Fatigue Crack Growth in Unidirectional Metal Matrix Composite

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SUMMARY

The weight function method was used to determine the effective stress intensity factor ($\Delta K_{\text{eff}}$) and the crack opening profile for a fatigue tested composite which exhibited fiber bridging. The bridging mechanism was modeled using two approaches; the crack closure approach and the shear lag approach. The numerically determined $\Delta K_{\text{eff}}$ values from both methods were compared and correlated with the experimentally obtained crack growth rates for SiC/Ti-15-3 [0]$_8$ oriented composites. The near crack tip opening profile was also determined for both methods and compared with the experimentally obtained measurements.

INTRODUCTION

The ability to predict fatigue crack growth behavior of metal matrix composites is of particular interest due to the presence of many cracklike defects which tend to be present in these materials. Experimental studies on a number of these composites (refs. 1 to 3) have shown that cracks tend to propagate in the metallic matrix leaving behind unbroken fibers which bridge the two cracked surfaces.

A number of researchers (refs. 3 to 6) have attempted to model crack bridging. The crack bridging model that has received the most attention for composite materials is the so-called shear lag model (refs. 3 to 5). This model is based on the relative sliding between the fiber and the matrix in the region where the interface shear stresses exceed the frictional shear forces. This model was developed for brittle matrix fiber composites which have purely frictional bonding between the fibers and matrix. This model was also applied to metal matrix composite systems (ref. 3). Another type of bridging model which may be applicable for these types of composites is based on the application of a closure pressure in the bridged zone proportional to the load carried by the bridging fibers (henceforth this model will be termed crack closure model). This methodology had been applied previously to model the effect of unbroken ligaments in a fast fracture test of steels (ref. 6).
In order to decide whether these types of models can represent the actual fatigue behavior, an experimental FCG test program was conducted. Testing was performed using a specially designed loading stage mounted inside an SEM. This apparatus permits an accurate measurement of the crack opening displacements, measurement of crack growth rates and identification of active fatigue failure mechanisms.

Both types of models were used to predict the fatigue crack growth behavior and the near tip crack displacements. Analytical predictions were compared to the test data.

EXPERIMENTAL

Fatigue crack growth tests were conducted on two single edge notch (SEN) metal matrix composite specimens. The test material was a SiC (SCS-6) fiber reinforced composite with a Ti-15V-3Cr-3Al-3Sn (Ti-15-3) matrix. The testing was performed on unidirectionally oriented specimens with the fiber direction being parallel to the loading axis and perpendicular to the starter notch. Companion testing was performed on compact tension (CT) specimens machined out of laminated, unreinforced matrix material which was processed in a similar manner as the composite.

RESULTS

The fatigue crack growth behavior of the SEN composite specimens consisted mostly of stable crack growth perpendicular to the loading axis. Crack growth was confined to the matrix ligaments only, leaving behind in its wake unbroken fibers bridging the two cracked surfaces (ref. 2). A plot of the crack length versus number of cycles (fig. 1) revealed a continuous decrease in the fatigue crack growth rate with increasing crack length. After growth around four row of fibers (approximately 1 mm of growth) the crack growth arrested.

As a first attempt, the fatigue crack growth rate of the composite was plotted as a function of the stress intensity factor range (ΔKapp) and is shown in figure 2. The stress intensity range was calculated by considering only the applied load and the crack length and without taking into consideration the fiber bridging of the cracked surfaces. The data show crack growth rate decreasing with an increasing ΔKapp. Also shown in the figure is the fatigue crack growth data of the unreinforced matrix material. The crack growth rates for the composite are at least a factor of 10³ lower than those of the unreinforced composite for the same ΔKapp.

ANALYSIS

Both of the models evaluated use fracture mechanics as the basis for calculating the effective crack driving force, ΔKeff. The effective stress intensity factors and the crack opening displacements were determined using the weight function method. The weight function used is based on the Bueckner (ref. 7) formulation for the stress intensity factor calculation of a single edge notch in a finite geometry specimen (fig. 3). The stress intensity factor, SIF, for an unbridged crack of length a is given by:
\[
K(a) = \sqrt{\frac{2}{\pi}} \int_{0}^{a} \frac{p(x)H(a,x)}{\sqrt{a-x}} \, dx
\]  

where \( H(a,x) = (1 + m_1(a-x)/a + m_2(a-x)^2/a^2) \) and where \( m_1 \) and \( m_2 \) are given by:

\[
m_1 = 0.6147 + 17.1844 (a/w)^2 + 8.7822 (a/w)^6
\]
\[
m_2 = 0.2502 + 3.2889 (a/w)^2 + 70.0444 (a/w)^6
\]

The unbridged displacement profile is given by:

\[
u(x) = \frac{2(1 - \nu_c)^2}{E_c \pi} \int_{0}^{a} \frac{H(\ell,x)}{\sqrt{\ell - x}} \left( \int_{0}^{1} \frac{p(x)H(\ell,x')}{\sqrt{\ell - x'}} \, dx' \right) \, d\ell
\]  

where \( E_c \) and \( \nu_c \) are the composite elastic modulus and Poisson's ratio, respectively.

For a partially bridged crack the distribution of the pressure function \( p(x) \) is divided into two parts:

\[
\text{unbridged region } p(x) = \sigma^\infty, \quad \text{for } 0 < x < a_0 \tag{3a}
\]
\[
\text{bridged region } p(x) = \sigma^\infty - c(x), \quad \text{for } a_0 < x < a \tag{3b}
\]

where \( a_0 \) and \( a \) are the unbridged crack length and the total crack length, respectively. The pressure distribution \( c(x) \) is an approximate closure pressure due to bridging. Equations (1) and (2) can be modified for a partially bridged crack by substituting the appropriate pressures given in equation (3).

Up to this point both models are identical. The primary difference between the two models is the manner by which they account for the closure pressure \( c(x) \). For the shear lag model, references 3 and 4, \( c(x) \) is a function of the fiber/matrix interfacial friction shear stress and is proportional to the square root of the displacement and given by:

\[
c(x) = 2 \left[ \frac{u(x) \tau \nu_f^2 E_f E_m}{R(1 - \nu_f)E_m} \right]^{1/2}
\]  

where

\[
\tau \quad \text{fiber/matrix interfacial friction stress}
\]
\[
E_f \quad \text{fiber modulus}
\]
\[
E_m \quad \text{matrix modulus}
\]
\[
\nu_f \quad \text{fiber volume fraction}
\]
\[
R \quad \text{fiber radius}
\]

By substituting equation (4) into equation (3b), the solution of the displacement field, equation (2), becomes an integral equation which has to be solved.
numerically using an iterative scheme with a small damping factor to guarantee convergence. After reaching a converged displacement profile, the composite SIF for the shear lag model is calculated using equation (1). Finally, the effective crack driving force is related to the composite SIF by:

$$K_{\text{eff}} = \frac{K_{\text{Em}}}{E_c}$$

(5)

For the crack closure model, $c(x)$ is equal to the stress carried by the fibers in the bridged region averaged over the total bridged area and is given by:

$$c(x) = \sigma^\infty \left( \frac{w}{w-a_0} + \frac{6w a_0 [0.5x(w-a_0) - (x-a_0)]}{(w-a_0)^3} \right)$$

(6)

Equation (6) represents the normal and bending stresses in the bridged fiber region and is valid only for a partially bridged crack. The closure load in equation (6) is independent of the displacement, hence the SIF for the closure model can be solved directly without an iterative procedure. The effective crack driving force, $K_{\text{eff}}$, for the closure model is calculated by substituting the results of equation (6) into equations (3) and (1).

The values of the parameter used for the composite system tested are given in table 1. The value of the interface shear stress $\tau$ depicted in the table is a calculated value from experimental indentation tests (ref 3).

DISCUSSION

The calculated results of the $\Delta K_{\text{eff}}$ values for both bridging models are given in figure 4 as a function of the crack length. Also shown is the $\Delta K_{\text{app}}$ for the unbridged case. As shown in the figure, the $\Delta K_{\text{eff}}$ values for both models decrease with an increase in the crack length. The decrease in the $\Delta K_{\text{eff}}$ values corresponds to the observed behavior where the crack growth rates decreased and finally crack growth stopped with an increase in the crack length. The $\Delta K_{\text{eff}}$ for the shear model is slightly lower than $\Delta K_{\text{eff}}$ for the closure model.

The experimentally obtained composite FCG data was replotted in terms of the effective SIF, $\Delta K_{\text{eff}}$, calculated using both crack bridging models. As shown in figure 2, the predicted crack growth rate data for both models are in agreement with the data trends exhibited by the unreinforced matrix material. An additional check of the accuracy of the bridging models are the numerically determined displacements near the crack tip. Both the predicted and observed displacements were plotted in figure 5 for a given crack length as a function of the distance behind the crack tip. The predicted crack tip displacement fields by the two methods are almost identical and are difficult to distinguish from each other. Excellent correlation is observed between the measured and numerically determined displacement range in the crack tip area. As the distance from the crack tip increases some disagreement is observed for both models.
SUMMARY OF RESULTS

Two fracture mechanics based fiber bridging models were successfully used in an attempt to describe experimentally observed fatigue crack growth behavior in a SiC/Ti-15-3 composite. The crack closure model and the shear lag model, in agreement with the experimentally observed behavior, predicted a decrease in the crack growth rates and eventual crack arrest with an increase in the crack length. Both models also predicted the near crack tip displacements. The advantage of the closure model is that no numerical iteration is required to obtain the solution. The success of these models points to the applicability of fracture mechanics to describe the fatigue behavior of certain composites.

REFERENCES


TABLE 1. - SiC/Ti-15-3 METAL MATRIX PARAMETERS ASSUMED

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>$b$</td>
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<td>Ec</td>
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</tr>
<tr>
<td>$\tau$</td>
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</table>

Figure 1. - Crack length versus number of cycles.

Figure 2. - Fatigue crack growth (FCG) data.

Figure 3. - Partially bridged single edge notch specimen.
Figure 4. Stress intensity factor versus crack length.

Figure 5. Crack opening displacements as a function of the distance to the crack tip.
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**Abstract**

The weight function method was used to determine the effective stress intensity factor (\(\Delta K_{eff}\)) and the crack opening profile for a fatigue tested composite which exhibited fiber bridging. The bridging mechanism was modeled using two approaches; the crack closure approach and the shear lag approach. The numerically determined \(\Delta K_{eff}\) values from both methods were compared and correlated with the experimentally obtained crack growth rates for SiC/Ti-15-3 [0]_s oriented composites. The near crack tip opening profile was also determined for both methods and compared with the experimentally obtained measurements.

**Key Words (Suggested by Author(s))**

Metal matrix composite; Fatigue crack growth; Fiber bridging; Effective stress intensity; Crack tip displacement