Bearing Elastohydrodynamic Lubrication: A Complex Calculation Made Simple

Erwin V. Zaretsky
Lewis Research Center
Cleveland, Ohio

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BEARING ELASTOHYDRODYNAMIC LUBRICATION: A COMPLEX CALCULATION MADE SIMPLE

Erwin V. Zaretsky
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

The lubricant elastohydrodynamic (EHD) film thickness formula is reduced to a simplified form whereby only the rolling-element bearing inside and outside diameters and speed (in revolutions per minute) and the lubricant type and viscosity (in centipoise) at temperature are required for its use. Additionally, a graph is provided for the first time that is based upon experimental data giving an EHD film reduction factor as a function of contact lubricant flow number. This reduction factor accounts for lubricant starvation within the Hertzian contact. A graph relating the ratio of minimum film thickness to composite surface roughness and a lubrication-life correction factor is also provided. The life correction factor is used to determine resultant bearing life.

INTRODUCTION

Until approximately 1960, the role of the lubricant between surfaces in rolling contact was not fully appreciated. Metal-to-metal contact accompanied by boundary lubrication was presumed to occur in all applications. However, the development of elastohydrodynamic (EHD) theory showed that most rolling-element bearings have a thin film separating the contacting bodies during motion. Because of the high pressures in the contact zone, the lubrication process is accompanied by some elastic deformation of the contact surfaces shown in figure 1, and thus, the process is referred to as elastohydrodynamic (EHD) lubrication. Grubin (1949) was among the first to identify this phenomenon, which also occurs for other oil-lubricated, rolling-contact machine elements such as gears.

Historically, elastohydrodynamic (EHD) lubrication may be viewed as one of the major tribological developments of the twentieth century. It not only revealed the existence of a previously unsuspected regime of lubrication in highly stressed, nonconforming machine elements, but it also brought order to the complete spectrum of lubrication regimes, ranging from boundary to hydrodynamic.

The most important practical aspect of elastohydrodynamic lubrication theory is the determination of the film thickness separating the contacting surfaces. The first satisfactory equation representing the minimum film thickness was reported by Grubin (1949). Other theories are those of Dowson and Higginson (1966) and Archard and Cowking (1965-66) and Hamrock and Dowson (1977). Hamrock and Dowson (1981) provide a complete discussion of EHD theory and give an excellent historical overview of its development. Their respective formulas are

\[ H = 1.95 \times 10^{-0.73w-0.091} \] (1)

\[ H = 1.6 \times 10^{-0.6w-0.13} \] (2)
Archard-Cowling (1965-66)  \[ H = 2.04 \left( 1 + \frac{2R_x}{3R_y} \right)^{-0.71} G^{0.74} \eta^{0.74} \eta^{-0.074} \]  

Hamrock-Dowson (1977)  \[ H = 3.63(1 - e^{-0.68 k})G^{0.49} \eta^{0.68} \eta^{-0.073} \]  

The variations in EHD theory and the resulting formulas are beyond the scope of this writing. However, suffice it to say, the variations between the EHD film thicknesses calculated with the different formulas are less than the variations between the various sets of experimental data used to verify the theories (Coy and Zaretsky, 1981). This is illustrated in figure 2, which shows the film thickness according to the theories of Hamrock and Dowson (1977), Grubin (1949), Cheng (1972), and Chiu (1974). A theoretical model for oil starvation in a rolling-element bearing was formulated by Chiu (1974). Figure 2 shows the results from Chiu's (1974) analysis and compares his experimental data with the experimental data of Coy and Zaretsky (1981). From the correlation between the experimental data of Coy and Zaretsky (1981) and the analytical predictions of Chiu's starvation theory, oil starvation in the Hertzian contacts appears to be the primary reason why the experimental data deviated from classical EHD theory (Coy and Zaretsky, 1981).

### SYMBOLS

- \( a \): major semi-axis of Hertzian contact, in.
- \( b \): minor semi-axis of Hertzian contact, in.
- \( E' \): reduced modulus of elasticity, \( \frac{1}{E'} = \frac{1}{2} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) \), psi
- \( E_1, E_2 \): Young's modulus for body 1 and body 2, psi
- \( F \): normal load, lb
- \( G \): materials parameter, \( \alpha E' \)
- \( G_0 \): 0.49
- \( H \): dimensionless film thickness, \( h_c/R_x \)
- \( h_c \): central or average film thickness, in.
- \( h_c' \): \( h_c \) multiplied by EHD film reduction factor (see fig. 4), in.
- \( ID \): inner-ring (bearing bore) diameter, in.
- \( k \): ellipticity ratio, \( a/b \)
- \( k_G \): lubricant flow factor, min/cP
\( k_h \)  
EHD film factor, in.\(1.36/1b^{0.68} \)

\( N \)  
speed, rpm

\( OD \)  
bearing outer diameter, in.

\( P \)  
bearing pitch diameter, in.

\( R_x \)  
equivalent radius in rolling direction, \( \frac{1}{R_x} = \frac{1}{R_{x1}} + \frac{1}{R_{x2}} \), in.

\( R_y \)  
equivalent radius perpendicular to rolling direction, \( \frac{1}{R_y} = \frac{1}{R_{y1}} + \frac{1}{R_{y2}} \), in.

\( U \)  
speed parameter, \( u_0/E'R_x \)

\( u \)  
average surface speed, in./sec

\( W \)  
load parameter, \( F/E'R_x^2 \)

\( x \)  
coordinate in rolling direction, in.

\( y \)  
coordinate in transverse direction, in.

\( \eta_0 \)  
absolute viscosity, cP

\( \alpha \)  
pressure-viscosity exponent, ps1^{-1}

\( \eta_0 \)  
absolute viscosity at temperature, lb-sec/in.\(^2 \)

\( \Lambda \)  
film parameter, \( h_c^i/\sigma \)

\( v_1, v_2 \)  
Poisson's ratio for body 1 and body 2

\( \sigma \)  
composite surface roughness, \( (\sigma_1^2 + \sigma_2^2)^{1/2} \), in.

\( \sigma_1, \sigma_2 \)  
root mean square (rms) surface roughnesses of contacting bodies, in.

\( \omega_1 \)  
inner-ring speed, rad/sec

1  
body 1

2  
body 2
EHD SIMPLIFIED

Film thickness calculations can be greatly simplified for first-order approximations. Referring to the Hamrock-Dowson (1977) formula, the ellipticity ratio $k$ for most bearings can be assumed to be approximately 7. Additionally, for most ball and roller bearing applications, the resultant maximum Hertzian stresses on the bearing raceways will be between 200 000 and 350 000 psi. As the maximum Hertzian stress is increased from 200 000 to 350 000 psi, the EHD film thickness is reduced only 9 percent. As a result, a maximum Hertzian stress level of 275 000 psi can be assumed as a representative stress. Based upon the assumption that the load parameter $W$ can be considered a constant,

$$W = 0.073 = 2.27$$

(5)

Values of the materials parameter $G$ are a function of lubricant chemistry. From equation (4),

$$\bar{G} = G^{0.49}$$

(6)

Representative values of $G$ and $\bar{G}$ are given in table I. The average surface velocity $u$ at the contact of the rolling element and the race can be written as follows:

$$u \approx \frac{p \omega_1}{\frac{\omega_1 (OD + ID)}{8}}$$

(7)

Using equation (7) in the speed parameter $U$ gives

$$U \approx \frac{\omega_1 (OD + ID)}{8E'R_x} \eta_0$$

(8)

For most bearings,

$$R_x = \frac{OD - ID}{8}$$

(9)

Combining equations (5), (6), (8), and (9) into the Hamrock-Dowson (1977) formula (eq. (4)), where $k = 7$ and $E' = 33 \times 10^6$ psi, gives

$$h_c \approx k_h (OD - ID)^{0.32} [\omega_1 (OD + ID)]^{0.68} \eta_0^{0.68} \bar{G}^{0.68}$$

(10)

For OD, ID, and $h_c$ in inches, $k_h = 7.85 \times 10^{-6}$ in.$1.36/1b0.68$.

To further simplify equation (10), the equation can be rewritten to reflect the rotational speed in revolutions per minute and the viscosity in centipoise,
where

$$h_c = k_h(OD - ID)^{0.32} [N(OD + ID)]^{0.68} Z_0^{0.68} G$$  \(11\)

For OD, ID, and \(h_c\) in inches, \(k_h = 3.8 \times 10^{-11}\); for OD and ID in millimeters and \(h_c\) in inches, \(k_h = 1.49 \times 10^{-12}\).

Representative temperature-viscosity curves for typical lubricants are given in figure 3. From these graphs and the bulk bearing operating temperature (known or assumed), a value of \(Z_0\) for a typical lubricant type can be obtained for use in equation (11).

**STRAVATION EFFECT**

As stated in reference to figure 2, it was concluded that the deviation between calculated and measured EHD film thicknesses can be attributed to lubricant starvation in the Hertzian contact zone. Starvation is caused by the inability of the lubricant to flow into the contact zone at a rate necessary to supply the volume required to establish the theoretical film thickness. Because the starvation effect occurs mostly at higher speeds, where film thickness is generally large, the reduction in film thickness by itself may not affect bearing performance. However, thinner films can shorten bearing life.

From figure 2, starvation becomes a factor to be considered when the contact lubricant flow number \((G \times U)\) becomes greater than \(2 \times 10^{-7}\). Below this value, the measured film thickness reasonably approximates that which is calculated.

The EHD film thicknesses of figure 2 calculated by using the Grubin (1949) and Hamrock and Dowson (1977) formulas are based upon the contact being fully flooded with lubricant. If the bearing itself is starved for oil because the oil supply is interrupted or because the oil cannot reach the bearing contact zone during extremely high-speed bearing operation, then additional reduction in film thickness can occur at all values of the lubricant flow number. This reduction is usually manifested by distress of the bearing raceway surfaces.

For the purpose of determining an EHD film reduction factor, the experimental data plotted in figure 2 were compared with the Hamrock-Dowson (1977) prediction. The resultant EHD film reduction factor was plotted against the contact lubricant flow number in figure 4.

The contact lubricant flow number is as follows.

$$G \times U = k_G \alpha Z_0 N (OD + ID)/(OD - ID)$$  \(12\)

For \(\alpha\) in square inches per pound, \(k_G = 1.52 \times 10^{-10}\) min/cP.

By using the value of \(G \times U\) from equation (12), the EHD film reduction factor can be obtained from figure 4. An estimate of the reduced EHD film thickness \(h_c'\) in the central region of the Hertzian contact zone is then found by multiplying the \(h_c\) (from eq. (11)) by the EHD film reduction factor.
BEARING LIFE EFFECTS

The measure of the effectiveness of the lubricant film is the lubricant film parameter or λ ratio (i.e., h_c/σ, the central film thickness multiplied by EHD film reduction factor divided by the composite surface roughness of the rolling-element surfaces). Usually the root mean square (rms) surface finishes of the contacting bodies σ_1 and σ_2 are used to determine the composite surface roughness as follows:

$$\sigma = \left(\sigma_1^2 + \sigma_2^2\right)^{1/2}$$

For most commercial off-the-shelf bearings, σ can be taken as 10 μm. For most aerospace bearings, σ can be taken as 5 μm. For very large industrial bearings, σ can be taken as 25 μm.

The lubricant film parameter or λ ratio can be used as an indicator of rolling-element performance and life. For λ < 1, surface smearing or deformation, accompanied by wear, will occur on the rolling surfaces. For 1 < λ < 1.5, surface distress may be accompanied by superficial surface pitting. For 1.5 < λ < 3, some surface glazing can occur with eventual failure caused by classical subsurface-originated rolling-element fatigue. At λ > 3, minimal wear can be expected with extremely long life; failure will eventually be by classical subsurface-originated rolling-element fatigue. The most expedient way of attaining a higher λ ratio is to reduce the bearing operating temperature and, thus, increase the lubricants viscosity. Another way is to select a lubricant with a higher viscosity at operating temperature and/or a larger pressure-viscosity coefficient. The most expensive way of attaining a higher λ ratio is to select a high-quality surface finish on the raceways and rolling elements.

The effect of film thickness on bearing life is shown in figure 5. The life factor obtained from this figure can be used to modify or adjust the bearing calculated life (Zaretsky, 1986).

REFERENCES


### TABLE I. PRESSURE-VISCOSITY COEFFICIENT AND MATERIALS PARAMETER FOR REPRESENTATIVE LUBRICANTS

<table>
<thead>
<tr>
<th>Type (specification)</th>
<th>Pressure-viscosity coefficient, ( a ) m²/N (psi⁻¹)</th>
<th>Materials parameter, ( G = a )</th>
<th>( G - 0.49 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40 °C (104 °F)</td>
<td>100 °C (212 °F)</td>
<td>150 °C (302 °F)</td>
</tr>
<tr>
<td>Automatic transmission</td>
<td>1.54x10⁻⁸</td>
<td>1.117x10⁻⁸</td>
<td>1.02x10⁻⁸</td>
</tr>
<tr>
<td>Fluid (GM6137-H)</td>
<td>1.08x10⁻⁸</td>
<td>0.81x10⁻⁸</td>
<td>0.70x10⁻⁸</td>
</tr>
<tr>
<td>Superrefined mineral oil</td>
<td>2.54x10⁻⁸</td>
<td>1.56x10⁻⁸</td>
<td>1.27x10⁻⁸</td>
</tr>
<tr>
<td>Type II ester (MIL-L-23699)</td>
<td>1.24x10⁻⁸</td>
<td>1.06x10⁻⁸</td>
<td>0.87x10⁻⁸</td>
</tr>
<tr>
<td>Diester (polyester) (MIL-L-7808)</td>
<td>0.99x10⁻⁸</td>
<td>0.83x10⁻⁸</td>
<td>0.60x10⁻⁸</td>
</tr>
<tr>
<td>Synthetic hydrocarbon</td>
<td>1.38x10⁻⁸</td>
<td>1.13x10⁻⁸</td>
<td>1.04x10⁻⁸</td>
</tr>
<tr>
<td>plus 20 percent polyester (MIL-L-2104C, MIL-L-46152)</td>
<td>0.95x10⁻⁸</td>
<td>0.79x10⁻⁸</td>
<td>0.52x10⁻⁸</td>
</tr>
<tr>
<td>Synthetic hydrocarbon</td>
<td>1.77x10⁻⁸</td>
<td>1.51x10⁻⁸</td>
<td>1.09x10⁻⁸</td>
</tr>
<tr>
<td>Mineral oil (MIL-L-6081)</td>
<td>1.17x10⁻⁸</td>
<td>1.04x10⁻⁸</td>
<td>0.75x10⁻⁸</td>
</tr>
</tbody>
</table>

The values given are representative values of \( a \) and may vary from batch to batch or from one lubricant brand to another.
Figure 1. - Elastohydrodynamic film between two bodies in rolling contact.

Figure 2. - Theoretical effect of kinematic starvation and inlet shear heating on film thickness and comparison with experiment (Coy and Zaretsky, 1981).

Figure 3. - Viscosity-temperature chart for representative lubricants. (Data for mineral oils courtesy of BP America, Inc., Cleveland, Ohio; synthetic lubricants from Roelands (1968).)
Figure 4. - Elastohydrodynamic (EHD) film reduction factor as a function of contact lubricant flow number.

Figure 5. - Lubricant-life correction factor as a function of lubricant film parameter (ratio of minimum film thickness to composite surface roughness).
The lubricant elastohydrodynamic (EHD) film thickness formula is reduced to a simplified form whereby only the rolling-element bearing inside and outside diameters and speed (in revolutions per minute) and the lubricant type and viscosity (in centipoise) at temperature are required for its use. Additionally, a graph is provided for the first time that is based upon experimental data giving an EHD film reduction factor as a function of contact lubricant flow number. This reduction factor accounts for lubricant starvation within the Hertzian contact. A graph relating the ratio of minimum film thickness to composite surface roughness and a lubrication-life correction factor is also provided. The life correction factor is used to determine resultant bearing life.