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Title of Research: "Physics of Systems Containing Neutron Stars"

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November 1989
A. Progress Report for Grant NAGW-1618

The years during which my research was supported by NASA saw the birth, within these research grants, of the accretion-spin-up model for Millisecond Pulsars (mPSRs), of the Beat-Frequency (BF) model for horizontal branch QPOs and of the wind-driven-accretion model for Very Low Mass X-ray Binaries (LMXBs) and millisecond pulsars (presentations 1, 2). All of these were important milestones on the road towards understanding the evolution from LMXBs to millisecond pulsars. Clearly, there is still a lot to be done, notably understanding of other branch QPOs and of the full machinery of QPO sources and understanding of magnetic field decay in neutron stars vs. the possibility of a low field at birth.

The following is a summary of work done during this period of March – October 1989. Three major topics have been extensively looked into during this time: The reported 2,000 Hz optical signal from the direction of SNR1987A, the possibility that neutron stellar surface magnetic fields do not decay except when the star is accreting and the 6 Hz QPOs of LMXBs; the latter is the major investigation topic at present.

1. A Neutron Star in SN 1987A? (Papers 1,2)

If the recently reported 0.5 ms period pulsed optical signal from the direction of SN 1987A originated in a young neutron star, its interpretation as a rotational period has difficulties. First, the upper bound on the present luminosity of 1987A will limit such a rotating star’s surface magnetic field to $\leq 10^9$G. Unless this field rises to $10^{12}$G in a time of $\sim 10^3$ yrs, either because of the emergence of a presently buried field or from magnetothermal generation, such a low field marks this event as very different from the Crab supernova as well as from those explosions responsible for the half-dozen other pulsar/supernova remnant associations and may have
important implications for our understanding of mPSRs. Second, such a high rotation rate (and without triaxial instability) may place too severe a constraint on the equation of state (EOS) of nuclear matter. In fact, none of the normal-matter EOSs known today can support both such rotation against equatorial break up and a slowly-rotating star against collapse. Only strange EOSs may do that, so one can choose between the 1987A pulsar being unique among neutron stars, or neutron stars being unique among other astronomical objects. In paper (1) we point out that there is a way out of this uncomfortable situation, namely, that a remnant radial oscillation of a neutron star, excited in the supernova event, may survive for several years and has the expected (gravitationally red-shifted) period. We show, that if a PSR 1987A is indeed producing the .5 ms signals it may very well be an ordinary, slow, pulsar, which is simply still so young that it is still vibrating.

To overcome difficulties in understanding the origin of the submillisecond optical pulses we applied a model similar to that of Kundt and Krotscheck for pulsed synchrotron emission from the Crab. The interaction of the expected ultrarelativistic 

\[ e^\pm \] pulsar wind, due to stellar vibration or rotation, with the pulsar dipole electromagnetic wave reflected from the walls of a "pulsar cavity" within the SN 1987A nebula can generate pulsed optical emission with efficiency at most \( \eta_{\text{max}} \approx 10^{-3} \). The maximum luminosity of the source is reproduced and other observational constraints can be satisfied for an average wind energy flow \( \approx 10^{38} \text{erg/(s steradian)} \) and for electron Lorentz factor \( \gamma \approx 10^5 \). This model applied to the crab yields pulsations of much lower luminosity and frequency (paper 2).

2. Decay of Neutron Stellar Magnetic Fields (Paper 3)

Theoretical calculations for the decay of neutron stellar core and crustal magnetic field find for their time scales, depending on assumptions on the detailed internal structure, values varying from \( \lesssim 10^6 \) yrs up to the age of the Universe.
There is, therefore, a good case to be made for determining field decay from observations. The possible existence of a pulsar in SNR1987A has reignited the debate on whether birth fields can be much below $10^{12}$ Gauss; at present, however, everything we know is consistent with this not being the case. The low fields of millisecond pulsars may, thus, be the consequence of general field decay in these (supposedly old) systems.

A recent γ-ray observation raises, however, the possibility that field decay is not a general occurrence in neutron stars. Cyclotron absorption lines at energies 20 keV and 40 keV from gamma-ray burst sources could indicate old neutron stars with $10^{12}$ G magnetic fields! Thus we set out to investigate the possibility that field decay only occurs in accreting neutron stars. Paper (3) reports the results of a phenomenological study of field decay according to

$$B = B_0 \left(1 + \frac{\Delta m}{m_B}\right)^{-1},$$

where $B$ and $B_0$ are the present and birth fields, $\Delta m$ the mass accreted and $m_B$ is a characteristic value for the accretion-induced field-decay process, $m_B \sim 10^{-4} M_\odot$. The study shows that the millisecond and accreting pulsar data are consistent with this process. More work is in progress (see below).

3. The 6 Hz QPO in LMXBs

This rather universal Normal Branch LMXB phenomenon has, so far, not found a completely satisfactory explanation. Present models connect it with near-Eddington accretion rates, $\dot{m} \sim \dot{m}_E$, an attractive idea which has some interesting consequences, but which seems to work only when $\dot{m}/\dot{m}_E$ is a few percent below unity. This may prove to be a limitation, in particular in view of evidence that the Normal Branch luminosity may, actually, be much farther away from Eddington (Mitsuda, private communication).
In preparation for investigating various classes of models we are developing computer codes for various-spectra photon transfer through various electron-spectra scattering clouds. We are using the CONVEX fast computer at Columbia.

B. Plans for Coming Year's Research Activity

At least at the beginning of the coming year, we plan to follow the research indicated in item (3) and (2) of the progress report (in that order of emphasis). When approaching Eddington luminosities, radiation pressure dominates over gas pressure close to the star and scattering optical depths become large, so that it is clear that any model for the matter dynamics and photon transfer must take that into account. We must learn, in item (3) how to properly take angular momentum into account (instead of just assuming free-fall), and how local oscillations can be excited. We want to see the role that the soft $\gamma$-rays we have suggested previously to be emitted in LMXBs play – perhaps in requiring cool scattering clouds.

As for magnetic field decay, the Beat Frequency model, which successfully accounts for Horizontal Branch QPOs in LMXBs, does point in the direction of matter-magnetic field interaction in the DMB (Disc-Magnetosphere Boundary) of LMXBs. We want to look at the effects of incoming accreted material on the stellar field (burying it? field reconnexions?)

When this research, which goes along items (1), (3) and (6) of section B of our original proposal (of May 1988) has been given satisfactory answers, it will be time to shift focus to items (2), (4), and (5) of the original proposal, namely, the self-excited companion winds and their role in the evolution of VLMXBs and of the windy radio pulsar 1957+20; by then, more data will be available from this pulsar, possibly permitting better understanding of the wind formation mechanism.
C. Papers and Major Presentations

PAPERS


MAJOR PRESENTATIONS


PAPER 1
Does SN 1987A Contain a Rapidly Vibrating Neutron Star?

Q. Wang, K. Chen, T. T. Hamilton, M. Ruderman and J. Shaham

If the recently reported 0.5 ms period pulsed optical signal from the direction of SN 1987A originated in a young neutron star, its interpretation as a rotational period has difficulties. First, the upper bound on the present luminosity of 1987A will limit such a rotating star’s surface magnetic field to \( \leq 10^9 \) G. Unless this field rises to \( 10^{12} \) G in a time of \( \sim 10^3 \) yrs, because of the emergence of a presently buried field or from magnetothermal generation, such a low field marks this event as very different from the Crab supernova as well as from those explosions responsible for the half-dozen other pulsar/supernova remnant associations. Second, such a high rotation rate without triaxial instability may place too severe a constraint on the equation of state of nuclear matter. Here we point out that a remnant radial oscillation of a neutron star, excited in the supernova event, may survive for several years and has the expected (gravitationally red-shifted) period. Heavy ions at the low density stellar surface, periodically shocked by the vibration, will efficiently produce sharp pulses of optical cyclotron radiation in a surface field of \( \sim 10^{12} \) G. These pulses may be only negligibly modulated by a (much slower) stellar rotation because of the nearly isotropic emission mechanism and the strong gravitational bending of light rays. We discuss below some details of this model. We do not discuss here a mechanism for the reported 8 hr modulation, which may be the result of timing noise in much the same way that spurious quasi-sinusoidal modulations have appeared in period timing analyses of older pulsars.
Neutron star vibrations have already been discussed at some length in the literature \(^7,8\). From dimensional considerations the fundamental radial mode period is expected to be of order \(P \sim (G\rho)^{-1/2} \sim 10^{-3} \) s with \(\rho\) the mean neutron star density. Typical neutron star models\(^8\) with \(M \sim 1M_\odot\) and \(R \sim 10^6\) cm give periods close to \(4 \times 10^{-4}\) s with no sensitivity to the exact central density. This vibrational period is close to that observed when corrected for the gravitational red shift \((4. \times 10^{-4}(1 - 2GM/Rc^2)^{-1/2} \sim 5 \times 10^{-4}\) s). Non-radial and higher order radial modes would be damped on timescales of \(< 1\) yr\(^7,8,9\) from gravitational, neutrino, and electromagnetic radiation. According to Finzi and Wolf\(^10\), the major damping source for the fundamental radial mode is the URCA neutrino emission process, which gives a damping time \(\sim 10^2\) yrs. However, this timescale could be reduced dramatically by two effects:

(1) "Exotic" enhancement of the weak interactions, the main source of the radial vibration damping. These include central \(\pi\)-condensates, or quark matter\(^11,12\). Confirmation of our model may rule out the presence of these in the putative neutron star produced by SN 1987A, unless the neutrino emission they can give is suppressed by superfluid energy gaps.

(2) Enhancement of gravitational radiation from coupling to non-radial vibrations. Such coupling will arise when the underlying spherical symmetry is broken, e.g., if the neutron star is rotating. For a neutron star with a uniform density, Chau\(^13\) calculated the rotation-dependent gravitational radiation damping time to be \(\sim 2 \times 10^3 P^4\) yrs, where \(P\) is the rotation period in seconds. Our model would then require a slowly rotating neutron star, with \(P \geq 10^{-1}\) sec. With such a period, a \(10^{12}\) G field does not cause the neutron star spin down power to exceed the current supernova luminosity.

The optical radiation cannot originate in a region larger than a light-travel size of 150 km. Furthermore, because the reported presence of strong first and
second harmonics indicates a sharp pulse, the size of the emission region should be much smaller than this, implying emission very close to the stellar surface. For a pulse luminosity $\geq 5 \times 10^{35} \text{erg s}^{-1}$ (18th magnitude at a distance of 55 kpc) any thermal emission must occur at a temperature $T > 10^6 K$; upper limits on the X-ray emission from the supernova$^{14}$ constrain the emission process to be nonthermal.

If a radiating particle of charge $Z$ emits energy $E$ per vibration period, the observed pulse luminosity from an optically thin surface region would require

$$E/Z \geq 8 \text{ MeV.} \quad (1)$$

If optically thick only at optical frequencies, it requires $E \geq 10^2 \text{ MeV}$. If electron synchrotron radiation were responsible for the optical emission, the magnetic field would have to be

$$B \sin \alpha \leq 3.5 \times 10^5 \left( \frac{8 \text{MeV}}{E} \right)^2 \epsilon \text{ Gauss} \quad (2)$$

to produce optical emission, where $\epsilon$ is a typical photon energy in eV and $\alpha$ is a typical electron pitch angle. This extreme constraint on $B$ suggests, instead, that the radiation arises from cyclotron radiation from stellar surface heavy ions, $Fe^{+Z}$ for example. These will produce optical cyclotron radiation (at much higher fields) with a typical photon energy of

$$\epsilon \sim 3B_{12} \frac{2Z}{A} \text{ eV,} \quad (3)$$

where $B_{12}$ is the magnetic field value in units of $10^{12}$ Gauss and $A$ is the atomic number of the ion. [While curvature radiation by an electron could fall in the optical band, this mechanism, generally, has a very low efficiency ($\sim \frac{e^2}{\hbar c} \frac{h \omega}{mc^2} \sim 10^{-7}$) compared to that of synchrotron emission.]

In this ion cyclotron emission model, condition (1) requires the energy of $Fe^{+26}$, for example, to be $\geq .2 \text{ Gev per ion}$; the column density is then $\sim 2 \times$
10^{22} E_{\text{Gev}}^{-1} \text{ cm}^{-2}. Fully stripped energetic but nonrelativistic Fe ions (or \text{He}^{++} or protons) can give strong cyclotron optical emission in a field \( B \sim 10^{12} \) Gauss. We propose that these ions can be given the needed velocities as the radial vibration steepens into a shock when it reaches the small densities and scale heights at the stellar surface. These strong shocks occur at 0.5 ms intervals, just after the surface reaches its maximum outward speed, and can accelerate particles to velocities of order \( 10^{10} \) cm/sec. Just after the shock the kinetic energy (\( \sim 4 \) Gev per ion) carried by ions will dominate that carried by electrons. Because of the short travel time for the shock passing through the surface of the neutron star and the short ion cyclotron lifetime (\( \leq 10^{-5} \) sec), a sharp pulse is expected within each cycle. Furthermore, since the emission is concentrated around the ion gyration frequency, the vibration shocked surface can gives a reasonably efficient conversion of internal vibration energy to optical radiation.

The total luminosity of SN 1987A sets a lower limit to the neutron star rotation period of \( \geq 20 B_{12} L_{38}^{-1} \) ms, where \( L_{38} \) is the supernova luminosity in \( 10^{38} \) erg/s. As noted above, our model requires \( P \geq 10^{-1} \) sec. As the cyclotron emission occurs in a magnetic field which varies over the surface of the star, one expects a modulation at the stellar rotation period. However, the amplitude of this modulation may be rather small because of the isotropic energy input from the vibration, the fairly isotropic geometry of cyclotron emission, and the strong gravitational bending of the emitted light rays.4,5

Future period observations should test our model. The period of the neutron-star vibration should not increase significantly with time although the luminosity will decrease as the vibration is damped; the rotation period of the star should be found to be \( \geq 10^{-1} \) s. The optical pulse spectrum should be significantly different from that of the Crab pulsar, which originates from a very different mechanism. The frequency corresponding to the peak emission in the SN 1987A
optical pulsar spectrum could be used to estimate the magnetic field at the surface of the neutron star (see Equation (3)). Observations in other wavelength bands are highly desirable. The detection of X-rays from the neutron star before the vibration dies out could provide important input to our understanding of the origin of the optical light.

We thank D. Helfand, J. Halpern and J. Applegate for many helpful discussions and R. Muller for an early communication to us of the results in ref 1. This is Columbia Astrophysics Laboratory contribution No. 371 and has been supported, in part, by NASA grants NAG8-497 (TTH and QW) and NAGW-567 (JS), and by National Science Foundation grant AST86-02831 (MR).


PAPER 2
ORIGIN OF PULSED EMISSION FROM
THE YOUNG SUPERNOVA REMNANT SN 1987A

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ABSTRACT

To overcome difficulties in understanding the origin of the submillisecond optical pulses from SN 1987A we apply a model similar to that of Kundt and Krotscheck for pulsed synchrotron emission from the Crab. The interaction of the expected ultrarelativistic $e^\pm$ pulsar wind with the pulsar dipole electromagnetic wave reflected from the walls of a “pulsar cavity” within the SN 1987A nebula can generate pulsed optical emission with efficiency at most $\eta_{\text{max}} \approx 10^{-3}$. The maximum luminosity of the source is reproduced and other observational constraints can be satisfied for an average wind energy flow $\approx 10^{38}\text{erg}/(\text{s}\cdot\text{steradian})$ and for electron Lorentz factor $\gamma \approx 10^5$. This model applied to the Crab yields pulsations of much lower luminosity and frequency.
1. Introduction

The strong luminosity (between 400 and 900 nm) and the short period ($P = 0.5\,\text{ms}$) of the reported optical pulsations from the young supernova remnant (SNR) SN 1987A (Kristian et al. 1989) raises problems for conventional models of pulsar optical emission. If relativistic beaming plays no dominant role, a rather small radiating area ($cP$) is implied, leading to an extraordinarily high optical brightness temperature ($kT_b >> 1\,\text{GeV}$). It has not been demonstrated how such emission may arise close to a neutron star. On the other hand, it is widely accepted that pulsars may give rise to a wind of relativistic electrons and/or positrons ($e^\pm$) (Rees and Gunn 1974, Kundt and Krotscheck 1977, Kennel and Coroniti 1984, Cheng, Ho and Ruderman 1986). As suggested by Kundt and Krotscheck for the Crab nebula, ultra relativistic $e^\pm$ may give rise to pulsed emission far from the stellar surface where the relativistic wind runs into the pulsar dipole electromagnetic wave reflected from the inner boundary of the surrounding nebula. The main point of our paper is that such a mechanism can account successfully for the periodicity of the modulated optical signal reported from SN 1987A and it alleviates the optical luminosity problem posed by observations.

During the January 18 observation the brightness of the detected pulsed signal varied from magnitude 17 to 16 reaching at its maximum 1% of the luminosity of the SN 1987A remnant (Middleditch 1989). Thus, the maximum "optical" pulsed luminosity of the source was $L_{\text{opt}} = 3 \cdot 10^{36}\,\text{erg s}^{-1} \times \Delta\Omega/4\pi$, where $\Delta\Omega$ is the solid angle into which the pulsed radiation was beamed. At the same time the luminosity of the remnant (SNR) was $L_{\text{SNR}} = 3 \cdot 10^{38}\,\text{erg/s}$ (Burki and Cramer 1989). Subsequent observations failed to detect the pulses at a limiting magnitude lower by 2 than the maximum observed (Kristian et al. 1989) and by 8 than that of the SNR (Ogelman et al. 1989). By the end of April 1989 the remnant bolometric luminosity decreased to $L_{\text{SNR}} = 1 \cdot 10^{38}\,\text{erg/s}$. If $L_P$ is the electromagnetic power of the pulsar and $\bar{L}_P$ is the time average (over several months) of this quantity, then the pulsed luminosity is $L_{\text{opt}} = \eta L_P$, where $\eta$ is the efficiency, while the SNR luminosity is $L_{\text{SNR}} = f \cdot \bar{L}_P + L_0(t)$, where $0 < f \leq 1$ and the last term ($L_0 \geq 0$) represents the luminosity the remnant would have if the pulsar had no power. At maximum brightness of the optical pulses $\eta \geq 3 \cdot 10^{-2} f(\bar{L}_P/L_P)(\Delta\Omega/4\pi)$. The large
value of the numerical coefficient constitutes the "optical luminosity problem."

Below, we find $\eta \lesssim 10^{-3}$. This implies that emission from the pulsar is beamed ($\Delta \Omega < 4\pi$), or the pulsar wind power is only sporadic ($\bar{L}_P \ll L_P$), or most ($L_P - fL_P$) of the pulsar spin-down power is either converted into kinetic energy of the nebula or reradiated at unobserved frequencies, (or all of the above). At any rate, we conclude that the pulsed-beam synchrotron emission model presented below can account for all observations if the relatively modest requirement $f(\bar{L}_P/L_P)(\Delta \Omega/4\pi) \lesssim 10^{-1}$ is met.

The cavity model is discussed in Section 3, while the constraints implied by the data on SN 1987A are considered in Sections 4 and 5.

2. Difficulties of magnetospheric models

Optical pulses from the Crab pulsar can originate in that neutron star's (outer) magnetosphere. But if the neutron star in SN 1987A is a weak-magnetic-field ($B_* < 10^9$ G) "millisecond" rotator (Kristian et al. 1989, Pacini, Bandiera and Salvati 1989), it is hard to understand how the optical pulses could arise by an analogous process in its magnetosphere.

Because the Crab pulsar spin rate $2\pi/P_{\text{Crab}} \approx 200\text{ s}^{-1} \approx 60$ times less than that of the 1987A neutron star, the emitting area (at the light cylinder radius) can be $\sim (60)^2$ times larger. In addition, the pulsed optical luminosity is an order of magnitude smaller in the Crab. The needed Crab optical brightness temperature is then $\sim 10^6$ eV, a value generally exceeded for synchrotron radiation of $e^\pm$ pairs created by $\gamma$-rays in the outer magnetosphere (Cheng, Ho and Ruderman 1986). Such emission mechanisms do not work for the pulsar in SN 1987A for two reasons.

i) A 10 GeV electron would give peak synchrotron radiation at photon energies above 100 MeV in the pulsar's magnetospheric field. The fraction of energy emitted into the optical band would then be very small, $\sim 10^{-5}$ of the total radiated synchrotron power.

ii) The detected neutrino burst confirmed that the neutron star in 1987A was formed hot, as expected (Hirata et al. 1987, Bionta et al. 1987). The present surface temperature of the star should be about $5 \cdot 10^6$ K. The whole magnetosphere between the surface of the star and the "light cylinder" (at $r_{lc} \equiv P/2\pi = 3 \cdot 10^6$ cm) should then be suffused with keV X-rays. In this (black body) X-ray flux, the mean free
path for inverse Compton scattering by GeV electrons is \( \sim 10^3 \text{cm} < r_{lc} \). Therefore effective potential drops along the field lines are limited to \( \Delta U \sim 10^9 \text{V} \) by pair plasma created by the Comptonized photons: \( e + X \rightarrow e + \gamma \) followed by \( \gamma + X \rightarrow e^+ + e^- \). On the other hand, magnetospheric currents cannot give magnetic fields exceeding that of the neutron star. This limits the current flow density along open field lines to the Goldreich-Julian value \( \tilde{j}_{\text{max}} = (2\pi|\tilde{B}|)^{-1}(\tilde{\Omega} \cdot \tilde{B}) \tilde{B} \) (Goldreich and Julian 1969), where \( |\tilde{\Omega}| = 2\pi/P \). The maximum power of those currents is \( L_c = \tilde{j}_{\text{max}} R_*^3 \Omega^{-1} \Delta U \). Clearly \( L_c > L_{\text{opt}} \) is needed, as the electrons cannot radiate more energy than they carry. For \( L_{\text{opt}} = 3 \cdot 10^{36} \text{erg/s} \), a minimum potential drop along \( \tilde{B} \) of \( \Delta U \geq 10^{14} \text{V} \) is required. This last value is hugely in excess of the \( 10^9 \text{V} \) value sustainable without electron pair avalanching. The magnetospheric accelerator would thus have been quenched long before it attains the required power.

It has also been suggested that the neutron star in SN 1987A is vibrating with the 0.5 ms period. Wang et al. 1989 proposed cyclotron radiation (in a \( B_* \approx 10^{12} \text{G} \) magnetic field) of ions powered by surface-penetrating shock waves as the mechanism for optical emission. However, it has not been shown how shocked ions could gain the necessary velocity perpendicular to \( \tilde{B} \) without being fragmented. Nor has it been shown how stellar vibration of reasonable amplitude could give rise to rapidly recurring shocks of requisite energy.

We conclude that an origin from within the stellar magnetosphere for the optical pulsations from SN 1987A has not been plausibly demonstrated for either the vibrational or the rotational model.

3. Pulsar cavities in supernova remnants

Far beyond the light cylinder of a pulsar in a vacuum, the spin-down power is carried largely in two forms (Rees and Gunn 1974, Kundt and Krotscheck 1977, Kennel and Coroniti 1984):

a) an ultrarelativistic \( e^\pm \) wind,

b) electromagnetic (EM) fields of the magnetic dipole radiation (from the perpendicular component of the pulsar dipole) and a possible toroidal magnetic field (from the spin-aligned part of the dipole) carried with the wind.

Most of the wind energy is probably due to acceleration of \( e^\pm \) by the very strong (time dependent) fields near the pulsar. For a rotating neutron star with a
non-spin-aligned dipole moment the pulsar spin frequency would be impressed on the electron wind when the electrons are ejected (in a particular direction) from the outer magnetosphere and when they are subsequently accelerated. The resulting \(e^\pm\) bunch structure would repeat at any (distant) point at the period \(P\) of the pulsar dipole radiation. If a similar electron injection and wind creation process were operative in a strongly pulsating neutron star a modulation at the vibration frequency of the magnetic dipole would also be expected.

When the pulsar is contained within a young SNR the large pressure from the pulsar wind and the radiation will create a "cavity" within the remnant. The pulsar cavity is terminated by a shock at radius \(d\) well within the outer nebula radius \(D\). When pulsar emission is the main source of nebular power (Rees and Gunn 1974)

\[
(d/D)^2 \sim \sigma \sim \left(\frac{d}{c}\right),
\]

where \(\sigma\) is the ratio of the pulsar outflow magnetic energy to the total energy density of the wind. For the Crab, Kennel and Coroniti obtain \(\sigma \sim 3 \cdot 10^{-3}\) and \(d_{\text{Crab}} \sim 3 \cdot 10^{17}\) cm, Kundt and Krotscheck find \(\sigma \sim 1\) and \(d_{\text{Crab}} \sim 10^{18}\) cm. Adopting similar values of \(\sigma\) for SN 1987A one would then infer a cavity radius \(d \sim 10^{16}\) cm in that SNR, smaller than that in the Crab by roughly the ratio of the SNR ages.

We do not expect this estimate to be accurate for such a young remnant. However, our model only requires that a cavity with radius \(d < D\) exist; for SN 1987A, \(D \approx 10^{16}\) cm at the epoch of interest (Papaliolios et al. 1989).

The outflowing ultrarelativistic bunches of \(e^\pm\) do not radiate significantly in the nearly comoving EM waves. To the extent that EM energy is backscattered at the cavity wall, they will, however, pass through a magnetic field which may be taken to be comparable with that of the preshock incident magnetic field

\[
B \sim B_{\text{EM}} \approx \left(\sigma L_P/cd^2\right)^{1/2} \sim 2 \cdot 10^{-2}\left(L_{35} \sigma^{-2} d_{15}^{-2}\right)^{1/2},
\]

This value of \(B_{\text{EM}}\) is similar to the one needed to understand the soft X-ray excess emission from SN 1987A, if one assumes equipartition in the nebula (Pacini 1989). If \(\omega_B \equiv eB/mc > 2\pi/P\), the \(e^\pm\) wind will lose energy in the cavity mostly by synchrotron radiation. Had \(\omega_B < 2\pi/P\) the dominant loss mechanism would have been inverse Compton scattering.
4. Pulsed emission from the SN 1987A

In a $B \sim 10^{-2}$G cavity field, the characteristic synchrotron emission frequency is $\sim 10^{16} \gamma_6^2$ Hz, giving optical radiation if $\gamma_6 \equiv \gamma/10^6 \sim 1/6$. The fraction of beam energy converted to such radiation in a $d = 10^{15}$ cm cavity is $\eta = \gamma \omega_B^2 (c^2/mc^4) d \sim 10^{-4}$ for the same values. Because the optical radiation is emitted almost exactly radially, to a distant observer the radiation would appear to be coming from the pulsar itself. Thus, cavity and beam parameters of Section 3. could easily give the kind of optical luminosity observed from SN 1987A if the wind power were $\sim 10^{40} \times (\Delta \Omega/4\pi)$—about ten times the spin-down power of the Crab pulsar\(^1\) if emission is isotropic.

Almost all of the beam power would ultimately be dissipated beyond the cavity boundary shock in the surrounding nebula where $B$ is expected to be $\sim 10^2$ times larger than in the cavity. Refer to Section 1. for a discussion of how the current upper limit on the bolometric luminosity of the nebula can be satisfied.

We must now ask what constraints are imposed on the model parameters by insisting that the observed optical (or near infrared) synchrotron light is pulsed with the $e^\pm$ wind frequency $1/P$. As shown in the next section, this approach yields for the various parameters values close to the ones adopted directly above. We find that the size of the nebula places an upper bound $\eta_{\text{max}} \lesssim 10^{-3}$ on the efficiency of radiation allowed by the model.

A critical assumption is that the relativistic electrons synchrotron radiate in an ordered EM field of wavelength $cP$. This guarantees that the deflection from the radial direction of the radiating $e^\pm$ never exceeds an angle ($\theta_0$, eq. [P10]) less

\(^1\) The expected pulsed cavity emission from the Crab can be scaled from that from SN 1987A. For the “optical” frequency $\omega_{\text{Crab}}/\omega_{1987} = [\gamma^2 B]_{\text{Crab}}/[\gamma^2 B]_{1987} \sim [\gamma^2 \sqrt{\sigma_L P/d}]_{\text{Crab}}/[\gamma^2 \sqrt{\sigma_L P/d}]_{1987}$. For comparable $\gamma$ and $\sigma_L P$, $\omega_{\text{Crab}} \sim \omega_{1987}/500$ or $\lambda(\text{Crab}) \sim 10^2 \mu$m. With similar approximations and assumptions the ratio of pulsed cavity emission luminosities from the Crab and SN 1987A is the ratio of the values of $\sigma_L P \gamma/d$, again corresponding to a reduction of about 500. Thus, the Crab’s pulsed cavity far IR luminosity would be $\sim 10^{33}$ erg/s. A bump of about this magnitude appears in the near ($\lambda \leq 3.5 \mu$m) IR pulse shape of the Crab (Middleditch, Pennypacker and Burns 1983).
than the critical one beyond which the pulses would be washed out. If, instead, the field had been a collection of randomly oriented domains of size $cP$ the average total deflection would have been too large, $\theta_0(d/cP)^{1/2}$.

5. Constraints on pulsed beamed synchrotron emission implied by the SN 1987A data.

By assumption, the optical signal is due to synchrotron radiation of relativistic $e^\pm$ (energy $\gamma mc^2$) in transverse magnetic field of strength $B$ alternating in direction with wavelength $cP$. Before entering an assumed emission zone of radial extent $l$, the electrons travel radially outwards a distance $d - l$ from the neutron star. The electrons radiate into a narrow forward cone of apex angle $\approx 1/\gamma$ about their instantaneous velocity direction, which is, itself, at an angle to the initial (radial) direction of flight. The latter angle is not greater than some maximum deflection angle $\theta_0$ (eq. [P10]). Thus, the cross-sectional area of the emission region seen by an observer at infinity is $\approx \pi b^2$, where

$$b \approx d\theta,$$  \hspace{1cm} (P1)

and

$$\theta \approx \theta_0 + 1/\gamma << 1.$$ \hspace{1cm} (P2)

For the purposes of computation we take the optical brightness temperature to be $kT_b = 10^3 \text{ GeV} \times (b^2/10^{12}\text{ cm}^2)^{-1}$ and the synchrotron frequency to be

$$\gamma^2 eB/mc = 2 \text{ eV}/\hbar,$$ \hspace{1cm} (P3)

(i.e. $\gamma^2 B/10^8\text{ G} = 2$) to obtain (nearly) optimum efficiency of optical detection. The maximum extent of the nebula, $D \approx 10^{16}$ cm places an upper bound on the size of the emitting region and its radial distance from the star: $l < D$, $d < D$.

**Notation**

- $\gamma$—initial Lorentz factor of the radiating electron
- $\theta$—maximum angle between line of sight and initial direction of electron motion
- $\theta_0$—maximum deflection angle of electrons
- $d$—radial distance from the neutron star to the emitting region
- $l$—radial extent of the emitting region
\( \pi b^2 \) — area of emission seen by observer

\( \Delta t \) — maximum allowed differential time of arrival (t.o.a.)

\( \alpha, \beta, \lambda, d_{16} \leq 1 \) — dimensionless parameters not greater than unity

\( \Gamma, G > 1 \) — dimensionless parameters greater than unity

**THE CONSTRAINTS**

A class of constraints is introduced by the requirement that the optical pulses not be washed out. Let the upper bound on the differential spread in time of arrival of all photons in a pulse be \( \Delta t = \alpha P/5 = \alpha \times 10^{-4} \text{s} \), i.e.,

\[
c\Delta t = \alpha \times 10^{6.5} \text{cm}, \quad \alpha \leq 1,
\]

Any initial spread in energies \( (mc^2 \Delta \gamma) \) of \( e^\pm \) leads to a constraint \( l \leq \gamma^3 (c\Delta t)/\Delta \gamma \), less stringent than the following. We define

\[
G = \frac{1}{2} (\beta^2 \gamma^2 + 1) \approx 1 + \frac{1}{2} \gamma^2 \theta_0^2 + \gamma \theta_0.
\]

The differential t.o.a. constraint from time of flight delay of the emitting \( e^\pm \) gives

\[
l = \lambda G^{-1} \gamma^2 (c\Delta t), \quad \lambda \leq 1,
\]

Differential t.o.a. because of different path lengths due to the transverse extent of the emitting region gives \( b = \beta \theta^{-1} (c\Delta t) \), and hence

\[
d = \beta \theta^{-2} (c\Delta t), \quad \beta \leq 1,
\]

(Strictly speaking \( \lambda + \beta \leq 1 \), but we are not concerned with factors of 2.) We note the following limits:

\[
\theta_0 \ll 1/\gamma \Rightarrow G = 1,
\]

\[
\theta_0 \sim 1/\gamma \Rightarrow G \sim 2,
\]

\[
\theta_0 \gg 1/\gamma \Rightarrow G = \frac{1}{2} \gamma^2 \theta_0^2.
\]

The inferred brightness temperature places a lower bound on the electron energy

\[
\gamma = 10^{5.3} \Gamma \beta^{-2} \alpha^{-2} \theta^2, \quad \Gamma > 1.
\]
The efficiency of conversion of the electron energy to optical is \( \eta \sim (\text{Synchrotron power}) \times (\gamma mc^2)^{-1} \times l/c \), i.e.,
\[
\eta = \frac{\alpha \lambda}{G \gamma} \times 10^{3.9}, \quad (P9)
\]
where eq. (P3) was used to eliminate \( B \).

Since the magnetic field traversed by the \( e^\pm \) alternates in direction, the appropriate expression for the deflection angle is \( \theta_0 \approx PeB/(2\pi \gamma mc) \), i.e.
\[
\gamma^3 \theta_0 \approx 10^{11.4}, \quad (P10)
\]
while the size of the nebula limits \( d \) and \( l \),
\[
l \leq d = d_{16} \times 10^{16} \text{cm}, \quad d_{16} < 1. \quad (P11)
\]

**RESTRICTIONS ON \( \gamma \)**

Consider the constraints (P6) to (P9) in the following two cases.

i) \( \theta_0 \lesssim \gamma^{-1} \)

From eq. (P10) this regime holds iff \( \gamma^2 \gtrsim 10^{11.4} \), i.e. \( \gamma \gtrsim 10^{5.7} \). However, the constraint (P6) with the subsidiary condition (P11) then gives a low efficiency, since
\[
l = \alpha \lambda \gamma^2 \times 10^{18.5} \text{cm}.
\]
Hence, \( \alpha \lambda \lesssim 10^{-2} d_{16} \) and therefore \( \eta \lesssim 10^{-4} d_{16} \).

ii) \( \theta_0 \gg \gamma^{-1} \)

From point i) above, this can only hold if \( \gamma < 10^{5.7} \). Now, from eqs. (P5) and (P10)
\[
\theta \approx (2G)^{1/4}/\gamma \approx 10^{11.4}/\gamma^3.
\]
Hence, using also eq. (P8),
\[
\gamma = \Gamma^4 (\alpha \beta)^{-1/3} \times 10^{4.0} < 10^{5.7}, \quad \text{i.e.,}
\]
\[
10^4 \leq \gamma < 10^{5.7}. \quad (P12)
\]
Now, from eqs. (P6), (P7) and (P9), respectively,
\[
l \approx (\alpha \lambda) \gamma^6 \times 10^{8.0} \text{cm} \approx (\lambda/\beta) \Gamma^4 (\alpha \beta)^{-5/7} \times 10^{8.0} \text{cm}, \quad (P13)
\]
\[
d \approx (\alpha \beta) \gamma^6 \times 10^{7.7} \text{cm} \approx \Gamma^4 (\alpha \beta)^{-5/7} \times 10^{7.7} \text{cm}, \quad (P14)
\]
\[
\eta \approx (\alpha \lambda) \gamma^3 \times 10^{-6.4} \approx (\lambda/\beta) \Gamma^{3/7} (\alpha \beta)^{1/7} \times 10^{-6.4}. \quad (P15)
\]
Recall that $\alpha$ is the precision with which fine structure can be observed in the optical pulses in units of $P/5 = 0.1\text{ ms}$ and $d_{16}$ is the maximum distance of the emitting region from the pulsar in units of $10^{16}\text{ cm}$. In principle, $\alpha$ is an observable number. The free parameters $\lambda, \beta$ (and $\Gamma$) were introduced to replace upper (lower) bounds with equalities.

To satisfy $l \leq d$ we must have $\lambda \leq \beta/2 \leq 10^{-0.3}$. But (P13) and (P11) require $(\alpha \lambda)^{\gamma} < 10^{8.0}d_{16}$, giving, upon substitution in eq. (P15), $\eta < 10^{-2.4}(\alpha \lambda d_{16})^{1/2}$. We conclude that this model allows a maximum efficiency of

$$\eta_{\text{max}} \approx 3 \cdot 10^{-3}(d_{16})^{1/2}, \quad (P16)$$

occurring for $\gamma = 10^{5.5}(d_{16})^{1/6}$ and the most favorable values possible of the free parameters ($\beta = 1, \lambda = 0.5$).

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Does mass accretion lead to field decay in neutron stars?
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The recent discovery\(^1,2\) of cyclotron lines from gamma-ray bursts indicates that the strong magnetic fields of isolated neutron stars might not decay. The possible inverse correlation\(^3\) between the strength of the magnetic field and the mass accreted by the neutron star suggests that mass accretion itself may lead to the decay of the magnetic field. We calculated the spin and magnetic field evolution of the neutron star under the hypothesis of the accretion-induced field decay. We show that the calculated results are consistent with the observations of binary and millisecond radio pulsars.

Whether neutron stellar magnetic fields decay or not is at present a controversial issue. Statistical analyses\(^4,5\) of \(\sim 400\) radio pulsars and the study\(^3\) of the origin and evolution of magnetized neutron stars in binary systems indicate that neutron stars are born with magnetic fields of \(10^{12}\)G which then decay with a time constant of \((5 - 10) \times 10^6\) yr. Estimates of age and magnetic field strength of binary millisecond radio pulsars, however, require\(^6\) that the field decay, if it occurs, should stop or proceed much more slowly on time scales of \(\gtrsim 10^9\) yr, at field strengths \(\lesssim 10^{10}\)G. Pulsar models in which magnetic fields do not decay but align with the rotation axis are shown to also be consistent with the observed properties of radio pulsars\(^7,8,9\).
Ohmic dissipation of electrical currents in the crust has been thought to be the physical cause for field decay\(^1\). However, recent calculations\(^1\) of the ohmic decay of dipolar magnetic fields have shown that the field does not decay exponentially as has been assumed in most statistical analyses of radio pulsars and that, if the field occupies the entire crust, it decays by only less than a factor of 100 in a Hubble time.

If the strong magnetic fields of neutron stars do not decay, then the origin of the weak magnetic fields of less than \(10^{10}\) G, found in binary and millisecond radio pulsars\(^1\), remains to be explained. Neutron stars in binary and millisecond radio pulsars are thought to be formed by the iron core collapse of massive stars or by the accretion-induced collapse of massive white dwarfs (for reviews see refs. 13 and 14). In one scenario for the weak field and rapid rotation of millisecond radio pulsars it is argued that, quite simply, white dwarfs\(^15,16,17\) or iron cores of massive stars\(^18\) with the appropriate field strength and angular momentum give birth to the observed millisecond radio pulsars after a collapse. In another scenario, mass accretion in low mass X-ray binaries, which are assumed to be progenitors of millisecond radio pulsars, is proposed to cause both the field decay\(^19\) and the spin-up\(^20\). In fact, Taam and van den Heuvel\(^3\) found a possible inverse correlation between the magnetic field strength and the estimated total mass of accreted matter for binary X-ray sources and for binary and millisecond radio pulsars. This inverse correlation supports the latter scenario. A study of the evolution of magnetic fields in the crust of a neutron star\(^21\) showed further that the inward heat flux caused by mass accretion powers thermomagnetic effects that could remove the strong magnetic field of a neutron star.

In the present paper we examine further the possibility that magnetic fields of neutron stars indeed decay only when neutron stars undergo mass accretion. We do that by considering gamma-ray bursts and the periods and field strengths of binary and millisecond radio pulsars.

The recent discovery\(^1,2\) of cyclotron absorption lines with energies 20 keV and 40 keV from gamma-ray bursts revealed that the central objects of gamma-ray bursters are
indeed strongly magnetized neutron stars with magnetic fields of $10^{12}$G as has already been suggested in some theoretical works$^{22,23}$. The statistical arguments suggest $\sim 10^7$yr (refs. 24 and 25) or $\gg 10^7$yr (ref. 26) for the age of neutron stars in gamma-ray bursters depending on the assumed distance. When combined with the high field strength, the latter age estimate contradicts the field decay hypothesis whereas the former does not.

We note that the absorption lines were seen only in a limited portion of the burst$^1$. This observed fact can also be used to test the field decay hypothesis, if the cyclotron absorption lines appear and disappear due to the rotation of the star, since the presence of the cyclotron lines depends sensitively on the configuration of the magnetic field relative to the line of sight$^{27}$. If this is the case, the 5–10s duration of the cyclotron absorption feature indicates the rotation period of $\gtrsim 10 - 20$s. This rotation period and the field strength put the gamma-ray burst sources into the category of the “turned-off” pulsars$^5$. If the spin-down of the neutron star is caused by the magnetic field, which decays exponentially on a time scale of $\tau_B$, the variation of the rotation period $P$ with time $t$ is represented by

$$P^2 = AB_0^2\tau_B[1 - \exp(-2t/\tau_B)] + P_0^2,$$

(1)

where $B_0$ and $P_0$ are the initial field strength and rotation period, respectively, and the constant $A = 9.8 \times 10^{-40}$sG$^{-2}$ for a stellar moment inertia of $10^{45}$ g cm$^2$ and radius of 10$^6$ cm. In order to attain the rotation period of $\gtrsim 10 - 20$ s from the initial value of maybe less than 1 s, magnetic braking requires the initial field strength of the neutron star to be larger than $\sim 2 \times 10^{13}$G if $\tau_B \sim 10^7$yr. This value of the initial field is on the high side, compared to the average values of $\sim 10^{12}$G indicated from the statistical analyses of radio pulsars. In view of that we would like to pursue here the other possibility, namely, that the magnetic fields of gamma-ray burst sources, which might be isolated neutron stars, do not decay.

As already mentioned, the weak magnetic fields of less than $10^{10}$G are found in binary and millisecond radio pulsars$^{12}$, whose magnetic fields are plotted against rotation periods
in Fig. 1. In the resurrected pulsar scenario\textsuperscript{20} the progenitors of millisecond radio pulsars are the low mass X-ray binaries and their rapid rotations are the consequence of the spin-up due to mass accretion in binary systems. Let us consider the weak magnetic field and the rapid rotation of these radio pulsars in terms of the hypothesis that mass accretion leads to both spin-up and field decay. We assume that the magnetic field decays with accretion as

\[ B = \frac{B_0}{1 + \Delta M / m_B} = \frac{B_0}{1 + \dot{M} t / m_B}, \tag{2} \]

where \( B \) is the field strength at time \( t \), \( \Delta M \) the accreted mass, \( m_B \) the mass constant for the field decay and \( \dot{M} \) the accretion rate. Equation (2) fits well the inverse correlation between the magnetic field strength and the estimated mass of the total accreted matter for binary X-ray sources and binary and millisecond radio pulsars\textsuperscript{3}. As already noted by Taam and van den Heuvel\textsuperscript{3}, this inverse correlation is also consistent with the simple field decay hypothesis if the larger amount of accreted matter is interpreted simply to reflect the greater age of the neutron star. Here, however, although we do not advocate any physical model leading to equation (2), we do assume that a direct relation between field loss and accreted mass exists. Note that equation (2) represents a change in the whole field, not just the component perpendicular to the rotation axis. Using the formula given by Ghosh and Lamb\textsuperscript{28} for the accretion torque, the variation of rotation period is described by

\[ \dot{P} = -0.11 I^{-1} (GM)^{3/7} (BR^3)^{2/7} n \dot{M}^{6/7} P^2, \tag{3} \]

where \( n \) is the dimensionless accretion torque (for details see ref. 28), \( R \) the radius, \( I \) the moment of inertia and \( G \) the gravitational constant. The calculated evolutionary tracks in the magnetic field versus rotation period diagram are illustrated by solid lines in Fig. 1. Rotation periods and magnetic fields become smaller with increasing time. The calculations were terminated when the inner edge of the accretion disk reached the surface of the neutron star. When the mass constant for the field decay is \( m_B \geq 10^{-3} M_\odot \), the evolution proceeds along the equilibrium rotation line\textsuperscript{28} (dash-dotted line in Fig. 1), where
the spin-up torque due to accreting matter and the spin-down torque due to magnetic field
are balanced. This is because the time scale of the field decay is longer than the spin-up
time scale. After mass accretion stops, stars move horizontally rightwards in Fig. 1. If
\( m_B \geq 10^{-4}M_\odot \), the calculated evolutionary tracks are consistent with \( B \) and \( P \) of binary
and millisecond radio pulsars. These evolutionary tracks are also approximately consistent
with \( B \) and \( P \) of low mass binary X-ray sources such as suggested from the beat frequency
model\(^{29}\) for the quasi-periodic X-ray oscillations (see ref. 30 for a review). Note that the
possible inverse correlation\(^3\) between \( B \) and \( \Delta M \) is represented well by equation (2) with
\( m_B \sim 10^{-4}M_\odot \). Hence, if we adopt \( m_B \sim 10^{-4}M_\odot \), the accretion-induced field decay
hypothesis represented by equation (2) is consistent with \( B \), \( P \), and \( \Delta M \) as observed or
estimated for binary and millisecond radio pulsars and binary X-ray sources.

The magnetic fields of single radio pulsars are in the range of \( 10^{11} - 10^{13} \)G with a
distribution peak around \( 10^{12} \)G. The accreted mass of interstellar matter onto a single
radio pulsar is negligible compared to \( m_B \sim 10^{-4}M_\odot \). Hence, the accretion-induced field
decay hypothesis argues that the present field strengths of single radio pulsars should be
equal to their initial values. This assertion, however, conflicts with the results of the
statistical analyses\(^4,5\) of radio pulsars, which suggest that surface fields decay on a time
scale of \( (5 - 10) \times 10^6 \)yr. Pulsar statistics depends, however, on the assumptions made for
the radio luminosity law, the braking index, the magnetic field evolution, the distribution
of initial field strength and so on. In order to resolve the above conflict, the statistical
analysis under the assumption of no field decay should be conducted, examining especially
the dependence on the assumptions for the above properties.

Mass accretion of neutron stars is likely to give rise to the inward heat flow through
the crust. The thermomagnetic effects in the crust due to this inward heat flux, such
as suggested by Blondin and Freese\(^{21}\), have been invoked as a possible mechanism of the
accretion- induced field decay. The formula of Blondin and Freese predicts, in its simplest
form, a stronger dependence of the field decay on time compared to equation (2). The field
decay formula due to the thermomagnetic effects, however, may depend on the structure of
the crust, the accretion process, and the history of the heat flux. More detailed study on the
thermomegnetic effects in the crust is important in order to resolve the controversial issue
of the field decay in neutron stars. Other possible mechanisms for the accretion-induced
field decay should also be explored.

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Figure Captions
Fig. 1. Evolutionary tracks (solid lines) of neutron stars in the magnetic field versus rotation period diagram. Rotation periods and magnetic fields become smaller with accreted mass and hence with increasing time. After mass accretion stops, stars move horizontally rightwards. The initial magnetic field is taken as $B_0 = 10^{12}$ G, the initial rotation period is chosen as $P_0 = 0.5$ or 100 s, and the mass accretion rate is fixed at $\dot{M} = 1.1 \times 10^{18}$ gm/s. The mass, radius, and moment of inertia of the neutron star are taken as $1.4 M_\odot$, 106 cm, and $10^{45}$ gm cm$^2$, respectively. The dash-dotted and dashed lines denote the equilibrium rotation$^{28}$ and pulsar death lines$^5$, respectively. The filled circles represent the positions of binary and millisecond radio pulsars in this diagram.