Examination of cloud radiance fields derived from satellite observations sometimes indicates the existence of a range of scales over which the statistics of the field are scale invariant. Many methods have been developed to quantify this scaling behavior in geophysics. The usefulness of such techniques depends both on the physics of the process being robust over a wide range of scales and on the availability of high resolution, low noise observations over these scales. The present paper applies these techniques (area perimeter relation, distribution of areas, estimation of the capacity, $d_0$, through box counting, correlation exponent ) to the high resolution satellite data taken during the FIRE experiment and provides initial estimates of the quality of data required by analyzing simple sets. We proceed by contrasting the results of the observed fields with those of images of objects with known characteristics (e.g. dimension) where the details of the constructed image simulate current observational limits. Throughout we shall speak of cloud elements and cloud boundaries; it should be clearly understood that by this we mean structures in the radiance field: all the boundaries considered here are defined by simple threshold arguments.

DATA

The satellite images considered here are Spot images (see Figure 1), covering approximately 60km by 70km, which were taken during the FIRE experiment on stratocumulus on June 7th, 8th and 19th in the panchromatic mode (10m resolution at visible wavelengths) and the multispectral mode (20m resolution at visible and near infrared wavelengths). The results presented here are for the 10m resolution images unless otherwise stated. Two of the scenes, those from the 7th and the 8th, show overcast conditions while the scene on the 19th is almost clear with only small cumulus clouds present. For each image, several thresholds are chosen and the corresponding binary images constructed. In order to test the reliability of the methods defined below, we construct similar binary images of known scaling structure. Our aim is to determine the effect of reasonable amounts of variation in the large scale structure; here we report initial studies considering only the observational effects on the common Koch island. This set is constructed by repeated replacement of a sample pattern at smaller and smaller scales and is shown in Figure 2. In this case we know the boundary is a homogenous fractal with dimension(s) equal to $\log(4)/\log(3)$.

INDIVIDUAL ELEMENT ANALYSIS

Historically, the first approach to analyzing the scaling behavior in cloud boundaries (Lovejoy, 1982) was through techniques which quantified the properties of individual cloud elements via area-perimeter studies. This approach is very appealing and we begin with it.

1) Area-Perimeter Studies

When self-similar objects are viewed under increasing magnification, details in the boundaries appear in a rate determined by the dimension of the boundary. In a similar way, one expects a relation between the area and perimeter of a collection of similar objects of different size all observed at the same resolution (Mandelbrot , 1982). In an image the area is simply defined as the number of pixel forming the cloud; the perimeter may be defined in one of two ways either (1) as the number of pixels on the cloud boundary, or (2) as the number of pixel edges on the cloud boundary.

To estimate the effects of finite resolution in the most optimal case, Koch islands of various sizes and orientations relative to a $4096 \times 4096$ grid were constructed and their areas and perimeters
were computed. The results are shown on a log-log plot of the square root of area versus the perimeter (Figure 3). The linear relation is evident. Note the variability due to the grid; this would be the expected error if every cloud had exactly the same macroscopic structure; it is, effectively, the smallest possible error level. A more complete study using clouds with homogeneous boundary dimensions but different macroscopic characteristics will be presented in a more detailed report. As expected the "scatter" in this area-perimeter graph is greater due to macroscopic structure in the observations.

The area and perimeter of the cloud elements from the SPOT images were computed for a variety of different subscenes of size 1024*1024 to 6000*7000 pixels. To avoid finite length scale effects clouds smaller than 16 pixels in area were not included in the analysis. Figure 3 shows the observed power law relation between these area and perimeters. The exponents found are almost independent of the threshold and of the size of the sub-scene and were generally similar for both the 10m and the 20m resolution images. For the stratocumulus deck this exponent is between (1.35-1.40); for the fair weather cumulus (19th) this exponent is between 1.30-1.32. A similar behavior was found by Cahalan and by Welch et al. using Landsat images. We do not, however, see the increase in these exponents with increasing threshold observed by Cahalan.

The variability found for clouds is larger than that for the Koch islands. This is in part due to the different macroscopic structure of the various cloud elements. We are also concerned that high ellipticity clouds are selectively removed from the sample because they are more likely to cross the boundary of the image. Such a bias could result in lowering the spread of observed areas for a given perimeter and producing a misleading result.

Using definition (2) for the perimeter results in changes as large as 0.8 in the estimated value of the power law exponent. This change remain if the inner cutoff (i.e. the size of the smallest clouds taken included in the estimate) is changed, such an effect does not occur in the scaling of the Koch island and indicate the presence of macroscopic effects.

i) The distribution of observed areas

For the SPOT scenes, taking reasonable thresholds, our observations are in agreement with those of Cahalan (1988) and Welch et al.(1988). Specifically, we observe that the rate at which the number of cells of a given surface area decreases with increasing surface area is well approximated by a power law over a range of scales compatible with the scene size. This result is clear on the 1024*1024 scenes but not so evident from the 2048 scene compare to the 6000 scene. Again in agreement with Cahalan, the fair weather cumulus fields appear, in general, less fragmented (exponent value smaller than 1) than the stratocumulus fields (exponent value larger than 1). The results obtain from the 3 simultaneous 20m resolution scenes appear quite consistent while they exist some differences between these results and those obtained with the 10m resolution scene. The extend to which these differences are due to resolution, threshold, and changes in cloud properties is not certain; however, resolution effect seems to dominate.

FULL FIELD ANALYSIS

Both the strength and the weakness of the area-perimeter method is that it does not consider the interaction of the different elements which compose the field. An alternative approach to individual element analysis is to consider the scaling properties of the boundary set of the entire field as a whole. Once the behavior of the full field is considered, the potential complexity of the structures observed increases greatly. As pointed out by Schertzer and Lovejoy(1989), the disparity in the scaling exponents reported by different groups may be due to differences in the absolute calibration and resolution of the instruments involved. Given a gridded field of fractal boundaries, it is not yet clear how much data is required to estimate the dimension. The range of scales, and hence the data requirements, will clearly depend on the complexity of the boundary. Below we give an indication of data range required by analysing a single Koch curve.
For the boundary sets, the estimated capacity or fractal dimension, $d_0$, is obtained by assuming that the number of boxes (aligned with the grid) required to cover the set is related to their size (side length) by a power law. The capacity is the exponent of this power law. When dealing with finitely resolved sets, it is useful to consider the variations of this exponent with length scale. We denote this quantity $d_0(I)$; $d_0$ is then the limiting value of $d_0(I)$ as $I$ tends to zero.

Figure 5 shows this function for a single Koch island which fills the initial frame. Note the slow transition to the asymptotic value; effectively scales greater than $2^{-6}$ times (or however many data points are bad) the size of the element must be treated as a transition zone. This makes the analysis of a field composed of a variety of distinct elements of various sizes difficult. For the SPOT images, for the cloud boundaries, the same estimation of $d_0$ has been done for both 1024, 2048 and 4096 square scenes (Fig 5). The continuity of the boundary on the grid can induce a value close to 1 for the smallest scales while the outer effects produce $d_0(I)=2$ for the largest. The lack of a range of scales with a flat plateau makes it impossible to reliably estimate $d_0$ from these curves. We note however, that near the smallest scales the slope is roughly independent of the threshold; as the sub-scene size increases this range of this scaling increases. This indicates that the technique might converge if observations over a slightly longer range of scales were available.

**POINT DISTRIBUTIONS: CORRELATION INTEGRAL**

The correlation integral (Grassberger and Procaccia, 1983) has become a standard measure of the geometry of scaling point sets. This quantity has been computed for the boundary sets. The integral is defined as

$$C_2(I) = \frac{\text{Number of pairs of points separated by less than } I}{\text{Total Number of Pairs of points}}.$$

When $C_2$ is a power law in $I$, $d_2$ quantifies this scaling in the limit of zero length scales. As with $d_0(I)$ we consider $d_2(I)$, the local slope of $C_2$, as a function of $I$.

For the cloud boundaries, at the smallest scales $d_2(I)$ shows variability due to finite (quantized) length of the smallest scales used, while at large scales finite size effects bias $d_2(I)$ to smaller values (see Smith, 1988). At intermediate length scales, some sub-scenes (and thresholds) show a region in which $d_2(I)$ varies around a mean value. In general, the mean value decreases as the threshold increases. However, this variation around a mean value is not always present, and increasing the size of the image considered does not always resolve this problem. In general, increasing the true area considered is not expected to clarify the analysis if it results in the inclusion of clouds with different scaling properties (or, for example, a cloud free region!). One method for improving the statistics is to analyse several images of similar radiation fields, for example of the same region at the same time of day on different days. Cumulating the data in this way (instead of increasing the area size) appears to yield good results. We are investigating whether this approach will make it possible to use observations with lower spatial resolution data sets which are extended in time.

**CONCLUSION**

We have quantified the scaling behavior of the terrestrial radiance field during the FIRE experiment using observations from the SPOT satellite, noting the constraints and uncertainty imposed by the range of scales available in the data set. SPOT provides data at 10 meter resolution with an outer length scale of 100 km - the longest scaling range yet considered from a single instrument. Nevertheless the effects of a finite range of length scales and the outer boundary cutoff are clearly visible in our results. An idea of the quantity and quality of data required to ascertain reliable estimates from grid point data has been determined by examining the scaling of simple sets. Finally, the correlation exponent has been computed for the radiation field data with promising results.
References


Fig 1. 7th of July SPOT scene

Fig 2. Koch Island
Fig 3: perimeter versus area for Koch islands (x symbol), for clouds extracted from a 4096 square scene on the 7th of July (+ symbol). For clarity, the Koch island results are displaced in x (area) by a factor of 2.

Fig 4: area distribution for a 4096 square scene on the 7th of July (+ curve) and the 3 corresponding 20m resolution scene (x, o and . symbols).

Fig 5: d0 versus scale eps for two 4096 square Koch islands (+ and x symbols), for one 4096 square Spot scene on the 7th of July for 3 different thresholds (o, ., . symbols).

Fig 6: d2 versus scale for a cloud scene on the 8th of July for 4 different thresholds.