NASA Contractor Report 4307

Unsteady Potential Flow Past a Propeller Blade Section

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CONTRACT NASI-19000
JULY 1990
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Prepared for
Langley Research Center
under Contract NAS1-19000
Summary

An analytical study has been conducted to predict the effect of an oscillating stream on the time dependent sectional pressure and lift coefficients of a model propeller blade. The assumption is that as the blade sections encounter a wake, the actual angles of attack vary in a sinusoidal manner through the wake, thus each blade is exposed to an unsteady stream oscillating about a mean value at a certain reduced frequency. On the other hand, an isolated propeller at some angle of attack can experience periodic changes in the value of the flow angle causing unsteady loads on the blades. Such a flow condition requires the inclusion of new expressions in the formulation of the unsteady potential flow around the blade sections. These expressions account for time variation of angle of attack and total shed vortices in the wake of each airfoil section. It was found that the final expressions for the unsteady pressure distribution on each blade section are periodic and that the unsteady circulation and lift coefficients exhibit a hysteresis loop.

Introduction

Propeller performance\textsuperscript{1} and radiated noise\textsuperscript{2} can be substantially altered by the exposure to a non-uniform freestream, such as the flow produced by the wake of an upstream wing or pylon. It has also been shown that an isolated propeller at some angle of attack is also exposed to the unsteady loads experienced by the propeller blade sections\textsuperscript{3}. While many researchers, motivated by unsteady flow encountered on helicopter rotor blades or inside turbomachinery, have studied the physics of airfoils exposed to nonuniform flow\textsuperscript{4-5}, the present analysis addresses the problem of
propeller aerodynamics from a technology base point of view. This analysis is centered on the application of ANOPP6 (Aircraft Noise Prediction Program), developed at NASA Langley Research Center, to the unsteady flow environment. While ANOPP is capable of coupling the aerodynamic calculations to the analytical noise prediction segment, only the aerodynamic computations leading to the unsteady blade section pressure distribution and their unsteady lift coefficients are considered here. The derivation of the equations for unsteady terms are described in the analysis section of the report and in the final portion of the analysis, closed form solutions have been presented for the special case of a thin airfoil. The thin airfoil approximation has been used in a hybrid propeller performance code developed by the present author. The above hybrid code has shown to be capable of computing the performance characteristics of advanced propellers in a very efficient manner with good accuracy. It is intended to extend the capabilities of the hybrid code to the unsteady environment by the use of the present two dimensional analysis in a future work.

**Symbols**

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<th>Symbol</th>
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<td>A</td>
<td>terms due to complex velocity potential in Z-plane</td>
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<tr>
<td>a</td>
<td>radius of the mapping circle, m/s (ft/s)</td>
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$C_t$ section lift coefficient

$C_p$ section pressure coefficient

$F$ complex velocity potential in $z$-plane, re $aV_\infty$

$M$ Mach number

$T$ transformation,

$\tau$ nondimensional time, $\frac{\tau a}{U_\infty}$

$V_\infty$ free stream velocity as seen by blade section, m/s (ft/s)

$\bar{w}$ complex velocity, m/s

$x$ vortex location in $Z$-plane, re $a$

$z$ complex plane of the airfoil, re $a$

$\alpha$ section angle of attack, deg.

$\Gamma$ unsteady circulation, re $aV_\infty$

$\gamma$ strength of the vortex per unit length of the wake

$\Delta \alpha$ increment of angle of attack

$\epsilon \theta - \xi$, rad.

$\xi$ angular measure in near circle, $\zeta'$ plane

$\zeta$ perfect circle, $\zeta = b e^{\Psi_0 + i\theta}$
\( \zeta' \) near circle, \( \zeta' = b \, e^{\psi + 2\pi i \zeta} \)

\( \theta \) angular measure in \( \zeta\)-plane

\( \rho \) location of vortex in the \( \zeta\)-plane

\( \phi \) nondimensional potential, \( \text{re} \, aV_\infty \)

\( \omega \) reduced frequency, \( \text{re} \, \frac{V_\infty}{a} \)

\( \Omega \) propeller angular velocity, rad./s

**Subscripts:**

le leading edge

te trailing edge

0 steady term

1 unsteady components multiplying \( \Delta \alpha \cos \omega t \)

2 unsteady components multiplying \( \Delta \alpha \sin \omega t \)

**Analysis**

The derivation of expressions for time dependent section pressure and lift coefficients are described in the following using the potential flow analysis. First,
expressions are described for a generic airfoil; next, special cases, such as a family of Theodorson’s airfoils and in particular flat plate approximations, are discussed.

a) Unsteady Potential Flow

It has been shown\(^1\), that when the blade sections of a propeller encounter a wake, the blade section angle of attack can be approximated to vary in a sinusoidal manner throughout the passage. Thus, each blade is exposed to an unsteady oncoming flow. This flow can be taken as a uniform stream oscillating about a mean value at a certain reduced frequency.

\[
\alpha = \alpha_0 + \Delta \alpha \sin \omega t
\]  

(1)

Such a flow condition requires inclusion of new expressions in the formulation of the unsteady potential flow around the blade sections. These expressions are needed to properly account for time variation of angle of attack and total shed vortices in the wake of each airfoil section. Therefore, the effect of the vortical wake behind each section is included in terms of the bound circulation which gives rise to the magnitude of the total lift.

It is, therefore, possible to extend the steady analysis to the problem of an oscillating flow about an airfoil. The unsteady potential flow is formulated for the perfect circle in the \(\zeta\)-plane. Theodorsen’s transformation is utilized to obtain the unsteady complex velocity in the \(Z\)-plane (see Fig. 1). The wake is modeled by distributed vortices composed of pairs of an isolated vortex and its image vortex outside and inside of the unit circle, respectively. Thus the complex velocity potential in the \(\zeta\)-plane is written:
\[ F(\zeta) = V_\infty \left( \zeta e^{-i\alpha} + \frac{a^2}{\zeta} e^{i\alpha} \right) + \frac{i}{2\pi} \ln \left( \frac{\zeta}{a} e^{-i\alpha} \right) \]  
\[ + i \int_{x_0}^{\infty} \gamma \left[ \ln \left( \frac{\zeta}{\alpha} \right) + \ln \left( \frac{\zeta}{\alpha} - \rho \right) - \ln \left( \frac{\zeta}{\alpha} - \frac{1}{\rho} \right) \right] d\xi \]  

where the above dimensional quantities can be normalized with respect to the mean free stream velocity, \( V_\infty \), and radius of the circle, \( a \):

\[ \Gamma = \frac{\Gamma}{a^2} V_\infty, \quad x = \frac{x}{a}, \quad \gamma = \frac{\gamma}{V_\infty} \text{ and } \zeta = \frac{\alpha}{a} \]

Therefore, the complex velocity in the \( \zeta \)-plane becomes

\[ w(\zeta) = \frac{\tilde{w}(\zeta)}{V_\infty} = ie^{-i\theta} \left[ 2\sin(\theta - \alpha) + \frac{\Gamma}{2\pi} + \int_{x_0}^{\infty} Q \cdot \gamma \, dx \right] \]  

with

\[ Q = \frac{2(1 - \rho \cos \theta)}{(1 + \rho - 2 \rho \cos \theta)} \quad (4) \]

Assuming that the distributed vortices all lie on the x-axis extending to infinity, the unsteady pressure coefficient can be written as

\[ C_p = 1 + \left\{ \frac{2}{V_\infty^2} \frac{\partial \phi}{\partial t} + \left| \frac{\overline{w}(z)}{V_\infty} \right|^2 \right\} \]  

\[ \quad (5) \]
with

\[ |w(z)|^2 = w(z) \cdot \overline{w(z)}, \]  
\[ T = \begin{bmatrix} \frac{d\zeta'}{dz} & d\zeta' \end{bmatrix} \]  

and

\[ w(z) = w(\zeta) \cdot T \]  

Note that \( \phi \) is the real part of the complex potential, \( F(\zeta) \). In the above expression \( \zeta' \) is the plane of the near circle obtained by the Joukowski transformation:

\[ z = \zeta' + \frac{b^2}{\zeta'} \]  

Since the instantaneous value of angle-of-attack varies in a sinusoidal manner, it is logical to assume that the circulation can be expressed in the following form:

\[ \Gamma(t) = \Gamma_0 + \Delta\alpha \Gamma_1 \cos\omega t + \Delta\alpha \Gamma_2 \sin\omega t \]  

This expression allows for any phase lead or lag in the final expression for circulation and, hence, lift value. Therefore, this is an adequate expression to describe the vorticity since it can be understood as the first term in a series representation for \( \Gamma \). Since for the range of velocity defects encountered, the values of \( \Delta\alpha \) remain very small, terms of order of \((\Delta\alpha)^2\) or higher can be neglected. With
this in mind, it can be shown that for trailing vortices to be left behind, as the airfoil moves forward, the strength of the distributed vortices can be related to the bound circulation so that:

\[
\gamma = \gamma_1 \Delta \alpha \cos \omega t + \gamma_2 \Delta \alpha \sin \omega t \quad (11)
\]

and

\[
\gamma_1 = -\omega \left( \Gamma_1 \sin \omega (x - x_{te}) + \Gamma_2 \cos \omega (x - x_{te}) \right) \quad (12)
\]

\[
\gamma_2 = \omega \left( \Gamma_1 \cos \omega (x - x_{te}) - \Gamma_2 \sin \omega (x - x_{te}) \right) \quad (13)
\]

Obviously, an expression can be found for \( \ddot{w}(\zeta) \) representing a steady term and the unsteady contributions by using Eq. (11) in Eq. (3) to obtain

\[
\frac{\ddot{w}(\zeta)}{V_\infty} = B_0 + B_1 \Delta \alpha \cos \omega t + B_2 \Delta \alpha \sin \omega t \quad (14)
\]

where:

\[
B_0 = i e^{-i \theta} \left[ 2 \sin(\theta - \alpha_g) + \frac{\Gamma_0}{2\pi} \right] \quad (15)
\]

\[
B_1 = i e^{-i \theta} \left[ \frac{\Gamma_1}{2\pi} - \omega \left( \Gamma_1 \cdot S_u + \Gamma_2 \bullet C_u \right) \right] \quad (16)
\]
\[ B_2 = i e^{-i \theta} \left[ \frac{\Gamma_2}{2\pi} - 2\cos(\theta - \alpha_0) + \omega \left( \Gamma_1 \cdot Cu - \Gamma_2 \cdot Su \right) \right] \]  

(17)

and terms \( Su \) and \( Cu \) are given as:

\[ Su = \int_{x_{te}}^{\infty} \sin(\rho) \cdot Q(\rho, \theta) \, dx \]  

(18)

\[ Cu = \int_{x_{te}}^{\infty} \cos(\rho) \cdot Q(\rho, \theta) \, dx \]  

(19)

Similarly, the following integrals are defined as:

\[ CNu = \int_{x_{te}}^{\infty} \cos(\rho) \cdot V(\rho, \theta) \, dx \]  

(20)

\[ SNu = \int_{x_{te}}^{\infty} \sin(\rho) \cdot V(\rho, \theta) \, dx \]  

(21)

where \( V \) is the argument of the expression:

\[ W_L = |W_L| e^{i\nu} \]  

(22)
These are used in the expression for time rate of change of potential:

$$\frac{\partial \phi}{\partial t} = \phi'_1 \Delta \alpha \cos \omega t + \phi'_2 \Delta \alpha \sin \omega t$$  \hspace{1cm} (23)

$$C_p = \phi'_1 = \omega \left\{ 2(\theta - \alpha_0) + \frac{\Gamma_0}{2\pi} - \frac{\Gamma_2}{2\pi} (\theta - \alpha_0) - \omega \left( \Gamma_1 \Delta \nu - \Gamma_2 \Delta \nu \right) \right\}$$  \hspace{1cm} (24)

$$\phi'_2 = \omega \left\{ \frac{\Gamma_1}{2\pi} (\theta - \alpha_0) - \omega \left( \Gamma_1 \Delta \nu - \Gamma_2 \Delta \nu \right) \right\}$$  \hspace{1cm} (25)

With eq. (15), (16), (17) and (7) let:

$$A_0 = B_0 T$$  \hspace{1cm} (26)

$$A_1 = B_1 T$$  \hspace{1cm} (27)

$$A_2 = B_2 T$$  \hspace{1cm} (28)

Then the final expression for the unsteady pressure distribution is obtained as a steady term plus unsteady components in the usual manner

$$C_p = C_{p0} + \Delta \alpha C_{p1} \cos \omega t + \Delta \alpha C_{p2} \sin \omega t$$  \hspace{1cm} (29)
with

\[ C_{p_0} = 1 - A_0 \bar{A}_0 \]  
\[ C_{p_1} = -2\phi_1 - A_0 \bar{A}_1 - \bar{A}_0 A_1 \]  
\[ C_{p_2} = -2\phi_2 - A_0 \bar{A}_2 - \bar{A}_0 A_2 \]

where \( \bar{A} \) stands for the conjugate of \( A \).

In the above expressions, values of the unsteady circulation components \( \Gamma_0, \Gamma_1, \Gamma_2 \) need to be specified before the pressure coefficients can be computed. The correct amount of the circulation is, however, related to the vorticity production and dissipation which itself is inherently a viscous phenomenon. For steady flow past thin airfoils at small angles of attack, the Kutta condition is merely a specification of the points of separation at the trailing edge. For unsteady flows, the rate of change of bound circulation must be related to the vorticity flux into the wake at the points of flow separation (on both upper and lower surfaces). This relation is known as the MRS (Moore, Rott, Sears) Criterion. This procedure would require computation on the viscous boundary layers on both surfaces up to the points of separation. As was indicated above for airfoils at moderate angle of attack, the Kutta condition can be understood as trailing edge separation where the removal of a singularity due to transformation specifies the circulation value uniquely. Therefore, an unsteady extension of the Kutta condition can be understood as trailing edge separation for all times. This requirement specifies the circulation components uniquely, when \( \tilde{w}(\xi) \) is forced to vanish at a point corresponding to the trailing edge.

\[ \Gamma_0 = 4\pi \sin(\alpha_0 - \theta_{TE}) \]  

11
\[
\Gamma_1 = \frac{2\omega \text{Cut} \cos(\theta_{TE} - \alpha_0)}{(1/2\pi - \omega \text{Suit})^2 + (\omega \text{Cut})^2}
\]

\[
\Gamma_2 = \frac{2(1/2\pi - \omega \text{Suit}) \cos(\theta_{TE} - \alpha_0)}{(1/2\pi - \omega \text{Suit})^2 + (\omega \text{Cut})^2}
\]

In this manner then, the components of the unsteady circulation can be used to obtain a final expression for the unsteady lift coefficient as an unsteady extension of Blasius relation:

\[
C_l = C_h' + \Delta\alpha C_l' \cos \omega t + \Delta\alpha C_l' \sin \omega t
\]

Indeed, the above expression will account for any phase lead or lag in the final expression for total lift on each blade section, to order of $\Delta\alpha$, as the sections experience the oscillating streams which are caused by blade wake encounter.

b) **Family of Theodorsen Airfoils**

In the previous section, the expressions for lift and pressure coefficients were obtained in terms of the mapped flow past the unit circle and the metrics of the mapping function. In numerical analysis these constitute the Jacobian of the transformation. In analytical methods, such as for a family of Theodorsen's airfoils, the airfoil is mapped into a near circle $\zeta'$ and, subsequently, the near circle is mapped into the unit circle. Thus, for a known airfoil section, the coordinates and leading edge radius of the airfoil are given and the task is to obtain parameters of the near circle and the radius of the circle in terms of the airfoil data.
It can be shown (ref. 9) that if

\[ z = \zeta' + \frac{b_2}{\zeta'} \]  

(37)

for the near circle:

\[ \zeta' = b \Psi e^{i\xi} \]  

(38)

and the perfect circle is described by:

\[ \zeta = b \psi_o e^{i\theta} = a e^{i\theta} \]  

(39)

One can write the deviation of the near circle from the perfect circle as:

\[ \varepsilon = \theta - \zeta \]  

(40)

In order to compute the deviation, \( \varepsilon \), and radius of the perfect circle, \( a \), in terms of \( z = x + iy \), the relation between \( \zeta \) and \( \zeta' \) is obtained by division of equation (38) by (39):

\[ \zeta' = \zeta e^{(\Psi - \Psi_o)} e^{i(\xi - \theta)} \]  

(41)

This can be expanded in a Fourier Expansion as:

\[ \zeta' = \exp \left( \sum_{n=1}^{\infty} \frac{(A_n + B_n)}{a^n} e^{i n \theta} \right) \]  

(42)

Thus, coefficient of the expansion can be written as:

\[ \frac{A_n}{a^n} = \frac{1}{\pi} \int_{0}^{2\pi} \Psi \cos n\theta \, d\theta \]  

(43)

\[ \frac{B_n}{a^n} = \frac{1}{\pi} \int_{0}^{2\pi} \Psi \sin n\theta \, d\theta \]  

(44)

\[ \Psi_o = \frac{1}{2\pi} \int_{0}^{2\pi} \Psi \, d\theta \]  

(45)
In an iterative manner, then, real and imaginary parts of (42) are computed for the following relations:

\[
\psi - \psi_0 = \sum_{n=1}^{\infty} \left( \frac{A_n}{a^n} \cos n\theta + \frac{B_n}{a^n} \sin n\theta \right) \quad (46)
\]

\[
\epsilon = \sum_{n=1}^{\infty} \left( \frac{B_n}{a^n} \cos n\theta - \frac{A_n}{a^n} \sin n\theta \right) \quad (47)
\]

In the limiting case for \( \theta = \pi \), the leading edge radius is obtained as:

\[
\bar{r}_{le} = \frac{2b \sinh^2 \psi}{\cosh \psi} \quad (48)
\]

and the Joukowski transformation parameter per chord length becomes:

\[
\bar{b} = \frac{1 - \frac{\bar{r}_{le}}{8}}{4} \quad (49)
\]

and the expression for the position of the isolated vortex, per radius of unit circle, becomes:

\[
\rho = \frac{\left( x + \sqrt{x^2 - 4b^2} \right)}{2} \quad (50)
\]

Note that in the above expression:

\[
b = \bar{b} \cdot c \quad (51)
\]
c) **Flat Plate Approximation**

In many cases, the airfoil sections for advanced turboprops are very thin airfoils and the calculations above can be approximated for a flat plate airfoil section. In this case, the limiting values for influence functions becomes:

\[ Q = \frac{1}{1 - \rho} \quad (52) \]

or

\[ Q = \frac{-4}{(x-2 + \sqrt{x^2 - 4})} \quad (53) \]

Let \( \eta = x-2 \) then:

\[ C_{u0} = -4 \int_{\eta}^{\infty} \frac{\cos (\omega \eta)}{\eta + \sqrt{\eta^2 + 4\eta}} \, d\eta \quad (54) \]

\[ S_{u0} = -4 \int_{\eta}^{\infty} \frac{\sin (\omega \eta)}{\eta + \sqrt{\eta^2 + 4\eta}} \, d\eta \quad (55) \]

Thus, the expression for the unsteady lift coefficient becomes:

\[ C_\ell (t) = \Gamma (t)/2 \quad (56) \]

and expressions for \( C_{\ell 0}, C_{\ell 1}, \) and \( C_{\ell 2} \) are written as:

\[ C_{\ell 0} = 2 \pi \sin \alpha_0 \quad (57) \]
Results and Discussions

The analysis described above has been employed to predict the steady state and unsteady lift coefficient components for blade sections of a straight bladed scale model (SR-2) propeller. The physical dimensions and the operating conditions are given in reference 1, and they are not repeated here. However, the practical application of the above analysis is shown by the variation of the unsteady lift coefficient for a value of mean angle-of-attack as described by reference 1. Accordingly, the section angle of attack varies in a sinusoidal form due to the passage of the propeller blade through the wake of the pylon. The pylon, in this case, is positioned upstream of the propeller in a so-called pusher configuration. The wake momentum deficit can cause an immediate change in angle of attack as it is seen by the propeller blade section. This change is periodic and can be approximated by sinusoidal behavior. Thus, the amplitude and reduced frequency of the motion are obtained from the kinematics of blade/wake interaction. A typical variation of angle of attack is illustrated in Fig. 2. It can be seen that the amplitude of oscillation is of order of 1°. Thus, the present analysis, which is restricted to small $\Delta\alpha$, is valid for this type of application.
Next, consider the unsteady lift behavior shown in Fig. 3 for a blade section at 37% radial distance from the propeller pitch axis. The lift coefficient was obtained by conformal mapping of Theodorsen's method. A modification of the aerodynamic module of ANOPP was required to obtain the unsteady components of lift, drag and pressure coefficients under an unsteady environment. Note that the hysteresis behavior of the unsteady lift coefficient is recovered by this analysis without inclusion of unsteady boundary layer analysis. This can be explained, as discussed earlier, by the fact that for airfoils at small angles of attack, the unsteady Kutta condition is understood as trailing edge separation and uniquely defines the magnitude of circulation and consequently the lift curve behavior. This has paramount implications in understanding structural and acoustic problems. Reference 2 points out that lift hysteresis of aerodynamic loads, obtained under nonuniform inflow conditions, increases loading noise and the associated radiated noise by a propeller blade.

Consider the results for the special case of a flat plate airfoil, described above under part c) of the analysis. This case is directly applicable to the work of reference 7. In this work, the present author has developed a code to predict the performance of advanced propellers by a simple strip method, where the induced blade velocities are obtained by an iterative procedure. Thus, sectional angle of attack and Mach numbers are obtained through direct computation of section aerodynamic characteristics using thin airfoil theory. This analysis has shown excellent agreement with wind tunnel results for advanced propellers, such as SR-7. In this case, the contribution of the thin airfoil to the unsteady lift coefficient can be approximated by equations 58-60. A parametric behavior of the unsteady lift coefficient is simulated in Fig. 4. The angle of attack was chosen to be the same as shown in Fig. 2. The values of unsteady lift coefficient, $C_l$, are plotted for various
reduced frequencies, $\omega$. It can be seen that the lift hysteresis loop becomes smaller with increasing value of $w$ and changes the inclination of the loop accordingly. In addition, Fig. 5 shows that for higher mean angle-of-attack the lift coefficient and the size of the hysteresis loop both decrease with $\alpha_0$ relative to their mean value. Furthermore, there is no apparent change in the direction of the loop. This is in accord with the explanations of reference 8, since the present analysis is limited by the potential flow assumption and cannot account for flow separation. Consequently, the above results do not apply to airfoils near stall. Nevertheless, the analysis captures the essence of excess loads encountered under time dependent flow conditions.

Conclusions

A method of solution has been developed to compute the unsteady potential flow past airfoil sections of arbitrary shape. Even though the study is limited to perturbations of angle of attack from a mean value, the analysis accounts for any lead or lag, of the order of small amplitudes, in the final expressions for the unsteady aerodynamic coefficients. These coefficients then properly represent the hysteresis behavior encountered in a periodic flow such as the flow over advanced propeller blades in an unsteady flow environment.

References


Fig. 1 Potential flow mapping for blade section
Fig. 2 Variation of angle of attack during the wake passage.
Fig. 3 Normalized lift hysteresis for a propeller blade section.

Span = .373
x_w = .1 c
\( \Omega = 11400 \) rpm
\( \alpha_o = 6.837 \) deg
\( c_{l_o} = .964 \)
Fig. 4 Unsteady Lift Curve

\[ \alpha_0 = 3 \quad c_{l0} = 0.33 \quad \omega = 0.05 - 0.5 \]
Fig. 5  Unsteady Lift Curve

\( \alpha_0 = 3 \quad c_{l0} = 0.657 \quad \omega = 0.05 - 0.5 \)
An analytical study has been conducted to predict the effect of an oscillating stream on the time dependent sectional pressure and lift coefficients of a model propeller blade. The assumption is that as the blade sections encounter a wake, the actual angles of attack vary in a sinusoidal manner through the wake, thus each blade is exposed to an unsteady stream oscillating about a mean value at a certain reduced frequency. On the other hand, an isolated propeller at some angle of attack can experience periodic changes in the value of the flow angle causing unsteady loads on the blades. Such a flow condition requires the inclusion of new expressions in the formulation of the unsteady potential flow around the blade sections. These expressions account for time variation of angle of attack and total shed vortices in the wake of each airfoil section. It was found that the final expressions for the unsteady pressure distribution on each blade section are periodic and that the unsteady circulation and lift coefficients exhibit a hysteresis loop.