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OBTAINING AN EQUIVALENT BEAM

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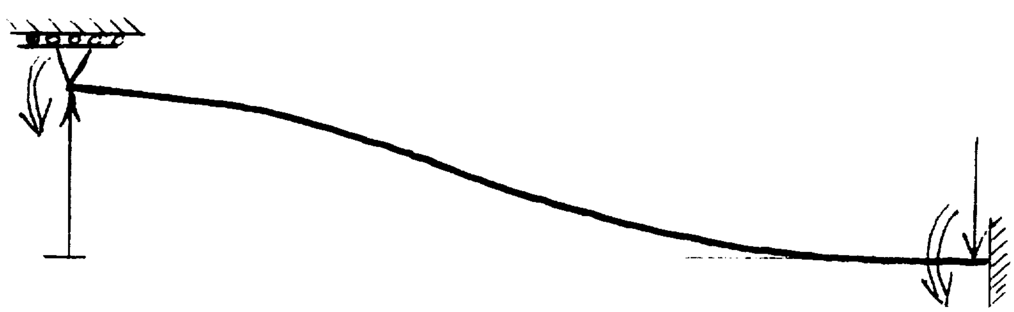
In modeling a complex structure I was faced with a component that would have logical appeal if it were modeled as a beam. It was a mast of a robot controlled gantry crane. The structure up to this point already had a large number of degrees of freedom, so the idea of conserving grid points by modeling the mast as a beam was attractive. I decided to make a separate problem of the mast and model it in three dimensions with plates then extract the equivalent beam properties by setting up the loading to simulate beam like deformations and constraints. The results could then be used to represent the mast as a beam in the full model. This seemed to be a straight forward approach, but it was sufficiently challenging that it merited publishing a paper on this topic.

The endeavor is to obtain the area **A**, the area moments of inertia **I1** and **I2**, and torsional area moment of inertia **J** of a prismatic beam that would be an equivalent of the crane mast over its full length. The detailed model involved about 4500 unconstrained degrees of freedom. The mast structure was essentially a hollow steel tube of square section with a cylindrical indentation along its length on one surface only. Complications that made it difficult to estimate equivalent properties analytically were the placement of two types of interior partial shear stiffeners at regular intervals along its length. These two different types of shear stiffeners alternated on opposite sides from each other most of the length. This posed no difficulty to model elastically in a three dimensional model. The interesting phase is the loading of the 3-D model in order to simulate beam action.

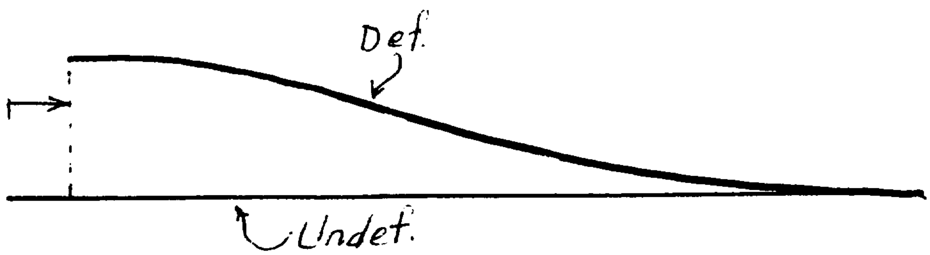
To put the problem in perspective, review for the moment, the definition of beam stiffness.

> DEFINITION: Beam stiffness is the array of forces produced at the six degrees of freedom on both ends when a single degree of freedom at one end is deformed a unit amount while enforcing all other eleven degrees of freedom at both ends to be zero.

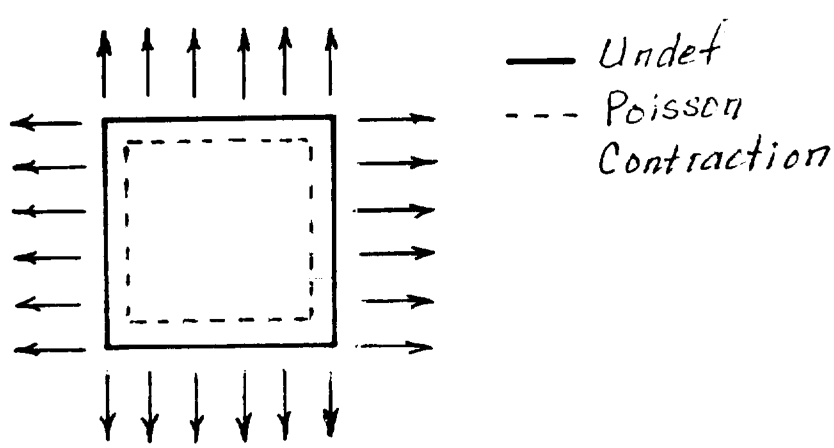
But the Bernoulli Euler formulation of the beam as used in finite element analysis programs does not faithfully follow this prescription of stiffness to the letter. For example, when one end is displaced a unit transversely, action is assumed to occur in - plane only. Diagrammatically the boundary conditions of the centroid of the B.E. formulation are indicated in the sketch.



Note that the length remains invariant, because its transversely deformed end is not constrained in the axial direction. In effect, with this B.E. approach, the end position contracts when bending deformation occurs. This is shown in exaggerated fashion in the sketch.



If the length is not allowed to deform, Poisson deformation does not occur and therefore needs no constraining force to inhibit Poisson deformation. But if the true definition of beam stiffness were adhered to in the finite element beam, the axial positions of the ends would be held to zero displacement and the beam would lengthen as transverse deformation occurs. Such axial stretching would result in Poisson contraction in both transverse directions. But if transverse translational deformations were held to zero, as the definition of stiffness demands, such constraints would exert forces to prevent Poisson contraction. For instance, the transverse forces at the end of a solid beam of square section with a full set of constraints applied would appear as sketched.



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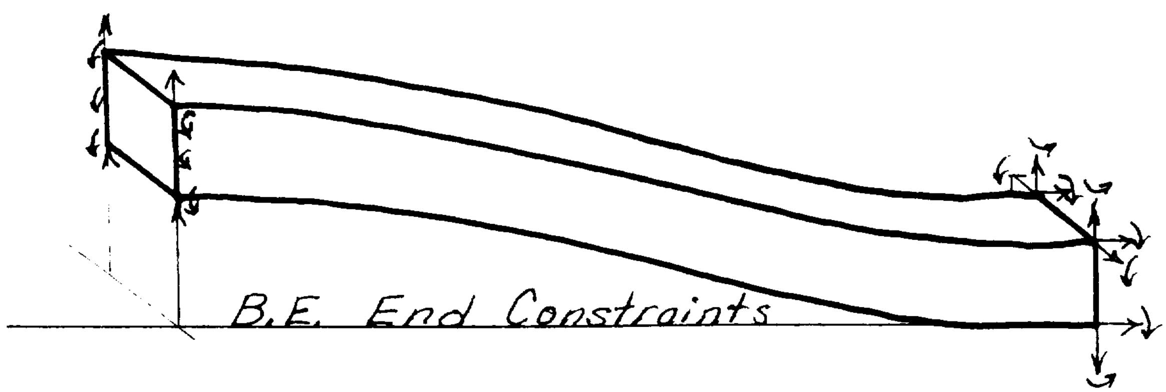
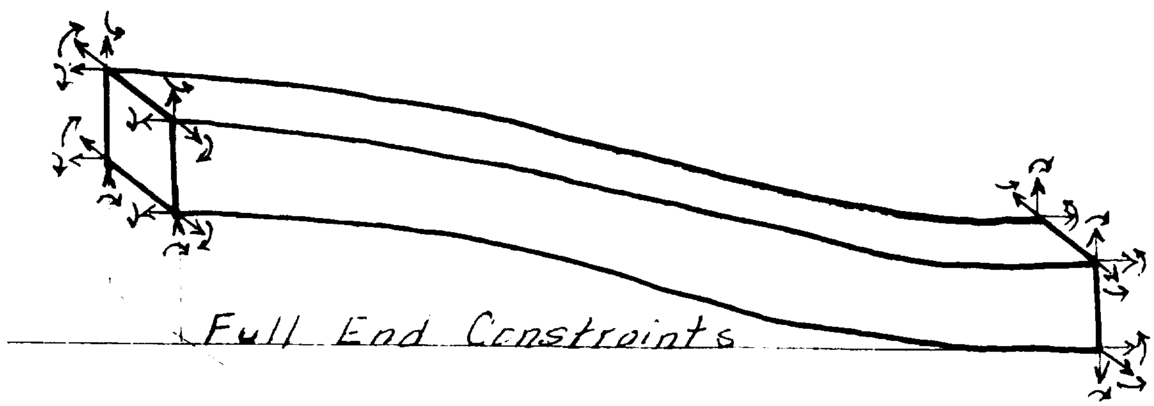
The dilemma now is to try to define what kind of equivalence should be sought. If A, I1, I2, and J were obtained with true stiffness constraints, would it be proper to operate as an equivalent beam according to those entries on the property card, so as to exclude bending/axial coupling even though such action was present during the sample run? Or would it be more proper to use only B.E. conditions to get the properties that will used as a B.E. beam? If the latter were chosen, the question arises as to how faithfully we would be representing equivalence to the true structure. Having some doubts as to how to proceed, I modeled the constraints in two different ways; with full end constraints and with B.E. end constraints and compared the results. The sketch shows the constraints imposed for the two models. One of the things to consider in the B.E. simulation is that the theory requires planes to remain plane in bending.





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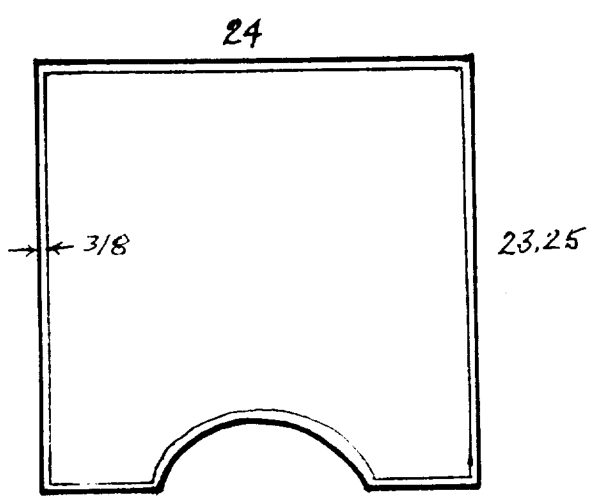
The next question is: After constraint forces are measured, will it be acceptable to derive sectional properties by substituting into the formulation based strictly on B.E.? That is to say, should the stiffness forces obtained on the left be equated to the B.E. formulas on the right? Just enough of the matrix on the right is shown to illustrate the problem.

$$
\begin{array}{cccccccccccc|cccccc}
K_{11} & & & & & & K_{17} & & & & & & EA/L & & & & & \vdots \\
 & K_{22} & & & & K_{26} & & K_{28} & & & & K_{2C} & & 12EI_z/L^3 & & & & \vdots \\
 & & K_{33} & & K_{35} & & & & K_{39} & & K_{3B} & & & & 12EI_y/L^3 & & & \vdots \\
 & & & K_{44} & & & & & & K_{4A} & & & & & & GJ/L & & \vdots \\
 & & K_{53} & & K_{55} & & & & K_{59} & & K_{5B} & & & & -6EI_y/L^2 & & & \vdots \\
 & K_{62} & & & & K_{66} & & K_{68} & & & & K_{CC} & & 6EI_z/L^2 & & & & \vdots
\end{array}
$$

Not having any reference to use for the fully coupled beam I chose to use B.E. formulation to evaluate sectional properties for both types of modeling.

The next question is: After accepting B.E. formulation, what basis should be used to reconcile differences in results of the methods? The reconciliation method is to use an estimation of the computed value of the section without the shear panels present as per the dimensions in the sketch.



COMPARISON OF PROPERTIES DERIVED FROM MODELS OF DIFFERENT CONSTRAINTS VS MANUAL CALCULATIONS

SOURCE	A	I1	I2	J
FULL CONSTRAINTS	36.72	2,444.47	2,605.14	INVALID
B.E. CONSTRAINTS	36.66	728.63	853.62	600.19
MANUAL	35.96	2,541.82	3,517.62	4,710.60

This exercise had some unexpected results. The whole purpose of the exercise was to get an equivalent beam by using a full 3-D model instead of making an analytical estimate because of the uncertainty in being able to represent the effect of the partial shear panels correctly. One expects that the effect of the shear panels is to stiffen the steel tube, but the 3-D results showed less stiffness than the manual check which neglected the panels. Why?

In going back to examine the axial displacements in the 3-D model using the B.E. constraints, it indicated that the end faces tilted instead of remaining perpendicular to the undeformed centroidal axis as the B.E. theory requires. The total burden of meeting the requirement of zero slope at the displaced end was put on the QUAD4 elements which formed the side panels of the steel tube. That is; the open ended tube had two surfaces that could carry such bending and two surfaces unable to carry in-plane shear about their normals. Even those that picked up such bending couldn't transmit this moment to the QUAD's on the perpendicular surface, so an inadequate moment developed to produce a net slope of zero at the centroid. By letting the end points displace axially they were in no position to create couples to satisfy the moments for zero slope. That is why the model with the B.E. constraints produced inadequately stiff sectional properties in bending and torsion.

Going back to the model with fully constrained ends, the explanation as to why this model was also inadequate for simulating an equivalent beam was this. Even though it did develop couples which formed the resisting moment for zero end slope by holding the axial displacements to zero; it still felt the deficiency of moments about the normals of side panels. In effect membrane action on corner displacements alone was not sufficient to represent the true structure without the help of the existing -- but unrepresented -- in plane shear from moments about the normals of the panels.

In the case of torsion the fully constrained model was invalid because it developed local equilibrium at the end undergoing unit rotation. The unit rotation about the axial direction for every end grid point was inhibited by the translational d.o.f.'s being held to zero. The deformation became a scalloped pattern instead of a uniformly rotated face. Representation of torsion with the B.E. model was also inadequate because it required, but didn't get, the assistance of the panels on all four sides to carry the rotation about their normals.

Does this mean that if no attempt were made to model the mast as an equivalent beam, but a full 3-D model were used, that the 3-D model would be invalid? Not at all. What it shows is that the 3-D model is ineffective in trying to conform to the requirements of an equivalent beam representation. If a full 3-D plate model were used in the complete representation of the crane structure, good results would be obtained.

Since the attempt is to economize on the size of the model, a better way to achieve the same results is to use sub-structuring and condense the mast to equivalent end boundary and intermediate mass points.

The spirit in which this paper is presented is to publish failures as well as successes to help analysts avoid retracing the ground that has already been plowed.