

## Detecting Perceptual Groupings in Textures By Continuity Considerations

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**Abstract.** This paper presents a generalization of the second derivative of a Gaussian  $D^2G$  operator to apply to problems of perceptual organization involving textures. Extensions to other problems of perceptual organization are evident and a new research direction can be established. The technique presented is theoretically pleasing since it has the potential of unifying the entire area of image segmentation under the mathematical notion of continuity and presents a single algorithm to form perceptual groupings where many algorithms existed previously. The eventual impact on both the approach and technique of image processing segmentation operations could be significant.

### Introduction

The notion of "continuity" provides a unifying framework for representing and solving the problems of perceptual organization. For example, the vertical lines in Figure 1 tend to be organized into three distinct groups based on proximity. From this example, one can infer the existence of a distance threshold among the lines. Lines whose spatial neighbors are within the distance threshold maintain continuity of distance relations and are grouped together. Lines which exceed the threshold cause a discontinuity in distance relationships and form another grouping. Hence, a notion of continuity/discontinuity can provide a convenient concept for formalizing the language and mathematical treatment of perceptual organization.

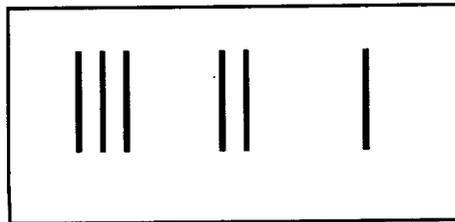


Figure 1. Continuity by proximity.

More generally, given a "space", the notion of continuity depends on a definition of a distance metric and "smoothness" over that space related to the distance metric. Discontinuities occur at places within the space which are not "smooth enough". Perceptual groups are homogeneous precisely because they are "continuous" with respect to some grouping property. In short, the elements grouped together may differ in exact detail from one another but only within limits. Thus, the goal perceptual organization is to locate discontinuities among the perceptual elements and, by doing so, isolate "continuous" groups of elements.

Unfortunately, as the paragraphs above imply, an operational definition of "continuity" can be elusive. Natural language lacks the precision to adequately define "enough" or "too much".

Euclidean geometry, on the other hand, can be too precise in characterizing the concerns of continuity and thus result in large descriptions consisting of a superposition of special cases.

This paper presents several problems of perceptual organization using a simple texture to illustrate the problems of determining continuity. The Laplacian of the Gaussian convolution operator  $D^2G$  is generalized to detect discontinuities within this context and thus form perceptual groupings.

### Perceptual Organization in Textures

Textures may be defined as a regularity of the spatial distribution of texture elements (or texels). For the sake of illustration, assume the texels are a short, equal length line segments, each associated with an angle it makes with an imaginary horizontal line oriented from the left of the image to the right. Consider the texture of the "wavy plane" in Figure 2. The texels are the line segments and their spatial distribution defines a specific texture. Textures may also define perceptual grouping: Places where the texels do not vary "smoothly" define a discontinuity and may indicate perceptual groupings.

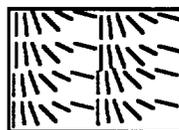


Figure 2. Lined Texture.

Notice how the perceptual groups differentiated by continuity of texel angle emerge from Figure 2. In some cases, the change among the texels is not drastic enough to cause a "discontinuity". However, in other cases, a group of texels will vary enough from their neighbors to form a new perceptual group. Even from this simple example, one can experience the detection of discontinuities among texel groups.

Clearly, the notion of continuity offers a useful conceptual framework for discussing perceptual grouping. However, to be applied, we must formalize the detection of texture discontinuities within a mathematical framework and derive an algorithm from the mathematical theory. This is done in the next section.

### Detecting Discontinuities

The literature of edge detection has documented many techniques for discontinuity detection with respect to intensity values[2,3]. A line is defined by its edge points which, in turn, lie on a "steep change" in intensity among the edge point and some of its neighbors. In fact, the line itself is often modeled as a step function and edge points detected by taking spatial derivatives in order to locate the maxima of the step function. The maxima of the step function is located by locating places in the image where second spatial derivative crosses zero.

One theoretically compelling technique to emerge from this approach has been the Laplacian of the Gaussian convolution operator  $D^2G$  [4]. The motivation for this operator is twofold: convolving the image with a Gaussian lessens intensity "noise" and a critical point (maximum or minimum) of the edge's step function will be found wherever the second spatial derivative (Laplacian) crosses zero. Furthermore, the  $D^2G$  is the least complex operator which is rotationally symmetric. In other words, the Gaussian convolution lessens the overall effect of relatively isolated intensity changes while the spatial Laplacian indicates areas where the intensity values change the "fastest". Because the operator is rotationally symmetric, edges from any angle are detectable. These areas of maximal change are prime candidates for "discontinuities". The theory of the  $D^2G$  operator is explained in Marr.

However, the  $D^2G$  operator has been applied only to detecting discontinuities in intensity values.  $D^2G$  could be generalized easily if one can find a way to transform other perceptual grouping cues into intensity values and then apply the  $D^2G$  operator to the resulting transform image. This was the approach taken for texture. From the examples presented, one can readily see that different textures have different properties and more complicated definitions of continuity. Nevertheless, once the texels are transformed to intensities, the  $D^2G$  operator can locate where the property changes the fastest. This then will locate a candidate for a discontinuity.

For illustration purposes, uniformly sized simple texels were selected and distinguished only by the angle each makes with a horizontal reference line placed at the bottom of the texel. For the sake of simplicity, a simple 10 digit code was used. Figure 3 illustrates this. For example a line with two vertical texels and two horizontal texels would be encoded as 5,5,0,0. Any other coding scheme which defines a metric space over the textures is acceptable. The metric space restriction is discussed below.

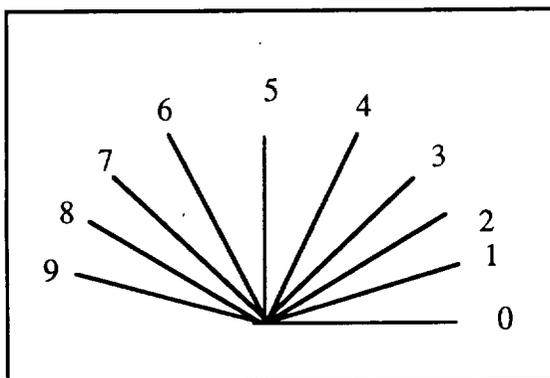


Figure 3. Texel properties.

There are two challenges in applying the  $D^2G$  operator to an image consisting of these texels: first, how to transform texels to intensity values associated with their individual texel distinctions yet indicative of the spatial distribution of texels, and, second, how to associate the transformed intensity value in the transform image to a texel in the original image? The texels were transformed into intensity values by mapping the texel's angle (0 - 180 degrees) to an intensity by using its code (0..9). Thus texels with similar angles have similar intensities. The second

problem, that of associating a texel to a transformed image is handled by making the transformed image's pixels represent a polygon which covers exactly one texel. In effect, the original image is overlaid with a grid of polygons or "tiled". The original image must be divided into tiles such that every texel is contained in exactly one section and that every pixel in the original image is accounted for by exactly one tile.

Once this tiling is accomplished, the texel must be transformed to an intensity value which satisfies the axioms of a metric space [1]. The metric space restriction insures that the distance metric between texels is continuous and that two texels that are far away in metric value are indeed different. For example, if the metric defined above had been extended to include a horizontal texel "10", a "0" and "10" would be far apart in the metric space but very close (indistinguishable!) visually. In short, the metric space restriction allows us to ignore "wraparound" phenomena; i.e. two objects deemed "far apart" by the metric are actually very close together because the objects are about to disappear from one end of the space and appear on the other end.

The transformed image is convolved with the  $D^2G$  operator and the zero crossings are noted: these candidates for the texel angle discontinuities. Since the mapping between the transformed image resolution and original image is uniform, the corresponding image pixels can be identified as candidate points of discontinuity.

## Results

This section presents texture images, their corresponding transform images, and the output of the transform image convolved with a  $D^2G$  operator with zero crossings marked. These zero crossings are candidates for discontinuities and hence, perceptual groupings. Space limitations restrict a full treatment of the experiments as well as the details of the  $D^2G$  operator. The standard deviation of the Gaussian used was three pixels; this parameter proved sufficient to detect sharp discontinuities on the simple textures used. Given below are the results of a simple texture. This example is intended to clearly demonstrate the technique and is not intended to limit the range of spatial scales or texture types amenable to processing.

Figure 4 shows a simple texture image with uniform length texels and Figure 5 shows the transformed intensity image. Finally, Figure 6 shows the results of convolving the transformed intensity image with the  $D^2G$  operator. Note the sharp lines locating where the texture became discontinuous or changed most rapidly.

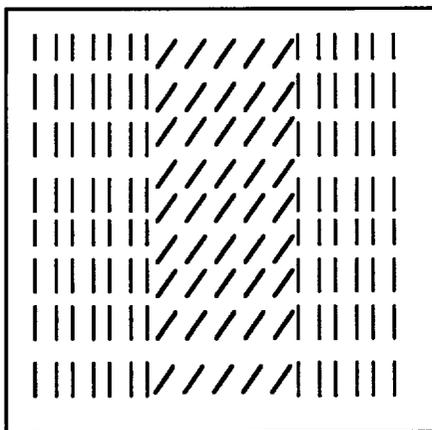


Figure 4. Texture Image.

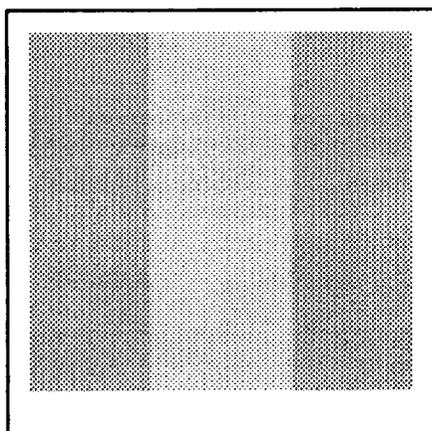


Figure 5. Transformed Intensity Image of Texture Image.

In Figure 6 the lines formed by the zero crossings indicate the distinct perceptual groupings. Note how the groupings formed by the zero crossing curves closely parallel intuition. Other, more comprehensive, experiments support these results and suggest the technique handles a variety of textures and spatial scales.

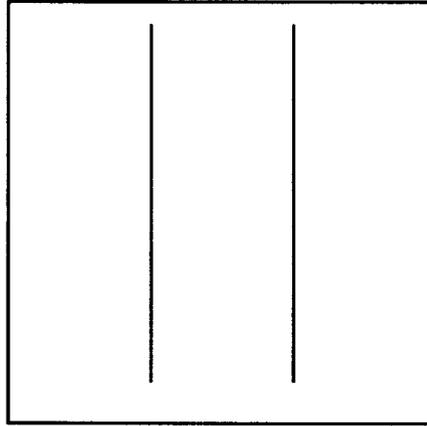


Figure 6. Zero Crossings of the Convolution.

## Conclusions

This section suggests how the  $D^2G$  operator can be extended to apply to distance or any other criteria which exhibits a "continuity". The experiments with the simple texture images suggest that continuity considerations can play a key role in determining perceptual groupings. First the property in question must be represented by an intensity value. This can be done by defining a metric space on the property and using the metric as an intensity value. There are no restrictions on the dimension of the metric. For example, there is no reason to prohibit a two-dimensional metric combining texel length and angle. Discontinuities could then represent breaks in texel lengths, texel angles, or both. Perceptual groupings can be formed from either of these and very strong groups can occur where both of these properties change the fastest. Research is already investigating multi-dimensional properties.

The resulting image is convolved with the  $D^2G$  operator and zero crossings noted. The  $D^2G$  operator is well-suited to detecting the discontinuities on various scales and is rotationally symmetric. Finally, the zero crossings represent where the property in question may be discontinuous. In fact, some psychophysical evidence suggests that the eye/brain may implement some form of  $D^2G$  convolution for the location of discontinuities in intensity value[3, 4]. It would certainly be theoretically pleasing to discover that discontinuities in other properties such as texture are located in essentially the same way.

## References

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