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Probabilistic Model for Fracture Mechanics Service Life Analysis

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-- Abstract --

The service longevity of complex propulsion systems -- such as the SSME -- can be at risk from several competing failure modes. Conventional life assessment practice focuses upon the most severely life-limited feature of a given component, even though there may be other, less severe, potential failure locations. Primary, secondary, tertiary failure modes, as well as their associated probabilities, must also be considered. Furthermore, these probabilities are functions of accumulated service time. Thus a component may not always succumb to the most severe, or even the most probable failure mode.

Propulsion system longevity must be assessed by considering simultaneously the actions of, and interactions among, life-limiting influences. These include, but are not limited to, high frequency fatigue (HFF), low cycle fatigue (LCF) and subsequent crack propagation, thermal and acoustic loadings, and the influence of less-than-ideal nondestructive evaluation (NDE). This paper provides an outline for a probabilistic model for service life analysis, and reports on progress towards its implementation. The work is being performed by Pratt & Whitney under NAS8-36901.

Introduction and Background

Present fracture mechanics analyses for SSME critical structural components may underestimate life by assuming the worst-case conditions to define single values of life-controlling parameters even though these parameters are subject to statistical variations. The probability of occurrence for any individual worst-case condition is very low, while that of a combination of worst-case conditions is infinitesimal. These life underestimates can result in inefficient use of material and/or excessive component weight causing a reduced payload capacity and an increased operations cost. SSME life analyses may also overestimate life by the assumption of greater material/structural capabilities and/or less severe operating conditions.

Advancing technology is imposing increasing demands on structural analysis methods. The previously acceptable technique of assuming worst-case conditions is no longer a viable method for analyzing SSME operating systems. A more accurate life assessment technique is needed which will account for errors in life estimates and also provide a tool for statistically assessing the level of risk created by engineering decisions involved in defining a system design.

The objective of this program is to develop a computer code for performing fracture mechanics calculations which consider distributions of major SSME life-controlling parameters, rather than the traditional single-valued, worst-case, estimates.

Probabilistic Modeling

The Space Shuttle Probabilistic Optimization Code, SSPOC, has as its objective estimating the uncertainty in SSME component cyclic longevity which necessarily results from the engineering uncertainties in life-controlling parameters. Only by understanding the probabilistic behavior of these components can realistic mission risk assessments be undertaken.

Among the necessary uncertainties which must be addressed by a probabilistic simulation are uncertainties in:

1. Initial material quality (IMQ)
2. Stress analysis variability
3. Crack initiation life (Low Cycle Fatigue)
4. Nondestructive evaluation (NDE)
5. Crack size vs. propagation life (a vs. N)
6. Crack initiation life (High Cycle Fatigue)
7. Mission severity

The following discussion illustrates how statistical uncertainties can be modeled algorithmically for four major elements: (1) LCF variability; (2) stress variability; (3) NDE variability; and (4) fracture mechanics crack propagation (a vs. N) variability. The examples are greatly simplified for expository purposes, while the actual SSPOC computer code is necessarily more complex. The underlying ideas are , however, very similar.

Modeling LCF Probabilistically

First consider that fatigue, even under well-controlled laboratory conditions, is a stochastic process. That is: even when stress, temperature, loading frequency and stress ratio, are known "exactly", the resulting fatigue life displays considerable variability. In an actual component these, and other, life-controlling parameters are ***not*** known exactly. Rather, parameters must be described as statistical distributions of possible values, some values being more likely than others. It is conventional statistical practice to describe these effects using some engineering model for the expected, or mean, behavior ***plus*** an error term to account for uncertainty in outcome. As an example, the expected fatigue life, N_f at a given stress, s, might be modeled using a simple inverse relationship between stress and the logarithm of life:

$$\log N_f = a + b(1/s) \qquad [1]$$

where a and b are s-N model parameters.

The uncertainty is then treated as a normal distribution of logarithmic errors, with zero mean, and some specified variance, σ^2 . (The zero mean implies that the model is expected to be correct, on the average.)

The resulting log life would then be:

$$\log N_f = a + b(1/s) + \varepsilon_i \qquad [2]$$

where ε_i is the normally distributed uncertainty. The following segments of pseudocode illustrate the implementation of these ideas.

For a given stress (and temperature, ...) fatigue life can be modeled algorithmically as follows:

Pseudocode

```
PROCEDURE "LIFECALC"
    FOR STRESS = s_i
        BEGIN:
            CALCULATE: expected behavior
                log N_f = a + b(1/s_i)
            CALCULATE: life model random error
                ε_i = N(0, σ_l^2)
            CALCULATE: individual life
                log N_i = N_f + ε_i
                N_i = 10^(log N_i)
                STORE the ith life, N_i, for further consideration
        END:
```

Now consider the introduction of an additional systematic uncertainty. Stress is not known exactly, so it too has an associated -- and of course different -- error structure. (Some individuals are uncomfortable with the word "error", and prefer to use "uncertainty". In this discussion we will use both terms interchangeably).

The uncertainty in exactly knowing stress influences the overall uncertainty in component life. The following segment of pseudocode illustrates how this effect can be modeled mathematically.

Pseudocode

```
PROCEDURE "STRESSCALC"
    FOR DESIGN STRESS = s_d
        BEGIN:
            CALCULATE: expected stress
                s = s_d
            CALCULATE: stress random error
                ε_i = N(0,σ_s^2)
            CALCULATE: individual stress
                s_i = s + ε_i
        END:
```

These two examples have been greatly simplified for purposes of explanation, but they illustrate how the SSPOC code treats both deterministic and random influences in overall component life uncertainty. The next segment of pseudocode shows how these two effects (life uncertainty at a known stress and uncertainty in knowing the stress) can be combined.

Psuedocode

```
PROCEDURE "FATIGUE LIFE"
    BEGIN:
        CALCULATE: individual stress
            CALL PROCEDURE "STRESSCALC"
    (we now have an individual value of s_i)
        CALCULATE: life at s_i
            CALL PROCEDURE "LIFECALC"
    (returns an individual value of N_i)
    END:
```

So far we've calculated a single fatigue initiation life which has been influenced by both stress and life model uncertainties. After accumulating these initiation cycles and exhausting its initiation life, the part will now contain a crack of a given size.

Modeling the Inspection Process

After having initiated a crack, a part does not necessarily immediately fail. Depending on its design it may survive several subsequent missions. (This is especially true of components designed using damage-tolerance concepts). Furthermore, if a component can be inspected between return-to-service intervals, damaged parts can be removed and replaced, and the overall system reliability thereby greatly improved. Inspection too is a stochastic process, and the probability of crack detection (POD) is a function of cracksize (a) (Thompson and Chimenti, editors, 1982-1986).

Recent advances in the application of statistical modeling techniques to NonDestructive Evaluation (NDE) have allowed incorporation of both left- and right-censored observations in modeling the Probability of Detection vs cracksize (POD vs. a) relationship (Annis and Erland, 1987). The log-logistic function is one example of a $POD = f(a)$ model.

$$POD = [e^{\alpha+\beta \ln(a)}]/[1 + e^{\alpha+\beta \ln(a)}] \qquad [3]$$

For a given cracksize, POD can be modeled as follows:

Pseudocode

```
PROCEDURE "NDECALC"
     FOR CRACKSIZE = a_i
          BEGIN:
               CALCULATE: POD_i
                    POD_i = f(a_i)
               INSPECT: inspect the part
                    GENERATE: random P_i uniformly distributed [0,1]
                    COMPARE: individual probability of detection with P_i
                    IF P_i < POD_i THEN retire part
                    ELSE return part to service
          END:
```

To summarize to this point, the pseudocode illustrates modeling the behavior of an individual life-limiting location on a single component: its life to initiate a crack of a given size, and the result of subsequent inspection. The following discussion continues the theme of stochastic modeling from Low Cycle Fatigue (LCF) into the area of crack propagation. The examples are again greatly simplified for expository purposes.

Fracture Mechanics Modeling

Crack propagation also, even under well-controlled laboratory conditions, is a stochastic process. Even when stress, temperature, loading frequency and stress ratio, are known "exactly", the resulting fracture mechanics (FM) life can display variability. As with LCF, in an actual component these life-controlling parameters are ***not*** known exactly. Again, these parameters may be described as statistical distributions of possible values, some values being more likely than others. As a greatly simplified example, the expected stress intensity K at a given stress, s, might be modeled using a simple relationship between stress and the stress intensity:

$$K = s\sqrt{(\pi a)} \qquad [4]$$

where a represents crack size.

(Remember that the ***uncertainty*** in stress was treated as a normal distribution of errors, with zero mean and some specified variance, σ^2 in the preceding discussion.)

Now consider the problem of relating stress intensity to fracture mechanics life. Perhaps the simplest relationship between stress intensity and fracture mechanics life is the Paris equation, a straight line in the $\log(da/dN)$ vs $\log(K)$ plane; that is:

$$\log(da/dN) = b_0 + b_1 \log(K) \qquad [5]$$

where da/dN is crack growth rate and b_0, b_1 are model parameters.

Substituting equation [4] into equation [5] results in a simple linear first order differential equation. Separating variables and integrating gives:

$$N_i = [a_f^{(1-b_1/2)} - a_i^{(1-b_1/2)}]/[b_0 s^{b_1} \pi^{b_1/2}(1 - b_1/2)] \qquad [6]$$

where a_f = final crack size and a_i = initial crack size. (As with LCF modeling, other, more complicated, models may be required in many cases.)

This equation then directly relates stress and fracture mechanics life. For a given component, design calculations can arrive at a nominal, or mean, predicted fracture mechanics life vs. cracksize curve. Then, given the design stress, parameters b_0 and b_1 can be determined.

Even when stress is known "exactly", as it is in laboratory conditions, predicted crack propagation life exhibits some variability, which can be considered as uncertainty in the fracture mechanics life prediction calculation. An analysis and comparison of predicted and actual lives from specimen tests and laboratory component tests may be used to estimate the distribution of Actual/Predicted (observed life divided by calculated life) lives. This uncertainty in AOVRP can be modeled as a normal distribution of logarithmic errors with mean μ and variance σ^2 .

The following pseudocode shows how the individual fracture mechanics life may be determined. Remember that the individual stress, s_i was previously found using "STRESSCALC". Note also that the FM life is directly dependent on applied stress.

Pseudocode

PROCEDURE "FMLIFE"
    FOR APPLIED STRESS = s_i
        BEGIN:
            CALCULATE: expected fracture mechanics life
                $N_i = [a_f^{(1-b_1/2)} - a_i^{(1-b_1/2)}]/[b_0\, s_i^{b_1}\pi^{b_1/2}(1 - b_1/2)]$
            CALCULATE: AOVRP random error due to life prediction uncertainty
                $AOVRP_i = N(\mu, \sigma^2)$
            CALCULATE: individual fracture mechanics life
                $N_i = N_i/AOVRP_i$
        END:

Tying It All Together

So far we've calculated a single fatigue initiation life which has been influenced by both stress and life model uncertainties, the probabilistic outcome of the inspection process, and, assuming the part was returned to service, the fracture mechanics life dependent on cracksize and stress. To examine the behavior of an entire population of components, the procedure (initiate a crack, inspect, return to service) must be repeated many times, but using ***different*** individual errors each time. For example:

Pseudocode

```
PROCEDURE "POPULATION"
    BEGIN:
        FOR N = 1 to 10,000
            CALL PROCEDURE "FATIGUE LIFE"
            CALL PROCEDURE "NDECALC"
            CALL PROCEDURE "FMLIFE"
    END:
```

Each call to "FATIGUE LIFE" returns an individual initiation life, and stores it for further statistical analysis; the calls to "NDECALC" and "FMLIFE" perform a similar function for inspection results and fracture mechanics life. Therefore procedure "POPULATION" has stored 10,000 (or whatever number is appropriate) individual component initiation and fracture lifetimes along with the corresponding inspection outcomes, each having been influenced by the various modeling uncertainties.

Of course there are many other parameters which can affect a component's cyclic longevity. The purpose of the forgoing discussion was to illustrate, in admittedly simplistic terms, how these many interacting life-controlling parameters can be modeled by first considering the individual contributions to overall uncertainty, and then combining these elemental results in a statistically correct form.

Again, it is very important not to lose sight of the goal: to model component lifetime uncertainty. Without special caution it is easy to become so involved in the potential intricacies of modeling an individual effect that the overall goal is compromised.

This paper discusses only the rudimentary aspects of probabilistic life modeling. Any real component may have several life-limiting locations, some of which may interact either simultaneously or sequentially. Intrinsic material quality (microstructural anomalies, poor weldments, etc.) can also influence system longevity. Although beyond the scope of this paper, SSPOC addresses these and other complex life-controlling effects, and can provide valid estimates of space shuttle hardware reliability.

Pratt & Whitney has nearly a decade of experience in probabilistic life assessment for gas turbine engines. As a result of the Retirement for Cause program, life cycle cost savings of $966 million are projected for the USAF F100-PW-100/200 engine systems over the period 1986 to 2005 (Harris, et. al, 1987). Maintenance intervals and risk analyses for the RFC program were based on simulations using the Probabilistic Life Analysis Technique (Watkins & Annis, 1985).

This program will build on P & W experience to develop a stand-alone probabilistic life analysis computer code, refined and tailored for SSME applications. A complete user's manual including tutorial training sessions at NASA-MSFC will be provided. Two SSME life analysis test cases using SSPOC will be completed. The software will then be installed at NASA-MSFC; software installation will be verified by reproducing the SSME life analysis test cases results at NASA-MSFC.

REFERENCES

1. Annis, C. and K. Erland (1987), ***Estimating the Probability of Crack Detection from Data with Right- and Left-Censored Observations***, submitted for publication, and offered as a working document for the Air Force NDE Reliability Team; to be presented at the American Statistical Association Annual Meeting, August 1988.

2. Harris et. al. (1987), ***Engine Component Retirement for Cause***, AFWAL-TR-87-4069, Vols 1, 2, and 3, August, 1987.

3. Thompson and Chimenti, editors (1982-1986) - ***Review of Progress in QUANTITATIVE NONDESTRUCTIVE EVALUATION***, Vol. I (1982), Vols. 2A,B (1983), Vols. 3A,B (1984), Vols. 4A,B (1985), Vols. 5A,B (1986), Plenum Press, New York.

4. Watkins, T. and C. G. Annis, ***Engine Component Retirement for Cause: Probabilistic Life Analysis Technique***,AFWAL-TR-85-4075, June, 1985.

The interested reader may find these texts helpful:

Fishman, George S. (1978) - ***Principles of Discrete Event Simulation***, New York: John Wiley.

Lawless, J.F. (1982) - ***Statistical Models and Methods for Lifetime Data***, New York: John Wiley.