

**RESOLUTION OF SEVEN-AXIS MANIPULATOR REDUNDANCY-  
A HEURISTIC ISSUE**

I. Chen

Computer and Information Sciences Department,  
Alabama A & M University,  
Normal, AL 35762, U.S.A.

**ABSTRACT**

This paper presents an approach for the resolution of the redundancy of a seven-axis manipulator arm from the AI and expert systems point of view. This approach is heuristic, analytical, and globally resolves the redundancy at the position level. When compared with other approaches, this approach has several improved performance capabilities, including singularity avoidance, repeatability, stability, and simplicity.

**1. INTRODUCTION**

With an addition of one more degree of freedom (d.o.f.), a yaw motion of the forearm, to the typical six d.o.f. articulated arms, the shoulder of the three-link manipulator will have pitch, yaw, and roll motions (Figure 1). These seven-axis articulated manipulator arms when compared with the typical six d.o.f. robot arms may have many advantages, including flexibility, manipulability, obstacle avoidance, singularity avoidance, stability, and optimal control. In addition, because of the similarity to human arm configuration, the seven-axis, three-link manipulator arm is the best candidate for the master-slave teleoperated robot useful in hazardous environments. However, the resolution of the kinematics, control, and dynamics with redundancy is not an easy task. With the redundant d.o.f., there will be an infinite number of arm configurations kinematically for each desired hand position. The motion that maximizes a specific performance index is the optimal motion for that special condition. However, the optimal solutions are not easily accessible due to a highly nonlinear relationship between the joint angle space and the hand position. Because of the complex nature of this problem, each approach seems to have its own drawbacks and limitations. In order to resolve this redundancy with the already complicated problem, the author employed ([5], [6]) human arm motion heuristics to analytically derive the

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inverse kinematics (see the Appendix). The purpose of this paper is to present this heuristic concept and to compare the approach with other existing methods.

## 2. STATEMENT OF THE APPROACH

By observing the human arm motions, two heuristic rules are concluded:

- 1) The travel distance of the wrist joint from a hand position to a new hand position should be a minimum for each arm motion.
- 2) The travel distance of the elbow joint from an arm position to a new arm position should be a minimum for each arm motion.

In addition, there are two meta-level rules:

- 1) The first rule can be applied only when the second rule is satisfied.
- 2) If the movement of the robot arm violates the joint limit constraints or obstacle constraints, then a configuration which satisfies the above two domain specific rules and the constraints is the solution.

The first two rules state the fact that both the joint travel distances have to be minimized in order to obtain the kinematic solution. The first meta rule defines the relationship between these two domain-specific rules. The first rule is subject to the constraint of the second rule, and the first rule will be applied only when the second rule is satisfied. The purpose is to minimize the use of the lower arm as much as possible since each movement of the lower arm causes the movement of the forearm and hand together. This consumes more energy because a bigger moment of inertia is involved in the movement. The same principle is also applied to the movement of forearm and hand. In other words, the travel distance of the wrist joint should also be maintained to a minimum to ensure the minimization of the energy consumption. The main idea is to minimize the lower link of the manipulator since the motion of the lower link causes movement of all the upper links together. A bigger movement of the lower link will in turn consume more energy because of the bigger moment of inertia involved in the motion. The motion with the least amount of energy consumption is the most comfortable to the body. Besides, for a given kinetic energy, the optimal arm motion which obeys these rules is the minimum-time motion and is the fastest way to reach the new hand position. The second meta rule governs the condition whenever the arm motions are prohibited because of the violation of the joint limit constraints or obstacle constraints. Although this strategy has not yet been developed, a suboptimal configuration, which relaxes the constraint free minimum-distance solution according to the imposed constraints, is accessible. Note that from the point of view of AI and expert systems the second meta rule is of second-order since it governs all the rest of the

rules, including the first meta rule. This relaxation strategy may be an iterative process by relaxing the related joint angles to the subsequent neighbors according to a prescribed small joint interval.

The motion control of this seven-axis manipulator arm may be restated as follows:

Minimize the following performance index,

$$J = K_1 * D_1^2 + k_2 * D_2^2 \quad (1)$$

subject to the kinematic equations of the manipulator arm and the joint limit and obstacle avoidance constraints.

Here,  $D_1$  is the elbow joint travel distance and  $D_2$  is the wrist joint travel distance.  $K_1$  and  $K_2$  are the corresponding weighting factors with  $K_1 \gg k_2$  to make sure that the minimization of the travel distance of the elbow joint  $D_1$  before minimizing the travel joint distance of the wrist,  $D_2$ . The squares of  $D_1$  and  $D_2$  assure that  $D_1$  and  $D_2$  will be positive throughout the converging process.

### 3. COMPARISONS

This approach is compared to other approaches in the following manner:

#### 3.1 INVERSE KINEMATIC METHOD VS. RESOLVED MOTION METHOD

Research has been carried out on the control of redundant arms mostly through a pseudoinverse matrix, also known as the Moore - Penrose generalized inverse. The instantaneous joint displacements are computed from the joint velocities by using the pseudoinverse  $J^+$  of the Jacobian matrix  $J$ . For a given hand position, the resolved motion methods cannot provide the corresponding joint angles. This implies that the resolved motion technique cannot directly map the workspace to the joint angle space of the arms. In other words, the approach is numerical and not analytical. Because of the instantaneous resolution of the redundancy, this type of technique also inherits other drawbacks (to be discussed later). The proposed approach has advantage over the resolved motion methods by resolving the redundancy analytically and globally ( not incrementally ). Benati's approach [2] is recursive, partly analytical, and partly numerical. The application of the heuristics rules to the derivation of the inverse kinematics of a seven-axis manipulator arm is shown in the Appendix.

#### 3.2 RESOLUTION LEVEL

This approach resolves the redundancy at the position level rather than resolving the redundancy at the velocity level or at

the acceleration level.

The resolved motion method has many interesting characteristics [4]. A desired performance criterion function can be incorporated in the general solution for the avoidance of joint limits ([13], [17]), the improvement of the repeatability for repeated end effector motions ([1], [11]), and the obstacle avoidance in work space (e.g. [7], [12], [14], [17]). Another advantage is the least square property [3] of the pseudoinverse method which minimizes the sum of squares of joint velocities which in turn approximately minimizes kinetic energy. However, this approach has an intrinsic inaccuracy because of the error accumulation of the linear approximation of the Jacobian matrix. Therefore, lack of repeatability is the major drawback of this method (e.g. [4], [8]). The approach involves the instantaneous resolution of the redundancy at the velocity level where the sum of the squares of the joint velocities is minimized. This means that the kinetic energy is approximately minimized. This method kinematically resolves the redundancy at the velocity level. Chang [4] proposed a method called the criterion function method which has resulted in improved repeatability for end effector motions. Many researchers (e.g. [8], [10], [16]) extended the kinematic resolution method from the velocity level to the acceleration level by incorporating the generalized inverse into dynamics. While this method resolves the redundancy at the kinetic level, it may lead to stability problems. Local tampering with the energetics of movement has led to global disaster [8].

### 3.3 SINGULARITIES AND AVOIDANCE

Some arm configurations are singular where the joint angles or instantaneous joint velocities are impossible to realize. For the resolved motion approach or other approaches, the joint velocities for some manipulator configurations sometimes approach mathematical infinity in the derivation. The joint velocities that close to the singular points are also too large and are very difficult to realize. This establishes forbidden regions around the singular points. A substantial fraction of the workspace is lost and the degree of freedom of the manipulator is functionally reduced ([9], [15]).

Unlike other approaches, no singularity and singularity avoidance consideration are necessary for this approach since the resolution of redundancy is at the position level, and joint angles are determined globally not incrementally (Refer to the Appendix.) This approach is a geometrical approach where only three inverse matrices appear in the derivation, two rotational inverse matrices,  $R_z^{-1}$  and  $R_x^{-1}$ , and one translation inverse matrix,  $T^{-1}$ . These three are nonsingular matrices since their determinants are non-zero.

That no singularity exists in this approach implies improvement of the performance capabilities of the manipulator arm because singular points functionally reduce manipulator degree of

freedom. Besides, there is no need to consider the singularity avoidance in the control or arm movement planning.

### **3.4 COMPLICATION OF APPROACH**

Another major attraction of this approach is the simplicity of the derivation (Refer to the Appendix.) The resolution of the redundancy at the position level does not require incorporation of the dynamics, pseudoinverse, etc. in the derivation. Compared to other approaches, the derivation is drastically reduced. In addition, this approach is analytical and geometrical. This means that the resolution of the redundancy is not incremental or local, but global. Thus, control and trajectory planning of the manipulator are much more simplified than those of the other approaches.

### **3.5 OPTIMALITY**

This approach geometrically applies the heuristic rules for the resolution of the redundancy. For the resolved motion method, the kinetic energy is approximately minimized; however, it has the drawback of poor repeatability. Other extended methods of this type which attempt to realize the energy minimization have other problems also, such as stability. It would be a significant research task to verify the assertion that heuristic application of these rules does result in minimization of the energy consumption for manipulator arm movements.

## **4. SUMMARY AND CONCLUSION**

Resolution of seven-axis manipulator redundancy is a very interesting and important research topic. Unlike most of the approaches, the redundancy is resolved through the implementation of heuristic rules in the derivation. It seems to the author that this approach resolves the very complicated seven-axis redundancy very easily and has no known drawbacks. In addition to this, although yet to be verified theoretically, this approach may have the most desirable feature of true minimization of the kinetic energy. Since the resolution of the redundancy is at the position level, the corresponding joint angles are derived analytically and geometrically. Thus, the resulting joint angles are determined globally not incrementally. This implies that, unlike other approaches, the joint angles for a given hand position are given directly or the joint angle space is a direct mapping from the workspace. Another very significant advantage of the approach is that there is no singular point in the derivation. Therefore, compared with other approaches, this approach has a greater degree of freedom functionally and has no singularity avoidance problem in the workspace.

## **5. APPENDIX**

Since the forward kinematics for redundant arms is always feasible, these heuristics are applied only to the derivation of inverse kinematics for the seven-axis anthropomorphic arms.

According to the arm motion characteristics as described above, the inverse kinematics of a two-link manipulator will be derived. The two-link strategy for lower arm and forearm is then extended to a three-link strategy with hand included ([5], [6]).

### 5.1 THE TWO-LINK STRATEGY

According to Figure 2, the arm positions are represented by the following three points: the origin of the coordinates  $P_0(0,0,0)$ ,  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$ . The point  $P_2'(x_2', y_2', z_2')$  represents the desired position. When  $P_2$  reaches  $P_2'$ , the collection of all possible locations of  $P_1$  forms a circle (denoted by  $C$ ) with center at  $P_c(x_c, y_c, z_c)$  and radius  $r$ . The objective is to find the point  $P_1^*(x_1^*, y_1^*, z_1^*)$  in  $C$  such that the distance between  $P_1$  and  $P_1^*$  is minimized. Homogeneous coordinates should be employed to simplify the required calculation. First of all,  $P_c P_2'$  coordinates of  $P_1^*$  with respect to the original coordinates.

$$[x_1^*, y_1^*, z_1^*, 1]^t = T^{-1} R_z^{-1} R_x^{-1} [x_1^{\wedge}, y_1^{\wedge}, z_1^{\wedge}, 1]^t$$

### 5.2 THE THREE-LINK EXTENSION

By referring to Figure 3, let  $L_p$  = distance ( $P_0, P_3^*$ ). If  $L_p > l_1 + l_2 + l_3$  or,  $L_p < l_1 - (l_2 + l_3)$  and  $l_1 > l_2 + l_3$ , then there is no solution. Evaluate  $d_1 = \text{ABS}(l_2 - l_3)$ ,  $d_2 = l_2 + l_3$ , and,  $D$  = distance ( $P_1, P_3^*$ ). If  $d_1 \leq D \leq d_2$ , then do not move  $P_1$ , i.e.,  $P_1 = P_1^*$ . Apply the "two-link" method (Section 5.1) to find  $P_2^*$ . If  $D < d_1$ , then apply the "two-link" method with length of link<sub>1</sub> =  $l_1$  and length of link<sub>2</sub> =  $\text{ABS}(l_2 - l_3)$  or  $d_1$  to find the new location  $P_1^*$ . If  $D > d_2$ , then apply the "two-link" method with length of link<sub>1</sub> =  $l_1$  and length of link<sub>2</sub> =  $l_2 + l_3$ , or  $d_2$  to find new  $P_1^*$ .

Because the joint limits are not imposed,  $P_1^* P_2^* P_3^*$  is always in a straight line for the last two cases..

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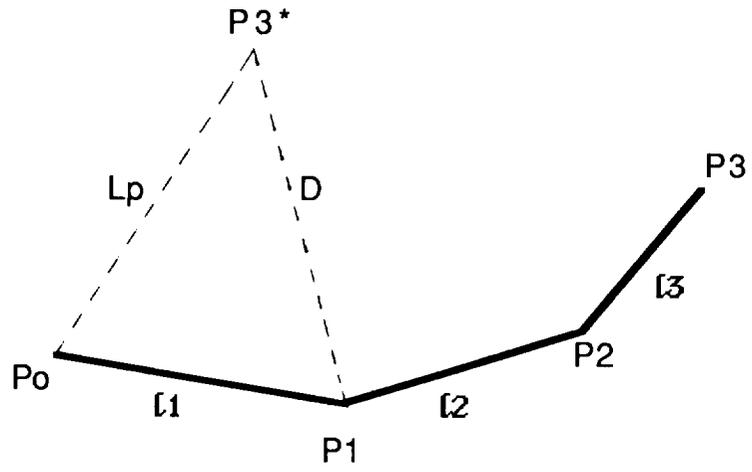


Figure 3.

