Big-Bang Nucleosynthesis Revisited

Keith A. Olive\textsuperscript{1}, David N. Schramm\textsuperscript{2}, Gary Steigman\textsuperscript{3}, and Terry P. Walker\textsuperscript{4}

Abstract

We compute the homogeneous big-bang nucleosynthesis yields of D, \(^3\)He, \(^4\)He, and \(^7\)Li, taking into account recent measurements of the neutron mean-life as well as updates of several nuclear reaction rates which primarily affect the production of \(^7\)Li. We discuss the extraction of primordial abundances from observation and the likelihood that the primordial mass fraction of \(^4\)He, \(Y_p\), is less than or equal to 0.24. Using the primordial abundances of D+\(^3\)He and \(^7\)Li we limit the baryon-to-photon ratio (\(\eta\) in units of \(10^{-10}\))\(\leq \eta_{10} \leq 4.3\) (or, in terms of the present mass density in baryons, \(1.8 \times 10^{-31} \leq \rho_B \leq 3.0 \times 10^{-31} \text{g/cm}^3\), for a microwave background temperature of 2.75 degrees) which we use to argue that baryons contribute between 0.02 and 0.11 to the critical energy density of the universe. An upper limit to \(Y_p\) of 0.24 constrains the number of light neutrinos to \(N_\nu \leq 3.4\), in excellent agreement with the LEP and SLC collider results. We turn this argument around to show that the collider limit of 3 neutrino species can be used to bound the primordial abundance of \(^4\)He: \(0.235 \leq Y_p \leq 0.245\)

11/89

\textsuperscript{1}School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455
\textsuperscript{2}University of Chicago, Chicago, IL 60637 and NASA/Fermilab Astrophysics Group, Fermi National Accelerator Laboratory, Batavia, IL 60510
\textsuperscript{3}Departments of Physics and Astronomy, The Ohio State University, Columbus, OH 43210
\textsuperscript{4}Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138
The last complete study of big-bang nucleosynthesis [1] concluded that the primordial abundances of D and $^3$He and $^7$Li require a baryon-to-photon ratio in the range $3 \leq \eta_{10} \leq 10$ (where $\eta_{10} \equiv 10^{10} \times n_B/n_\gamma$) and that an upper limit of 0.254 to the primordial mass fraction of $^4$He ($Y_p$) marginally allowed, following the argument of Steigman, Schramm, and Gunn[2], one additional light two-component neutrino, thus constraining the number of light neutrinos: $N_\nu \leq 4$. Since that time, the limit to the number of neutrinos has been reexamined (still finding $N_\nu \leq 4$) [3], and the upper limit to $\eta_{10}$ from primordial $^7$Li has been slightly tightened ($\eta_{10} \leq 5 - 6.4$)[4],[5]. Recent measurements of the neutron mean-life using ultra-cold neutrons [6], along with a new compilation of the nuclear reaction cross sections relevant to the big-bang reaction network[7], modify the predicted yields of primordial elements as a function of $\eta_{10}$ and $N_\nu$—the changes in nuclear physics primarily affect $^7$Li production and the decrease in the neutron mean-life (and its uncertainty) is reflected in the $^4$He yield. In this letter we discuss these changes and their effects as well as provide an assessment of the most reasonable current limits to the primordial abundances. In particular we note that the observational data on $^4$He have improved somewhat and seem to constrain $Y_p$ to a lower value than that used by Yang et al.[1]. 1 We also discuss the relationship between the primordial nucleosynthesis and collider limits to the number of neutrinos and show that the LEP and SLC results[9] can be used to constrain the primordial abundance of $^4$He.

We have updated the big-bang nuclear reaction network using the cross sections of Caughlan and Fowler[7] and find that $^7$Li is the only element whose primordial abundance changes noticeably. Low-$\eta$ production of $^7$Li decreases due to a decrease in $^4$He($t,\gamma$)$^7$Li and an increase in $^7$Li($p,\alpha$)$^4$He. The high-$\eta$ $^7$Li production increases due to an increase in D(d,n)$^3$He. There is also a slight decrease in D+$^3$He production for all values of $\eta$ due to increases in d-d reactions. In fig. 1 and fig. 2, we show the big-bang yields of $^7$Li, D, and $^3$He (by number relative to hydrogen) for $N_\nu = 3$ or 4 and a neutron mean-life of 889.8 seconds ($\tau_\frac{4}{3} = 10.28$ minutes) as a function of $\eta$ (variations in these primordial abundances due to allowed changes in the neutron mean-life are negligible).

Although primordial $^4$He shows no effects of the changes in the nuclear reaction network, it is sensitive to changes in weak interaction physics. Using ultra-cold neutrons (i.e., neutrons with kinetic energies less than the neutron scattering potential of their storage

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1 For a more detailed discussion of the current status of the standard big-bang nucleosynthesis model, see Walker, Steigman, Schramm and Olive [8].
vessel[10]), Mampe et al.[6] have measured the mean-life of stored neutrons to be $887.6 \pm 3$ sec. When combined with earlier measurements of the neutron mean-life[11], we find for a world average:

$$\tau_n = 889.8 \pm 4.4 \text{ sec}, \quad (1)$$

or a reduction in the neutron half-life to $\tau_\frac{1}{2} = 10.28 \pm 0.05$ minutes. The amount of $^4$He produced in the big-bang depends on the neutron-to-proton ratio when weak interactions freeze-out. A decrease in the neutron half-life is equivalent to an increase in the weak interaction rates, thus leading to a decrease in the neutron-to-proton ratio at freeze-out and resulting in less primordial $^4$He. The change in the primordial mass fraction of $^4$He due to changes in the neutron half-life is given by $\Delta Y_p \approx 0.2(\tau_n - 889.8 \text{ sec})/889.8 \text{ sec}$. Note that $\pm 2\sigma$ changes in $\tau$ lead to $\Delta Y_p \approx \pm 0.002$. Variations of this magnitude in $Y_p$ are also found if radiative and Coulomb corrections to the tree-level weak rates are included[12].

The accuracy of the measured neutron mean-life requires that we include these higher order corrections to the weak rates which we have done$^2$. In fig. 3 we show the mass fraction of $^4$He as a function of $\eta$ for $\tau_n = 889.8 \pm 8.8$ seconds and $N_\nu = 3$ and 4. A good fit (for $2.5 \leq \eta_{10} \leq 10$) to primordial $^4$He production in the standard big-bang is

$$Y_p = 0.228 + 0.010 \ln \eta_{10} + 0.012(N_\nu - 3) + 0.185\left(\frac{\tau_n - 889.8}{889.8}\right), \quad (2)$$

where $\tau_n$ is the neutron mean-life measured in seconds. We can rewrite this as a limit on the number of light neutrinos $^3$:

$$N_\nu = 3.0 - 0.8 \ln \eta_{10} + 19\left(\frac{Y_p - 0.228}{0.228}\right) - 15\left(\frac{\tau_n - 889.8}{889.8}\right). \quad (3)$$

Since the discovery of Li in Pop.II stars[13], controversy has surrounded the primordial abundance of $^7$Li. Subsequent observations of Li in many more metal-poor galactic halo stars[14] exhibit a remarkably narrow ($\text{Li}/\text{H}$) plateau corresponding to $12^+ \log(\text{Li}/\text{H})$

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$^2$ Dicus et al.[12] found that evaluating the n→p rates numerically, including Coulomb, zero and finite temperature radiative corrections, corrections to the electron mass, and correct evaluation of the difference in neutrino and photon temperature leads to a net decrease in $^4$He production of 0.0022 for all values of $\eta$ with half the decrease coming from the numerical evaluation of the rate integrals. See Ref.[8] for further discussion.

$^3$ $Y_p$ actually depends on the number of relativistic degrees of freedom accesible at the epoch of primordial nucleosynthesis, which we assume here are due only to light ($\lesssim 1$ MeV) particles allowed by the standard model (i.e., photons, electrons, and $N_\nu$ neutrino species).
= 2.07 ± 0.08 for effective temperatures in the range 6350 ≥ T_{eff}(°K) ≥ 5500. Adoption of the plateau halo stars as representing the primordial $^7$Li abundance requires a mechanism to increase the disk abundance of Li to $10^{-9}$ over the history of the galaxy. Two such mechanisms (production in Type II supernovae shocks passing thru hydrogen envelopes[16] or via neutrino synthesis in supernovae cores[17]) may be able to account for the disk $^7$Li enhancement. The best argument, however, for believing that the Pop.II abundances do in fact represent primordial $^7$Li is the presence of the narrow $^7$Li plateau. Stars in the plateau cover a range of temperatures and metallicities sufficiently diverse that it seems highly unlikely that a substantial depletion mechanism independent of stellar properties can be found. The 2-$\sigma$ upper limit from the plateau stars to $12 + \log(^7\text{Li}/H)$ is 2.23 ($^7\text{Li}/H \leq 1.70 \times 10^{-10}$) and bounds the baryon-to-photon ratio from above (see Fig. 1):

$$\eta_{10} \leq 4.3,$$

for $N_{\nu} = 3$. From a recent study of depletion models[5] it was concluded that $12 + \log(^7\text{Li}/H) \leq 2.21(2-$\sigma$)$ for models with standard isochrones and at most $12 + \log(^7\text{Li}/H) \leq 2.36 (2-$\sigma$)$ for models including diffusion. Thus even with some depletion we have the constraint $\eta_{10} \leq 4.8$, for $N_{\nu} = 3$.

Since primordial D and $^3$He are monotonically decreasing functions of the baryon-to-photon ratio, an upper limit to their primordial abundance constrains $\eta_{10}$ from below[1]. Deuterium is easily converted to $^3$He at temperatures above $6 \times 10^5$°K via $D(p,\gamma)^3$He and, since the pre-main sequence Sun is fully convective, the observation of $^3$He implanted by the solar wind in gas-rich meteorites[18], on lunar foils[19], and in lunar soils[20], is a measure of the sum of pre-solar D and $^3$He: $10^4(y_{23}/y_4)_\odot = 4.03 \pm 0.19$ (where $y_{23}$ is the sum of D and $^3$He relative to hydrogen, by number, and $y_4$ is the ratio, by number, of $^4$He to hydrogen). Adopting $y_{4\odot} = 0.10 \pm 0.01$, we find conservatively:

$$3.11 \leq 10^5 y_{23\odot} \leq 5.06.$$  

Depending on the history of $^3$He destruction and production in stellar environments, $y_{23\odot}$ can be used to constrain $y_{23p}$. If some of the pre-solar gas has been through several

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4 The standard big-bang makes relatively little $^6$Li, and the isotopic shift in the Li resonance doublet is so small that, until recently, only the total Li abundance has been measured which we assume to be predominantly $^7$Li. There is now evidence supporting this assumption in even the hottest of the Pop.II stars[15].

5 We choose values consistent with current solar models ($Y_\odot = 0.28 \pm 0.02$).
generations of stellar processing and the surviving fraction of $^3$He per generation is defined as $\epsilon_3$, then, ignoring stellar $^3$He production, we can write:

$$\psi_{23p} < A_\odot^{(\epsilon_3-1)} \frac{X_\odot}{X_p} \psi_{23p},$$

(6)

where we define an astration factor $A_\odot \equiv (X_\odot/X_p)(\psi_{23\odot}/\psi_{23p})$ and we assume that some fraction of the gas goes thru several generations of stars instantaneously. Studies of stellar and galactic chemical evolution models indicate[21] that $\epsilon_3 \geq 1/4$, and constraints from the overproduction of heavy elements[22] require $A_\odot \geq 1/3$. With $\epsilon_3 = 1/4$, equation (6) yields

$$10^5 \psi_{23p} \leq 10.9.$$

(7)

As promised, the upper limit to D + $^3$He leads to a lower bound on the baryon-to-photon ratio (see Fig. 2):

$$\eta_{10} \geq 2.6,$$

(8)

for $N_p = 3$.

The limits on the baryon-to-photon ratio from D+$^3$He and $^7$Li can be expressed in terms of the ratio of baryon energy density-to-critical energy density of the Universe ($\Omega_B = \rho_B/\rho_c$, where $\rho_c = 10.5h^2\text{keV cm}^{-3}$, and $h$ is the Hubble constant measured in units of $100 \text{ km/sec/Mpc}$) using $\Omega_B h^2 = (3.73 \times 10^{-3})\eta_{10} T_{2.75}^{-3}$, where $T_{2.75}$ is the microwave background temperature measured in units of $2.75^\circ\text{K}$.[7] The limits on $\eta$ from big-bang nucleosynthesis given in equations (4) and (8) can be rewritten as

$$9.7 \times 10^{-3} \leq \Omega_B h^2 T_{2.75}^{-3} \leq 1.6 \times 10^{-2}$$

(9)

for $N_p = 3$. For $h \geq 0.4$, the upper limit on $\Omega_B$ from primordial nucleosynthesis is 0.11. For an $\Omega_{total} = 1$ universe older than 10 billion years, $h \leq 0.65$, which implies $\Omega_B \geq 0.02$. This upper bound is the principal argument for non-baryonic dark matter. The difference between the lower bound and the amount of visible matter argues that a significant fraction of baryons are unlit.

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6 This analysis improves on that given in Yang et al.[1] who assume a single generation of stellar processing. In fact, the single generation assumption would yield a very similar limit: $10^5 \psi_{23p} < 10.5$ for $\epsilon_3 \geq 1/4$.

7 Recent measurements[23] of the microwave background indicate a temperature of $2.78 \pm 0.01^\circ\text{K}$.
Having derived a lower bound on $\eta_{10}$ of 2.6 from D+$^3$He and taking a 2-$\sigma$ lower bound to the neutron mean-life we find (using equation (3) )

$$N_\nu \leq 3.4 + 20\left(\frac{Y_p^{\text{max}} - 0.240}{0.240}\right),$$

(10)

where $Y_p^{\text{max}}$ is the 2-$\sigma$ upper limit to the mass fraction of primordial $^4$He allowed by observation. The size of the numerical coefficient on the $Y_p$ term represents the sensitivity of primordial $^4$He to the expansion rate. In order to extract the primordial abundance of $^4$He from observations, the $^4$He contribution from stellar nucleosynthesis must be determined. Peimbert and Torres-Peimbert\[24\] suggested that such a subtraction might be accomplished by correlating the observed $^4$He data with some indicator of metallicity (Z),

$$Y = Y_p + \frac{dY}{dZ}Z,$$

(11)

and extrapolating to zero metallicity. Highly-ionized HII regions in low-metallicity galaxies are the measurement of choice and oxygen is the traditional Z indicator. While the oxygen seen in these objects is produced by stars having masses $\geq 12M_\odot$, the stellar $^4$He comes from all stars $\geq 2M_\odot$ leading to the conclusion that oxygen may not be a good diagnostic of stellar $^4$He production\[25\]. Nitrogen and carbon are produced in intermediate mass stars and therefore may more closely track stellar $^4$He production than does oxygen \[26\]. We have compiled a ($Y,N,O$) data set with 36 objects (33 objects reviewed by Pagel\[27\] plus 3 others) and a ($Y,C$) data set consisting of 6 objects (5 objects considered by Steigman, Gallagher, and Schramm\[26\] and 1 other). A linear least-squares fit to these data yields the following:

$$Y = (0.226 \pm 0.005) + [(1.6 \pm 0.4) \times 10^2]\frac{O}{H}$$

$$= (0.230 \pm 0.004) + [(3.0 \pm 0.7) \times 10^3]\frac{N}{H}$$

$$= (0.234 \pm 0.008) + [(2.6 \pm 2.3) \times 10^2]\frac{C}{H}$$

(12)

The errors in the fits are 1-$\sigma$ with respect to the quoted statistical errors in the observations. However, because some galaxies are reported with what appear to be underestimated errors (systematic errors are generally not included), the fits are quite sensitive to the individual data points. Fluctuations in the fits lead one to believe (this is discussed in great detail in \[8\] ) that a fair estimate of the primordial helium abundance is $Y_p = 0.23 \pm 0.01$. For
\( Y_p^{max} = 0.240 \), the upper bound to the number of light neutrino species from primordial nucleosynthesis is (from eqn.(10) )

\[
N_\nu \leq 3.4. \tag{13}
\]

The cosmological limit is in remarkably good agreement with the results of the LEP and SLC \( e^+e^- \) colliders which, when combined in weighted average, find \( N_\nu \leq 3.6 \) at the 2-\( \sigma \) level. Of course, the \( Z^0 \) experiments and the cosmological limits on \( N_\nu \) are complementary. The width of the \( Z^0 \) receives contributions from anything that couples to the \( Z^0 \) and has a mass \(< M_Z/2\), whereas the cosmological limit is sensitive to particles which contribute to the energy density of the universe during primordial nucleosynthesis.

In addition to relativistic neutrinos with \( m \leq \text{MeV} \), the energy density at this epoch can also be affected by slightly non-relativistic neutrinos having masses from a few to a few tens of MeV or by superweakly interacting particles. Since the experimental number is in such good agreement with the cosmological prediction, significant exotica are unlikely. However, one loophole worth noting is that the experimental limit on the \( \nu_\tau \) mass \(< 35 \text{ MeV} \) could allow the \( \nu_\tau \) to be incorrectly assessed in cosmological counting while still fully contributing to the \( Z^0 \) counting.

To summarize, we have shown that recent changes in nuclear cross sections and the neutron mean-life produce some modest changes in the standard big-bang nucleosynthesis predictions for \( D, ^3\text{He}, ^4\text{He}, \text{and } ^7\text{Li} \). The neutron mean-life is now measured to such a high degree of accuracy that uncertainties in the standard big-bang results are due in large part to changes in nuclear physics. For example, if we assume that there are only 3 light neutrinos, the standard big-bang predicts a \( ^4\text{He} \) abundance in the range (see Fig. 3)

\[
0.235 \leq Y_p \leq 0.245, \tag{14}
\]

where we use the constraints on the baryon-to-photon ratio from \( D + ^3\text{He} \) and \(^7\text{Li} \):

\[
2.6 \leq \eta_{10} \leq 4.3. \tag{15}
\]

In order to limit the number of light neutrinos, we need to extract the primordial abundance of \(^4\text{He} \) from measurements which are surely contaminated by stellar nucleosynthesis. Following tradition, we have tried to fit the observations of \(^4\text{He} \) in metal-poor highly ionized objects to a linear dependence on metallicity, using either carbon, nitrogen, or oxygen as the tracers of stellar \(^4\text{He} \) production. Surprisingly, we find that fits all indicate that \( Y_p = 0.23 \pm 0.01 \). We cannot infer a "2-\( \sigma \) upper-limit" to \( Y_p \) because large systematic errors dominate. If indeed the upper limit to primordial \(^4\text{He} \) is 0.24, then big-bang nucleosynthesis constrains the number of light neutrino species: \( N_\nu \leq 3.4. \)
Acknowledgements

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References


Figure Captions

Fig. 1  The big-bang yield of $^7$Li, by number relative to hydrogen, as a function of the baryon-to-photon ratio in units of $10^{-10}$, $\eta_{10}$, for 3 (solid line) or 4 (dashed line) light neutrino species and a neutron mean-life of 889.8 seconds.

Fig. 2  As in Fig. 1 for D, $^3$He, and D+$^3$He.

Fig. 3  The primordial mass fraction of $^4$He as a function of the baryon-to-photon ratio in units of $10^{-10}$, $\eta_{10}$ for 3 or 4 light neutrino species and a neutron mean-life of 889.8 seconds (solid lines). We also show the 3 neutrino yield for 2-$\sigma$ variations in the neutron mean-life (dash-dot lines).
Primordial $^7\text{Li}$

$\tau_s = 689.8$ sec

$N_e = 4$
$N_e = 3$

$(\text{H})N/((\text{He})N)$
Figure 2

Primordial D and $^3$He

$\tau_\ast = 888.8$ sec

D + $^3$He

$^3$He

D