravity loads are significantly distant from our traditional data base whereas all other zero-G structures can profit to a larger degree from available mass produced systems.

The rapid growth of the composite materials industry now closely followed by explosive developments in ceramics would appear to herald the demise of metals but for the new processes emerging in this field: super plastic forming, rapid solidification and mechanical alloying. These changes and applications to 3-D trusses will certainly parallel the historical transition from wood to metal described in this paper. Nodes will be a product of these metallic processes and the strut and ties effected by composites.

Materials will be the engine driving us from this point of departure away from the traditional solutions and redirecting our
efforts toward the application of these lighter and stronger elements. Composites have been costly due to labor intensive layering processes but with the production of the composite B-2 stealth bomber automation has been utilized and portends well for improved economy in the composite industry. Similarly, new processes for composite metals will greatly affect the nodal geometry on the heavily loaded aerobrake truss. Furthermore, "smart" materials will ultimately play a part in the aerobrake and other structures in space with built-in sensors and micro computer transponders.
2. Development of New Joint Concepts for Use in Aerobrake Structures

2-1 General Description of Aerobrake Structure

The proposed Pathfinder mission will utilize a large area, hard surface aerobrake to decelerate a manned spacecraft on entry into the Martian atmosphere. The following description focuses on and is restricted to possible member connections for a three-dimensional (3-D) truss support of the aerobrake surface used to decelerate a manned spacecraft on a Mars orbital atmosphere entry. Such a truss will have a plan view spanning approximately 120 feet between opposite, parallel, faceted perimeter elements. This 3-D truss is described by two slightly concentric levels of structural elements separated but connected by vertical and diagonal members forming the depth of the 3-D truss. The rectilinear strut and ties are 10 feet in length with diagonals correspondingly slightly longer than 14 feet. Strut and ties, connections and nodes are designed for an ultimate concentric load
of 150,000 pounds. These truss elements are graphite reinforced epoxy pipes of approximately 6 inches in diameter. The pipe ends are capped with titanium fittings for their connection to nodes made of the same material. Other metals with equally satisfying properties can be employed in place of titanium. Optimized truss assembly in a zero-G space environment requires minimizing extravehicular activity (EVA), manual labor and tools. All strut and tie members may be designed for length adjustments during installation, replacement or adjustment. And, they can be configured in such a fashion as to provide a more dense package by shaping them into cones then stacking them one within the other for trans shipment and dispersement. There is no provision for adjustment due to rotational misalignment at the nodal points except in scheme #7, the pinned injection connection. All connections are mechanical except #7 which is made rigid by injecting an epoxy compound. This connection may be classified as a bonded joint. Seven different nodal schemes are proposed in this study with their narrative and graphic descriptions provided within the body of this document.
The titanium end fittings of the 6-inch diameter graphite reinforced epoxy pipe member are shaped into a wide flat hook which grips onto an identically shaped hook formed onto a stub extension from the node element. After both sides of the connection are mated, a circular lock-ring is advanced along the axis of the tube toward the node over the interlocking hooks. The lock-ring prevents separation and rotation of the pipe member with respect to the nodal points. The lock-ring is kept in place by a retainer button which is a temperature resistant rubber plug. This spring plug is of a rubber type (EPDM) which remains elastic at very low temperatures. This connection has the simplest geometry for two interlocking members.

Assembly in space does not require any tools and can be performed with one hand. The locking ring has an exterior, annular, concave depression which allows for a firm grip with the thumb and fingers of a space glove. The diameter of the locking ring is sized so that a hand-span provides a firm grip. The
cylindrical stub extensions from each nodal point spring from a solid plate which is in the plane of the upper and lower chords of the 3-D truss. A maximum of nine possible stub extensions would occur for a conventional space truss at each of these nodes.

Additionally, within the defined interstitial space, diagonal and vertical pipe elements span between node grids. The node element is a precision casting of titanium. The indicated mass can be reduced by a more accurate stress analysis permitting material reduction between the stub extensions. The end fittings of the pipe members can be detailed with a threaded connection making possible a length adjustment of the member in a fashion similar to that proposed for scheme #2.
The end fittings of pipe members are shaped into a slotted double claw which is slipped over a hammerhead shaped extension projecting from the node. After vertical or horizontal insertion of the hammerhead into the double claw, opening of the connection is prevented by threading a locking ring over the hammerhead. With this ring torqued against the hammerhead loosening of the locking ring is prevented. A second lock ring, working in tandem, allows for a length adjustment of the pipe member. The relatively small thickness of the hammerhead extending from the nodal casting allows for a small and therefore lightweight nodal point. With the strut and tie members engaging their end claws over the hammerhead it is necessary to turn the axis of the hammerhead 90 degrees from the diagonal in order to provide clearance. The tight, tapered fit of the hammerhead into the claw gives sufficient rotational restraint to the node when fully inserted. The lock rings have spaced, cylindrical sockets serving as locking keys for small tool torquing points along the ring perimeter.
This connection uses the most elementary principle proposed—simply a pin connecting two plates. In order to avoid eccentricity the end fitting of the truss member is placed between two close parallel plates thereby straddling the single plate projecting from the node. A pin is inserted, after aligning the holes, creating an efficient double shear pin connection. The pin has a conical shape in order to facilitate its' alignment for insertion into the holes. The position of the pin is secured by tightening a pre-installed safety bolt.

The function of the pin is to eliminate a hinge which could result in a stability problem at the nodal point if members under compression are misaligned when double hinged connections tend to form at pins in the opposite member ends. Consequently, it is necessary to lock the pin connection against rotation after assembly is completed. The locking may be done even when in a slightly misaligned rotational position by providing a fill-plate between hinge plate and nodal surface which, due to its beveled ends and twin screws, can be adjusted for any misalignment.
The major characteristic of this concept is that the total joint can be assembled at a remote location. The metal end fittings of the pipe members are detailed in such a way that only a single large bolt makes the connection. This bolt is threaded through a hole in the hollow, spherical node and into the end of the pipe member. If adjustment in length of the member is required the pipe can be threaded along its axis as in scheme #2 or remain retracted with length adjustment accommodated by the tandem threaded rings. The single connection bolts are simply tightened from inside the node plate to a specified torque. A simple, preset torque tool for tensioning these closely spaced bolts could be advantageous. Despite its apparent size the hollow node is lightweight. The wall thickness of the sphere has to be determined by a thorough finite element analysis or by empirical methods. The node is open, flattened and reinforced by a perimeter stiffening ring with its flush surface facing the outside of the trussing. This flush surface provides areas where heat resistant atmospheric entry panels or other equipment may be attached.
This concept is the opposite of Scheme #4. The single connection bolt is attached to the pipe end fitting. The bolt can be retracted in order to shorten the member between nodal points for assembly. After alignment, the bolt is advanced into a solid node along its threaded axis from the pipe end fitting. This arrangement probably results in the smallest and lightest nodal point, however, the threads within the node have to be cast producing a possible fabrication problem.
Scheme #6 - "Nodal Tree"

Scheme #6 has two distinguishing features. First, half-length pipes are all preassembled at the node point and positioned for extending the truss matrix by coupling contiguous arms at their mid span with the adjustable slip-joints. Preassembly of the node members allows a reduction of the EVA splicing operation by as much as 50%. The second advantage resulting from half-length pipe is that these pieces can be made in a conical shape and stacked one within the other thereby reducing storage volume and effecting more efficient shuttle transport and container packaging for EVA assembly.

The construction sequence could follow this scenario: multi axial half members are threaded and locked in a focused pattern to the nodal joints within the space shuttle. Then, the preassembled nodes, with radiating half-length spokes, are moved to their final location at the expanding truss perimeter where these barbed-like pieces require only pipe connections instead of pipes and nodes during EVA. The member connection at mid span is done by a simple
splice and locking ring. In the scheme shown on drawing S6-2, a complete symmetrical sleeve is placed over symmetrical member fittings and a locking bar is lowered by turning a pin screw. In the scheme shown on drawing S6-3, the locking of the splice ring is done by inserting an additional locking ring.
2-2-7  **Scheme #7 - "Injected Connection"**

Basically, with this scheme, a loose initial connection between the nodal point and the members is made. This connection is then made rigid by injecting a high strength epoxy filler. Moreover, the epoxy filler can be dissolved, when desired, by application of heat. A male extension at the end of the member is inserted into a receptacle attached to the nodal point. Their relative position is then fixed by locking a ring over the connection. The very loose fit of the connection allows for dimensional correction of the trussing in length and rotation. The total trussing can be erected and adjusted to a final shape before the epoxy filler is injected. After injection, the joint is rigidly held into its position within the final truss matrix.
2 - 3. MATERIAL SPECIFICATIONS

TITANIUM ALLOY CASTING (Ti-2-8-4-4)

Titanium alloy casting is proposed for the fabrication of joint connections for the aerobrake support trusses.

Ultimate strength: \( F_u = 193 \) ksi

Yield strength: \( F_y = 180 \) ksi

Young's Modulus: \( E = 15,000 \) ksi

Density: \( \rho = 0.174 \) lb/in\(^3\)

Thermal conductivity: 3.6 BTU/HR/FT\(^2\)/\(^\circ\)F at 70\(^\circ\)F

Coeff. of thermal expansion: 5.4 \( \times 10^{-6} \) in/in/\(^\circ\)F

GRAPHITE EPOXY

Graphite epoxy material is proposed for the fabrication of the aerobrake support truss members.

Young's Modulus: \( E = 40,000 \) ksi

Density: \( \rho = 0.063 \) lbm/in\(^3\)

Ultimate strength: \( F_u = 350 \) ksi

Max. working stress: \( F_s = 100 \) ksi
2.4. MEMBER AND JOINT LOADING

**WORKING LOADS**

- **Axial Tension** = 107 kips
- **Axial Compression** = 107 kips

**SAFETY FACTORS**

- **For Ultimate Load** = 1.4
- **For Yield Load** = 1.1

**MAXIMUM DESIGN LOADS**

- **Ultimate** $P_u = 1.4 \times 107 = 150.0$ kips
- **Yield** $P_y = 1.1 \times 107 = 118.0$ kips

**ALLOWABLE STRESSES AT ULTIMATE**

- **Bending Stress** = $0.66 \times (1.4) P_y$
  - $= 0.924 \times P_y$
  - $= 0.924 \times F_y$

- **Shear Stress** = $0.4 \times (1.4) P_y$
  - $= 0.56 \times P_y$
  - $= 0.56 \times F_y$
2-5. CALCULATION OF TUBULAR TRUSS MEMBERS.

Material: GRAPHITE EPOXY

Typical length = 10 ft.

Diagonal length = \(10\sqrt{2} = 14.14 \text{ ft.}\)

Pu design = 150 kips.

Area: pipe diameter = 6''

pipe wall thickness = 0.14''

\[
\begin{align*}
\text{A} &= \pi \left(\frac{d}{2}\right)^2 \\
&= \pi \left(\frac{6}{2}\right)^2 \\
&= 11.07 \text{ in}^2
\end{align*}
\]

\[
E = 40,000 \text{ KSI}
\]

FULLER FORMULA

\[
P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{71^2 \times 40,000 \times (11.07)}{(14.14 \times 12)^2}
\]

\[
P_{cr} = 152.0 \text{ kips} > 150 \text{ kips}
\]

O.K.
Symmetrical Buckling Under Uniform Axial Compressive Load (Wall Crippling)


Chapter II: Buckling of Shells
Equation (4)

\[ N_{cr} = \frac{D (\frac{m^2 \pi^2}{L^2} \pm \frac{E t L^2}{2 \pi^2 D m^2 \pi^2})}{\pi^2} \]  \( (1) \)

Where: \( N_{cr} \) = Axial compressive force per unit circumferential length of wall.

\[ D = \frac{Et^3}{12(1-v^2)} \]

or:

\[ t = \text{Wall Thickness} \]
\[ L = \text{Pipe Length} \]
\[ m = \text{Number of half sine waves in the axial direction} \]
\[ E = \text{Young's Modulus} \]
\[ v = \text{Poisson's Ratio} \]
\[ a = \text{Radius of the pipe} \]
\[ L_m = \text{Length of half sine waves} \]

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\[ N_{CR} = D \left( \frac{m^2 \tau^2}{L^2} + \frac{E + L^2}{a^2 D m^2 \tau^2} \right) \]

**NCR minimum can be obtained by**

\[ \frac{d N_{CR}}{d L} = 0 \]

\[ \frac{d N_{CR}}{d L} = -2 D \frac{m^2 \tau^2}{L^3} + \frac{2 L E + a^2 m^2 \tau^2}{a^2 m^2 \tau^2} = 0 \]

\[ L^4 = \frac{D a^2 m^2 \tau^2}{E} \]

\[ L^4 = \frac{L a^2 m^4 \tau^4}{E} = \frac{E + a^2 m^4 \tau^4}{12(1 - \nu^2)} E \]

\[ L^4 = \frac{a^2 m^4 \tau^4}{12(1 - \nu^2)} \]

\[ L = m \tau^4 \sqrt{\frac{a^2 m^4 \tau^4}{12(1 - \nu^2)}} \quad \text{or} \quad \frac{L}{m^4} = \tau^4 \sqrt{\frac{a^2 m^4 \tau^4}{12(1 - \nu^2)}} \quad -(2) \]

**NCR minimum when** \[ \frac{L}{m^4} = \tau^4 \sqrt{\frac{a^2 m^4 \tau^4}{12(1 - \nu^2)}} \]

Substitute equation (1) into equation (2).

\[ N_{CR} = D \left( \frac{\pi^2}{\pi^2 \sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}}} + \frac{E + \pi^2}{a^2 \tau^2} \sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}} \right) \]

\[ = \frac{D}{\sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}}} + \frac{E + \pi^2}{a^2 \tau^2} \sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}} \]

\[ = (D) \frac{12(1 - \nu^2)}{a^2 + t^2} \sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}} + \frac{E + \pi^2}{a^2 \tau^2} \sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}} \]

\[ = \frac{E + \pi^2}{12(1 - \nu^2)} \frac{12(1 - \nu^2)}{a^2 + t^2} \sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}} + \frac{E + \pi^2}{a^2 \tau^2} \sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}} \]

\[ = \frac{2 E + \pi^2}{a^2} \sqrt{\frac{a^2 + t^2}{12(1 - \nu^2)}} \]

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\[ N_{CR} = \frac{2E_t}{a^2} \sqrt{ \frac{a^2 + t^2}{12(1 - \nu^2)} } \]
\[ = \frac{2E_t^2}{a} \sqrt{ \frac{1}{12(1 - \nu^2)} } \]
\[ = \frac{E_t^2}{a} \sqrt{ \frac{1}{3(1 - \nu^2)} } \]
\[ = \frac{E_t^2 \sqrt{3(1 - \nu^2)}}{a \sqrt{3(1 - \nu^2)}} \times \frac{a}{\sqrt{3(1 - \nu^2)}} \]
\[ N_{CR} = \frac{E_t^2}{a \sqrt{3(1 - \nu^2)}} \]

\[ \sigma_{CR} = \frac{N_{CR}}{t} = \frac{E_t}{a \sqrt{3(1 - \nu^2)}} \]

Assuming \( \nu = 0 \) (conservative).

\[ \sigma_{CR} = \frac{10^4,000 (0.14)}{3 \sqrt{3(1 - 0)}} = 1077 \text{ KSI} \geq \sigma_u = 193 \text{ KSI} \]

Cross-sectional area \( A = \pi (3)^2 - \pi (2.86)^2 = 2.577 \text{ in}^2 \)

\[ P_{CR} = 1077 (2.577) = 2775 \text{ KIPS} \geq P_{CR} \]

Euler buckling governs!
2-6. **Calculation of Proposed Joints.**

I. **Joint Scheme 1.**

![Diagram of joint scheme 1 with labeled dimensions and section A-A]

**Section A-A**

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\[ R_S = \sqrt{2^2 - (0.125)^2} = 1.996'' \]

\[ \alpha = \cos^{-1}\left( \frac{0.125}{2.0} \right) = 86.42^\circ = 1.508 \text{ rad} \]

\[ y = \frac{2\pi}{3} \left( \frac{\sin^3\alpha}{\alpha - \sin\alpha \cos\alpha} \right) = \frac{2\pi}{3} \left( \frac{\sin^3 86.42}{1.508 - \sin 86.42 \cos 86.42} \right) = 0.92'' \]

\[ y'_1 = 0.92 - 0.125 = 0.795'' \]
\[ y'_2 = 2 - 0.92 = 1.08'' \]

\[ A = \pi^2 \left( \alpha - \sin\alpha \cos\alpha \right) = \pi^2 \left( 1.508 - \sin 86.42 \cos 86.42 \right) = 5.783 \text{ in}^2 \]

\[ I_x = \frac{\pi^4}{4} \left( \alpha - \sin\alpha \cos\alpha + 2 \sin^3\alpha \cos\alpha \right) = \frac{\pi^4}{4} \left( 1.508 - \sin 86.42 \cos 86.42 + 2 \sin^3 86.42 \cos 86.42 \right) = 6.28 \text{ in}^4 \]

\[ I_x = I_0 + Ay'^2 \]

\[ I_0 = 6.28 - (5.783)(0.92)^2 = 1.385 \text{ in}^4 \]
**SHEAR**

Check shear at shear plane B-B.

Shear plane area \( = 2(1.996)(1) = 3.992 \text{ in}^2 \).

\[
T_{\text{MAX}} = \frac{150 (1.5)}{3.992} = 56.36 \text{ KSi} < 0.56 F_y = 100.8 \text{ KSI}.
\]

**BEARING STRESSES AT C-C.**

Bearing area \( = 0.25 (2)(1.996) = 0.998 \text{ in}^2 \).

\[
\sigma_{\text{BEARING (u)}} = \frac{150}{0.998} = 150.3 \text{ KSI} < 193 \text{ KSI}.
\]

\[
\sigma_{\text{BEARING (y)}} = \frac{118}{0.998} = 118.24 \text{ KSI} < 180 \text{ KSI}.
\]

**BENDING MOMENT**

Bending moment at C = 150 (0.92) = 138 Kips-\text{in}.

Tension / compression force at C = 150 Kips.

\[
M_1 = \pm \frac{138 (0.795)}{1.385} \pm \frac{150}{5.783} = \pm 105.15 \text{ KSI}.
\]

\[
M_2 = \pm \frac{138 (1.08)}{1.385} \pm \frac{150}{5.783} = \pm 133.55 \text{ KSI}.
\]

\[
< F_y = 180 \text{ KSI}., \text{ O.K.}
\]

**ORIGINAL PAGE IS OF POOR QUALITY**
CHECK TENSIONAL CAPACITY AT SECTION D-D.

CROSS SECTIONAL AREA = \( \pi (2^2 - 1.4^2) \) = 6.41 \( \text{in}^2 \).

ULTIMATE LOAD CAPACITY = 6.41 (193) = 1237 kips < 1500 kips. O.K.

YIELD LOAD CAPACITY = 6.41 (180) = 1153 kips < 1180 kips. O.K.
II. **JOINT SCHEME 2**

**TENSION AND COMPRESSION OF PART I**

**TENSION**

Cross-sectional area = 3.5 x 0.625 = 2.187 in²

Ultimate tension capacity = 2.187 x 193 = 422.59 Kips
> Pu = 150 Kips
  O.K.

Yield tension capacity = 2.187 x 180 = 393.66 Kips
> Py = 118 Kips
  O.K.
COMPRESSION

EULER LOAD

\[ P_0 = \frac{\pi^2 EI}{(KL)^2} \]

where:
\[ I_{x-x} = \frac{1}{12} (3.5)(0.625)^3 = 0.0712 \text{ in}^4 \]
\[ E_s = 150,000 \text{ ksi} \]
\[ K = 1.0 \]

\[ P_0 (W) = \frac{\pi^2 (150,000)(0.0712)}{(5.5)^2} = 348.4 \text{ kips} \]

\[ W > P_0 = 150 \text{ kips} \]

O.K.
**UNIFORM BENDING PRESSURE**

\[ \frac{150}{2(0.5)(3.5)} = 42.86 \text{ ksi} \leq F_y = 180 \text{ ksi} \]

O.K.

**SHEAR**

SHEAR PLANE on A-A:

\[ A_v = 2(3.5)(\frac{1}{2})(1.25 + 0.875) = 7.437 \text{ in}^2 \]

\[ T_u = \frac{150 \times 1.5}{7.437} = 30.25 \text{ ksi} \leq 0.56 F_y = 100.8 \text{ ksi} \]

O.K.
BENDING

Bending moment at section A-A.

\[ M = \frac{150}{2} \left( 0.1875 + \frac{0.5}{2} \right) = 32.812 \text{ k' in.} \]

Average thickness \( t = \frac{(1.25 + 0.875)}{2} = 1.0625 \text{"} \)

\[ W = \frac{t}{6} (3.5)(1.0625)^2 = 0.658 \text{ in}^3 \]

\[ \sigma_{\text{max}} = \frac{32.812}{0.658} = 49.86 \text{ ksi} < \sigma_y = 180 \text{ ksi}. \]

O.K.
SHEAR AND BENDING OF PART III

SECTION 2-2

\[
\text{SHEAR} \quad \frac{Pq}{A} = \sqrt{2.375^2 - 1.125^2} = 2.09"
\]

Shear plane area B-B = 2(2.09)(1.25) = 5.225 in²

\[
T_u = \frac{150 \times 1.5}{2 \times 5.225} = 21.53 \text{ KSI} < 0.56 \bar{\sigma}_y = 108.8 \text{ KSI}, \quad \text{O.K.}
\]
BENDING

SECTION B-B

\[ M = (150/3) (0.125 + 0.5/2) = 28.125 \text{ k-in} \]

\[ W = \frac{1}{6} (2 \times 1.09)(1.25)^2 = 1.088 \text{ k-in} \]

\[ T_{\text{max}} = \frac{28.125}{1.088} = 25.85 \text{ ksi} < F_y = 180 \text{ ksi}, \text{ O.K.} \]

SECTION C-C

\[ \alpha = \cos^{-1} \left( \frac{1.125}{2.375} \right) = 61.73^\circ = 1.077 \text{ rad} \]

\[ y = \frac{2}{3} \left( \frac{\sin^3 \alpha}{\alpha - \sin \alpha \cos \alpha} \right) = \frac{2(2.375)}{3} \left( \frac{\sin \alpha}{1.077 - \sin \alpha \cos \alpha} \right) = 1.64'' \]

\[ y_1 = 0.375'', \quad y_2 = 0.515'' \]

\[ A = \frac{n^2}{4} (\alpha - \sin \alpha \cos \alpha) = \frac{2.375^2}{4} (1.077 - \sin \alpha \cos \alpha) = 3.922 \text{ in}^2 \]

\[ I_x = \frac{n^4}{4} (\alpha - \sin \alpha \cos \alpha + 2 \sin^3 \alpha \cos \alpha) \]

\[ = \frac{2.375^4}{4} (1.077 - \sin \alpha \cos \alpha + 2 \sin^3 \alpha \cos \alpha) = 10.396 \text{ in}^4 \]

\[ I_0 + A y^2 = I_x \]

\[ I_0 = 10.396 - 3.722 (1.64)^2 = 0.385 \text{ in}^4 \]
\[ M_{c-e} = \left( \frac{150}{2} \right) \left( 0.5/2 + 0.125 + 0.515 \right) = 66.75 \text{ k-in} \]

\[ T_{c-e} = 150 \text{ k} \text{ (tension)} \]

\[ \sigma_1 = -\frac{66.75 \left( 0.735 \right) + 150}{0.385} = -87.13 \text{ ksi} < F_Y = 180 \text{ ksi} \]

(Compression) OK.

\[ \sigma_2 = +\frac{66.75 \left( 0.515 \right) + 150}{0.385} = 129.6 \text{ ksi} < F_Y = 180 \text{ ksi} \]

(Tension) OK.
III. Joint Scheme 3.
**PLATE A**

Plate A subjected to tension force.

Net cross-sectional area = $2.5 \times 0.625 = 1.562$ in$^2$.

**Ultimate Load**

Pu capacity = $1.562 \times 193 = 301.46$ kips > 150 kips O.K.

**Yield Load**

Py capacity = $1.562 \times 180 = 281.16$ kips > 118 kips O.K.

**Bearing Stresses in the Hole**

Bearing area $A = 1.5 \times 0.625 = 0.937$ in$^2$.

$P_{bearing (x)} = 0.937 \times 193 = 180.94$ kips > 150 kips O.K.

$P_{bearing (y)} = 0.937 \times 180 = 168.66$ kips > 118 kips O.K.
PLATE B
Plate B subjected to tension force.

PLATE THICKNESS = 1/2"

Net cross-sectional area = 2.5(0.5) = 1.25 \text{in}^2.

**ULTIMATE LOAD**
Pu capacity = 1.25 \times (193) = \frac{241.25 \text{ kips}}{0.75} > \frac{150}{2} = 75 \text{ kips} \text{ O.K.}

**YIELD LOAD**
Py capacity = 1.25 \times (100) = \frac{225 \text{ kips}}{0.5} > \frac{118}{2} = 59 \text{ kips} \text{ O.K.}

**BEARING STRESSES IN THE HOLE**

Bearing area \ A = 1.5 \times (0.5) = 0.75 \text{ in}^2.

P_{bearing (u)} = 0.75 \times (193) = \frac{144.75 \text{ kips}}{0.75} > \frac{150}{2} = 75 \text{ kips} \text{ O.K.}

P_{bearing (y)} = 0.75 \times (100) = \frac{135 \text{ kips}}{2} = 59 \text{ kips}
SHEAR IN THE PIN

\[
\frac{P}{2} \leq \frac{P}{2} \leq \frac{P}{2}
\]

\[
\frac{1}{2}
\]

Single shear plane area = \( \pi (0.75)^2 = 1.767 \text{ in}^2 \).

\[
T_v \text{ average} = \frac{150}{2(1.767)} = 42.44 \text{ ksi}.
\]

\[
T_v \text{ max} = \frac{4}{3} \times T_v \text{ ave.} = \frac{4}{3}(42.44) = 56.58 \text{ ksi}.
\]

\[
< 0.56 F_y = 108.8 \text{ ksi}.
\]

O.K.
IV. JOINT SCHEME 4

[Diagram showing a joint scheme with dimensions and notes]
SCREW THREAD

MAJOR DIA. \( D = 2\frac{1}{4}'' \)

MINOR DIA. \( \frac{1}{2}'' \)

PITCH DIA.

ASSUME 3 THREADS IN 1.375''

\( \frac{3}{1.375} = 2.182 \) THREADS IN 1 INCH

\[ \text{PITCH} = p = \frac{1}{2.182} = 0.458'' \]

FOR GENERAL PURPOSE ACME THREADS:

BASIC THREAD HEIGHT \( \ell_1 = 0.5p = 0.229'' \)
STRESS AREA OF GENERAL PURPOSE ACME THREADS.

\[
\text{STRESS AREA} = \pi \left( \frac{E_s + K_s}{4} \right)^2
\]

WHERE:
- \( E_s \) = MIN. PITCH DIAMETER
- \( E_s' \) = MAX. PITCH DIAM. - TOLERANCE FROM TABLE 5
- MAX. PITCH DIAM. = \( E' \) - ALLOWANCE FROM TABLE H
- \( E = \) BASIC PITCH DIAM. = \( D - 0.5\)"
- \( K_s = \) MIN. MINOR DIAMETER
- \( K_s' = \) MAX. MINOR DIAMETER - 1.5 \times PITCH DIAMETER TOLERANCE FROM TABLE 5
- MAX MINOR DIAMETER = \( K - 0.01 \)
  (FOR < 10 THREADS PER INCH)
- \( K = \) BASIC MINOR DIAMETER = \( D - 1\)
- \( D = \) BASIC MAJOR DIAMETER

ASSUME CLASS OF THREAD 2-C:

REFERENCE: MACHINERY'S HANDBOOK: AMERICAN STANDARD FOR UNIFIED SCREW THREADS.

FROM TABLE 11:
- \( D = 2\frac{1}{8}'' \) TO \( 2\frac{3}{8}'' \) : PITCH DIAMETER ALLOWANCE = 0.012

FROM TABLE 5:
- \( D = 2\frac{1}{4}'' \) : PITCH DIAMETER TOLERANCE = 0.009"

\[E = D - 0.5 = 2.25 - 0.5(0.458) = 2.02''\]

MAX. PITCH DIAM. = \( E - 0.012 = 2.02 - 0.012 = 1.999''\)

\[E_s = 2.008 - 0.009 = 1.999''\]

\[K = D - 1 = 2.25 - 0.458 = 1.792''\]

MAX. MINOR DIAM. = \( K - 0.01 = 1.792 - 0.01 = 1.782''\)

\[K_s = 1.782 - 1.5(0.009) = 1.768''\]
**Stress Area**

\[ \text{Stress Area} = \pi \left( \frac{1.99 + 0.768}{4} \right)^2 = 2.773 \text{ in}^2. \]

**Bearing Stress**

\[ \text{Bearing Stress} (\sigma) = \frac{150}{2.773} = 54.1 \text{ ksi} \quad \text{<} \quad \sigma_y = 180 \text{ ksi} \quad \text{O.K.} \]

**Shear Area of General Purpose Acme Threads**

**Shear Area**

\[ \text{Shear Area} = \pi K_m \left[ 0.5 + \frac{1}{p} \tan 45^\circ \left( E_s - K_m \right) \right] \]

**WHERE:**

- \( K_m \) = Max. Minor Diameter of Internal Thread
- \( K_m \) = Min. Minor Diameter + 0.05 \( p \)
  \( (\geq 0.005) \)
  Min. Minor Dia. = \( K = D - p \)

\[ K_m = 2.25 - 0.458 + 0.05 (0.458) \]
\[ = 1.815. \]

\[ \text{Shear Area} = \pi (1.815) \left[ 0.5 + \frac{1}{0.458} \tan 45^\circ (1.99 - 1.815) \right] \]
\[ = 3.414 \text{ in}^2. \]

\[ T (\sigma) = \frac{150}{3.414} = 44.04 \text{ ksi} < 0.56 \sigma_y = 100.8 \text{ ksi} \quad \text{O.K.} \]
CHECK TENSION CAPACITY OF MINOR DIAMETER

CROSS SECTIONAL AREA = \( \pi (0.75)^2 = 1.767 \text{ in}^2 \).

TENSION CAPACITY \((u) = 1.767 \times 193 = 341 \text{ kips} \geq 150 \text{ kips} \) O.K.

TENSION CAPACITY \((y) = 1.767 \times 180 = 318 \text{ kips} \geq 118 \text{ kips} \) O.K.

NODE RING- SUBJECTED TO TENSION OR COMPRESSION

REF: DESIGN OF WELDMENTS BY OMER W. BLODGETT, SECT. 7.6

A. TANGENTIAL TENSILE/COMPRESSIVE FORCE IN RING:

\[ T = K_1 P_u \]
Compute \( K_1 \) from: Case 4 + Case 2

\[
\begin{align*}
T_{max} &= \text{Tension or Compression Force in the Ring} \\
\sum T &= 1.207 \times 150 = 181.05 \text{ kips.}
\end{align*}
\]

**Ring Cross-Section**

<table>
<thead>
<tr>
<th>SECTION</th>
<th>A</th>
<th>X</th>
<th>AX</th>
<th>( A (x-x)^2 )</th>
<th>( I_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 × 2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1.97</td>
<td>1.0</td>
</tr>
<tr>
<td>2 × 2.8</td>
<td>5.6</td>
<td>1</td>
<td>5.6</td>
<td>3.67</td>
<td>1.87</td>
</tr>
<tr>
<td>2 × 5</td>
<td>10</td>
<td>2.5</td>
<td>25</td>
<td>4.76</td>
<td>20.83</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
2A &= 18.6 \\
25 &= 33.6 \\
10.40 &= 10.40 \\
23.70 &= 23.70
\end{align*}
\]

\[
x_2 = x = \frac{33.6}{18.6} = 1.81 \quad \text{in.}, \quad x_1 = 5 - 1.81 = 3.19 \quad \text{in.}
\]

\[
I_y = 10.40 + 23.70 = 34.1 \quad \text{in}^4
\]

\[
S_1 = 10.69 \quad \text{in}^3
\]

\[
S_2 = 18.84 \quad \text{in}^3
\]
**Tensile / Compressive Stress Tangentially in the Ring**

\[ \frac{181.05}{18.6} = 9.73 \text{ ksi} \]

\[ \text{OK} \]

**Node Ring Subjected in Plane Bending**

\[ M = K_2 P_u R \]

Compute \( K_2 \) from: CASE 4 + CASE 2

\[ \text{Max. } K_2 = 0.253 \]

\[ \text{Max. } M = 0.253 (150)(7.35 + 1.81) = 347.62 \text{ k-in} \]

\[ \sigma_1 = \pm \frac{347.62}{10.62} = \pm 32.52 \text{ ksi} \]

\[ \sigma_2 = \pm \frac{347.62}{18.84} = \pm 18.45 \text{ ksi} \]

\[ \{ \sigma_1, \sigma_2 \} < f_y = 180 \text{ ksi} \]

OK.
PUNCHING SHEAR AT THE NODE-JOINT INTERSECTION

\[ \text{Shear Area} = 2\pi (1.5)(2) = 18.85 \text{ in}^2. \]

\[ T = \frac{150}{18.85} = 7.96 \text{ ksi} < 0.56 F_y = 100.8 \text{ ksi}. \]

**Bearing Stress at A-A**

\[ A = \pi (2^2 - 1.5^2) = 5.49 \text{ in}^2. \]

\[ \text{Bearing Stress} = \frac{150}{5.49} = 27.32 < F_y = 180 \text{ ksi}. \]
I. \text{JOINT SCHEME 5}

**BOLT MEMBER**

Effective diameter = $1\frac{1}{2}''$

Effective cross sectional area = $\pi (0.75^2) = 1.767 \text{ in}^2$
TENSION LOAD CAPACITY

\[ P_u = 1.767 \times 193 = 341.0 \text{ kips} > 150 \text{ kips} \quad \text{O.K.} \]

\[ P_\gamma = 1.767 \times 100 = 318 \text{ kips} > 118 \text{ kips} \quad \text{O.K.} \]

COMPRESSION

APPLY EULER FORMULA

\[ P_{cr} = \frac{\pi^2 E I}{(KL)^2} \]

WHERE:

\[ I = \frac{\pi R^4}{4} = \frac{\pi (0.75)^4}{4} = 0.248 \text{ in}^4 \]

\[ E = 15,000 \text{ ksi} \]

\[ K = 0.5 \]

\[ P_{cr} = \frac{\pi^2 (15,000) (0.248)}{(0.5 \times 4.5)^2} = 7252 \text{ kips} > 150 \text{ kips} \quad \text{O.K.} \]
BEARING STRESS AT SECTION A-A.

Bearing area = \( \pi (2^2 - 1.15^2) \) = 8.41 in².

Max. bearing stress = \( \frac{150}{8.41} \) = 17.83 ksi, \( < F_y = 180 \) ksi.

O.K.

SCREW THREAD CALCULATION
(SEE JOINT SCHEME 4 CALCULATION).
VI. JOIN'T SCHEME 6

A. NODE JOINT CONNECTION

SECTION A-A

ELEVATION
**SECTION B-B**
(Locked Position)

**BEARING STRESS ON TEETH A**

Bearing area:

\[
\text{Bearing area} = 6 \left[ \frac{15}{360} \left\{ \pi (2^2 - 1.5^2) \right\} \right] = 1.37 \text{ in}^2
\]

Bearing stress:

\[
\text{Bearing stress} = \frac{150}{1.37} = 107.49 \text{ ksi} \quad \text{less than} \quad f_y = 180 \text{ ksi}
\]

O.K.
**Shear on Tooth Intersection A-B**

Shear area = $6 \left[ \frac{15}{360} (2 \times 7 \times 1.5) \right] (\phi) = 4.71 \text{ in}^2$

$T_{\text{max}} = \frac{150}{4.71} = 31.85 \text{ ksi} < 0.56 F_y = 103.8 \text{ ksi}$

O.K.

**Axial Tension and Compression Capacity**

**End Fitting**

Cross sectional area = $\pi \left( \frac{1.5^2 - 1^2}{2} \right) = 3.93 \text{ in}^3$

$P_u = 3.93 (193) = 758.5 \text{ kips} \gg 150 \text{ kips}$

O.K.

**Node Extension Pipe**

Cross sectional area = $6 \left[ \frac{30}{360} \{ \pi (2^2 - 1.5^2) \} \right] = 2.75 \text{ in}^2$

$P_u = 2.75 (193) = 530.75 \text{ ksi} \gg 150 \text{ kips}$

O.K.
B. MIDSPAN JOINT ALTERNATE 1.

8''

1/2''

1/2''

LOCK BAR

\( R = 3/4'' \)

\( R = H'' \)

\( 0.75'' \)

CONICAL GRAPHITE EPOXY TUBE

MIDSPAN JOINT TUBE FITTING

SPlice RING

SECTION 8-0
SECTION A-A

BEARING STRESS ON THE TEETH INTERFACE (T)

Bearing area = \[ b \left[ \frac{30}{360} \left\{ \pi \left( R^2 - r^2 \right) \right\} \right] = 10.3 \text{ in}^2 \]

Bearing stress = \[ \frac{150}{10.3} = 14.56 \text{ ksi} \ll F_Y = 18.0 \text{ ksi} \]

O.K.

ORIGINAL PAGE IS OF POOR QUALITY.
SHEAR ON SECTION C-C.

Shear area = 6\left[ \frac{30}{960} \left\{ 271 (4) \right\} \right] (2) = 25.13 \text{ in}^2.

\[ \text{\( L_{\text{max.}} = \frac{150}{25.13} = 5.97 \text{ ksi} \leq 0.56 F_y = 100.8 \text{ ksi}, \)} \]

TENSION / COMPRESSION CAPACITY OF TUBE FITTING:

Cross-sectional area = 71 (4^2 - 3.25^2) = 17.08 \text{ in}^2.

\[ \text{\( P_u = 17.08 (198) = 3296 \text{ Kips} \geq 150 \text{ Kips}, \)} \]

O.K.
C. Midspan Joint Alternate 2

SECTION A-A

LOCK RING

SCREWED PIN

CONICAL GRAPH EPOXY TUBE

MID SPAN PIPE FITTING

SPlice RING

ORIGINAL PAGE IS OF POOR QUALITY
BEARING STRESS ON TEETH T

Bearing area = \( 6 \left[ \frac{\pi}{3 \cdot 6^2} \cdot \{ \frac{71}{5} \cdot (5^2 - 4^{1/2}) \} \right] = 6.22 \text{ in}^2. \)

Bearing stress = \( \frac{150}{6.22} = 24.11 \text{ KSI} \leq F_y = 180 \text{ KSI}; \)  
                   OK.
CHECK SHEAR ON PLANE E-E

Shear area = 6 \left[ \frac{\pi}{3 \sqrt{10}} \right] \left( 2.77 \times 4.5^2 \right) (1) = 11.78 \text{ in}^2.

I_{max} = \frac{150}{11.78} = 12.73 \text{ ksi} \ll 0.56 F_y = 100.8 \text{ ksi}, \text{ O.K.}

CHECK SPICE RING AND PIPE FITTING AXIAL TENSION

SPICE RING-

Cross sectional area = \pi \left( \frac{5.5^2}{2} - \frac{5^2}{2} \right) = 16.49 \text{ in}^2.

P_u = 16.49 \times (193) = \frac{3182}{0.43} \text{ kips} \ll 150 \text{ kips}, \text{ O.K.}

PIPE FITTING-

Cross sectional area = \pi \left( \frac{4^2 - 3.25^2}{2} \right) = 17.08 \text{ in}^2.

P_u = 17.08 \times (193) = \frac{3296}{0.43} \text{ kips} \ll 150 \text{ kips}, \text{ O.K.}
SECTION 2

ORIGINAL PAGE IS OF POOR QUALITY

\[ \phi = 3^{1/4} \]

\[ \phi = 2'' \]

BAR G

NODE JOINT
BEARING STRESSES ON TEETH A, B, C DUE TO AXIAL TENSION OR COMPRESSION FORCES

Bearing area = \( \frac{160}{360} \left[ \pi (3.525^2 - 2.5^2) \right] = 9.62 \text{ in}^2 \)

Bearing stress \( (\sigma) = \frac{150}{9.62} = 15.59 \text{ ksi} < \sigma_y = 180 \text{ ksi} \)

O.K.

SHEAR ON TEETH AT SECTION D-D, E-E & F-F

Shear area = \( \frac{160}{3.60} \left[ 2.7 \times 2.5 \times 0.75 \right] = 5.23 \text{ in}^2 \)

\( T_{\text{max}} = \frac{150}{5.23} = 28.68 \text{ ksi} < 0.56 \sigma_y = 100.8 \text{ ksi} \)

O.K.

CHECK BAR G ON TENSION AND COMPRESSION CAPACITY

TENSION

Cross sectional area = \( \pi (1)^2 = 3.14 \text{ in}^2 \)

\( P_u = 3.14 \times 193 = 606 \text{ kips} > 150 \text{ kips} \)

O.K.

\( P_y = 3.14 \times 180 = 565 \text{ kips} > 118 \text{ kips} \)

O.K.
COMPRESSION

\[ L = 5' \quad k = 0.5 \]

EULER FORMULA

\[ P_{cr} = \frac{\pi^2 E I}{(KL)^2} \]

\[ \text{WHERE:} \quad I = \frac{\pi R^4}{4} = \frac{\pi (1')^4}{4} = 0.785 \text{ in}^4 \]

\[ E = 15,000 \text{ KSI} \]

\[ K = 0.5 \]

\[ P_{cr} = \frac{\pi^2 (15,000)(0.785)}{(0.5 \times 5)^2} = \frac{18,594}{1.25} = 14,875 \text{ KIPS} \]

\[ \text{>> 150 KIPS} \]

\[ \text{O.K.} \]
2-7. WEIGHT OF PROPOSED JOINTS

SCHEME 1 WEIGHT CALCULATION

ASSEMBLY PLAN

MODE POINT (TITANIUM CASTING)
SPACE TRUSS CHORD MEMBER

SECTION A-A
**Section B-B**

**Single Joint Connection**

**Item 1**

\[
V_1 = \pi (2)^2 (6.312) - \pi (0.5^2 + 1.4^2) (0.5) (2.312) = 71.29 \text{ m}^3
\]

\[
\pi (3)^2 (3) - \pi (1.4^2 + 2.5^2) (0.5) (3) = 46.13 \text{ m}^3
\]

\[
- \pi (3^2 - 2.5^2) (1) = -8.64 \text{ m}^3
\]

\[
- \pi (3^2 - 2.7^2) (1.5) = -8.06 \text{ m}^3
\]

\[
V_1 = 160.72 \text{ m}^3
\]

**Item 2**

\[
V_2 = \pi (2.375^2 - 2^2) (3) = 15.46 \text{ m}^3
\]

**Total Volume of Single Joint**

\[
116.180 \text{ m}^3
\]
Volume of node plate:

\[ V = \frac{1}{2} (8)(6)^2 \sin \frac{360}{8} + (4)(4)(3.5) + (0.5)(3.5)^2(4)(2) \]

\[ V = \frac{206.82}{m^3} \]

Total volume of one connection with 9 joints:

\[ = 206.82 + 9(116.18) = 1252.044 \text{ m}^3 \]

Total weight:

\[ = 1252.44 \times 0.174 = 217.92 \text{ Lbs} \]
SCHEME 2 - WEIGHT CALCULATION

SECTION 2-2

SECTION 1-1
SINGLE JOINT CONNECTION

ITEM (1)

\[ V_1 = \left( \frac{\pi}{4} \cdot 3^2 - \frac{\pi}{4} \cdot 2.375^2 \right) 1.0 = 10.55 \text{ m}^3 \]

ITEM (2)

\[ V_2 = \frac{10.55}{\text{m}^3} \]

ITEM (3)

\[ V_3 = \pi (3^2 - 2.375^2)(1.5) = 13.90 \text{ m}^3 \]

ITEM (4)

\[ V_4 = (\pi \times 2.375^2)(3.125) - (\pi \times 1.125^2)(1.375) - (\pi \times 0.5^2)(1.75) = 48.53 \text{ m}^3 \]
ITEM 5

\[ V_5 = 2\pi \left( 2.375 \right)^2 \left( 1.877 - \sin 61.73 \cdot \cos 61.73 \right) \left( 1.25 \right) = 9.30 \text{ m}^3 \]

ITEM 6

\[ V_6 = 2\pi \left( 2.375 \right)^2 \left( 1.358 - \sin 77.85 \cdot \cos 77.85 \right) \left( 1.25 \right) = 16.24 \text{ m}^3 \]

ITEM 7

\[ V_7 = \left( 1.3 + 0.275 \right) \left( 4.56 \right) \left( 1.2 \right) = 9.92 \text{ m}^3 \]

ITEM 8

\[ V_8 = \left( 0.625 \right) \left( 3.5 \right) \left( 4.25 \right) = 9.30 \text{ m}^3 \]

\[ V_{\text{joint}} = \sum_i V_i = 128.29 \text{ m}^3 \]

**Spherical segment**

\[ V = \frac{4}{3} \pi \left( 4.5 \right)^3 - \frac{4}{3} \left( 2.75 \right)^3 \left( 3 - 4.5 - 2.75 \right) = 296.57 \text{ m}^3 \]

**Total** \[ V = 9(128.29) + 296.57 = 1451.18 \text{ m}^3 \]

**Total Weight** \[ = 1457.18 \left( 0.174 \right) = 252.50 \text{ LBS} \]
SCHEME 3  WEIGHT CALCULATION
SINGLE JOINT CONNECTION

ITEM 0
\[ V_1 = (6)(0.625)(4) - \pi (0.75)^2(0.625) - \pi (0.25)^2(0.625) = 13.77 \text{ m}^3 \]

ITEM 2
\[ V_2 = (2)(4.25 + 1.625)(\pi)(4)(0.5) - \pi (0.75)^2(1) = 15.78 \text{ m}^3 \]

ITEM 3
\[ V_3 = (2 + 1.625)(\pi)(4)(0.375) - \pi (0.25)^2(0.375) = 2.65 \text{ m}^3 \]

ITEM 4
\[ V_4 = \pi (2.5)^2(2.25) - \pi (1.75^2 + 1^2)(\pi)(1.375) = 35.40 \text{ m}^3 \]

ITEM 5
\[ V_5 = \pi (3^2 - 2.5^2)(1.0) = 8.64 \text{ m}^3 \]

ITEM 6
\[ V_6 = \pi (3^2 - 2.5^2)(2) - \pi (3^2 - 2.812^2)(1.5) = 12.13 \text{ m}^3 \]

ITEM 7
\[ V_7 = \pi (0.75^2)(2.75) + (1.75 \times 2 + 0.5 \times \pi \times 1^2) \times 0.25 + (2)(\frac{1}{2})(0.25) = 6.88 \text{ m}^3 \]

ITEM 8
\[ V_8 = \pi (0.5)^2(0.25) + 1 \times 1 \times 0.25 = 0.15 \text{ m}^3 \]

ITEM 9
\[ V_9 = \pi (0.25)^2(1.25) + \pi (0.5)^2(0.25) + 1 \times 1 \times 0.25 = 0.69 \text{ m}^3 \]

\[ V \text{ single joint} = \sum_{i=1}^{9} V_i = 96.60 \text{ m}^3 \]

\[ V \text{ sphere} = \frac{4}{3} \pi (1.45)^3 = 381.7 \text{ m}^3 \]

TOTAL WEIGHT = \[ 9(96.6) + 381.7 \] \times 0.174 = \[ 217.78 \text{ LBS-M} \]
SCHEME 4 WEIGHT CALCULATION

ITEM 1.

\[ V_1 = \pi (2)^2 (2.5) - \pi (0.75)^2 (1.5) - \pi (3^2 - 2.44^2) (2) - \pi (1.375^2 + 2^2) (0.5)(1) \]

\[ V_1 = 48.89 \text{ in}^3 \]

\[ V_1 = 39.64 \text{ in}^3 \]
ITEM 2.

\[ V_2 = 71 (0.75^2 \cdot 4.75) + 71 (2^2 \cdot 0.5) + (3.25)(2)(0.5) = 17.93 \text{ in}^3 \]

\[ V \text{ Single Joint} = 39.64 + 17.93 = 57.57 \text{ in}^3 \]

VOLUME OF NODE HOUSE

\[ V \text{ Node} = 71 (9.35^2 - 7.35^2)(9.3) + 71 (12.35^2 - 9.35^2)(2) = 1384.87 \]

\[ + [71(5.75^2 + 10^2)(0.5) - 71(3^2 + 7.25^2)(0.5)]4.25 = 477.32 \]

\[ + 71(3.75^2 + 5.75^2)(0.5)(2) \]

\[ - 9 \left[ 71(1^2 + 1.5^2)(0.5)(2) \right] = 148.05 \]

\[ V \text{ Node} = 1918.35 \]

TOTAL WEIGHT = \[ 9 (57.57) + 1918.35 \] \( 0.174 \)

\[ = 423.95 \text{ lbs} \]
SCHEME 5 WEIGHT CALCULATION

WEIGHT OF SINGLE JOINT CONNECTION

ITEM 1
\[ V_1 = \pi \left( 4 \right)^2 \left( 9.5 \right) = 29.84 \text{ in}^3 \]

ITEM 2
\[ V_2 = \pi \left( 2^2 - 1^2 \right) \left( 1 \right) = 9.42 \text{ in}^3 \]

ITEM 3
\[ V_3 = \pi \left( 3^2 - 1^2 \right) \left( 1 \right) = 25.13 \text{ in}^3 \]
ITEM 4
\[ V_H = \pi (2^2 - 1^2) (1) = 9.42 \text{ in}^3. \]

ITEM 5
\[ V_5 = \pi (1.5^2 - 1^2) (0.75) = 2.94 \text{ in}^3. \]

ITEM 6
\[ V_6 = \pi (2.875^2) (5) - \pi (2)^2 (3.75) - \pi (1.25^2)(1.75) = 76.58 \text{ in}^3. \]
\[ V_6 = \pi (2.875^2 - 2.625^2) \times 2.75 = -11.88 \text{ in}^3, \]
\[ V_{\text{SINGLE JOINT}} = \frac{6}{2} \text{ITEM} = 141.45 \text{ in}^3. \]

\[ V_{\text{SOLID NODE}} = \frac{4}{3} \pi (5)^3 - \frac{4}{3} (3)^2 (3 \times 5 - 3) = 410.5 \text{ in}^3. \]

VOLUME OF CONNECTION WITH NINE JOINTS
\[ = 141.45 \times 9 + 410.5 = 1683.55 \text{ in}^3. \]

\[ \rho = \frac{0.174}{1 \text{ in}^3}. \]

TOTAL WEIGHT = 0.174 (1683.55) = 292.94 lbs
SCHEME 6 WEIGHT CALCULATION

A NODE JOINT CONNECTION

SECTION A-A

ELEVATION
SECTION B-B
(LOCKED POSITION)

NODE JOINT
**SINGLE JOINT**

**ITEM 1**

\[ V_1 = \pi (15^2 - 1^2)(4) + 6 \left[ \frac{15}{360} \{ \pi (2^2 - 1.5^2) \} \right](2) = 18.46 \text{ in}^3 \]

\[ + \pi (2^2 - 1^2)(1) + \pi (1.725^2 - 1^2)(1.5) = 18.73 \text{ in}^3 \]

\[ V_1 = 37.19 \text{ in}^3 \]

**ITEM 2**

\[ V_2 = \pi (2^2 - 1.5^2)(6) - 6 \left[ \frac{15}{360} \{ \pi (2^2 - 1.5^2) \} \right](2 + 4) = 24.74 \text{ in}^3 \]

\[ V \text{ single joint} = 61.93 \text{ in}^3 \]

**VOLUME OF NODE HOUSE**

\[ V = \frac{1}{2} \left[ \frac{4}{3} \pi \{ (6)^3 - (4)^3 \} \right] + \pi (9^2 - 4^2)(2) + \pi (6^2 - 4^2)(3) \]

\[ = 915.25 \text{ in}^3 \]

**TOTAL WEIGHT**

\[ = [9(61.93) + 915.25] \times 0.174 = 256.23 \text{ lbs} \]
B. MIDSPAN JOINT ALTERNATE 1.

SECTION B-B

CONICAL GRAPHITE
FIBER TUBE
MIDSPAN JOINT
TUBE FITTING
SPICE RING

R = 3 1/4"
R = 4"
R = 1/4"
R = 5/2"
R = 4 3/4"

0.6"

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SINGLE JOINT

ITEM 1

\[ V_1 = \pi (\frac{3}{8} - 3\frac{1}{4}) (8) + 6 \left[ \frac{30}{300} \left\{ \pi \left( \frac{1}{4} \cdot 95.5^2 - 4^2 \right) \right\} \right] (4) + \pi \left( 3.74^2 - 3.25^2 \right) (8) \]

= 210.17 in³

ITEM 2

\[ V_2 = \pi (5.5^2 - 4.75^2) (8) + 6 \left[ \frac{30}{300} \left\{ \pi \left( \frac{1}{4} \cdot 95.5^2 - 4^2 \right) \right\} \right] (4) = 234.44 in³ \]

TOTAL WEIGHT = (210.17 + 234.44)(0.174) = 77.36 LBS

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C. MIDSPAN JOINT ALTERNATE 2

SECTION A-A

LOCK RING

SCREWED PIN

MIDSPAN PIPE FITTING

CONICAL GRAY TUBE

SPICE RING

R= 4"
R= 3/4"
R= 5"
R= 4/2"
R= 5/2"

1/2"
1/2"
1"
1"
1/2"
1/2"

3/8"

0.6"
SINGLE JOINT

\[ V_1 = \pi (4.5^2 - 3.25^2)(3) + \pi (4^2 - 3.25^2)(7) + \pi (3.74^2 - 3.25^2)(3) = \frac{243.16}{\text{in}^3} \]

\[ V_2 = \pi (5.5^2 - 4^2)(1.5) + \pi (5.5^2 - 4.5^2)(3) + \pi (5.5^2 - 5^2)(2) + 6 \left[ \frac{5}{360} \{ \pi (5^2 - 4.5^2) \} \right](1) + 6 \left[ \frac{30}{360} \{ \pi (5^2 - 4.5^2) \} \right](1) = \frac{203.89}{\text{in}^3} \]

\[ V_3 = \pi (4.5^2 - H^2)(3.5) + 6 \left[ \frac{25}{360} \{ \pi (5^2 - 4.5^2) \} \right](1) = \frac{52.95}{\text{in}^3} \]

TOTAL WEIGHT = \( (243.16 + 203.89 + 52.95)(0.174) = 86.86 \text{ lbs} \)
SCHEME 7 WEIGHT CALCULATION

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SINGLE JOINT

ITEM ①
\[ V_1 = \pi (1)^2 (b) + \frac{4}{3} \pi (1.75)^3 = 41.30 \ \text{m}^3. \]

ITEM ②
\[ V_2 \rightarrow \pi (3.75)(3.75) - \frac{4}{3}(1)^2 (3 \times 2 - 1) - \pi (2)^2 (1) = 77.62 \ \text{m}^3 \]
\[ \rightarrow \frac{1}{2} \left[ \pi (2.5^2 - 2^2) \right] (1.5)^2 (2.5^2 - 2^2) (0.87 - 5 \sin 50^\circ \cos 50^\circ) (1.5) = 8.86 \ \text{m}^3 \]
\[ \rightarrow \frac{1}{2} \left[ \pi (3^2 - 2.5^2) \right] (0.75) \]
\[ V_2 = 90 \ \text{m}^3 \]

ITEM ③
\[ V_3 \rightarrow \pi (3.625^2 - 1^2)(1) + \pi (3.625^2 - 3^2)(2) = 64.16 \ \text{m}^3 \]
\[ \rightarrow \frac{1}{2} \left[ \pi (3^2 - 2.5^2) \right] (0.75) = 3.24 \ \text{m}^3 \]
\[ V_3 = 67.4 \ \text{m}^3 \]

\[ V_1 + V_2 + V_3 = 198.7 \ \text{m}^3. \]
ALLOW 10% FOR EPOXY FILLER
\[ V \text{ SINGLE JOINT} = 1.1 (198.7) = 218.6 \ \text{m}^3 \]

VOLUME OF NODE JOINT HOUSE:
\[ V = \frac{1}{2} \left[ \frac{4}{3} \pi (b^3 - h^3) \right] + \pi (11^2 - h^2)(2) + \pi (6^2 - h^2)(2) \]
\[ = 1103.75 \ \text{m}^3 \]

TOTAL WEIGHT = [(9 (218.6) + 1103.75) (0.174) = 534.4 \ \text{lbs}]
GRAPHITE - EPOXY PIPE

Density: \( \rho = 0.063 \text{ lb/ft}^3 \)
Diameter: 6"
Wall thickness: 0.14"

\[ A = 2.577 \text{ ft}^2 \]

1. For 10 ft long pipe:

\[ V = 10(12)(2.577) = 309.24 \text{ ft}^3 \]

\[ \text{WEIGHT} = 309.24(0.063) = 19.48 \text{ lbs} \]

2. For 14.14 ft long pipe:

\[ V = 14.14(12)(2.577) = 437.26 \text{ ft}^3 \]

\[ \text{WEIGHT} = 437.26(0.063) = 27.55 \text{ lbs} \]
### Summary of Joint Weight Quantity

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Nodal Joints Weight (Lbs)</th>
<th>Midspan Joints Weight (Lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>217.92</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>252.50</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>217.70</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>424.00</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>292.94</td>
<td>-</td>
</tr>
<tr>
<td>6 (ALT. 1)</td>
<td>256.23</td>
<td>77.36</td>
</tr>
<tr>
<td>6 (ALT. 2)</td>
<td>256.23</td>
<td>86.86</td>
</tr>
<tr>
<td>7</td>
<td>534.40</td>
<td>-</td>
</tr>
</tbody>
</table>

Diagram showing nodal and midspan joints with dimensions.
# Heavily Loaded Joints for Assembling Aerobrake Support Trusses

Hannskarl Bandel, Nils Olsson, and Boris Levintov

The major emphasis of this study was to develop erectable joints for large aerobrake support trusses. The truss joints must be able to withstand the large forces experienced by the truss during the aero-pass, as well as be easily assembled and disassembled on orbit by astronauts or robots. Other important design considerations include; strength, stiffness, and allowable error in strut length. Six mechanical joint designs, as well as a seventh joint design, where a high strength epoxy is injected to make the connection rigid, are presented.