Sensor Performance Analysis

H. E. Montgomery,
H. Ostrow,
and G. M. Ressler
Sensor Performance Analysis

H. E. Montgomery
and H. Ostrow
Goddard Space Flight Center
Greenbelt, Maryland

G. M. Ressler
Ressler Associates, Inc.
Laurel, Maryland
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Symbols</td>
<td>v</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. POWER AND SIGNAL</td>
<td>4</td>
</tr>
<tr>
<td>3. SPECTRAL RADIANCE</td>
<td>7</td>
</tr>
<tr>
<td>3.1 Visible and Shortwave (SWIR) Spectral Radiance</td>
<td>7</td>
</tr>
<tr>
<td>3.2 Infrared Spectral Radiance</td>
<td>7</td>
</tr>
<tr>
<td>4. DWELL TIME</td>
<td>9</td>
</tr>
<tr>
<td>4.1 Spinning Mirror</td>
<td>9</td>
</tr>
<tr>
<td>4.2 Rocking Mirror</td>
<td>15</td>
</tr>
<tr>
<td>4.3 Linear Array</td>
<td>16</td>
</tr>
<tr>
<td>5. NOISE</td>
<td>17</td>
</tr>
<tr>
<td>5.1 Visible and SWIR Noise</td>
<td>17</td>
</tr>
<tr>
<td>5.1.1 Visible and SWIR Detector Noise Sources</td>
<td>17</td>
</tr>
<tr>
<td>5.1.2 Visible and SWIR System Noise</td>
<td>20</td>
</tr>
<tr>
<td>5.2 Infrared Noise</td>
<td>21</td>
</tr>
<tr>
<td>5.2.1 Infrared Detector Noise</td>
<td>21</td>
</tr>
<tr>
<td>5.2.2 Infrared System Noise</td>
<td>26</td>
</tr>
<tr>
<td>6. FIGURES OF MERIT</td>
<td>27</td>
</tr>
<tr>
<td>6.1 Noise Equivalent Delta Reflectance</td>
<td>27</td>
</tr>
<tr>
<td>6.2 Noise Equivalent Delta Temperature</td>
<td>28</td>
</tr>
<tr>
<td>7. MODULATION TRANSFER FUNCTION (MTF)</td>
<td>30</td>
</tr>
<tr>
<td>7.1 Total MTF</td>
<td>30</td>
</tr>
<tr>
<td>7.2 Component MTF</td>
<td>30</td>
</tr>
<tr>
<td>7.2.1 Optical Aperture MTF</td>
<td>30</td>
</tr>
<tr>
<td>7.2.2 Detector Aperture MTF</td>
<td>32</td>
</tr>
<tr>
<td>7.2.3 Satellite Motion MTF</td>
<td>32</td>
</tr>
<tr>
<td>7.2.4 Charge Diffusion MTF</td>
<td>33</td>
</tr>
<tr>
<td>7.2.5 Charge Transfer MTF</td>
<td>33</td>
</tr>
<tr>
<td>7.2.6 Satellite Jitter MTF</td>
<td>34</td>
</tr>
<tr>
<td>8. REFERENCES</td>
<td>35</td>
</tr>
<tr>
<td>APPENDIX A—DETECTOR IRRADIANCE</td>
<td>A-1</td>
</tr>
<tr>
<td>APPENDIX B—COMPUTED EARTH/ATMOSPHERE RADIANCES</td>
<td>B-1</td>
</tr>
<tr>
<td>APPENDIX C—SENSOR FIELD OF VIEW (FOV)</td>
<td>C-1</td>
</tr>
<tr>
<td>APPENDIX D—VIEW FACTOR FOR A SINGLE DETECTOR VIEWING A CIRCULAR BACKGROUND</td>
<td>D-1</td>
</tr>
<tr>
<td>APPENDIX E—VIEW FACTOR FOR A DETECTOR IN AN N BY M ARRAY VIEWING A RECTANGULAR BACKGROUND</td>
<td>E-1</td>
</tr>
<tr>
<td>APPENDIX F—COMPUTATION OF GAMMA</td>
<td>F-1</td>
</tr>
<tr>
<td>APPENDIX G—NOISE EQUIVALENT DELTA TEMPERATURE</td>
<td>G-1</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Overlap factor across track, given by Equation (C-9) in Appendix C.</td>
</tr>
<tr>
<td>A₀</td>
<td>Sensor entrance aperture area (cm²).</td>
</tr>
<tr>
<td>A_D</td>
<td>Detector area (m²).</td>
</tr>
<tr>
<td>A_I</td>
<td>The area of the ground resolution element (km²), used in Equation (A-2).</td>
</tr>
<tr>
<td>A_m</td>
<td>A function used in the MTF_{OA} computation, Equation (7-2).</td>
</tr>
<tr>
<td>B</td>
<td>Overlap factor along track, given by Equation (4-9).</td>
</tr>
<tr>
<td>B(λ)</td>
<td>Planck's spectral distribution of radiation (W/cm²-sr-μm) from a blackbody, given by Equation (3-5).</td>
</tr>
<tr>
<td>B'(λ)</td>
<td>Planck's spectral distribution of radiation (p/sec-cm²-sr-μm) from a blackbody, given by Equation (3-7).</td>
</tr>
<tr>
<td>B_m</td>
<td>A function used in the MTF_{OA} computation, Equation (7-2).</td>
</tr>
<tr>
<td>c</td>
<td>Speed of light = 2.998 × 10¹⁰ (cm/sec).</td>
</tr>
<tr>
<td>C</td>
<td>A constant used in Equation (2-8).</td>
</tr>
<tr>
<td>C₁</td>
<td>2πhc² = A constant used in the computation of B(λ).</td>
</tr>
<tr>
<td>C₀</td>
<td>Output capacitance.</td>
</tr>
<tr>
<td>C_m</td>
<td>A function used in the MTF_{OA} computation, Equation (7-2).</td>
</tr>
<tr>
<td>C'₁</td>
<td>2πc = A constant used in the computation B'(λ).</td>
</tr>
<tr>
<td>C₂</td>
<td>hc/k_B = A constant used in the computation of B(λ) and B'(λ).</td>
</tr>
<tr>
<td>d_C</td>
<td>Distance covered cross track during the time required to map the Earth t_M, given by Equation (C-6).</td>
</tr>
<tr>
<td>D</td>
<td>Aperture diameter of the sensor, used in Equation (A-7).</td>
</tr>
<tr>
<td>D*</td>
<td>Specific detectivity (laboratory or handbook value), defined in Equation (5-20).</td>
</tr>
<tr>
<td>D*_{BLIP}</td>
<td>Background-limited value of D*, given by Equation (5-33).</td>
</tr>
<tr>
<td>d</td>
<td>Photodetector depletion region depth, used in Equation (7-19).</td>
</tr>
<tr>
<td>d_m</td>
<td>Distance moved by the satellite along the ground track during one scan period, given by Equation (4-7).</td>
</tr>
<tr>
<td>d_C</td>
<td>The extent imaged along the ground track at nadir during one scan mirror period.</td>
</tr>
</tbody>
</table>
given by Equation (4–6).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_S</td>
<td>Width of square detector (μm).</td>
</tr>
<tr>
<td>dλ</td>
<td>Differential of wavelength λ.</td>
</tr>
<tr>
<td>E(λ)</td>
<td>Scene spectral irradiance (W/cm²-μm) into the detector, given by Equation (2-10).</td>
</tr>
<tr>
<td>E'(λ)</td>
<td>Scene spectral irradiance (p/sec-m²-μm) into the detector, given by Equation (2-6).</td>
</tr>
<tr>
<td>E_BG</td>
<td>The irradiance at the infrared detector from the scene and the background, given by Equation (5-25).</td>
</tr>
<tr>
<td>E_G</td>
<td>Silicon band gap (e–v) used in Equation (5-6a).</td>
</tr>
<tr>
<td>F_M</td>
<td>A factor used in Equation (4-5) to distinguish between a 45-degree scan mirror and a paddle scan mirror.</td>
</tr>
<tr>
<td>E_λ0</td>
<td>Spectral irradiance (mw/cm²-μm) of the direct sunlight above the atmosphere, used in Equation (B-2).</td>
</tr>
<tr>
<td>E_λS</td>
<td>The solar irradiance (mw/cm²) of the direct sunlight at sea level, used in Equation (B-2).</td>
</tr>
<tr>
<td>f</td>
<td>Focal length (cm), used in Equation (A-6).</td>
</tr>
<tr>
<td>F_A</td>
<td>View factor for a detector in an n by m array viewing a rectangular background, given by Equation (E-1), in Appendix E.</td>
</tr>
<tr>
<td>F_L</td>
<td>View factor of the detector in the laboratory where D* was measured, given by Equation (5-19).</td>
</tr>
<tr>
<td>F_C</td>
<td>View factor for a cold shielded detector viewing the scene through a circular aperture, given by Equation (5-29).</td>
</tr>
<tr>
<td>f_N</td>
<td>The f-number (nd) of the optical system, given by Equation (A-9).</td>
</tr>
<tr>
<td>g_m</td>
<td>Transconductance.</td>
</tr>
<tr>
<td>J_0</td>
<td>Zeroth-order Bessel function, used in Equation (7-26).</td>
</tr>
<tr>
<td>J_DC</td>
<td>Dark current density (a/cm²), given by Equation (5-6a).</td>
</tr>
<tr>
<td>H</td>
<td>Satellite height (km).</td>
</tr>
<tr>
<td>h</td>
<td>Planck's constant = 6.626 × 10⁻³⁴ (W-sec²).</td>
</tr>
<tr>
<td>k</td>
<td>Spatial frequency (cycles/mm).</td>
</tr>
<tr>
<td>k₀</td>
<td>1/2λf_N, used in the MTF_{OA} computations and given by Equation (7-12).</td>
</tr>
</tbody>
</table>
\( k_B \) = Boltzmann's constant \( = 1.380 \times 10^{-23} \text{ (W-sec/K)} \).

\( k_C \) = \( 2k_o \) = Cutoff frequency, used to compute \( \text{MTF}_{OA} \) in Equation (7-13).

\( L \) = A function used in the \( \text{MTF}_{CT} \) and given by Equation (7-22).

\( L_0 \) = Diffusion length, used in Equation (7-22).

\( L(\lambda) \) = Scene spectral radiance \( \text{(W/cm}^2\text{-sr-μm)} \), given by Appendix B for the visible bands and by Equation (3-2) for the infrared bands.

\( L'(\lambda) \) = Scene spectral radiance \( \text{(p/sec-cm}^2\text{-sr-μm)} \), given by Equation (2-12).

\( L^N_A \) = Spectral radiance \( \text{(W/cm}^2\text{-sr-μm)} \) from the atmosphere observed by the sensor when viewing along the nadir direction, used in Equation (B-4).

\( L^N \) = Total spectral radiance \( \text{(W/cm}^2\text{-sr-μm)} \) observed by the sensor when viewing along the nadir direction, given by Equation (B-4).

\( L_S \) = Spectral radiance \( \text{(W/cm}^2\text{-sr-μm)} \) observed by the sensor which comes from the surface of the Earth, used in Equation (B-4).

\( M \) = Mass of the Earth \( \text{(kg)} \).

\( M_{CT} \) = Total number of charge transfers, used in Equation (7-23).

\( M_g \) = Number of gate transfers.

\( \text{MTF} \) = Modulation transfer function, given by Equation (7-1).

\( \text{MTF}_{CT} \) = Charge transfer MTF, given by Equation (7-23).

\( \text{MTF}_{DA} \) = Detector aperture MTF, given by Equation (7-16).

\( \text{MTF}_{OA} \) = Optical aperture MTF, given by Equation (7-2).

\( \text{MTF}_{SJ} \) = Satellite jitter MTF, given by Equation (7-26).

\( \text{MTF}_{SM} \) = Satellite motion MTF, given by Equation (7-17).

\( m \) = Mass of the electron \( = 9.1 \times 10^{-31} \text{ (kg)} \).

\( m_p \) = Number of clock phases for readout, used in Equation (7-24).

\( m_S \) = Number of stages, detectors or picture elements, used in Equation (7-24).

\( N_{BT} \) = Bulk trap noise \( \text{(e)} \), given by Equation (5-3).

\( N_{CT} \) = Charge transfer noise \( \text{(e)} \), given by Equation (5-11).
\begin{align*}
N_{DC} & = \text{Dark current noise (e), given by Equation (5-5).} \\
N_{DCS} & = \text{Dark current noise (e) for a Schottky barrier detector, given by Equation (5-9).} \\
N_{DET} & = \text{Detector noise (e), used in Equations (5-1) and (5-16).} \\
\text{NEE} & = \text{Noise equivalent electrons (e), given by Equation (5-21).} \\
\text{NEP} & = \text{Noise equivalent power (W), given by Equation (5-20).} \\
\text{NE\Delta T} & = \text{Noise equivalent delta temperature (K), given by Equation (6-5).} \\
\text{NE\Delta \rho} & = \text{Noise equivalent delta reflectance (\text{n}_d), given by Equation (6-1).} \\
N_{M} & = \text{Multiplexer noise (e).} \\
N_{OA} & = \text{Output amplifier noise (e), given by Equation (5-4).} \\
N_{OD} & = \text{Other detector noise (e), given by Equation (5-17).} \\
N_{OS} & = \text{Other system noise (e).} \\
N_{p} & = \text{Photon noise (e), given by Equation (5-2) for the visible domain and by Equation (5-23) for the infrared domain.} \\
N_{PL} & = \text{Photon noise (e) under laboratory conditions, given by Equation (5-18).} \\
N_{Q} & = \text{Quantization noise (e), given by Equation (5-12) for the visible domain and by Equation (5-34) for the infrared domain.} \\
N_{T} & = \text{Thermal (Johnson) noise (e), given by Equation (5-8).} \\
N_{TOT} & = \text{Total noise from all sources (e), given by Equation (5-1).} \\
N_{SYS} & = \text{System noise (e), used in Equation (5-1).} \\
n & = \text{Size distribution function for aerosol particles, used in Equation (B-3).} \\
n_{D} & = \text{Number of detectors per spectral band.} \\
n'_{D} & = \text{Number of detectors along an array, used to compute the charge transfer noise in Equation (5-11).} \\
n_{E} & = \text{Number of resolution elements along a scan line, given by Equation (4-1).} \\
n_{f} & = \text{Number of facets in a 45-degree scan mirror.} \\
n_{p} & = \text{The number of phases used to transfer charge along a detector array, used in Equation (5-11).}
\end{align*}
\( n_s \) = A factor used in Equation (4-14) to distinguish between imaging in one scan mirror direction and imaging in both scan mirror directions.

\( n_{SS} \) = Density of surface states, used in Equation (5-3).

\( P_D \) = Detector pitch (\( \mu \)m).

\( Q \) = Number of bits used in the analog-to-digital converter.

\( q \) = Charge of an electron = \( 1.602 \times 10^{-19} \) (Coul).

\( R \) = Resistance (ohms).

\( R(\lambda) \) = Detector current responsivity (A/W), given by Equation (2-3).

\( R_c \) = \( 4\pi qmk^2/h^3 \) = Richardson constant (a/cm\(^2\)-K\(^2\)), used in Equation (5-9a).

\( R_e \) = 6378.165 (km) = Radius of the Earth.

\( R_s \) = Distance (km) from the center of the Earth to the satellite, given by Equation (B-2).

\( r \) = Radius of aerosol particle, used in Equation (B-3).

\( S \) = Signal (A) from the detector, given by Equation (2-2).

\( S_F \) = Saturation factor, defined by Equation (5-14).

\( S' \) = Signal (e) from the detector, given by Equation (2-7).

\( S'_{SAT} \) = The saturation signal (e), given by Equation (5-13).

\( S_d \) = Distance (km) from the satellite to the point on the Earth that corresponds to the maximum scan angle, given by Equation (C-3).

\( S_w \) = Swath width (km) on the Earth, given by Equation (C-5).

\( S_o \) = Percentage overlap along track, used in Equation (4-10).

\( S'_{o} \) = Percentage overlap across track, used in Equation (C-9).

\( T \) = Blackbody temperature (K).

\( T_{BG} \) = Background temperature for the sensor (K).

\( T_{BGL} \) = Background temperature (K) when \( D^* \) is measured in the laboratory.

\( T_S \) = Earth's surface temperature (K).

\( T_A \) = Earth's atmospheric temperature (K).

\( t_A \) = Active scan time, which is that part of the scan mirror period when data are being acquired, given by Equation (4-4).
\( t_I \) = Detector integration time (sec), given by Equation (2-8).

\( t_D \) = Detector dwell time (sec), given by Equations (4-3) and (4-12) for a spinning scan mirror; Equations (4-14) and (4-17) for a rocking scan mirror; and by Equation (4-18) for a linear array.

\( t_M \) = Period of the scan mirror (sec), given by Equation (4-11) for a spinning mirror and by Equation (4-16) for a rocking scan mirror.

\( t_{MAP} \) = Time to map the Earth (days).

\( t_S \) = Satellite orbital period (sec), given by Equation (C-8).

\( V_{SUB} \) = The velocity (km/sec) of the subsatellite point, given by Equation (4-8).

\( V_I \) = Image velocity (km/sec), given by Equation (7-18).

\( z \) = A curve-fitted function used to compute the silicon absorption coefficient, given by Equation (7-21).

\( \alpha \) = Instantaneous angular field of view (\( r \)), given by Equation (4-2).

\( \alpha_a \) = Silicon absorption coefficient (nd), used in Equation (7-19).

\( \alpha_M \) = A material-dependent factor used in Equation (5-6a).

\( \beta \) = \( D_o/D \), used in the MTF\(_{OA}\) computations, given by Equation (7-15).

\( \gamma \) = \((S/N) \cdot \text{NE\( \Delta \rho \)}\) and is given by Equation (6-2).

\( \gamma_o \) = \((S/N) \cdot \text{NE\( \Delta \rho \)}\) along nadir and is given by Equation (6-3).

\( \Delta f \) = Electrical bandwidth (Hz), given by Equation (5-22).

\( \Delta \lambda \) = \( \lambda_2 - \lambda_1 \) = Spectral bandpass (\( \mu m \)).

\( \delta_A \) = Optical thickness (nd) of the aerosols, used in Equation (B-1).

\( \delta_G \) = Optical thickness (nd) of the absorbing gases, used in Equation (B-1).

\( \delta_R \) = Optical thickness (nd) due to Rayleigh scattering, used in Equation (B-1).

\( \delta_T \) = Optical thickness (nd).

\( \delta_{TN} \) = Optical thickness (nd) in the nadir direction, used in Equations (6-4) and (B-1).

\( \eta \) = Detector quantum efficiency (nd).

\( \eta_M \) = A material-dependent carrier recombination factor, used in Equation (5-6a).

\( \epsilon \) = Charge transfer inefficiency (nd), used to compute MTF\(_{CT}\).
\( \varepsilon_A \) = Emissivity of the atmosphere (nd).

\( \Theta \) = Field-of-view (FOV) angle (deg) subtended by the swath width at the satellite. It is given by Equation (4-8) for a linear array and by Appendix C [Equation (C-1)] for whiskbroom (scanning) systems.

\( \Theta_M \) = Maximum satellite angular movement (rad) used to compute jitter MTF in Equation (7-26).

\( \theta \) = Optics half-cone angle (deg), given by Equation (A-10).

\( \theta_{BG} \) = The full-cone angle (deg) of the background used in Equation (5-29).

\( \theta_z \) = Solar zenith angle (deg), angle between Earth normal and Sun direction, used in Table B-2.

\( \kappa \) = Scan efficiency (nd), given by Equation (4-5) for spinning mirrors.

\( \pi \) = The well-known ratio between the circumference and the diameter of a circle, 3.14159 (nd).

\( \lambda \) = Center wavelength of a spectral band (\( \mu m \)).

\( \Lambda \) = \( k / k_0 \), used in the MTF\( _{OA} \) computations.

\( \lambda_1 \) = Lower wavelength of a spectral band (\( \mu m \)).

\( \lambda_2 \) = Upper wavelength of a spectral band (\( \mu m \)).

\( \mu \) = \( GM \), the product of the gravitational constant \( G \) and the mass of the Earth \( M \)

\( = 3.98603 \times 10^5 \) (\( km^3 \cdot sec^{-2} \)).

\( \rho \) = Reflectance of the Earth's surface (nd).

\( \tau \) = Transmittance (nd) of the atmosphere in the visible spectral region, given by Equation (B-2).

\( \tau' \) = Total number of electrons produced by an infrared detector from the scene and the background, given by Equation (5-24).

\( \tau_A \) = Transmittance (nd) of the atmosphere, used for the infrared spectral region.

\( \tau_{AN} \) = Optical transmittance (nd) along the nadir direction, given by Equation (6-4).

\( \tau_0 \) = Optical transmittance (nd) from the sensor entrance aperture to the detector, used in Equation (5-26).
\( \Phi \) = Power (W) incident on the detector, given by Equation (2-1).

\( \Phi' \) = Photon flux incident on the detector (p/sec).

\( \Phi_S \) = Angle (deg) subtended at the satellite by ground swath, given by Equation (C-4).

\( \phi_C \) = Cone angle (deg) for detector view, used in Equation (5-19).

\( \phi_f \) = The angle between the normal to the differential area \( dA_B \) and the line between the center of the detector and the center of the area \( dA_B \), used in Appendix F.

\( \phi_m \) = A function that when multiplied by the charge of an electron \( q \), gives the work function of the metal in the semiconductor. It is used in Equation (5-9a).

\( \phi' \) = The angle (deg) between the line of sight and the surface normal, used in Equation (6-2).

\( \psi \) = A function used in the MTF_{OA} computation, given by Equation (7-10).

\( \Omega \) = The effective solid angle (sr) through which the detector receives energy from the resolution element, given by Equation (5-27).

\( \Omega_o \) = Effective solid angle (sr) subtended by the entrance aperture at the subsatellite point, used in Equation (A-7).

\( \Omega_{BG} \) = Effective solid angle (sr) of the background, given by Equation (5-28).
1. **INTRODUCTION**

The purpose of this paper is to present an analytic model of an imaging sensor system so that: (1) sensor performance predictions can be made; (2) design tradeoffs and sensitivity analyses can be rapidly performed; and (3) insight into various aspects of imaging sensor performance can be obtained. The model is applicable to image sensors which operate from the visible through the thermal infrared spectral regions.

The design of sensors for remote observation of the Earth from polar orbiting satellites takes about a decade and occurs in five different stages. These stages are:

- **Definition of Scientific Requirements.** During this stage, scientific working groups formulate the scientific requirements (e.g., spatial, temporal, and spectral resolution; measurement accuracy, etc.)

- **Preliminary Design.** During this stage a "paper design" is developed. (Determination is made of such parameters as f-number, detector size, and number of detectors) that permit the sensor to meet the scientific requirements (e.g., signal-to-noise ratio, noise-equivalent delta temperature, or noise-equivalent delta reflectance).

- **Feasibility Studies (Phase B).** During this stage, engineering feasibility is established without regard to optimization of the design.

- **Design Studies (Phase B).** During this phase, the design is optimized, and an in-depth analysis is performed on each subsystem (e.g., optics, focal plane, cooler, electronics, mechanical systems) and a credible cost estimate is produced.

- **Flight Hardware Design, Development, Test and Integration into the Space Platform (Phase C/D).** During this stage, flight hardware is designed, developed, and tested to prove that it meets the specification, and is integrated into the space platform.

Only Preliminary Design (the second stage) is addressed in this document. Derivations or references are given for all the equations to make it easy to change the theory as required in future applications.

The spectral range is limited to 0.4 to 15.0 μm which is generally appropriate for studies of the Earth.
and its environment. The types of scanners include the “pushbroom” and two different kinds of “whiskbrooms.” A substantial portion of the analysis presented herein has been incorporated into a self-documented Lotus 1-2-3 spreadsheet.

Analytic (as opposed to statistical) methods are used in the model. Analyses are carried out at very low spatial (and therefore temporal) frequencies in order to simplify the computations. High spatial frequencies are used only to determine the MTF of the sensor. Carrying out the analyses at low spatial frequencies also avoids the necessity of working in the frequency domain which generally involves Fourier transformations or convolutions in the time domain, and would therefore make the sensor model very complex. To further simplify the analyses, it is also assumed that the sensors have narrow spectral bandwidths so that those parameters that are spectrally dependent (e.g., detector responsivity) can be reduced to a constant.

The model assumes that the sensor is a linear system from the optical input through the electronic signal processing. This assumption is satisfied if (1) the optical system is not dominated by diffraction effects; (2) incoherent detection methods are employed; (3) the various noise sources are additive in an RSS sense; (4) the imaging process is spatially invariant; and (5) the electronic signal processing constitute linear operations. These assumptions are all valid for the types of imaging sensors that the model is presently being applied to.

Appropriate scene radiance levels must be assigned in order to assess sensor system performance. In the model described in this document, tables are included which allow a user to determine radiances at the top of the Earth-atmosphere system as a function of wavelength. These tables apply to the visible and near IR spectral regions. The origin of the tables is discussed in Appendix B. For the thermal infrared region, radiances are directly computed.

Section 2 addresses the power incident on the detector during an observation interval and the signal coming out the detector. These are written as functions of the irradiance at the detector and as a function of the scene radiance.
Section 3 provides a description of the model used for the scene radiance.

Section 4 derives the equations for detector dwell time for four different scanning configurations which include: a spinning 45-degree scan mirror, a spinning paddle mirror, a rocking scan mirror, and a linear array-pushbroom.

Section 5 is devoted to the major noise sources associated with visible and infrared detectors.

Section 6 presents definitions and derivations of various "figures of merit" including Noise Equivalent Delta Reflectance (NEΔρ) and Noise Equivalent Delta Temperature (NEΔT).

Section 7 develops the Modulation Transfer Function (MTF) for the optical aperture, the detector aperture, satellite motion, charge diffusion, charge transfer, and satellite jitter.

The Appendixes treat many of the equations in a tutorial manner.
2. **POWER AND SIGNAL**

When viewing the scene, the power (flux) into the detector is given by

\[
\Phi = A_D \int_{\lambda_1}^{\lambda_2} E(\lambda) \, d\lambda \quad [W],
\]

(2-1)

where \( A_D \) = the area of the detector (cm\(^2\));

\( E(\lambda) \) = scene spectral irradiance at the detector (W/cm\(^2\)-\(\mu m\));

\( \lambda \) = center wavelength of spectral band (\(\mu m\)); and

\( \Delta \lambda = \lambda_2 - \lambda_1 \) = spectral bandpass (\(\mu m\)).

The signal out of the detector is given by

\[
S = A_D \int_{\lambda_1}^{\lambda_2} E(\lambda)R(\lambda) \, d\lambda \quad [A],
\]

(2-2)

where \( R(\lambda) \) is the detector current responsivity and is given by

\[
R(\lambda) = \frac{q\eta}{hc} \lambda \quad [A/W],
\]

(2-3)

where \( q \) = the charge of an electron = 1.60 \times 10^{-19} \text{ [coul]};

\( \eta \) = the detector quantum efficiency [nd] (assumed to be constant over the spectral bandpass \( \Delta \lambda \));

\( h \) = Planck’s constant = 6.63 \times 10^{-34} \text{ [Wsec\(^2\)]}; and

\( c \) = the speed of light = 3.00 \times 10^{10} \text{ [cm/sec]}.

Substituting Equation (2-3) into Equation (2-2) gives

\[
S = \frac{A_D q\eta}{hc} \int_{\lambda_1}^{\lambda_2} E(\lambda) \lambda \, d\lambda \quad [A].
\]

(2-4)
Equation (2-4) may be written in terms of the spectral photon irradiance as

\[ S = A_D \eta \int_{\lambda_1}^{\lambda_2} E'(\lambda) \, d\lambda \quad [A], \quad (2-5) \]

where the scene spectral photon irradiance \( E'(\lambda) \) at the detector is given by

\[ E'(\lambda) = \frac{\lambda}{hc} E(\lambda) \quad [\text{p/sec-cm}^2\mu\text{m}] \quad (2-6) \]

The signal out of the detector is given by

\[ S' = \frac{t_1}{q} S \quad [\text{e}], \quad (2-7) \]

where the integration time \( t_1 \) is given by

\[ t_1 = C t_D \quad [\text{sec}], \quad (2-8) \]

where \( C \) is an input constant and \( t_D \) is the sensor dwell time, which is described in detail in Section 4.

Substituting Equation (2-5) into Equation (2-7) gives

\[ S' = t_1 A_D \eta \int_{\lambda_1}^{\lambda_2} E'(\lambda) \, d\lambda \quad [\text{e}] \quad (2-9) \]

The scene spectral irradiance at the detector is related to the scene spectral radiance by the following two equations given in terms of watts and photons, respectively. (See Appendix A.) The first equation is

\[ E(\lambda) = \frac{\tau_o \pi}{4 f_N^2} L(\lambda) \quad [\text{W/cm}^2\mu\text{m}], \quad (2-10) \]

where \( \tau_o = \) the optical transmittance [nd] from the sensor aperture to the detector, and

\[ f_N = f\text{-number of the optics [nd]} \]

The second equation is

\[ E'(\lambda) = \frac{\tau_o \pi}{4 f_N^2} L'(\lambda) \quad [\text{p/sec-cm}^2\mu\text{m}] \quad (2-11) \]
Also, for completeness, note that

\[ L'(\lambda) = \frac{\lambda}{hc} L(\lambda) \quad [\text{p/cm}^2\text{-sec-sr-\textmu m}] \quad (2-12) \]

For the visible and shortwave infrared (SWIR) wavelengths, it is assumed that the spectral bandpass \( \Delta\lambda \) is small and that the scene spectral photon irradiance \( E'(\lambda) \) varies slowly over \( \Delta\lambda \). In this case, the integration in Equations (2-1) and (2-4) can then be approximated by \( E'\Delta\lambda \), where \( E' \) is the average value of \( E(\lambda) \) over the spectral bandpass \( \Delta\lambda \).
3. **SPECTRAL RADIANCE**

In this section, we will show how to obtain the spectral radiance for the visible and SWIR spectral wavelengths (0.4 to 2.2 μm) and the infrared spectral wavelengths (2.2 to 15 μm).

3.1 **Visible and Shortwave Infrared (SWIR) Spectral Radiance**

A table of values of the scene spectral radiance at the sensor aperture is given in Appendix B as a function of $\rho$, $\theta_z$, and $\lambda$

where $\rho = $ Earth's surface reflectance (nd), and $\theta_z = $ solar zenith angle (deg).

The scene spectral radiance $L(\lambda)$ is obtained from the tables by trilinear interpolation at the desired values of $\rho$, $\theta_z$, and $\lambda$. Although the values of scene spectral radiance in the tables were computed for a nadir viewing sensor, we assume that the values are independent of viewing angle. (See Equation C-4).

3.2 **Infrared Spectral Radiance**

The total scene spectral radiance in the infrared is given by

$$L(\lambda) = \tau_A B(\lambda, T_S) + \epsilon_A B(\lambda, T_A) \quad [W/cm^2-sr-\mu m]$$  \hspace{1cm} (3-2)

where $\tau_A$ is the atmospheric transmittance for an optical depth $\delta$, given by

$$\tau_A = e^{-\delta}$$  \hspace{1cm} (3-3)

and where $\epsilon_A$ is the atmospheric emissivity given by

$$\epsilon_A = 1 - \tau_A$$  \hspace{1cm} (3-4)

The quantities $B(\lambda, T_S)$ and $B(\lambda, T_A)$ are the spectral radiances of the Earth's surface at temperature $T_S$ and the atmosphere at temperature $T_A$, respectively, and are given by Planck's equation (Hudson, 1969, p. 35) for a blackbody at temperature $T,$
\[ B(\lambda, T) = \frac{C_1}{\pi \lambda^5} \frac{1}{\exp \left( \frac{C_2}{\lambda T} \right) - 1} \quad \text{[w/cm}^2\text{-sr-\mu m]} \quad (3-5) \]

where Hudson's equation (2-8) has been divided by \( \pi \) to convert to radiance, and where

- \( C_1 = 2\pi hc^2 = 3.74 \times 10^4 \ [W\cdot\mu m^4/cm^2] \);
- \( h = \) Planck's constant = \( 6.63 \times 10^{-34} \ [W\cdot sec^2] \);
- \( c = \) speed of light = \( 2.998 \times 10^{10} \ [cm/sec] \);
- \( C_2 = \frac{hc}{k_B} = 1.44 \times 10^4 \ [\mu m\cdot K] \);
- \( k_B = \) Boltzmann's constant = \( 1.38 \times 10^{-23} \ [W\cdot sec/K] \); and
- \( T = \) Blackbody temperature [K].

The total scene spectral photon radiance is given by

\[ L'(\lambda) = \tau_A B'(\lambda, T_S) + e_A B'(\lambda, T_A) \quad \text{[p/sec-cm}^2\text{-sr-\mu m]}, \quad (3-6) \]

where the surface spectral photon radiance \( B'(\lambda, T_S) \) and the atmospheric spectral photon radiance \( B'(\lambda, T_A) \) are found by evaluating the following equation (Hudson, 1969, p. 38) at \( T_S \) and \( T_A \), respectively:

\[ B'(\lambda, T) = \frac{C_1'}{\pi \lambda^4} \frac{1}{\exp \left( \frac{C_2}{\lambda T} \right) - 1} \quad \text{[p/sec-cm}^2\text{-sr-\mu m]} \quad (3-7) \]

where

\[ C_1' = 2\pi c = 1.88 \times 10^{23} \ [p\cdot sec^{-1}\cdot cm^{-2}\cdot \mu m^3] \]
4. **Dwell Time**

As the satellite moves in orbit, it images along scan lines perpendicular to the ground track. Let the extreme ends of the scan lines on the Earth subtend an angle $\Theta$ at the satellite. This angle is called the field-of-view (FOV). See Figure 1 for an illustration of the geometry involved and refer to Appendix C for a discussion of the relations between the parameters shown in the figure. The number of angular resolution elements $n_E$ is given by

$$n_E = \frac{\Theta}{\alpha} \text{ [nd]}$$  \hfill (4-1)

and (as can be seen from Figure 1) the instantaneous angular field of view $\alpha$ (geometric only) is given by

$$\alpha = \frac{d_S}{f} \text{ [rad]}$$  \hfill (4-2)

where $d_S = \text{ detector width (mm)}$, and $f = \text{ focal length of the optical system (mm)}$.

A scan line can be formed by a spinning mirror, a rocking mirror, or a linear array. Each of these are discussed in the following sections.

4.1 **Spinning Mirror**

Two types of spinning mirrors are discussed in this section: the 45-degree scan (faceted) mirror and the paddle scan mirror (Figures 2a and 2b). The axis of rotation for a 45-degree scan mirror is parallel to the sensor's optical axis, and a change in the angle of rotation of $\theta$ will produce an equal change in the line-of-sight angle. The axis of rotation for a paddle scan mirror is perpendicular to the sensor's optical axis, and therefore, a change in the angle of rotation of $\theta$ will result in a $2\theta$ change in the line-of-sight angle.

The dwell time for a spinning mirror is given by

$$t_D = \frac{t_A}{n_E n_f} \text{ [sec]},$$  \hfill (4-3)
$d_s = \text{Width of square detector (\(\mu\)m)}$

$f = \text{Focal length (cm)}$

$H = \text{Satellite height (km)}$

$R_e = \text{Radius of Earth (km)}$

$R_s = \text{Earth-to-satellite distance (km)}$

$S_d = \text{Maximum scan angle distance (km)}$

$\alpha = \text{Instantaneous angular FOV (deg)}$

$\Theta = \text{Swath width FOV (deg)}$

$\Phi_s = \text{Ground swath angle (deg)}$

$S_w = \text{Swath width (km)}$

Figure 1. FOV and IFOV Geometry
*NOTE: AS MIRROR ROTATES THROUGH ANGLE $\theta$ THE LINE OF SIGHT ALSO ROTATES THROUGH AN ANGLE $\theta$

Figure 2a. Scan Mirror Geometry for 45-Degree Faceted Scan Mirror

*NOTE: AS THE MIRROR ROTATES THROUGH ANGLE $\theta$ THE LINE OF SIGHT ROTATES THROUGH AN ANGLE $2\theta$

Figure 2b. Scan Mirror Geometry for Paddle Scan Flat Mirror
where \( n_f \) = the number of facets,
\[
\begin{align*}
    n_f \geq 1 & \text{ for a 45-degree scan mirror, and} \\
    n_f = 1 & \text{ for a paddle scan mirror.}
\end{align*}
\]

The active scan time \( t_A \), which is that part of the scan mirror period during which data are acquired, is given by
\[
t_A = \kappa t_M \quad \text{[sec]},
\]
where the scan mirror period \( t_M \) is the time for the spinning scan mirror to make a complete revolution, and the scan efficiency \( \kappa \) is given by
\[
\kappa = \frac{n_f \Theta}{2\pi F_M} \quad \text{[nd]},
\]
where \( \Theta = 1 \) for a 45-degree scan mirror,
\( F_M = 2 \) for a paddle scan mirror.

During each scan, an area on the ground is imaged. The extent imaged along the ground track at nadir (Figures 3 and 4) is
\[
d_C = n_f n_D \alpha H \quad \text{[km]},
\]
where \( H \) = the satellite height [km], and
\( n_D \) = the number of detectors per spectral band.

Also, during each scan, the satellite moves a distance \( d_m \) measured along the ground track, which is given by
\[
d_m = t_M V_{SUB} \quad \text{[km]},
\]
where \( V_{SUB} \) is the speed of the subsatellite point along the ground track and is given by
\[
V_{SUB} = \frac{R_e (\mu)^{1/2}}{(R_e + H)^{3/2}},
\]
where
\[
\mu = GM = 3.98603 \times 10^5 \text{ (km}^2\text{-sec}^{-2})
\]
where \( G \) is the universal gravitational constant, and \( M \) is the mass of the Earth.

The overlap factor \( B \) is given by
\[
B = \frac{d_C}{d_m} \quad \text{[nd]},
\]
12
Figure 3. Ground Track and Scanning Geometry

\[ d_m = \text{Ground distance moved per scan period (m)} \]
\[ d_c = \text{Extent imaged along ground track per scan period (m)} \]
\[ \Theta = \text{Swath width FOV (deg)} \]
\[ H = \text{Satellite height (km)} \]
\[ S_w = \text{Swath width} \]
\[ V_{\text{SUB}} = \text{Subsatellite velocity (km/sec)} \]
$n_f = 4$ = Number of facets on scan mirror

$d_c = \text{Extent imaged along ground track per scan (m)}$

Figure 4. Example of Scanning Geometry for a 45-Degree Faceted Scan Mirror
where \( B = 1 \) for contiguous coverage, \( B > 1 \) for overlap, and \( B < 1 \) for incomplete coverage (overlap). In terms of the percentage of overlap \( S_o \) we may write

\[
B = 1 + \frac{S_o}{100} \quad [\text{nd}] .
\]  

(4-10)

Using Equations (4-7), (4-9), and (4-6), one obtains

\[
t_M = \frac{n_F n_D \alpha H}{BV_{SUB}} \quad [\text{sec}] .
\]

(4-11)

However, from Equations (4-1), (4-3), (4-4), and (4-5), \( t_D \) may be written as

\[
t_D = \frac{\alpha}{2\pi F_M} t_M \quad [\text{sec}] .
\]

(4-12)

From Equation (4-12) it can be seen that \( t_D \) is independent of the field of view \( \Theta \), and therefore, changing the swath width does not change the dwell time. Substituting Equation (4-11) into (4-12) yields

\[
t_D = \frac{\alpha}{2\pi} \frac{n_F n_D \alpha H}{BV_{SUB} F_M} \quad [\text{sec}] .
\]

(4-13)

4.2 Rocking Mirror

During a complete cycle, the rocking mirror rotates back and forth through an angle that covers the FOV \( \Theta \) twice—once in each direction. For this case, the dwell time is

\[
t_D = \frac{t_A}{n_S n_E} \quad [\text{sec}]
\]

(4-14)

where \( n_S = 1 \), for imaging in the forward scan direction only, and

\[
n_S = 2, \text{ for imaging in both directions.}
\]

The active scan time \( t_A \) is given by Equation (4-4). However, in this case, the scan efficiency \( \kappa \) depends on the sensor design parameters; e.g., mirror turnaround time, value of \( n_S \), and mirror inertia. The scan period \( T_M \) in this case is the time for the rocking mirror to do a forward scan and retrace to its initial position.
During each scan period an area on the ground is imaged. The extent imaged along the ground track at nadir (Figure 3) is

\[ d_C = n_S n_D \alpha H \text{ [km]} \tag{4-15} \]

Use of Equations (4-7), (4-9), and (4-15) results in

\[ t_M = \frac{n_S n_D \alpha H}{BV_{SUB}} \text{ [sec]} \tag{4-16} \]

Using Equations (4-4), (4-14), and (4-16) gives

\[ t_D = \frac{k n_D \alpha H}{n_E BV_{SUB}} \text{ [sec]} \tag{4-17} \]

### 4.3 Linear Array

In this case, no scan mirror is used, and the number of angular resolution elements \( n_E \) is equal to the number of detectors in the cross-track direction. The dwell time for a linear array is, therefore,

\[ t_D = \frac{\alpha H}{V_{SUB}} \text{ [sec]} \tag{4-18} \]

and the field of view \( \Theta \) is given by

\[ \Theta = n_E \alpha \text{ [rad]} \tag{4-19} \]
5. NOISE

The total sensor noise is composed of detector noise and system noise. The detector noise is composed of a number of different noises that depend on the material composition of the detector and whether it is a single detector or is used in an array.

The total sensor noise is given by

\[ N_{TOT} = \left( N_{DET}^2 + N_{SYS}^2 \right)^{1/2}, \]  

where \( N_{DET} \) = root-mean-square detector noise, and \( N_{SYS} \) = root-mean-square system noise.

The following sections present a discussion of these various noise sources.

5.1 Visible and SWIR Detector Noise

Here we will describe the various types of noise encountered in visible and SWIR detectors.

5.1.1 Visible and SWIR Detector Noise Sources

5.1.1.1 Photon Noise

Photon noise or shot noise is due to the random arrival of photons at the detector. Because the incident photon flux follows a Poisson distribution, the photon noise is given by (Levi, 1968, p. 153)

\[ N_p = (S')^{1/2} \times [e], \]  

where the signal \( S' \) is the mean number of electrons produced by the photons arriving at the detector and is given by Equation (2-9).

5.1.1.2 Bulk Trap Noise

Bulk trap noise occurs in CCD focal plane arrays and arises from the random trapping and emission from interface or bulk states (Dereniak, 1984, p. 243) and is given by

\[ N_{BT} = \left( M_g k_B T A_D n_{SS} \times n \times 2 \right)^{1/2} \times [e], \]  

where \( M_g \) = the number of gate transfers;
\( n_{SS} \) = the density of surface states;
\( A_D \) = the detector area;
\[ k_B > \text{Boltzmann's constant; and} \]
\[ T = \text{Temperature [K].} \]

Typical values of \( N_{BT} \) are:
- Surface Channel CCD−1000 electrons
- Buried Channel CCD−100 electrons.

5.1.1.3 Output Amplifier Noise

The output amplifier noise is associated with the amplifier that buffers the signal from the focal plane and is generally a metal oxide semiconductor field effect transistor (MOSFET) of a given transconductance. An expression that can be used to compute this noise is (Dereniak, 1984, p. 244)

\[
N_{OA} = \left( \frac{8C_o k_B T \Delta f}{3q^2 g_m} \right)^{1/2} \text{[e]}, \tag{5-4}
\]

where \( C_o \) = the output capacitance [\( \mu \)farad];
\( \Delta f \) = the electrical bandwidth [Hz];
\( g_m \) = the transconductance of the MOSFET [mhos];
\( q \) = the charge of an electron [coul]; and
\( k_B \) = Boltzmann’s constant.

5.1.1.4 Dark Current Noise

Dark current or thermal generation noise is associated with charge carriers that are thermally generated to bring the CCD potential well into thermal equilibrium. Dark current root-mean-square (rms) noise is given by (Honeywell, 1986, p. 5-26)

\[
N_{DC} = \left( \frac{2J_{DC} A_D t_I}{q} \right)^{1/2} \text{[e]}, \tag{5-5}
\]

where \( J_{DC} \) = dark current density at temperature \( T \) [a/cm²];
\( A_D \) = detector area [cm²];
\( t_I \) = integration time [sec]; and
\( q \) = electron charge [coul].

The dark current density \( J_{DC} \) is given by
\begin{equation}
J_{DC} = \alpha_M T^3 \exp \left( -\frac{qE_g}{\eta_M k_B T} \right)
\end{equation}

where \( E_g = \) silicon band gap = 1.12 eV;
\( k_B = \) Boltzmann's constant = \(8.62 \times 10^{-5}\) [eV/K];
\( T = \) temperature [K];
\( \eta_M = \) a material-dependent carrier recombination factor; for silicon \( \eta_M = 2 \); and
\( \alpha_M = \) a material-dependent factor, typically \( \alpha_M = 1.1 \times 10^{-6} \) [A/cm\(^3\)-K\(^3\)].

5.1.1.5 Johnson (Thermal) Noise

The thermal motion of electrons in a resistor gives rise to voltage fluctuations across the resistor leads. These fluctuations are known as Johnson or thermal noise. The noise current is given by (Dereniak, 1984, p. 39)

\begin{equation}
\begin{aligned}
 i_{\text{rms}} &= \left( \frac{4k_B T \Delta f}{R} \right)^{1/2} \text{[A]}, \\
\end{aligned}
\end{equation}

where \( \Delta f = \) the effective bandwidth of the circuit [Hz] and \( R = \) the resistance [\(\Omega\)].

It follows that the noise in electrons is given by

\begin{equation}
N_T = \frac{t_1}{R} \left( \frac{4k_B T \Delta f}{q} \right)^{1/2} \text{[e]}.
\end{equation}

5.1.1.6 Schottky Noise

Electrons in the semiconductor of an M-S (metal-semiconductor) junction may overcome the potential barrier to reach the metal and produce a noise current. The noise current is called Schottky barrier noise and is given by (Yang, 1978, p. 130)

\begin{equation}
I_o = A_D R_C T^2 \exp \left( -\frac{q\phi_m}{k_B T} \right) \text{[A]},
\end{equation}

where \( R_C = \frac{4\pi q m_k^2}{h^3} = \) Richardson's constant = 120 [A/cm\(^2\)-K\(^2\)].
The Schottky noise current can be converted to electrons and is given by

\[ N_{DCS} = \left( \frac{I_0 \tau_1}{q} \right)^{1/2} \text{ [e]}. \] (5-9b)

### 5.1.1.7 Charge Transfer Noise

Charge transfer or transfer inefficiency noise is associated with CCD structures and occurs because of the random amount of charge lost by a signal upon transfer and the amount of charge introduced to a signal upon entering a well. The noise \( N_{CT} \) associated with a single well (Dereniak, 1984, p. 242) is given by

\[ N_{CT} = \left( 2eS' \right)^{1/2} \text{ [e]}, \] (5-10)

where \( e \) = the transfer efficiency [nd].

If the number of detectors is \( n_D' \), and the number of phases to transfer the charge is \( n_p \), then the total number of wells is \( n_p n_D' \). Hence the total charge transfer noise is given by

\[ N_{CT} = \left( 2e n_D' n_p S' \right)^{1/2} \text{ [e]}. \] (5-11)

### 5.1.2 Visible and SWIR System Noise

#### 5.1.2.1 Quantization Noise

The quantization noise \( N_Q \) is given by (Montgomery, 1978, p. B-1)

\[ N_Q = \frac{S_{SAT}'}{12^{1/2} 2^Q} \text{ [e]}, \] (5-12)

where \( Q \) is the number of bits used in the analog-to-digital (A/D) converter. From Equation (2-9) one obtains for the visible and SWIR...
\[ S'_{\text{SAT}} = t_1 A_D \eta \bar{E}'_{\text{SAT}} \Delta \lambda \ [e], \quad (5-13) \]

where \( S'_{\text{SAT}} \) is the signal that would result if the detector were receiving the saturation irradiance. This is the flux that produces a signal level that just causes the A/D converter to saturate.

The saturation irradiance \( \bar{E}'_{\text{SAT}} \) is given by

\[ \bar{E}'_{\text{SAT}} = E'_M S_F \ [\text{p/sec-cm}^2-\mu\text{m}], \quad (5-14) \]

where, from Equation (2-11), \( E'_M \) is given by

\[ E'_M = \frac{\tau_0 \pi}{4f^2_N} L'_M \ [\text{p/sec-cm}^2-\mu\text{m}] \quad (5-15) \]

When the saturation factor \( S_F \) is multiplied by the maximum expected scene irradiance \( E'_M \) at the detector, an irradiance \( \bar{E}'_{\text{SAT}} \) will be produced that will just saturate the A/D converter.

5.1.2.2 Other System Noise

When system noises are from unknown sources, they are designated as other system noise.

5.2 Infrared Detector Noise

5.2.1 Infrared Detector Noise Sources

The detector noise is composed of two parts, photon noise and other noise. It is given by

\[ N_{\text{DET}} = \left( N_p^2 + N_{\text{OD}}^2 \right)^{1/2} \ [e] \quad (5-16) \]

We will calculate the photon noise \( N_p \) on the basis of a cold shielded and cold filtered detector. The other detector noise \( N_{\text{OD}} \), will be estimated from laboratory values of \( D^* \). We will estimate the other detector noise from \( D^* \) first.

5.2.1.1 Other Detector Noise

Consider the detector that will be used in the sensor (same area and electrical bandwidth), but with background temperature and viewing angles identical to those used in the laboratory measurement of \( D^* \).

The other detector noise \( N_{\text{OD}} \) is given by
The other detector noise $N_{OD}$ is assumed to be independent of the level of cold shielding and cold filtering. The photon noise under laboratory conditions is

$$N_{PL} = \left( t_F \pi \eta A_D \int_0^{\lambda_c} B'(T_{BGL}, \lambda) d\lambda \right)^{1/2} [e],$$  

(5-18)

where $\lambda_c$ is the detector cutoff wavelength, $T_{BGL}$ is the background temperature in the laboratory, $B'(T_{BGL}, \lambda)$ is defined by Equation (3-7), and $F_L$ is the view factor in the laboratory ($F_L = 1$ when viewing $180^\circ$, or $2\pi$ steradians). In general

$$F_L = \sin^2 \left( \frac{\phi_C}{2} \right) \text{ [nd]}$$  

(5-19)

where $\phi_C$ is the full-cone view angle.

The detector noise equivalent power is given by

$$NEP = \left( \frac{A_D \Delta f}{D^*} \right)^{1/2} [W],$$  

(5-20)

where $A_D = \text{ detector area [cm}^2\text{]}$; and $D^* = \text{ the laboratory value of specific detectivity [cm-Hz}^{1/2}/\text{W]}\text{].}$

The number of noise equivalent electrons $NEE$ is given by

$$NEE = \frac{R_{t_I}}{q} \frac{NEP}{[e]}$$  

(5-21)

and the effective noise bandwidth is given by

$$\Delta f = \frac{\beta}{2t_I} \text{ [kHz]},$$  

(5-22)

where $\beta$ is the ratio of noise bandwidth to information bandwidth, and $t_I$ is the integration time given by Equation (2-8).
5.2.1.2 Photon Noise (Infrared)

The photon noise is given by (Levi, 1968, p. 153)

\[ N_p = (\tau')^{1/2} \]  \[ \text{[e]} \]  \[ (5-23) \]

where the total number of electrons produced by the scene and background is given by

\[ \tau' = t_A A_D \eta \left[ \int_{\lambda_1}^{\lambda_2} E'(\lambda) \, d\lambda + \int_{\lambda_1'}^{\lambda_2'} E_{BG}'(\lambda) \, d\lambda \right] \]  \[ \text{[e]} \]  \[ (5-24) \]

where \( \lambda_1 \) to \( \lambda_2 \) is the spectral bandpass of the scene photon spectral irradiance \( E'(\lambda) \), which is given by Equation (2-11), and \( \lambda_1' \) to \( \lambda_2' \) is the spectral bandpass of the background photon spectral irradiance \( E_{BG}'(\lambda) \), which is governed by the cold filter.

The background spectral irradiance \( E_{BG}'(\lambda) \) is given by

\[ E_{BG}'(\lambda) = \left[ 2 \epsilon_o + (\epsilon_{BG} - \epsilon_o) \tau_{CF} \right] B'(\lambda, T_{BG}) \]  \[ \text{[p/sec-cm}^2-\mu m] \]  \[ (5-25) \]

and the emissivity of the optics is given by

\[ \epsilon_o = 1 - \tau_o \]  \[ \text{[nd]} \]  \[ (5-26) \]

where \( B'(\lambda, T_{BG}) \) = the background spectral photon radiance, [p/sec-cm²-sr-μm] obtained from Equation (3-7);

\( T_{BG} \) = the background temperature [K] of any radiant energy source other than the ground resolution element; and

\( \tau_{CF} \) = optical transmission [nd] of the cold filter.

The effective solid angle \( \Omega \) (Figure 5) through which the detector receives energy from the ground resolution element is given by

\[ \Omega = \frac{\pi}{4f^2 N} \]  \[ \text{[sr]} \]  \[ (5-27) \]

The background effective solid angle \( \Omega_{BG} \) includes the effective solid angle \( \Omega \) plus a little more for tolerance purposes. Ideally, they would be identical.
Solid Angles
\[ \Omega' = 2\pi(1 - \cos \theta) \]
\[ \Omega'_{BG} = 2\pi(1 - \cos \theta_{BG}) \]

Effective Solid Angles
\[ \Omega = \pi \sin^2 (\theta) = \frac{\pi}{4f^2} \]
\[ \Omega_{bg} = \pi \sin^2 (\theta_{bg}) \]

Area of sensor entrance aperture (cm²) \( A_o \)
Diameter of sensor entrance aperture (cm) \( D \)
Focal length (cm) \( f \)
Optics half-cone angle (deg) \( \theta \)
Background half-cone angle (deg) \( \theta_{BG} \)
Optics effective solid angle (sr) \( \Omega \)
Background effective solid angle (sr) \( \Omega_{bg} \)
Optics solid angle (sr) \( \Omega' \)
Background solid angle (sr) \( \Omega'_{bg} \)

Figure 5. Background Viewing Geometry
The effective solid angle of the background $\Omega_{BG}$ is given by

$$\Omega_{BG} = \pi F_S \ [sr], \quad (5-28)$$

where the view factor $F_S$ is defined as

$$F_S = \begin{cases} F_C & \text{for a cold shielded detector viewing the scene through a circular aperture} \ (\text{Appendix D}), \text{ and} \\ F_A & \text{for a detector array surrounded by a cold fence} \ (\text{Appendix E}). \end{cases}$$

It follows that:

$$F_C = \sin^2 \left( \frac{\theta_{BG}}{2} \right) \ [sr], \quad (5-29)$$

where $\theta_{BG}$ is the full-cone angle [deg] of the background.

From Equation (5-20) one can write

$$D^*_S = \frac{\left( A_D \Delta \ell \right)^{1/2}}{\text{NEP}} \ [\text{cm-Hz}^{1/2}/\text{W}] \quad (5-30)$$

and

$$\text{NEP} = \frac{q}{R(\lambda) t_1 \left[ N_p^2 + N_{OD}^2 \right]^{1/2}} \ [\text{W}] \quad (5-31)$$

Substituting Equation (5-31) into Equation (5-30) and setting $N_{OD} = 0$ yields

$$D_{BLIP}^*(\lambda) = \frac{R(\lambda) t_1 \left( A_D \Delta \ell \right)^{1/2}}{qN_p} \ [\text{cm-Hz}^{1/2}/\text{W}] \quad (5-32)$$

where $D_{BLIP}^*$ is the background-limited photon (BLIP) value of $D^*$.

Substituting Equations (2-3), (5-21), (5-23), and (5-24) into Equation (5-32) gives

$$D_{BLIP}^*(\lambda) = \frac{\lambda}{h c} \left( \frac{\eta}{2} \right)^{1/2} \left( \int \frac{\lambda_2^2 E(\lambda) \ d\lambda}{\lambda_1} + \int \frac{\lambda_2' E_{BG}(\lambda) \ d\lambda}{\lambda_1'} \right)^{-1/2} \ [\text{cm-Hz}^{1/2}/\text{W}] \quad (5-33)$$

Equation (5-33) includes only the photon noise and is for a photovoltaic detector. For a photoconductive detector there is an additional term due to generation-recombination noise, and it reduces $D_{BLIP}^*$ by a factor of $(2)^{1/2}$. 

25
5.2.2 Infrared System Noise

5.2.2.1 Quantization Noise

The quantization noise is given by (Montgomery, 1978, p. B-1)

\[
N_Q = \frac{S'_{SAT}}{(2)^{1/2} 2^Q} \ [e],
\]  

(5-34)

where \( Q \) is the number of bits used in the A/D converter.

The infrared saturation signal is given by

\[
S'_{SAT} = \int_{\lambda_1}^{\lambda_2} E'_{SAT}(\lambda) \ d\lambda \ [e],
\]  

(5-35)

where the infrared saturation spectral photon irradiance \( E'_{SAT}(\lambda) \) is given by (see Appendix A)

\[
E'_{SAT}(\lambda) = \frac{\tau_0 \pi}{4 f_N^2} L'_{SAT}(\lambda) \ [\text{p/sec-cm}^2 \cdot \mu\text{m}]
\]  

(5-36)

and where the infrared saturation radiance \( L'_{SAT}(\lambda) \) is the appropriate value to just saturate the A/D converter.

5.2.2.2 Other Infrared System Noise

When the system noises are from unknown sources, they are designated as other system noise.
6. FIGURES OF MERIT

One important figure of merit that we will discuss here is the signal-to-noise ratio SNR, which is easily obtained by using the results of Section 2 for the signal $S'$ and Section 5 for the noise $N_{TOT}$. We will also discuss a figure of merit used in the visible and SWIR bands, the Noise Equivalent Delta Reflectance (NEΔρ), and for the infrared we shall discuss the Noise Equivalent Delta Temperature (NEΔT).

6.1 Noise Equivalent Delta Reflectance

When visible sensors are assessed with respect to surface observations such as reflectance, the SNR is not always a convenient figure of merit. Users of space or airborne remote sensor data are frequently concerned with the characterization of ground targets through the measurement of variations in target reflectance. Because the variations of interest are often small in magnitude and are difficult to measure precisely, there is considerable interest in the definition and measurement of the capability of the sensor to respond to small reflectance changes. This capability, related to sensitivity, is often described in terms of Noise Equivalent Delta Reflectance (NEΔρ), which is the minimum detectable variation in reflectance, and is sometimes preferred by the science user community over the spatial resolution of the system.

For the visible and SWIR bands, NEΔρ is the amount by which ρ would need to change to cause the signal to change by an amount equal to the noise, or it is the smallest change in reflectance between two adjacent surface elements that can be resolved by the sensor. In this section we compute NEΔρ from the SNR.

The figure of merit NEΔρ is given by

$$NEΔρ = \frac{L}{\frac{S^2}{N}} \left(\frac{dL}{dρ}\right) = \frac{γ}{\frac{S}{N}} \text{ [nd]}, \quad (6-1)$$

where $γ$ (see Appendix F) is

$$γ = \frac{γ_o}{τ_{AN}(sec φ' - 1)} \text{ [nd]} \quad (6-2)$$
and where

\[
\gamma_0 = \frac{\rho}{1 - \frac{L_{AN}^N}{L_N}} \quad [\text{nd}].
\]  

(6-3)

Also, \( \phi' \) is the angle between the line of sight and the surface normal (see Figure 6) and \( L_{AN}^N \) and \( L_N \) are the atmospheric and scene radiances respectively, from the tables in Appendix B.

The atmospheric optical transmission in the nadir direction \( \tau_{AN} \) is given by

\[
\tau_{AN} = e^{-\delta_{TN}} \quad [\text{nd}],
\]  

(6-4)

where the total nadir optical thickness \( \delta_{TN} \) is obtained from the tables in Appendix B.

6.2 Noise Equivalent Delta Temperature

For the infrared bands NEAT is given by

\[
\text{NEAT} = \frac{L'}{\tau_A \left( \frac{S}{N} \right) \left( \frac{dL'S}{dT_S} \right)} \quad [\text{K}]
\]  

(6-5)

where \( L' = \) the scene photon radiance \([p/\text{sec-cm}^2\cdot \text{sr}]\),

\( \tau_A = \) atmospheric transmission \([\text{nd}]\),

\( S = \) signal \([\text{e}]\),

\( N = \) noise \([\text{e}]\), and

\( dL'S/dT_S = \) the differential change in surface radiance with respect to surface temperature \([p/\text{sec-cm}^2\cdot \text{sr} \cdot \text{K}]\).

(See Appendix G for a detailed derivation.)
\[ \phi' = \sin^{-1} \left[ (1 + \frac{H}{R_e}) \sin \phi \right] \]

- \( \phi \) = Sensor view angle (deg)
- \( \phi' \) = Angle between line of sight and the surface normal (deg)
- \( H \) = Height
- \( R_e \) = Radius of Earth

Figure 6. Satellite/Scene Viewing Geometry
7. MODULATION TRANSFER FUNCTION (MTF)

If the sensor were to scan a very low spatial frequency (sinusoidal variation in radiance) an amplitude variation $\Delta S(0)$ in signal would result. At spatial frequency $k$ [cycles/mm] an amplitude variation of $\Delta S(k)$ would be obtained. The ratio of $\Delta S(k)$ to $\Delta S(0)$ is the Modulation Transfer Function (MTF) of the sensor; i.e.,

$$MTF = \frac{\Delta S(k)}{\Delta S(0)} \quad [nd] \quad (7-1)$$

7.1 Total MTF

For a linear system, the total modulation transfer function is the product of the modulation transfer functions of the individual elements of the system. There are many different MTFs associated with a sensor system, which typically might include:

- $MTF_{OA}$ = Optical Aperture MTF
- $MTF_{DA}$ = Detector Aperture MTF
- $MTF_{SM}$ = Satellite Motion MTF
- $MTF_{CD}$ = Charge Diffusion MTF
- $MTF_{CT}$ = Charge Transfer MTF
- $MTF_{SJ}$ = Satellite Jitter MTF.

The total or system MTF is the product of the component MTFs; i.e.,

$$MTF = MTF_{OA} \cdot MTF_{DA} \cdot MTF_{SM} \cdot MTF_{CD} \cdot MTF_{CT}$$

7.2 Component MTFs

In this section we list the MTF equations and their reference sources.

7.2.1 Optical Aperture MTF

The diffraction MTF is given by (O’Neill, 1955 and 1956)

$$MTF_{OA} = \frac{A_m + B_m + C_m}{(1 - \beta^2)}$$ \quad (7-2)

where
\[ A_m = \frac{2}{\pi} \left\{ \cos^{-1} \left( \frac{\Lambda}{2\beta} \right) - \left( \frac{\Lambda}{2\beta} \right)^2 \right\}^{1/2} \] for \( 0 \leq \frac{\Lambda}{2} \leq 1 \) \hspace{1cm} (7-3)

and

\[ A_m = 0 \] for \( \frac{\Lambda}{2} > 1 \), \hspace{1cm} (7-4)

where

\[ B_m = \frac{2\beta^2}{\pi} \left\{ \cos^{-1} \left( \frac{\Lambda}{2\beta} \right) - \left( \frac{\Lambda}{2\beta} \right)^2 \right\}^{1/2} \] for \( 0 \leq \frac{\Lambda}{2\beta} \leq 1 \) \hspace{1cm} (7-5)

and

\[ B_m = 0 \] for \( \frac{\Lambda}{2\beta} > 1 \) \hspace{1cm} (7-6)

and where

\[ C_m = -2\beta^2 \] for \( 0 < \frac{\Lambda}{2} \leq \frac{(1 - \beta)}{2} \) \hspace{1cm} (7-7)

\[ C_m = -2\beta^2 + \frac{2\beta}{\pi} \sin \psi + \left( \frac{1 + \beta^2}{\pi} \right) \psi - 2 \left( \frac{1 - \beta^2}{\pi} \right) \tan^{-1} \left[ \left( \frac{1 + \beta}{1 - \beta} \right) \tan \frac{\psi}{2} \right] \] for \( \frac{1 - \beta}{2} \leq \frac{\Lambda}{2} \leq \frac{1 + \beta}{2} \) \hspace{1cm} (7-8)

and

\[ C_m = 0 \] for \( \frac{\Lambda}{2} > \frac{1 + \beta}{2} \), \hspace{1cm} (7-9)

and \( \psi \) is given by

\[ \psi = \cos^{-1} \left( \frac{1 + \beta^2 - \Lambda^2}{2\beta} \right) \] \hspace{1cm} (7-10)

with

\[ \Lambda = \frac{k}{k_0} \] \hspace{1cm} (7-11)

where \( k \) is the spatial frequency [cycles/mm] measured in the image plane and
The modulation transfer function $MTF_{OA} = 0$ for $\Lambda = 2$ at the cutoff frequency when $k = k_C$, where

$$k_C = 2k_0 \quad [\text{cycles/mm}],$$

and where

$$f_N = \frac{f}{D} \quad [\text{nd}],$$

where $\lambda$ is the wavelength, $f$ is the focal length of the optics, and $D$ is the diameter of the optics; and

$$\beta = \frac{D_0}{D} \quad [\text{nd}],$$

where $D_0$ is the diameter of any obscuration.

Note that the quantity $k_0$ given in Equation (7-12) is not defined in O'Neill's paper but must be defined this way to be consistent with his Figure 3 in which the MTF goes to zero at $\Lambda = 2.0$.

### 7.2.2 Detector Aperture MTF

The detector aperture MTF is given by (Jensen, 1968, p. 27)

$$MTF_{DA} = \left| \frac{\sin(\pi kd_S)}{\pi kd_S} \right|$$

where $d_S = \text{the detector width} [\text{mm}]$, and

$k = \text{spatial frequency in the image plane} [\text{cycles/mm}]$.

### 7.2.3 Satellite Motion MTF

The MTF due to linear image motion is given by (Jensen, 1968, p. 117)

$$MTF_{SM} = \left| \frac{\sin(\pi kV_I t_1)}{\pi k V_I t_1} \right|$$

where the image velocity $V_I$ is given by

$$V_I = \frac{fV_{SUB}}{H} \quad [\text{km/sec}],$$

(7-18)
where \( V_{\text{SUB}} \) = the subsatellite point velocity [km/sec];
\( k \) = spatial frequency in the image plane [cycles/mm]; and
\( t_1 \) = integration time [sec].

### 7.2.4 Charge Diffusion MTF

The charge diffusion MTF is given by (Jespers, 1975, p. 519)

\[
\text{MTF}_{\text{CD}} = \exp \left( -\frac{\alpha_a d}{1 + \alpha_a L} \right)
\]

where \( d \) = photodetector depletion region depth (typical value = 5 \( \mu \)m).

The silicon absorption coefficient \( \alpha_a \) is a function of wavelength and temperature and is given by

\[ \alpha_a = 10^z \text{ [cm}^{-1}\text{]} \]  \hspace{1cm} (7-20)

where, after curve fitting to Jespers' Figure 25 for silicon, we have

\[ z = 2.897652 - 4.044143 (\lambda - 0.82) - 5.219219 (\lambda - 0.82)^2 \]
\[ - 3.828495 (\lambda - 0.82)^3 + 22.16724 (\lambda - 0.82)^4 \]  \hspace{1cm} (7-21)

where \( \lambda \) = wavelength [\( \mu \)m],
\( L_0 \) = diffusion length (typical value = 50 \( \mu \)m),
and where

\[ L = \left[ \frac{L_0^2}{1 + (2\pi k L_0)^2} \right]^{1/2} \]  \hspace{1cm} (7-22)

### 7.2.5 Charge Transfer MTF

The MTF due to inefficiency in charge transfer in a CCD detector is given by (Jespers, 1976, p. 520)

\[
\text{MTF}_{\text{CT}} = e^{-M_{\text{CT}}} e \left[ 1 - \cos \left( \frac{\pi k}{k_{\text{max}}} \right) \right] \]  \hspace{1cm} (7-23)
where $e$ is the charge transfer inefficiency and where the number of charge transfers $M_{CT}$ is given by

$$M_{CT} = m_S m_p,$$

(7-24)

where $m_S = \text{the number of stages, detectors, or picture elements, and}$

$m_p = \text{the number of clock phases for readout.}$

The Nyquist spatial frequency $k_{max}$ is given by

$$k_{max} = \frac{1}{2P_D},$$

(7-25)

where $P_D$ is the detector pitch.

### 7.2.6 Satellite Jitter MTF

The satellite jitter MTF is given by (Jensen, 1968, p. 124)

$$MTF_{SJ} = J_0(2\pi kf\Theta_M)$$

(7-26)

where $\Theta_M = \text{maximum satellite angular movement [rad]},$

$J_0 = \text{zeroth order Bessel function, and}$

$f = \text{focal length (mm).}$
8. REFERENCES


APPENDIX A

DETECTOR IRRADIANCE

The objective of this appendix is to show the relationship between the scene spectral radiance and the irradiance on the detector.

When a sensor images an area on the surface of the Earth called the instantaneous field of view (IFOV), it also receives energy from the intervening atmosphere. The scene spectral radiance $L(\lambda) \ [W/cm^2-sr-\mu m]$ is defined as the combined spectral radiance from the atmosphere and the IFOV area $A_1$ as viewed from the sensor. The power that the area $A_1$ and the intervening atmosphere radiate through the solid angle $\Omega_o$ into the sensor entrance aperture and into the detector is then

$$\Phi = \int_{\lambda_1}^{\lambda_2} \frac{\phi(\lambda)}{\lambda} \ d\lambda \ [W] \quad (A-1)$$

where

$$d\phi(\lambda) = \tau_o \Omega_o A_1 L(\lambda) \ d\lambda \ [W] \quad (A-2)$$

and where $\tau_o$ = the sensor optical transmission. However, by definition, for a detector of area $A_D$, Equation (A-2) may also be written in terms of the detector spectral irradiance $E(\lambda)$ [W/cm$^2$-\mu m] as

$$d\phi(\lambda) = A_D E(\lambda) \ d\lambda \ [W] \quad (A-3)$$

By comparing Equations (A-2) and (A-3), one can express the scene spectral irradiance $E(\lambda)$ in terms of the scene spectral radiance $L(\lambda)$ and sensor design-related parameters $A_D$, $A_1$, and $\Omega_o$ as

$$E(\lambda) = \frac{\tau_o \Omega_o A_1}{A_D} L(\lambda) \ [W/cm^2-\mu m] \quad (A-4)$$

Now, from Figure A-1,

$$A_1 = \alpha^2 H^2 \quad (A-5)$$
$A_d = \text{Detector area (}\mu\text{m}^2\),}

$A_o = \text{Area of sensor entrance aperture (cm}^2\text{),}

$A_I = \text{Area of ground resolution element (cm}^2\text{),}

$D = \text{Diameter of sensor entrance aperture (cm),}

$f = \text{Effective focal length (cm),}

$H = \text{Satellite height (km),}

$\alpha = \text{Instantaneous angular FOV (as in previous figs) (rad),}

$\theta = \text{Half-cone angle of optics (deg),}

$\Omega = \text{The effective solid angle through which the detector receives energy (sr),}

$\Omega_o = \text{Solid angle subtended by the sensor entrance aperture at the ground resolution element (sr),}

$\Omega_i = \text{Solid angle subtended by the ground resolution element at the sensor entrance aperture (sr),}

$\Omega_d = \text{Solid angle subtended by the detector at the entrance aperture (sr).}

Figure A-1. Radiometric Geometry
where \( \alpha = \) the sensor instantaneous field of view (IFOV), and 
\[ H = \] the distance from the satellite to the area \( A_I \).

Also, for a square detector

\[ A_D = \alpha^2 f^2, \quad (A-6) \]

where \( f \) is the sensor optics effective focal length, which represents the optical path length and is approximately equal to the geometric focal length for small optical convergence angles (<10°), and

\[ \Omega_o = \frac{A_o}{H^2} = \frac{\pi D^2}{4H^2}, \quad (A-7) \]

where \( D = \) aperture diameter of the sensor.

Using Equations (A-5), (A-6) and (A-7), one can write

\[ \frac{A_I \Omega_o}{A_D} = \frac{\pi}{4f^2 N}, \quad (A-8) \]

where the sensor optics f-number \( f_N \) is given by

\[ f_N = \frac{f}{D}. \quad (A-9) \]

The half-cone angle \( \theta \) of the optics is given by

\[ \sin \theta = \frac{D}{2f}. \quad (A-10) \]

It is also given by

\[ \tan \theta = \frac{D}{2f'}. \quad (A-11) \]

The f-number \( f_N \) is related to the approximate f-number \( \left( \frac{f'}{D} \right) \) by the expression

\[ f_N = \left( \frac{f'}{D} \right) \left[ 1 + \left( \frac{1}{2} \frac{f'}{f} \right)^2 \right]^{1/2} \quad (A-12) \]

Substituting Equation (A-8) into Equation (A-4) (See Figure A-2) gives
$f_N = F\text{-NUMBER}$

$$f_N = \frac{f}{D} = \frac{1}{2 \sin \theta}$$

$$= \frac{f'}{D} \left[1+\left(1+\frac{1}{2 \frac{f'}{D}}\right)^2\right]^{1/2}$$

$$\Delta = \left(\frac{f'_N}{f} - \frac{f'}{D}\right)/100 f_N$$

Figure A-2. Comparison of $f_N$ and $f'/D$
E(λ) = \frac{\pi r_o}{4f^2N} L(\lambda) \quad [W/cm^2-\mu m]. \quad (A-13)

REFERENCES

Fraser, Unpublished Memorandum, 1981.

APPENDIX B
COMPUTED EARTH ATMOSPHERE RADIANCES

With the permission of the authors, the following discussion has been taken from a NASA unpublished report (Mattoo, 1984) and a memorandum (Fraser, 1981).

The computed radiances at the top of the Earth-atmosphere system are tabulated for 15 wavelengths. The computations are made for plane-parallel models of the Earth-atmosphere system; that is, the optical properties of the models do not vary in a horizontal plane. However, the vertical profiles of the concentrations of the atmospheric constituents are arbitrary, but realistic. The ground reflects light according to Lambert's law, which implies that the radiance of the reflected light is constant, independent of direction.

Only one atmospheric model is used, and it contains the standard dry gas and the gases that absorb in the spectral bands of interest. The variable trace gases are 316 Dobson units of O₃ and 2.5 cm of H₂O. The model also contains particulates, but not liquid or ice clouds.

The average normal optical thickness of each constituent is given for each spectral band in Table B-1. The total nadir optical thickness ($\delta_{TN}$) at a wavelength equals the sum of the optical thicknesses of the constituents:

$$\delta_{TN} = \delta_R + \delta_G + \delta_A ,$$  \hspace{1cm} \text{(B-1)}

where $\delta_R$ = the scattering optical thickness (Rayleigh) of the dry atmosphere, $\delta_A$ = the optical thickness of the aerosols (particulates), and $\delta_G$ = the optical thickness of the absorbing gases.

The total optical transmission $\tau$ for the direct sunlight is given by

$$\tau = \frac{E_{\lambda_5}}{E_{\lambda_0}} = \exp(-\delta_{TN} \sec \theta_z)$$ \hspace{1cm} \text{(B-2)}

where $E_{\lambda_5}$ and $E_{\lambda_0}$ are the spectral irradiance of the direct sunlight at sea level and above the atmos-
Table B-1. Optical Thicknesses for Scattering by Molecules ($\delta_R$), Absorption by Gases ($\delta_G$), and Scattering from Aerosols ($\delta_A$)

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.4</th>
<th>0.44</th>
<th>0.48</th>
<th>0.52</th>
<th>0.56</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_R$</td>
<td>0.3637</td>
<td>0.2451</td>
<td>0.1713</td>
<td>0.1234</td>
<td>0.0912</td>
</tr>
<tr>
<td>$\delta_G$</td>
<td>0.0000</td>
<td>0.0007</td>
<td>0.0048</td>
<td>0.0151</td>
<td>0.0296</td>
</tr>
<tr>
<td>$\delta_A$</td>
<td>0.3600</td>
<td>0.3273</td>
<td>0.3000</td>
<td>0.2796</td>
<td>0.2571</td>
</tr>
<tr>
<td>$\delta_{TN}$</td>
<td>0.7237</td>
<td>0.5731</td>
<td>0.4761</td>
<td>0.4181</td>
<td>0.3779</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.62</th>
<th>0.66</th>
<th>0.70</th>
<th>0.74</th>
<th>0.82</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_R$</td>
<td>0.0603</td>
<td>0.0468</td>
<td>0.0368</td>
<td>0.0294</td>
<td>0.0195</td>
</tr>
<tr>
<td>$\delta_G$</td>
<td>0.0325</td>
<td>0.0169</td>
<td>0.0135</td>
<td>0.0145</td>
<td>0.0560</td>
</tr>
<tr>
<td>$\delta_A$</td>
<td>0.2370</td>
<td>0.2228</td>
<td>0.2087</td>
<td>0.1988</td>
<td>0.1794</td>
</tr>
<tr>
<td>$\delta_{TN}$</td>
<td>0.3298</td>
<td>0.2865</td>
<td>0.2590</td>
<td>0.2427</td>
<td>0.2549</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.88</th>
<th>1.05</th>
<th>1.25</th>
<th>1.60</th>
<th>2.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_R$</td>
<td>0.0147</td>
<td>0.0072</td>
<td>0.0036</td>
<td>0.0013</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\delta_G$</td>
<td>0.0039</td>
<td>0.0000</td>
<td>0.0092</td>
<td>0.0066</td>
<td>0.0608</td>
</tr>
<tr>
<td>$\delta_A$</td>
<td>0.1694</td>
<td>0.1420</td>
<td>0.1193</td>
<td>0.0932</td>
<td>0.0682</td>
</tr>
<tr>
<td>$\delta_{TN}$</td>
<td>0.1880</td>
<td>0.1492</td>
<td>0.1321</td>
<td>0.1011</td>
<td>0.1294</td>
</tr>
</tbody>
</table>
phere, respectively, and \( \theta_z \) is the solar zenith angle. The values of \( E_{\lambda_0} \) are given in Table B-2. The quantity \( E_{\lambda_0} \) is the solar irradiance on a square centimeter of surface perpendicular to the solar rays.

The aerosols (particulates) are assigned properties of those occurring in continental regions. The size distribution function \((n)\) of the particle radius \((r)\), which is the number of particles per cubic centimeter of air per micrometer of radius, decreases very rapidly with increasing radius:

\[
n \sim r^{-4} .
\]

The index of refraction of the particulates is \( m = 1.4300 - 0.0035i \). The nadir radiances of the model are not sensitive to the vertical profile of the particulate concentration. Here, a realistic profile with a high concentration near the ground is assumed.

The computations are made with a computer code developed by Dr. J. V. Dave of IBM as modified by R. S. Fraser of NASA. The equation of radiative transfer is solved numerically by a procedure that iteratively accounts for successive scatterings of light from the atmosphere and the ground. The polarization characteristics are not accounted for, and as a result, the computed radiances are in error by a few percent.

The input parameters for the computations include the model parameters: the vertical profiles of the concentrations of the gases and aerosols (particulates), the scattering and absorption optical thickness, the gaseous absorption coefficients, and 10 values of the surface reflectance \( \rho \). The volume extinction, scattering and absorption coefficients and scattering phase function of the particulates are computed according to the Mie theory by a separate code, and are part of the input.

The nadir spectral radiance at the top of the atmosphere \( L^N \) can be expressed as follows:

\[
L^N = L_S e^{-\delta_T N} + L^N_A , \tag{B-4}
\]

where the first term on the right-hand side of the equation gives the radiance at the surface (ground) \( L_S \) (assumed to be Lambertian) attenuated by the atmosphere; the second term gives the radiance of just the atmosphere \( L^N_A \), or path radiance. The radiances \( L^N \) and \( L^N_A \) are given in Table B-2, where
\[ \lambda = \text{the center wavelength [mm]} \]
\[ \theta_z = \text{the solar zenith angle [rad]} \]
\[ \rho = \text{the surface reflectance [nd]} \]
\[ \delta_{TN} = \text{the total optical thickness [nd]} \]
\[ E_{\lambda o} = \text{the solar spectral irradiance [mW/cm}^2\text{-}\mu\text{m]} \]
\[ L^N = \text{the total nadir spectral radiance [mW/cm}^2\text{-sr} \text{-} \mu\text{m]} \text{ and} \]
\[ L^N_A = \text{the atmospheric path spectral radiance [mW/cm}^2\text{-sr} \text{-} \mu\text{m]} \]
Table B-2. Reflected Solar Spectral Radiance

\[ \lambda = 0.400 \mu \text{m}, \quad E_{\lambda_0} = 165.400 \text{ mW/cm}^2\text{-}\mu\text{m}, \quad \delta_{TN} = 0.72 \]

<table>
<thead>
<tr>
<th>( \theta_z )</th>
<th>( \rho )</th>
<th>0.00</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( L_N )</td>
<td>( L_A )</td>
<td>( L_N )</td>
<td>( L_A )</td>
<td>( L_N )</td>
<td>( L_A )</td>
</tr>
<tr>
<td>60.0</td>
<td>5.126</td>
<td>5.126</td>
<td>5.263</td>
<td>5.181</td>
<td>5.821</td>
<td>5.407</td>
</tr>
<tr>
<td>80.0</td>
<td>2.552</td>
<td>2.552</td>
<td>2.584</td>
<td>2.565</td>
<td>2.710</td>
<td>2.615</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \theta_z )</th>
<th>( \rho )</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( L_N )</td>
<td>( L_A )</td>
<td>( L_N )</td>
<td>( L_A )</td>
<td>( L_N )</td>
<td>( L_A )</td>
</tr>
<tr>
<td>0.0</td>
<td>18.848</td>
<td>12.695</td>
<td>22.944</td>
<td>14.741</td>
<td>27.298</td>
<td>17.045</td>
</tr>
<tr>
<td>60.0</td>
<td>9.602</td>
<td>7.120</td>
<td>11.276</td>
<td>7.967</td>
<td>13.056</td>
<td>8.918</td>
</tr>
<tr>
<td>80.0</td>
<td>3.570</td>
<td>2.996</td>
<td>3.950</td>
<td>3.186</td>
<td>4.355</td>
<td>3.399</td>
</tr>
</tbody>
</table>
Table B-2. Reflected Solar Spectral Radiance (Continued)

\( \lambda = 0.440 \, \mu m, \quad E_{\lambda_0} = 177.300 \, \text{mW/cm}^2\cdot\mu\text{m}, \quad \delta_{TN} = 0.57 \)

<table>
<thead>
<tr>
<th>( \theta_z )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>80.0</td>
<td>2.269</td>
<td>2.269</td>
<td>2.307</td>
<td>2.280</td>
<td>2.460</td>
<td>2.329</td>
<td>2.656</td>
<td>2.393</td>
<td>3.062</td>
<td>2.536</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \theta_z )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>19.254</td>
<td>11.133</td>
<td>24.051</td>
<td>13.223</td>
<td>29.084</td>
<td>15.549</td>
<td>42.823</td>
<td>22.520</td>
<td>58.490</td>
<td>31.420</td>
</tr>
<tr>
<td>10.0</td>
<td>18.884</td>
<td>10.917</td>
<td>23.590</td>
<td>12.967</td>
<td>28.527</td>
<td>15.248</td>
<td>42.003</td>
<td>22.084</td>
<td>57.371</td>
<td>30.813</td>
</tr>
<tr>
<td>80.0</td>
<td>3.487</td>
<td>2.698</td>
<td>3.932</td>
<td>2.880</td>
<td>4.400</td>
<td>3.084</td>
<td>5.675</td>
<td>3.701</td>
<td>7.129</td>
<td>4.497</td>
</tr>
</tbody>
</table>
Table B-2. Reflected Solar Spectral Radiance (Continued)

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.00</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( L^N )</td>
<td>( L_N^A )</td>
<td>( L^N )</td>
<td>( L_N^A )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>5.287</td>
<td>5.287</td>
<td>5.797</td>
<td>5.438</td>
<td>7.854</td>
</tr>
<tr>
<td>60.0</td>
<td>3.530</td>
<td>3.530</td>
<td>3.748</td>
<td>3.594</td>
<td>4.628</td>
</tr>
<tr>
<td>80.0</td>
<td>2.144</td>
<td>2.144</td>
<td>2.195</td>
<td>2.159</td>
<td>2.400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( L^N )</td>
<td>( L_N^A )</td>
<td>( L^N )</td>
<td>( L_N^A )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>21.418</td>
<td>10.661</td>
<td>27.213</td>
<td>12.870</td>
<td>33.237</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>$\rho$</td>
<td>$\lambda$</td>
<td>$E_{\lambda_0}$</td>
<td>$L_N$</td>
<td>$L_A$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>3.565</td>
<td>3.672</td>
<td>3.891</td>
</tr>
<tr>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>3.567</td>
<td>3.591</td>
<td>3.618</td>
</tr>
<tr>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>3.569</td>
<td>3.594</td>
<td>3.621</td>
</tr>
<tr>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
<td>3.571</td>
<td>3.597</td>
<td>3.624</td>
</tr>
<tr>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>3.573</td>
<td>3.600</td>
<td>3.627</td>
</tr>
</tbody>
</table>

$E_{\lambda_0} = 183,400$ mW/cm²-μm.

For $\lambda = 0.520$ μm,
Table B-2. Reflected Solar Spectral Radiance (Continued)

\[ \lambda = 0.560 \, \mu m, \quad E_{\lambda_0} = 183.000 \, \text{mW/cm}^2\cdot\mu\text{m}, \quad \delta_{TN} = 0.38 \]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( 0.00 )</th>
<th>( 0.01 )</th>
<th>( 0.05 )</th>
<th>( 0.10 )</th>
<th>( 0.20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( \mathbb{L}_N )</td>
<td>( \mathbb{L}_A )</td>
<td>( \mathbb{L}_N )</td>
<td>( \mathbb{L}_A )</td>
<td>( \mathbb{L}_N )</td>
</tr>
<tr>
<td>0.0</td>
<td>2.753</td>
<td>2.753</td>
<td>3.223</td>
<td>2.865</td>
<td>5.115</td>
</tr>
<tr>
<td>50.0</td>
<td>1.965</td>
<td>1.965</td>
<td>2.244</td>
<td>2.032</td>
<td>3.369</td>
</tr>
<tr>
<td>60.0</td>
<td>1.798</td>
<td>1.798</td>
<td>2.002</td>
<td>1.847</td>
<td>2.822</td>
</tr>
<tr>
<td>70.0</td>
<td>1.599</td>
<td>1.599</td>
<td>1.722</td>
<td>1.629</td>
<td>2.215</td>
</tr>
<tr>
<td>80.0</td>
<td>1.161</td>
<td>1.161</td>
<td>1.207</td>
<td>1.173</td>
<td>1.390</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( 0.30 )</th>
<th>( 0.40 )</th>
<th>( 0.50 )</th>
<th>( 0.75 )</th>
<th>( 1.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( \mathbb{L}_N )</td>
<td>( \mathbb{L}_A )</td>
<td>( \mathbb{L}_N )</td>
<td>( \mathbb{L}_A )</td>
<td>( \mathbb{L}_N )</td>
</tr>
<tr>
<td>0.0</td>
<td>17.405</td>
<td>6.659</td>
<td>22.555</td>
<td>8.226</td>
<td>27.847</td>
</tr>
<tr>
<td>80.0</td>
<td>2.581</td>
<td>1.553</td>
<td>3.080</td>
<td>1.709</td>
<td>3.592</td>
</tr>
</tbody>
</table>
Table B-2. Reflected Solar Spectral Radiance (Continued)

\[ \lambda = 0.620 \text{ \( \mu \)m}, \quad E_{\lambda_0} = 172.400 \text{ mW/cm}^2\cdot\text{\( \mu \)m}, \quad \delta_{TN} = 0.33 \]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \theta_z )</th>
<th>0.00</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L^N )</td>
<td>( L_A^N )</td>
<td>( L^N )</td>
<td>( L_A^N )</td>
<td>( L^N )</td>
<td>( L_A^N )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>1.973</td>
<td>1.973</td>
<td>2.428</td>
<td>2.069</td>
<td>4.258</td>
<td>2.464</td>
</tr>
<tr>
<td>10.0</td>
<td>1.905</td>
<td>1.905</td>
<td>2.351</td>
<td>1.999</td>
<td>4.149</td>
<td>2.386</td>
</tr>
<tr>
<td>40.0</td>
<td>1.520</td>
<td>1.520</td>
<td>1.855</td>
<td>1.591</td>
<td>3.203</td>
<td>1.881</td>
</tr>
<tr>
<td>50.0</td>
<td>1.389</td>
<td>1.389</td>
<td>1.661</td>
<td>1.446</td>
<td>2.757</td>
<td>1.683</td>
</tr>
<tr>
<td>60.0</td>
<td>1.285</td>
<td>1.285</td>
<td>1.485</td>
<td>1.327</td>
<td>2.288</td>
<td>1.500</td>
</tr>
<tr>
<td>70.0</td>
<td>1.166</td>
<td>1.166</td>
<td>1.288</td>
<td>1.192</td>
<td>1.776</td>
<td>1.299</td>
</tr>
<tr>
<td>80.0</td>
<td>0.877</td>
<td>0.877</td>
<td>0.922</td>
<td>0.887</td>
<td>1.102</td>
<td>0.926</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( L^N )</td>
<td>( L_A^N )</td>
<td>( L^N )</td>
<td>( L_A^N )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>16.067</td>
<td>5.299</td>
<td>20.977</td>
<td>6.619</td>
<td>25.999</td>
</tr>
<tr>
<td>80.0</td>
<td>2.261</td>
<td>1.205</td>
<td>2.744</td>
<td>1.335</td>
<td>3.237</td>
</tr>
</tbody>
</table>
Table B-2. Reflected Solar Spectral Radiance (Continued)

\( \lambda = 0.660 \, \mu m, \quad F_{\lambda_0} = 156.400 \, mW/cm^2-\mu m, \quad \delta_{TN} = 0.29 \)

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( 0.00 )</th>
<th>( 0.01 )</th>
<th>( 0.05 )</th>
<th>( 0.10 )</th>
<th>( 0.20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( \text{L}^N )</td>
<td>( \text{L}_A^N )</td>
<td>( \text{L}^N )</td>
<td>( \text{L}_A^N )</td>
<td>( \text{L}^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>1.549</td>
<td>1.549</td>
<td>1.053</td>
<td>1.634</td>
<td>3.725</td>
</tr>
<tr>
<td>10.0</td>
<td>1.489</td>
<td>1.489</td>
<td>1.916</td>
<td>1.573</td>
<td>3.628</td>
</tr>
<tr>
<td>30.0</td>
<td>1.321</td>
<td>1.321</td>
<td>1.691</td>
<td>1.394</td>
<td>3.177</td>
</tr>
<tr>
<td>40.0</td>
<td>1.184</td>
<td>1.184</td>
<td>1.506</td>
<td>1.247</td>
<td>2.800</td>
</tr>
<tr>
<td>50.0</td>
<td>1.088</td>
<td>1.088</td>
<td>1.352</td>
<td>1.140</td>
<td>2.410</td>
</tr>
<tr>
<td>60.0</td>
<td>1.021</td>
<td>1.021</td>
<td>1.212</td>
<td>1.059</td>
<td>2.000</td>
</tr>
<tr>
<td>70.0</td>
<td>0.953</td>
<td>0.953</td>
<td>1.074</td>
<td>0.977</td>
<td>1.559</td>
</tr>
<tr>
<td>80.0</td>
<td>0.768</td>
<td>0.768</td>
<td>0.815</td>
<td>0.777</td>
<td>1.003</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( 0.30 )</th>
<th>( 0.40 )</th>
<th>( 0.50 )</th>
<th>( 0.75 )</th>
<th>( 1.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( \text{L}^N )</td>
<td>( \text{L}_A^N )</td>
<td>( \text{L}^N )</td>
<td>( \text{L}_A^N )</td>
<td>( \text{L}^N )</td>
</tr>
<tr>
<td>30.0</td>
<td>12.733</td>
<td>3.820</td>
<td>16.689</td>
<td>4.807</td>
<td>20.727</td>
</tr>
<tr>
<td>40.0</td>
<td>11.123</td>
<td>3.358</td>
<td>14.570</td>
<td>4.216</td>
<td>18.086</td>
</tr>
<tr>
<td>60.0</td>
<td>7.044</td>
<td>2.339</td>
<td>9.132</td>
<td>2.858</td>
<td>11.263</td>
</tr>
<tr>
<td>80.0</td>
<td>2.215</td>
<td>1.083</td>
<td>2.717</td>
<td>1.208</td>
<td>3.229</td>
</tr>
</tbody>
</table>
Table B-2. Reflected Solar Spectral Radiance (Continued)

\[ \lambda = 0.700 \ \mu m, \quad E_{\lambda 0} = 140.900 \ \text{mW/cm}^2\cdot\mu\text{m}, \quad \delta_{TN} = 0.26 \]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \theta_z )</th>
<th>( L_N )</th>
<th>( L_A N )</th>
<th>( L_N )</th>
<th>( L_A )</th>
<th>( L_N )</th>
<th>( L_A )</th>
<th>( L_N )</th>
<th>( L_A )</th>
<th>( L_N )</th>
<th>( L_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.201</td>
<td>1.201</td>
<td>1.599</td>
<td>1.273</td>
<td>3.202</td>
<td>1.571</td>
<td>5.221</td>
<td>1.959</td>
<td>9.313</td>
<td>2.789</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>1.151</td>
<td>1.151</td>
<td>1.543</td>
<td>1.222</td>
<td>3.118</td>
<td>1.515</td>
<td>5.103</td>
<td>1.897</td>
<td>9.125</td>
<td>2.713</td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>1.140</td>
<td>1.140</td>
<td>1.513</td>
<td>1.208</td>
<td>3.010</td>
<td>1.486</td>
<td>4.896</td>
<td>1.849</td>
<td>8.718</td>
<td>2.625</td>
<td></td>
</tr>
<tr>
<td>30.0</td>
<td>1.018</td>
<td>1.018</td>
<td>1.358</td>
<td>1.080</td>
<td>2.728</td>
<td>1.334</td>
<td>4.453</td>
<td>1.666</td>
<td>7.949</td>
<td>2.376</td>
<td></td>
</tr>
<tr>
<td>40.0</td>
<td>0.907</td>
<td>0.907</td>
<td>1.205</td>
<td>0.962</td>
<td>2.400</td>
<td>1.184</td>
<td>3.905</td>
<td>1.473</td>
<td>6.957</td>
<td>2.093</td>
<td></td>
</tr>
<tr>
<td>50.0</td>
<td>0.836</td>
<td>0.836</td>
<td>1.080</td>
<td>0.880</td>
<td>2.059</td>
<td>1.062</td>
<td>3.293</td>
<td>1.299</td>
<td>5.793</td>
<td>1.805</td>
<td></td>
</tr>
<tr>
<td>60.0</td>
<td>0.790</td>
<td>0.790</td>
<td>0.972</td>
<td>0.823</td>
<td>1.702</td>
<td>0.959</td>
<td>2.621</td>
<td>1.136</td>
<td>4.484</td>
<td>1.513</td>
<td></td>
</tr>
<tr>
<td>70.0</td>
<td>0.749</td>
<td>0.749</td>
<td>0.862</td>
<td>0.769</td>
<td>1.314</td>
<td>0.851</td>
<td>1.885</td>
<td>0.957</td>
<td>3.040</td>
<td>1.185</td>
<td></td>
</tr>
<tr>
<td>80.0</td>
<td>0.623</td>
<td>0.623</td>
<td>0.669</td>
<td>0.632</td>
<td>0.850</td>
<td>0.666</td>
<td>1.079</td>
<td>0.711</td>
<td>1.543</td>
<td>0.807</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \theta_z )</th>
<th>( L_N )</th>
<th>( L_A N )</th>
<th>( L_N )</th>
<th>( L_A )</th>
<th>( L_N )</th>
<th>( L_A )</th>
<th>( L_N )</th>
<th>( L_A )</th>
<th>( L_N )</th>
<th>( L_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>13.479</td>
<td>3.692</td>
<td>17.719</td>
<td>4.671</td>
<td>22.037</td>
<td>5.726</td>
<td>33.184</td>
<td>8.718</td>
<td>44.864</td>
<td>12.242</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>13.220</td>
<td>3.601</td>
<td>17.388</td>
<td>4.563</td>
<td>21.633</td>
<td>5.601</td>
<td>32.590</td>
<td>8.542</td>
<td>44.071</td>
<td>12.007</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>12.609</td>
<td>3.469</td>
<td>16.571</td>
<td>4.383</td>
<td>20.604</td>
<td>5.370</td>
<td>31.017</td>
<td>8.166</td>
<td>41.927</td>
<td>11.459</td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>8.338</td>
<td>2.357</td>
<td>10.929</td>
<td>2.954</td>
<td>13.568</td>
<td>5.598</td>
<td>20.379</td>
<td>5.425</td>
<td>27.515</td>
<td>7.577</td>
<td></td>
</tr>
<tr>
<td>0.60</td>
<td>6.380</td>
<td>1.924</td>
<td>8.311</td>
<td>2.370</td>
<td>10.277</td>
<td>2.850</td>
<td>15.352</td>
<td>4.213</td>
<td>20.670</td>
<td>5.817</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>4.216</td>
<td>1.434</td>
<td>5.413</td>
<td>1.704</td>
<td>6.632</td>
<td>1.995</td>
<td>9.779</td>
<td>2.824</td>
<td>13.077</td>
<td>3.803</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>2.015</td>
<td>0.912</td>
<td>2.496</td>
<td>1.024</td>
<td>2.986</td>
<td>1.146</td>
<td>4.250</td>
<td>1.490</td>
<td>5.574</td>
<td>1.894</td>
<td></td>
</tr>
</tbody>
</table>
Table B-2. Reflected Solar Spectral Radiance (Continued)

\( \lambda = 0.740 \, \mu m, \quad E_{\lambda_0} = 128.300 \, \text{mW/cm}^2\text{-}\mu m, \quad \delta_{TN} = 0.24 \)

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.00</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( L^N )</td>
<td>( L^N_A )</td>
<td>( L^N )</td>
<td>( L^N_A )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>0.956</td>
<td>0.956</td>
<td>1.322</td>
<td>1.019</td>
<td>2.793</td>
</tr>
<tr>
<td>10.0</td>
<td>0.913</td>
<td>0.913</td>
<td>1.273</td>
<td>0.975</td>
<td>2.719</td>
</tr>
<tr>
<td>20.0</td>
<td>0.907</td>
<td>0.907</td>
<td>1.249</td>
<td>0.966</td>
<td>2.623</td>
</tr>
<tr>
<td>30.0</td>
<td>0.804</td>
<td>0.804</td>
<td>1.118</td>
<td>0.858</td>
<td>2.375</td>
</tr>
<tr>
<td>40.0</td>
<td>0.714</td>
<td>0.714</td>
<td>0.987</td>
<td>0.761</td>
<td>2.086</td>
</tr>
<tr>
<td>50.0</td>
<td>0.658</td>
<td>0.658</td>
<td>0.882</td>
<td>0.697</td>
<td>1.784</td>
</tr>
<tr>
<td>60.0</td>
<td>0.625</td>
<td>0.625</td>
<td>0.793</td>
<td>0.654</td>
<td>1.466</td>
</tr>
<tr>
<td>70.0</td>
<td>0.599</td>
<td>0.599</td>
<td>0.704</td>
<td>0.617</td>
<td>1.126</td>
</tr>
<tr>
<td>80.0</td>
<td>0.508</td>
<td>0.508</td>
<td>0.550</td>
<td>0.515</td>
<td>0.718</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( L^N )</td>
<td>( L^N_A )</td>
<td>( L^N )</td>
<td>( L^N_A )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>12.206</td>
<td>3.110</td>
<td>16.079</td>
<td>3.952</td>
<td>20.018</td>
</tr>
<tr>
<td>40.0</td>
<td>9.113</td>
<td>2.322</td>
<td>12.005</td>
<td>2.950</td>
<td>14.946</td>
</tr>
<tr>
<td>60.0</td>
<td>5.774</td>
<td>1.612</td>
<td>7.547</td>
<td>1.998</td>
<td>9.349</td>
</tr>
<tr>
<td>70.0</td>
<td>3.825</td>
<td>1.217</td>
<td>4.936</td>
<td>1.458</td>
<td>6.065</td>
</tr>
<tr>
<td>80.0</td>
<td>1.793</td>
<td>0.753</td>
<td>2.236</td>
<td>0.849</td>
<td>2.686</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\theta_z$</td>
<td>$L_N$</td>
<td>$L_N^A$</td>
<td>$L_N$</td>
<td>$L_N^A$</td>
</tr>
<tr>
<td>-------</td>
<td>-----------</td>
<td>-------</td>
<td>---------</td>
<td>-------</td>
<td>---------</td>
</tr>
<tr>
<td>0.0</td>
<td>0.591</td>
<td>0.591</td>
<td>0.878</td>
<td>0.635</td>
<td>2.028</td>
</tr>
<tr>
<td>10.0</td>
<td>0.562</td>
<td>0.562</td>
<td>0.843</td>
<td>0.605</td>
<td>1.973</td>
</tr>
<tr>
<td>20.0</td>
<td>0.560</td>
<td>0.560</td>
<td>0.827</td>
<td>0.601</td>
<td>1.899</td>
</tr>
<tr>
<td>30.0</td>
<td>0.489</td>
<td>0.489</td>
<td>0.733</td>
<td>0.527</td>
<td>1.711</td>
</tr>
<tr>
<td>40.0</td>
<td>0.428</td>
<td>0.428</td>
<td>0.639</td>
<td>0.460</td>
<td>1.489</td>
</tr>
<tr>
<td>50.0</td>
<td>0.390</td>
<td>0.390</td>
<td>0.562</td>
<td>0.416</td>
<td>1.254</td>
</tr>
<tr>
<td>60.0</td>
<td>0.367</td>
<td>0.367</td>
<td>0.494</td>
<td>0.386</td>
<td>1.003</td>
</tr>
<tr>
<td>70.0</td>
<td>0.345</td>
<td>0.345</td>
<td>0.422</td>
<td>0.357</td>
<td>0.731</td>
</tr>
<tr>
<td>80.0</td>
<td>0.271</td>
<td>0.271</td>
<td>0.299</td>
<td>0.275</td>
<td>0.410</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\theta_z$</th>
<th>$L_N$</th>
<th>$L_N^A$</th>
<th>$L_N$</th>
<th>$L_N^A$</th>
<th>$L_N$</th>
<th>$L_N^A$</th>
<th>$L_N$</th>
<th>$L_N^A$</th>
<th>$L_N$</th>
<th>$L_N^A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>9.359</td>
<td>2.087</td>
<td>12.363</td>
<td>2.668</td>
<td>15.410</td>
<td>3.291</td>
<td>23.219</td>
<td>5.040</td>
<td>31.314</td>
<td>7.075</td>
<td>0.30</td>
</tr>
<tr>
<td>30.0</td>
<td>7.945</td>
<td>1.760</td>
<td>10.500</td>
<td>2.253</td>
<td>13.092</td>
<td>2.782</td>
<td>19.733</td>
<td>4.269</td>
<td>26.617</td>
<td>5.998</td>
<td>0.60</td>
</tr>
<tr>
<td>40.0</td>
<td>6.909</td>
<td>1.532</td>
<td>9.130</td>
<td>1.960</td>
<td>11.382</td>
<td>2.420</td>
<td>17.155</td>
<td>3.712</td>
<td>23.178</td>
<td>5.215</td>
<td>0.70</td>
</tr>
<tr>
<td>50.0</td>
<td>5.668</td>
<td>1.290</td>
<td>7.477</td>
<td>1.639</td>
<td>9.311</td>
<td>2.014</td>
<td>14.013</td>
<td>3.067</td>
<td>18.686</td>
<td>4.891</td>
<td>0.80</td>
</tr>
<tr>
<td>60.0</td>
<td>4.251</td>
<td>1.028</td>
<td>5.582</td>
<td>1.285</td>
<td>6.931</td>
<td>1.561</td>
<td>10.390</td>
<td>2.335</td>
<td>13.976</td>
<td>3.235</td>
<td>0.90</td>
</tr>
<tr>
<td>70.0</td>
<td>2.704</td>
<td>0.745</td>
<td>3.513</td>
<td>0.901</td>
<td>4.333</td>
<td>1.068</td>
<td>6.434</td>
<td>1.537</td>
<td>8.613</td>
<td>2.082</td>
<td>1.00</td>
</tr>
<tr>
<td>80.0</td>
<td>1.121</td>
<td>0.415</td>
<td>1.412</td>
<td>0.471</td>
<td>1.707</td>
<td>0.531</td>
<td>2.464</td>
<td>0.699</td>
<td>3.249</td>
<td>0.895</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table B-2. Reflected Solar Spectral Radiance (Continued)

$\lambda = 0.820 \ \mu m$, 
$E_{\lambda_0} = 107.500 \ \text{mW/cm}^2\cdot \mu \text{m}$, 
$\delta_{TN} = 0.25$
Table B-2. Reflected Solar Spectral Radiance (Continued)

\( \lambda = 0.880 \, \mu m, \quad E_{\lambda_0} = 96.300 \, mW/cm^2-\mu m, \quad \delta_{TN} = 0.19 \)

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.00</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( L^N )</td>
<td>( L^N_A )</td>
<td>( L^N )</td>
<td>( L^N_A )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>0.524</td>
<td>0.524</td>
<td>0.810</td>
<td>0.565</td>
<td>1.959</td>
</tr>
<tr>
<td>10.0</td>
<td>0.497</td>
<td>0.497</td>
<td>0.778</td>
<td>0.537</td>
<td>1.908</td>
</tr>
<tr>
<td>20.0</td>
<td>0.497</td>
<td>0.497</td>
<td>0.765</td>
<td>0.536</td>
<td>1.841</td>
</tr>
<tr>
<td>30.0</td>
<td>0.435</td>
<td>0.435</td>
<td>0.680</td>
<td>0.470</td>
<td>1.666</td>
</tr>
<tr>
<td>40.0</td>
<td>0.382</td>
<td>0.382</td>
<td>0.597</td>
<td>0.413</td>
<td>1.461</td>
</tr>
<tr>
<td>50.0</td>
<td>0.352</td>
<td>0.352</td>
<td>0.530</td>
<td>0.378</td>
<td>1.244</td>
</tr>
<tr>
<td>60.0</td>
<td>0.341</td>
<td>0.341</td>
<td>0.475</td>
<td>0.360</td>
<td>1.014</td>
</tr>
<tr>
<td>70.0</td>
<td>0.338</td>
<td>0.338</td>
<td>0.424</td>
<td>0.350</td>
<td>0.769</td>
</tr>
<tr>
<td>80.0</td>
<td>0.310</td>
<td>0.310</td>
<td>0.346</td>
<td>0.315</td>
<td>0.492</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( L^N )</td>
<td>( L^N_A )</td>
<td>( L^N )</td>
<td>( L^N_A )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>60.0</td>
<td>4.444</td>
<td>0.994</td>
<td>5.847</td>
<td>1.247</td>
<td>7.269</td>
</tr>
<tr>
<td>70.0</td>
<td>2.966</td>
<td>0.756</td>
<td>3.864</td>
<td>0.918</td>
<td>4.775</td>
</tr>
<tr>
<td>80.0</td>
<td>1.416</td>
<td>0.487</td>
<td>1.794</td>
<td>0.555</td>
<td>2.177</td>
</tr>
</tbody>
</table>
Table B-2. Reflected Solar Spectral Radiance (Continued)

\( \lambda = 1.050 \, \mu m, \quad E_{\lambda_0} = 63.300 \, mW/cm^2-\mu m, \quad \delta_{TN} = 0.15 \)

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \theta_z )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
<th>( L^N )</th>
<th>( L_A^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00</td>
<td>0.01</td>
<td>0.05</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.50</td>
<td>0.75</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.263</td>
<td>0.263</td>
<td>0.465</td>
<td>0.287</td>
<td>1.273</td>
<td>0.386</td>
<td>2.289</td>
<td>0.514</td>
<td>4.336</td>
<td>0.786</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.247</td>
<td>0.247</td>
<td>0.446</td>
<td>0.271</td>
<td>1.241</td>
<td>0.368</td>
<td>2.240</td>
<td>0.494</td>
<td>4.254</td>
<td>0.762</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>0.249</td>
<td>0.249</td>
<td>0.438</td>
<td>0.272</td>
<td>1.196</td>
<td>0.365</td>
<td>2.148</td>
<td>0.485</td>
<td>4.066</td>
<td>0.740</td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>0.214</td>
<td>0.214</td>
<td>0.388</td>
<td>0.235</td>
<td>1.083</td>
<td>0.320</td>
<td>1.957</td>
<td>0.430</td>
<td>3.718</td>
<td>0.664</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.186</td>
<td>0.186</td>
<td>0.338</td>
<td>0.204</td>
<td>0.949</td>
<td>0.279</td>
<td>1.717</td>
<td>0.375</td>
<td>3.264</td>
<td>0.581</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.172</td>
<td>0.172</td>
<td>0.298</td>
<td>0.187</td>
<td>0.805</td>
<td>0.249</td>
<td>1.441</td>
<td>0.329</td>
<td>2.724</td>
<td>0.499</td>
<td></td>
</tr>
<tr>
<td>0.60</td>
<td>0.168</td>
<td>0.168</td>
<td>0.264</td>
<td>0.180</td>
<td>0.649</td>
<td>0.227</td>
<td>1.133</td>
<td>0.288</td>
<td>2.109</td>
<td>0.417</td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>0.171</td>
<td>0.171</td>
<td>0.234</td>
<td>0.179</td>
<td>0.484</td>
<td>0.209</td>
<td>0.799</td>
<td>0.249</td>
<td>1.432</td>
<td>0.333</td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>0.166</td>
<td>0.166</td>
<td>0.194</td>
<td>0.170</td>
<td>0.303</td>
<td>0.183</td>
<td>0.440</td>
<td>0.200</td>
<td>0.716</td>
<td>0.237</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_z )</td>
<td>( L^N )</td>
<td>( L_A^N )</td>
<td>( L^N )</td>
<td>( L_A^N )</td>
<td>( L^N )</td>
</tr>
<tr>
<td>0.10</td>
<td>6.288</td>
<td>1.050</td>
<td>8.344</td>
<td>1.360</td>
<td>10.422</td>
</tr>
<tr>
<td>0.20</td>
<td>6.003</td>
<td>1.015</td>
<td>7.962</td>
<td>1.311</td>
<td>9.940</td>
</tr>
<tr>
<td>0.30</td>
<td>5.497</td>
<td>0.917</td>
<td>7.295</td>
<td>1.188</td>
<td>9.111</td>
</tr>
<tr>
<td>0.40</td>
<td>4.828</td>
<td>0.803</td>
<td>6.407</td>
<td>1.041</td>
<td>8.004</td>
</tr>
<tr>
<td>0.50</td>
<td>4.020</td>
<td>0.683</td>
<td>5.330</td>
<td>0.880</td>
<td>6.654</td>
</tr>
<tr>
<td>0.60</td>
<td>3.094</td>
<td>0.557</td>
<td>4.090</td>
<td>0.708</td>
<td>5.097</td>
</tr>
<tr>
<td>0.70</td>
<td>2.073</td>
<td>0.424</td>
<td>2.720</td>
<td>0.522</td>
<td>3.374</td>
</tr>
<tr>
<td>0.80</td>
<td>0.995</td>
<td>0.277</td>
<td>1.277</td>
<td>0.320</td>
<td>1.562</td>
</tr>
</tbody>
</table>
Table B-2. Reflected Solar Spectral Radiance (Continued)

<table>
<thead>
<tr>
<th>( \lambda = 1.250 , \mu \text{m} )</th>
<th>( \theta )</th>
<th>( L_{\lambda} )</th>
<th>( L^*_{\lambda} )</th>
<th>( L^N_{\lambda} )</th>
<th>( L^A_{\lambda} )</th>
<th>( L^N_{\lambda} )</th>
<th>( L^A_{\lambda} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.139</td>
<td>0.279</td>
<td>0.840</td>
<td>1.544</td>
<td>0.285</td>
<td>0.614</td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.130</td>
<td>0.267</td>
<td>0.820</td>
<td>1.512</td>
<td>0.274</td>
<td>0.609</td>
</tr>
<tr>
<td>0.05</td>
<td>0.05</td>
<td>0.135</td>
<td>0.263</td>
<td>0.788</td>
<td>1.448</td>
<td>0.287</td>
<td>0.601</td>
</tr>
<tr>
<td>0.10</td>
<td>0.10</td>
<td>0.127</td>
<td>0.251</td>
<td>0.763</td>
<td>1.386</td>
<td>0.280</td>
<td>0.597</td>
</tr>
<tr>
<td>0.20</td>
<td>0.20</td>
<td>0.120</td>
<td>0.240</td>
<td>0.742</td>
<td>1.320</td>
<td>0.273</td>
<td>0.593</td>
</tr>
<tr>
<td>0.50</td>
<td>0.50</td>
<td>0.110</td>
<td>0.225</td>
<td>0.700</td>
<td>1.260</td>
<td>0.266</td>
<td>0.589</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>0.100</td>
<td>0.212</td>
<td>0.657</td>
<td>1.200</td>
<td>0.260</td>
<td>0.585</td>
</tr>
</tbody>
</table>

\( E_{\lambda/o} = 46.400 \, \text{mW/cm}^2-\mu\text{m} \)

\( \delta_{TN} = 0.13 \)
Table B-2. Reflected Solar Spectral Radiance (Continued)

\[ \lambda = 1.600 \, \mu\text{m}, \quad E_{\lambda_0} = 24.900 \, \text{mW/cm}^2-\mu\text{m}, \quad \delta_{TN} = 0.10 \]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \theta_z )</th>
<th>0.00</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( L_N^N )</td>
<td>( L_A^N )</td>
<td>( L_N^N )</td>
<td>( L_A^N )</td>
<td>( L_N^N )</td>
<td>( L_A^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>0.054</td>
<td>0.054</td>
<td>0.130</td>
<td>0.060</td>
<td>0.436</td>
<td>0.084</td>
</tr>
<tr>
<td>10.0</td>
<td>0.050</td>
<td>0.050</td>
<td>0.125</td>
<td>0.056</td>
<td>0.426</td>
<td>0.080</td>
</tr>
<tr>
<td>20.0</td>
<td>0.051</td>
<td>0.051</td>
<td>0.122</td>
<td>0.056</td>
<td>0.409</td>
<td>0.080</td>
</tr>
<tr>
<td>30.0</td>
<td>0.042</td>
<td>0.042</td>
<td>0.108</td>
<td>0.048</td>
<td>0.372</td>
<td>0.069</td>
</tr>
<tr>
<td>40.0</td>
<td>0.036</td>
<td>0.036</td>
<td>0.094</td>
<td>0.040</td>
<td>0.326</td>
<td>0.059</td>
</tr>
<tr>
<td>50.0</td>
<td>0.033</td>
<td>0.033</td>
<td>0.081</td>
<td>0.036</td>
<td>0.274</td>
<td>0.052</td>
</tr>
<tr>
<td>60.0</td>
<td>0.032</td>
<td>0.032</td>
<td>0.069</td>
<td>0.035</td>
<td>0.217</td>
<td>0.047</td>
</tr>
<tr>
<td>70.0</td>
<td>0.034</td>
<td>0.034</td>
<td>0.058</td>
<td>0.036</td>
<td>0.155</td>
<td>0.044</td>
</tr>
<tr>
<td>80.0</td>
<td>0.035</td>
<td>0.035</td>
<td>0.046</td>
<td>0.036</td>
<td>0.089</td>
<td>0.039</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( L_N^N )</td>
<td>( L_A^N )</td>
<td>( L_N^N )</td>
<td>( L_A^N )</td>
<td>( L_N^N )</td>
</tr>
<tr>
<td>0.0</td>
<td>2.367</td>
<td>0.257</td>
<td>3.149</td>
<td>0.335</td>
<td>3.936</td>
</tr>
<tr>
<td>10.0</td>
<td>2.327</td>
<td>0.250</td>
<td>3.095</td>
<td>0.326</td>
<td>3.870</td>
</tr>
<tr>
<td>20.0</td>
<td>2.220</td>
<td>0.241</td>
<td>2.953</td>
<td>0.314</td>
<td>3.690</td>
</tr>
<tr>
<td>30.0</td>
<td>2.036</td>
<td>0.217</td>
<td>2.709</td>
<td>0.285</td>
<td>3.387</td>
</tr>
<tr>
<td>40.0</td>
<td>1.790</td>
<td>0.190</td>
<td>2.382</td>
<td>0.249</td>
<td>2.979</td>
</tr>
<tr>
<td>50.0</td>
<td>1.492</td>
<td>0.161</td>
<td>1.984</td>
<td>0.210</td>
<td>2.480</td>
</tr>
<tr>
<td>60.0</td>
<td>1.148</td>
<td>0.130</td>
<td>1.524</td>
<td>0.168</td>
<td>1.903</td>
</tr>
<tr>
<td>70.0</td>
<td>0.767</td>
<td>0.098</td>
<td>1.014</td>
<td>0.123</td>
<td>1.263</td>
</tr>
<tr>
<td>80.0</td>
<td>0.363</td>
<td>0.064</td>
<td>0.474</td>
<td>0.075</td>
<td>0.586</td>
</tr>
</tbody>
</table>
Table B-2: Reflected Solar Spectral Radiance (Continued)

<table>
<thead>
<tr>
<th>( \lambda = 2-200 \mu m )</th>
<th>( \delta_{TN} = 0.13 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{No} = 8,300 \text{ mW/cm}^2 \mu m )</td>
<td>( L_{A} )</td>
</tr>
<tr>
<td>( \theta_{z} )</td>
<td>( L_{A}^{N} )</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00011</td>
</tr>
<tr>
<td>0.01</td>
<td>0.00010</td>
</tr>
<tr>
<td>0.02</td>
<td>0.00009</td>
</tr>
<tr>
<td>0.03</td>
<td>0.00008</td>
</tr>
<tr>
<td>0.04</td>
<td>0.00007</td>
</tr>
<tr>
<td>0.05</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.06</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.07</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.08</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.09</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.10</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.11</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.12</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.13</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.14</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.15</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.16</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.17</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.18</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.19</td>
<td>0.00006</td>
</tr>
<tr>
<td>0.20</td>
<td>0.00006</td>
</tr>
</tbody>
</table>

B-19
APPENDIX C

SENSOR FIELD OF VIEW

Each time the satellite completes an orbit around the Earth, it maps out a swath on the Earth's surface of width $S_w$. The angle subtended at the satellite by $S_w$ is the sensor field of view $\Theta$. Referring to Figure 1 and using the law of cosines, one can write

$$\Theta = 2 \cos^{-1}\left(\frac{R_S^2 + S_d^2 - R_e^2}{2S_d R_S}\right) \text{ [rad]}, \quad (C-1)$$

where

$$R_S = H + R_e \text{ [km]} \quad (C-2)$$

and

$$S_d = \left[R_S^2 + R_e^2 - 2R_SR_e \cos\left(\frac{\Phi_S}{2}\right)\right]^{1/2} \text{ [km]} \quad (C-3)$$

From Figure 1, it can also be seen that

$$\Phi_S = \frac{S_w}{R_e} \text{ [rad]} \quad (C-4)$$

The swath width $S_w$ is given by

$$S_w = \frac{A_d C}{N_S} \text{ [km]} \quad (C-5)$$

where $d_C$ is the cross-track distance covered by the sensor and, for total coverage at the equator, is given by

$$d_C = 2\pi R_e \text{ [km]} \quad (C-6)$$

Now, if the total time required to map the Earth $t_{MAP}$ is known, then the total number of swaths $N_S$ required to map the Earth is

$$N_S = \frac{t_{MAP}}{t_S} \text{ [nd]} \quad (C-7)$$

where $t_S$ is the satellite's orbital period or the time required to map one swath and is given by

C-1
Finally, the cross-track overlap factor $A$ is given by,

$$A = \left( 1 + \frac{S'_o}{100} \right) \text{ [nd]} ,$$

where $S'_o$ is the percentage overlap across the ground track.
APPENDIX D

VIEW FACTOR FOR A SINGLE DETECTOR VIEWING A CIRCULAR BACKGROUND

The view factor $F_C$ for a detector that is receiving radiation from a circular background can be defined as

$$F_C = \frac{\Phi'}{\pi A_D L'_{\Delta \lambda}} \ [\text{nd}] , \quad (D-1)$$

where $\Phi'$ = the photon flux into the detector [p/sec],

$A_D$ = the area of the detector [cm$^2$],

and

$$L'_{\Delta \lambda} = \int_0^{\lambda_c} B'(\lambda, T_{BG}) \ d\lambda \ [\text{p/sec-cm}^2\text{-sr}] , \quad (D-2)$$

where $B'(\lambda, T_{BG}) = $ Planck's function evaluated at the background temperature $T_{BG}$, and

$\lambda_c$ = the detector cutoff wavelength.

If a detector of area $A_D$ views a source of radiance $L'_{\Delta \lambda}$ with area $dA'$ through a solid angle $\Omega$, the flux into the detector is

$$d\Phi' = L'_{\Delta \lambda} \Omega \ Cos(\phi'_c) \ dA' \ [\text{p/sec}] . \quad (D-3)$$

The angle $\phi'_c$ is shown in Figure D-1.

However,

$$dA' = 2\pi r' \ dr' \ [\text{cm}^2] \quad (D-4)$$

and

$$\Omega = \frac{A_D \ Cos(\phi'_c)}{S^2} \ [\text{sr}] . \quad (D-5)$$

Substituting Equations (D-4) and (D-5) into Equation (D-3) gives
a = Circular background radius (cm)
$A_d$ = Area of detector ($\mu$m)
d = Distance between the detector and the circular background (cm)
$\text{d}A' = 2\pi r' dr'$ = Area of elemental annular ring of radius $r'$ ($cm^2$)
$S$ = Distance from center of detector to elemental area on circular background (cm)
$\phi_c$ = Half angle subtended by the circular background at the center of the detector (deg)
$\phi_{c'}$ = Half angle subtended by the elemental circular area $dA'$ at the center of the detector (deg)
$\text{d}\Omega$ = Elemental solid angle subtended by the detector at a point on the elemental area $dA'$ (sr)

Figure D-1. View Factor Geometry for a Single Detector Viewing a Circular Background
\[ d\Phi' = \frac{2\pi r A_D L' \Delta_L \cos^2(\phi_c') dr'}{S^2} \quad [\text{p/sec}] \]  

(D-6)

However, from Figure D-1 we see that

\[ \cos \phi_c' = \frac{d}{S} = \frac{d}{(d^2 + r'^2)^{1/2}} \quad [\text{rad}] \]  

(D-7)

and substituting Equation (D-7) into Equation (D-6) gives

\[ d\Phi' = 2\pi d^2 A_D L' \Delta_L \frac{r' dr'}{(d^2 + r'^2)^2} \quad [\text{p/sec}] \]  

(D-8)

It follows that

\[ \Phi' = 2\pi d^2 A_D L' \Delta_L \int_0^a \frac{r' dr'}{(d^2 + r'^2)^2} \quad [\text{p/sec}] \]  

(D-9)

However,

\[ \int_0^a \frac{r' dr'}{(d^2 + r'^2)^2} = \frac{1}{2} - \frac{1}{2(d^2 + a^2)} \]  

(D-10)

Therefore, substituting Equation (D-10) into Equation (D-9) gives

\[ \Phi' = \pi A_D L' \Delta_L \left( 1 - \frac{d^2}{d^2 + a^2} \right) \quad [\text{p/sec}] \]  

(D-11)

However,

\[ \frac{d}{(d^2 + a^2)^{1/2}} = \frac{d}{S_a} = \cos \phi_c \quad [\text{nd}] \]  

(D-12)

Therefore, by substituting Equation (D-12) into Equation (D-11) we obtain

\[ \Phi' = \pi A_D L' \Delta_L \sin^2 \phi_c \quad [\text{p/sec}] \]  

(D-13)
APPENDIX E

VIEW FACTOR FOR A DETECTOR IN AN n X m ARRAY VIEWING
A RECTANGULAR BACKGROUND

The view factor $F_A$ for a detector in an $n$ by $m$ array viewing a rectangular background (Figure E-1) is given by

$$F_A = \frac{\Phi'}{\pi A_D L'_\Delta \lambda}$$  \hspace{1cm} (E-1)

where $\Phi'$ is the total number of photons per second entering the detector from the background through a rectangular aperture, given by

$$\Phi' = L'_\Delta \lambda \int_0^{A_B} \Omega_D \cos \phi_f \, dA_B \quad [p/sec] \hspace{1cm} (E-2)$$

and, as in Appendix D, $L'_\Delta \lambda$ is

$$L'_\Delta \lambda = \int_0^{\lambda_c} B'(\lambda,T_{BG}) \, d\lambda \quad [p/sec-cm^2-sr]$$

where the photon radiance $L'_\Delta \lambda$ is given by Equation (E-2) and

where $B'(\lambda,T_{BG}) = \text{the Planck function evaluated at the background temperature } T_{BG}$

$[p/sec-cm^2-sr-\mu m]$;

$\Omega_D = \text{the solid angle subtended by the detector at an arbitrary point on the rectangular aperture through which the background is viewed}$;

$dA_B = \text{the differential area on the background}$; and

$\phi_f = \text{the angle between the normal to the differential area } dA_B \text{ and the line between the center of the detector and the center of the differential area } dA_B$.

If $D$ is the distance between the center of the detector and the differential area $dA_B$ then
Figure E-1. Geometry for a Detector in n x m Array Viewing a Rectangular Background
\[
\Omega_d = \frac{A_d \cos(\phi_f)}{D^2} \qquad [\text{sr}],
\]  

(E-3)

where

\[
\cos(\phi_f) = \frac{Z}{D} \qquad [\text{nd}].
\]  

(E-4)

To evaluate the quantities \(D\), \(\phi_f\), and \(z\) in Equations E-3 and E-4, it is convenient to define two sets of coordinates. The two systems \((x,y,z)\) and \((x',y',z')\) are shown in Figure E-1. The \(n \times m\) detector array lies in the \(x\)-\(y\) plane and the plane of the rectangular aperture, through which the background is viewed, is a distance \(H_f\) from the \(x\)-\(y\) plane.

Consider an \(n \times m\) array of detectors, and let \(p\) be the pitch (distance between detector centers) in both directions (Figure E-1). Let the detector array be symmetrically surrounded by very cold sides (fence) of length \(L_f\), width \(W_f\), and height \(H_f\). The length and width are adjusted for a given height so that the edge detectors in the array can just accommodate the optical bundle which is defined by half the bundle cone angle \(\theta\). The relationships for the length \(L_f\) and width \(W_f\) of the fenced area that surrounds the detector array are

\[
L_f = np + 2H_f \tan(\theta) \quad [\text{m}],
\]  

(E-5)

and

\[
W_f = mp + 2H_f \tan(\theta) \quad [\text{m}],
\]  

(E-6)

where \(n = \) the number of detectors along the length of the detector array, and

\(m = \) the number of detectors along the width of the detector array.

Equations (E-5) and (E-6) follow because

\[
\sin(\theta) = \frac{1}{2f_n}. \quad \text{(E-7)}
\]

The coordinates of detector \(i, j\) with respect to the edges of the fenced area are given by
\[ x_d = H_f \tan(\theta) - \frac{p}{2} + ip \]  
(E-8)

and

\[ y_d = H_f \tan(\theta) - \frac{p}{2} + jp \]  
(E-9)

where \( i = \) the number of detectors along the fence length, \((i = 1, ..., n)\)
and \( j = \) the number of detectors along the fence width, \((j = 1, ..., m)\).

The coordinates of the arbitrarily placed differential area \( dA_B \) on the surface formed by the top edges of the fence are \((x,y,z)\). The coordinates with respect to the center of the detector \( i, j \) are given by

\[ x' = x - x_d \]  
(E-10)

\[ y' = y - y_d \]  
(E-11)

and

\[ z' = z \]  
(E-12)

where

\[ z = H_f \]  
(E-13)

and

\[ D = (x'^2 + y'^2 + z'^2)^{1/2} \]  
(E-14)

By substituting Equations (E-10) through (E-13) into Equation (E-14), one gets

\[ D = \left[ \left( x - x_d \right)^2 + \left( y - y_d \right)^2 + z^2 \right]^{1/2} \]  
(E-15)

Letting

\[ dA_b = dx \, dy \]  
(E-16)

and substituting Equations (E-3), (E-4), (E-13), and (E-16) into Equation (E-2) results in

\[ \Phi' = A_D H_f^2 B' \Delta \lambda \int_0^{W_f} \int_0^{L_f} \frac{dx \, dy}{D^4} \]  
[p/sec]  
(E-17)

E-4
Substituting Equation (E-17) into Equation (E-1) yields

\[ F_A = \frac{H_f^2}{\pi} \int_0^{W_f} g(y) \, dy , \tag{E-18} \]

where

\[ g(y) = \int_0^{L_f} \frac{dx}{D^4} . \tag{E-19} \]

Equation (E-19) may also be written as

\[ g(y) = \int_0^{L_f} \frac{dx}{X^2} , \tag{E-20} \]

where

\[ X = ax^2 + bx + c \tag{E-21} \]

\[ a = 1 \tag{E-22} \]

\[ b = -2x_d \tag{E-23} \]

and

\[ c = x_0^2 + (y - y_d)^2 + z^2 . \tag{E-24} \]

It follows from Equations (E-22) through (E-24) that

\[ 4ac - b^2 = 4(y - y_d)^2 + 4z^2 . \tag{E-25} \]

Now, Equation (E-25) reveals that

\[ 4ac > b^2 . \tag{E-26} \]

Therefore, Equation (E-20) may be written (Dwight, 1947, p. 33, Equations 160.01 and 160.02) as
$$\int \frac{dx}{x^2} = \frac{2ax + b}{r} + \frac{4a}{R_2} \tan^{-1} \left( \frac{2ax + b}{R_1} \right), \quad (E-27)$$

where

$$r = 4ac - b^2 \quad (E-28)$$

and

$$R_1 = r^{1/2} = \left( 4ac - b^2 \right)^{1/2} \quad (E-29)$$

and

$$R_2 = rR_1 = r^{3/2} \quad (E-30)$$

Applying Equation (E-27) to Equation (E-28), one gets

$$g(y) = \frac{2aL_f + b}{r \left( aL_f^2 + bL_f + c \right)} + \frac{4a}{R_2} \tan^{-1} \left( \frac{2aL_f + b}{R_1} \right)$$

$$- \frac{b}{r c} - \frac{4a}{R_2} \tan^{-1} \left( \frac{b}{R_1} \right) \quad (E-31)$$

Use of Equation (E-31) in Equation (E-18) and numerical integration enable the form factor $F_A$ to be computed.
APPENDIX F

COMPUTATION OF $\gamma$

Substituting Equations (2-11) and (2-12) into Equation (2-9) yields

$$S' = t_1 \tau_0 A_D \eta \left( \frac{\lambda}{hc} \right) \left( \frac{\pi}{4f^2 N} \right) \int_{\lambda_1}^{\lambda_2} L(\lambda) \, d\lambda \quad [e] . \quad (F-1)$$

Therefore, the signal equation for the visible and SWIR bands is given by

$$S' = t_1 \tau_0 A_D \eta \left( \frac{\lambda}{hc} \right) \left( \frac{\pi}{4f^2 N} \right) L \Delta \lambda \quad [e] \quad (F-2)$$

where we have replaced the integral in Equation (F-1) by $L \Delta \lambda$ because $L(\lambda)$ varies slowly over the spectral bandpass $\Delta \lambda$. Taking the differential of Equation (F-2) with respect to the scene radiance, one obtains

$$dS' = t_1 \tau_0 A_D \eta \left( \frac{\lambda}{hc} \right) \left( \frac{\pi}{4f^2 N} \right) dL \, \Delta \lambda \quad [e] . \quad (F-3)$$

Dividing Equation (F-2) by (F-3) gives

$$\frac{S}{N} = \frac{S'}{dS'} = \frac{L}{dL} \quad [nd] \quad (F-4)$$

where the signal $S = S'$, and the noise $N = dS'$.

However,

$$dL = \left( \frac{dL}{d\rho} \right) d\rho \quad [W/cm^2-sr-\mu m] . \quad (F-5)$$

Substituting Equation (F-5) into Equation (F-4) gives

$$\frac{S}{N} = \frac{L}{\left( \frac{dL}{d\rho} \right) d\rho} \quad [nd] . \quad (F-6)$$

F-1
Letting

\[ \gamma \equiv \frac{L}{\left( \frac{dL}{d\rho} \right)} \quad \text{[nd]} \quad \text{(F-7)} \]

\[ d\rho = \text{NEA} \Delta \rho \quad \text{(F-8)} \]

and substituting Equations (F-7) and (F-8) into Equation (F-5), one obtains

\[ \frac{S}{N} = \frac{\gamma}{\text{NEA} \Delta \rho} \quad \text{(F-9)} \]

However,

\[ L = L_S^S + L_A^S \quad \text{[W/cm}^2\text{-sr-um]} \quad \text{(F-10)} \]

where the surface spectral radiance \( L_S^S \) has the functional form

\[ L_S^S = \rho K_1 \quad \text{[W/cm}^2\text{-sr-um]} \quad \text{(F-11)} \]

and the atmospheric spectral radiance is a constant with respect to \( \rho \) and is given by

\[ L_A^S = K_2 \quad \text{[W/cm}^2\text{-sr-um]} \quad \text{(F-12)} \]

The superscript \( S \) denotes that the sensor observes these spectral radiances along a slanted path. (See Figure 6.) Since the scene spectral radiances are assumed to be equal in the normal and slant directions, no superscript is used.

Substituting Equations (F-11) and (F-12) into Equation (F-10) gives

\[ L = K_1 \rho + K_2 \quad \text{[W/cm}^2\text{-sr-um]} \quad \text{(F-13)} \]

Differentiating Equation (F-13) with respect to \( \rho \) results in

\[ \frac{dL}{d\rho} = K_1 \quad \text{[W/cm}^2\text{-sr-um]} \quad \text{(F-14)} \]

Solving for \( K_1 \) in Equation (F-11) and substituting it into Equation (F-14) results in

\[ \frac{dL}{d\rho} = \frac{L_S^S}{\rho} \quad \text{[W/cm}^2\text{-sr-um]} \quad \text{(F-15)} \]
Substituting Equation (F-15) into Equation (F-7) gives

\[ \gamma = \left( \frac{L}{L_S^S} \right) \rho \quad [\text{nd}] . \]  

(F-16)

The spectral radiances \( L \) and \( L_S^S \) are observed along the line-of-sight direction with angle of \( \phi \) and a line-of-sight surface-normal angle \( \phi' \) (Figure 6). These angles are related by

\[ \phi' = \sin^{-1} \left[ \left( 1 + \frac{H}{R_c} \right) \sin \phi \right] \quad [\text{rad}] . \]  

(F-17)

The surface spectral radiance \( L_S^S \) is given by

\[ L_S^S = \frac{E_S}{\pi} \rho \tau_{AN} \sec(\phi') \quad [\text{W/cm}^2\text{-sr-\mu m}] \]  

(F-18)

where \( E_S = \) the irradiance at the surface of the Earth \([\text{W/cm}^2\text{-\mu m}]\)

The atmospheric transmission along the nadir direction is given by

\[ \tau_{AN} = e^{-\delta_o} \quad [\text{nd}] , \]  

(F-19)

where \( \delta_o \) is the optical depth along the nadir direction. (See Appendix B)

The surface spectral radiance \( L_S^N \) observed along the nadir direction (for which \( \phi' = 0 \)) is given by

\[ L_S^N = \frac{E_S}{\pi} \rho \tau_{AN} \quad [\text{W/cm}^2\text{-sr-\mu m}] . \]  

(F-20)

Dividing Equation (F-18) by Equation (F-19) gives

\[ \frac{L_S^S}{L_S^N} = \tau_{AN}^\sec(\phi' - 1) \quad [\text{W/cm}^2\text{-sr-\mu m}] . \]  

(F-21)

Substituting Equation (F-21) into Equation (F-16) we obtain

\[ \gamma = \frac{\gamma_o}{\tau_{AN}^{\sec(\phi' - 1)}} \quad [\text{nd}] \]  

(F-22)
where

$$\gamma_o = \frac{L_p}{L_S} \quad [\text{nd}] .$$  \hspace{1cm} (F-23)

We assume the total spectral radiance $L$ is the same along the nadir and slant directions; hence

$$L = L_S^N + L_A^N \quad [\text{W/cm}^2\text{-sr-um}] \quad (F-24)$$

or

$$L_S^N = L - L_A^N \quad [\text{W/cm}^2\text{-sr-um}] . \quad (F-25)$$

Substituting Equation (F-25) into Equation (F-23) gives

$$\gamma_o = \left[ \frac{L}{L - L_A^N} \right] \rho \quad [\text{nd}] \quad (F-26)$$

or

$$\gamma_o = \left[ \frac{\rho}{1 - \frac{L_A^N}{L}} \right] \quad [\text{nd}] . \quad (F-27)$$
APPENDIX G

NOISE EQUIVALENT DELTA TEMPERATURE

Equation (2-9) enables the signal to be written as

\[ S' = t_1 \Delta D \eta E' \Delta \lambda \quad [e] , \]  \hspace{1cm} (G-1)

where

\[ E' \Delta \lambda = \int_{\lambda_1}^{\lambda_2} E'(\lambda) \, d\lambda \quad [\text{p/sec-cm}^2] \]  \hspace{1cm} (G-2)

However,

\[ E'(\lambda) = \frac{\pi r_0}{4f^2 N} L'(\lambda) \quad [\text{p/sec-cm}^2-\mu \text{m}] \]  \hspace{1cm} (G-3)

and

\[ L'(\lambda) = \tau A B'(\lambda,T_s) + \epsilon A B'(\lambda,T_A) \quad [\text{p/sec-cm}^2-\text{sr-um}] \]  \hspace{1cm} (G-4)

Therefore,

\[ E' \Delta \lambda = \frac{\pi r_0}{4f^2 N} L' \Delta \lambda \quad [\text{p/sec-cm}^2] \]  \hspace{1cm} (G-5)

where

\[ L' \Delta \lambda = \int_{\lambda_1}^{\lambda_2} L'(\lambda) \, d\lambda \quad [\text{p/sec-cm}^2-\text{sr}] \]  \hspace{1cm} (G-6)

Equation (G-4) can be written as

\[ L' \Delta \lambda = \tau A L' S \Delta \lambda + \epsilon A L' A \Delta \lambda \quad [\text{p/sec-cm}^2-\text{sr}] \]  \hspace{1cm} (G-7)

where

\[ L' S \Delta \lambda = \int_{\lambda_1}^{\lambda_2} B'(\lambda,T_s) \, d\lambda \quad [\text{p/sec-cm}^2-\text{sr}] \]  \hspace{1cm} (G-8)
Taking the differential of Equation (G-1), one obtains
\[ dS' = (t_1 A_D \eta) \, dE' \Delta \lambda \]  \hspace{1cm} (G-10)

Replacing \( dS' \) by \( N \), and \( dE' \Delta \lambda \) by \( \text{NEI} \) results in
\[ N = (t_1 A_D \eta) \, \text{NEI} \] \hspace{1cm} (G-11)

where \( N \) = the total noise \( [e] \), and \( \text{NEI} \) = the noise equivalent photon irradiance into the detector \( [\text{p/sec-cm}^2] \).

Taking the differential of Equation (G-5) gives
\[ dE' \Delta \lambda = \frac{\pi \tau_0}{4f^2 N} \, dL' \Delta \lambda \] \hspace{1cm} (G-12)

Replacing \( dE' \Delta \lambda \) by \( \text{NEI} \), and \( dL' \Delta \lambda \) by the noise equivalent photon radiance (NEPR) gives
\[ \text{NEI} = \frac{\pi \tau_0}{4f^2 N} \, \text{NEPR} \] \hspace{1cm} (G-13)

Taking the differential of Equation (G-7) gives
\[ dL' \Delta \lambda = \tau_A \, dL' S \Delta \lambda + \varepsilon_A \, dL' \Delta \lambda \] \hspace{1cm} (G-14)

However, since we are not interested in perturbations due to changes in the atmosphere, we assume \( L' \Delta \lambda \) is to be constant. Equation (G-14) then becomes
\[ dL' \Delta \lambda = \tau_A \, dL' S \Delta \lambda \] \hspace{1cm} (G-15)
Replacing $dL'_{\Delta \lambda}$ with NEPR, and $dL'_{S \Delta \lambda}$ with the surface NEPR, NEPR$_S$ in Equation (G-15), gives

$$\text{NEPR} = \tau_A \text{NEPR}_S \quad [\text{p/sec-cm}^2\text{-sr}] .$$

(\text{G-16})

However, by definition,

$$dL'_{S \Delta \lambda} = \frac{dL_{S \Delta \lambda}}{dT_S} \quad [\text{p/sec-cm}^2\text{-sr}]$$

(\text{G-17})

or

$$dT_S = \frac{dL'_{S \Delta \lambda}}{\left(\frac{dL_{S \Delta \lambda}}{dT_S}\right)} \quad [\text{K}] .$$

(\text{G-18})

Replacing $dT_S$ with NEDT and $dL'_{S \Delta \lambda}$ with NEPR$_S$ in Equation (G-18) yields

$$\text{NEAT} = \frac{\text{NEPR}_S}{\left(\frac{dL'_{S \Delta \lambda}}{dT_S}\right)} \quad [\text{K}]$$

(\text{G-19})

and

$$\frac{dL'_{S \Delta \lambda}}{dT_S} = \int_{\lambda_1}^{\lambda_2} \frac{dB(\lambda,T_S)}{dT_S} \ d\lambda \quad [\text{p/sec-cm}^2\text{-K}] ,$$

(\text{G-20})

where the Planck function is given by

$$B'(\lambda,T_S) = \frac{C_1'}{\lambda^4} \left[\frac{1}{\exp \left( \frac{C_2}{\lambda T_S} \right) - 1} \right] \quad [\text{p/sec-cm}^2\text{-sr-\mu m}] .$$

(\text{G-21})

Differentiating Equation (G-21) with respect to $T_S$ yields

$$\frac{dB'(\lambda,T_S)}{dT_S} = \frac{C_2 \lambda^2 \left( B'(\lambda,T_S) \right)^2 \exp \left( \frac{C_2}{\lambda T_S} \right)}{C_1 T_S^2} \quad [\text{p/sec-cm}^2\text{-sr-\mu m-K}] .$$

(\text{G-22})
Equation (G-19) may be written in terms of signal-to-noise ratio (S/N) as follows. Dividing Equation (G-1) by Equation (G-11) gives

\[
\frac{S}{N} = \frac{E' \Delta \lambda}{NE} \quad [\text{nd}] .
\]  

Substituting Equations (G-5) and (G-13) into Equation (G-23) and replacing \(dL' \Delta \lambda\) with NEPR gives

\[
\frac{S}{N} = \frac{L' \Delta \lambda}{NEPR} \quad [\text{nd}] .
\]  

Substituting Equation (G-16) into Equation (G-24) gives

\[
\frac{S}{N} = \frac{L' \Delta \lambda}{\tau A \cdot NEPR_S} \quad [\text{nd}]
\]  

or

\[
NEPR_S = \frac{L' \Delta \lambda}{\tau A \left( \frac{S}{N} \right)} \quad [\text{p/sec-cm}^2\text{-sr}]
\]  

and, finally, by substituting Equation (G-26) into Equation (G-19), one obtains

\[
NE\Delta T = \frac{L' \Delta \lambda}{\tau A \left( \frac{S}{N} \right) \left( \frac{dL' S}{dT S} \right)} \quad [\text{K}].
\]
This paper describes the theory and develops the equations required to design and analyze the performance of electro-optical sensor systems that operate from the visible through the thermal infrared spectral regions. Methods to compute essential optical and detector parameters, signal-to-noise ratio, MTF, and figures of merit such as NEΔp and NEΔT are developed. A set of atmospheric tables are provided to determine scene radiance in the visible spectral region. The Planck function is used to determine radiance in the infrared. The equations developed in the paper have been incorporated in a spreadsheet so that a wide variety of sensor studies can be rapidly and efficiently conducted.