OPTICAL RATE SENSOR ALGORITHMS

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ABSTRACT

Optical sensors, in particular CCD arrays, will be used on Space Station to track stars in order to provide inertial attitude reference. Algorithms are presented to derive attitude rate from the optical sensors. The first algorithm is a recursive differentiator. A variance reduction factor (VRF) of 0.0228 was achieved with a rise time of 10 samples. A VRF of 0.2522 gives a rise time of 4 samples. The second algorithm is based on the direct manipulation of the pixel intensity outputs of the sensor. In 1-dimensional simulations, the derived rate was within 0.07% of the actual rate in the presence of additive Gaussian noise with a SNR of 60 dB.
INTRODUCTION

Optical sensors will be of great use on Space Station as part of the on-board guidance, navigation, and control system. These sensors can be used to track stars to provide inertial attitude reference. The information may also be used by astronomical experiments to allow reference to a known star catalog to provide precise pointing of instruments. Optical sensors can be used on Space Station to keep track of other nearby objects such as incoming orbiters. Co-orbiting platforms may also use optical sensors for their GN&C needs.

Because optical sensors will provide attitude information to Space Station, it would be useful if attitude rate could be derived from these sensors. An optical rate sensor is currently being developed in the Avionic Systems Division at Johnson Space Center. The sensor will look at stars to obtain both attitude information and to derive attitude rate. A Videk Megapixel camera is supported by a Compac Deskpro 386 for data capture and processing. The Videk camera uses a Kodak charge coupled device (CCD) array. This array is made up of 1320 horizontal by 1035 vertical pixels. The pixels are 6.8 microns square and the array is full fill. The camera is capable of 7 still frames per second and has a grey scale of 256 levels.

RATE PROCESSING ALGORITHMS

Centroiding/Differentiator Approach

The centroids of the star images are obtained for each camera frame. The centroids are in field coordinates which can later be translated to inertial attitude. To obtain rate, the centroids are processed by a discrete time differentiator Previous analysis of this problem showed that a nonrecursive differentiator with low systematic errors to meet Space Station requirements had the following parameters:

100 Hertz sample rate decimated to 10 Hertz
Input bandlimited to 0.5 Hertz
Filter order of 127
Linear frequency response to 0.01 Hertz [1]

This nonrecursive differentiator has several drawbacks; the filter order is
high and, therefore, the delay in obtaining a an estimate of the derivative for a particular sample point is excessive. Also, the useful frequency range is severely limited compared to the sampling frequency.

In reworking the centroiding/differentiator approach, consideration was given to the type of data processed. Space Station orbital pitch rate is relatively constant at 240 degrees per hour. We want to track small, slow variations to this basic rate while smoothing out high frequency noise. A heuristic design approach was taken in which a nonrecursive differentiator was modified with feedback from the previous outputs. This new, recursive design provides noise reduction and small time delays.

The difference equation for a first order, nonrecursive, backward difference differentiator is given below:

\[ y(n) = x(n) - x(n - 1) \]

This difference equation is modified to provide feedback as follows:

\[ y(n) = K(x(n) - x(n - 1)) + (1 - K)(y(n - 1)) \]

A second order, nonrecursive difference equation is modified in a similar manner:

\[ y(n) = K(x(n) - x(n - 2))/2 + (1 - K)(y(n - 1) + y(n - 2))/2 \]

K is a constant chosen to be between 0 and 1. Values of K close to 1 give greater weight to the inputs; this gives a steady state frequency response closer to the ideal. Values of K close to 0 give greater weight to the previous outputs; this gives more high frequency noise attenuation.

Recursive Differentiator Results

A set of 10,000 random numbers with a Gaussian distribution were processed by the recursive differentiators. The variance of the output was computed, giving the variance reduction factor. It should be noted that the first order, nonrecursive differentiator causes the variance to increase by a factor of two.
Another set of inputs consisting of 100 samples with a constant rate of 1.0, followed by 100 samples with a constant rate of 2.0 was applied to the recursive differentiators. The rise time, in samples, from time of change in the input to the time that the output reaches 90% of its final value was found. The above results are summarized in Table 1 and in Figures 1 and 2. The solid line represents the ideal response; the dotted, dashed and dot-dash curves are for K equal 0.75, 0.50, and 0.25 respectively. Satisfactory results are obtained for a first order recursive differentiator, obviating the need for a second order filter with the added time delay and computational expense of a second order filter.

Image Processing Approach

In the image processing approach, the rate is derived by looking directly at the variation of intensity of the pixel outputs. The output of a CCD array is the two-dimensional, spatially sampled version of the transfer function of a pixel convolved with the input image field [3]. A specialized image field was used where all of the energy of the source is concentrated in the area of a single pixel. If the pixel is moved across the array at a constant rate, an intensity profile for a pixel $I_p(t)$ as a continuous function of time, is given below, and shown in Figure 3, where $R$ is the rate in $t^{-1}$.

$$I_p(t) = \begin{cases} 
I_{\text{max}}Rt & 0 < t < 1/R \\
I_{\text{max}}(2 - Rt) & 1/R < t < 2/R \\
0 & \text{elsewhere}
\end{cases}$$

Note that the slope of the above curve is $I_{\text{max}}R$. The rate cannot be obtained directly from the slope, but must be normalized by the maximum intensity of the image. The system is discrete not only spatially, but also in time since images are processed on a frame by frame basis. Figure 4 shows a set of intensity profiles for 5 adjacent pixels with the image moving at a rate of 0.2930 pixels per frame and a maximum intensity of 1.0000.
Image Processing Rate Algorithm

In the absence of noise, the algorithm used to derive rate directly from variations in pixel intensities is given as follows:

1) Obtain the frame to frame intensity differences for the pixels of interest.
2) Make a histogram of the absolute values of these differences.
3) Find the difference magnitude with the highest frequency of occurrence. This value gives the raw rate which must be normalized.
4) Find the sums of intensities of adjacent pixels taken two at a time for a given frame. Form a collection of these intensity sums for several frames.
5) Make a histogram of these sums. The sum with the highest frequency of occurrence is \( I_{\text{max}} \).
6) Take the raw rate found in step 3) and divide by the maximum intensity, \( I_{\text{max}} \), found in step 5). The result is the derived rate.

In the presence of noise, modifications must be made to steps 3) and 5) above. When the histograms are formed in each of these cases, the peak is found, and the weighted average of that peak and nearby values is computed in order to derive rate in step 6).

Examples

Three examples are presented below with and without noise:

1) The rate is \( R_1 = 0.2930 \) pixels per frame. The maximum intensity is \( I_{\text{max}} = 1.0000 \).
2) The rate is \( R_1 = 0.2930 \) pixels per frame. The maximum intensity is \( I_{\text{max}} = 0.8410 \).
3) The rate is \( R_2 = 0.2464 \) pixels per frame. The maximum intensity is \( I_{\text{max}} = 1.0000 \).

The noise was additive Gaussian random noise with a standard deviation of 0.001. This corresponds to the dark current noise of the CCD camera. The bins for the histograms have a resolution of 0.004 and range from 0.0 to 1.0. This is to approximately correspond with the 256 grey levels. The cumulative sums and differences for 5 pixels were used for a total of 9 frames for each pixel.
The results of these simulations are summarized in Table 2.

CONCLUSIONS

Two algorithms have been developed for obtaining rate information from optical sensors. Both approaches depend on having the camera frames accurately time-tagged. The recursive differentiator gives rapid rise times and significant variance reduction. The usefulness of the approach depends on how accurately the image centroids can be obtained.

The image processing approach has been shown to give good results for the one-dimensional simulations performed. An interesting finding is that noise, of the magnitude expected to be generated by the camera, can actually improve the accuracy of the results. This is because the noise distributes values across several bins in the histogram. The weighted average of the noisy histogram peaks gives sub-bin resolution. The accuracy of the computed rate can be improved by having a greater number of grey levels. A twofold improvement in grey levels can presently be obtained by cooling the CCD array.

The modifications to the algorithm to make it applicable to the two-dimensional case are straightforward. Modifications to allow for an image which is not exactly of one pixel width are also straightforward. Open issues involve optimizing for frame rate, processing time, array size, pixel size, number of grey levels, number of pixels and frames to be processed at one time, and noise performance. The algorithm will soon be applied to data obtained in laboratory simulations of a star moving at a constant rate.
REFERENCES


### TABLE 1

<table>
<thead>
<tr>
<th>ORDER</th>
<th>K</th>
<th>VRF</th>
<th>RISE TIME (samples)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>0.75</td>
<td>0.9059</td>
<td>3</td>
</tr>
<tr>
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<td>0.3351</td>
<td>4</td>
</tr>
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<tr>
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<td>0.75</td>
<td>0.2522</td>
<td>4</td>
</tr>
<tr>
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<td>0.1006</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.0228</td>
<td>10</td>
</tr>
</tbody>
</table>

### TABLE 2

| EXAMPLE | NOISE | ACTUAL $I_{\text{max}}$ | $|\text{DIFFERENCE}|$ | INTENSITY SUMS | DERIVED RATE |
|---------|-------|--------------------------|-----------------|----------------|--------------|
| 1       | NO    | 0.2930 1.0000            | 0.2920          | 1.0000         | 0.2920       |
|         | YES   |                          | 0.2928          | 1.0000         | 0.2928       |
| 2       | NO    | 0.2930 0.8410            | 0.2480          | 0.8400         | 0.2952       |
|         | YES   |                          | 0.2464          | 0.8409         | 0.2930       |
| 3       | NO    | 0.2464 1.0000            | 0.2480          | 1.0000         | 0.2480       |
|         | YES   |                          | 0.2463          | 0.9998         | 0.2463       |
FIGURE 1
First Order Recursive Differentiators

FIGURE 2
Second Order Recursive Differentiators
FIGURE 3

Intensity Profile

Imax

FIGURE 4

Intensity Profiles for 5 Pixels: \( R = 0.2930 \) pixels/frame; \( Imax = 1.0 \)