In principle, gravity gradiometers are immune to the effects of acceleration and vibrations. In real instruments, scale factor errors and structural compliance lead to undesired instrument outputs. This paper will describe the instruments and the fundamental sources of the problems, calculate the magnitude of the effects, demonstrate the need for isolation in the Shuttle (indeed, almost any spacecraft), and briefly describe the JPL eddy current isolation technique and its current development status.

The work I am going to report on today is generally in connection with the NASA program on gravity gradiometry and, referring to Figure 1, I'll give a few principles. The arrow symbol indicates an accelerometer and points along the sensitive axis. When you take two accelerometers and separate them on the ends of a rigid bar, you have a gradiometer. There are two ways in which you can build an accelerometer. The first arrangement shown in Figure 1 is a diagonal component gradiometer. Its output axes are in the same direction as the displacement, and so gives one of the diagonal components of the tensor gradient. You can also generate a cross-component device by changing the output axis as in the lower arrangement, and, in fact, instruments of both types exist today, or are under development. That is about all I am going to say about accelerometers. If you want to know more about gradiometers, consult H. J. Paik's paper in this workshop.
ACCELEROMETERS, GRAVIMETERS, and GRADIOMETERS

Figure 1.
I will discuss some of the reasons we are interested in acceleration and vibration. I will start with one of the most serious problems, acceleration. Suppose I built a general gradiometer of the type shown in Figure 2. It does not matter which type I have, and we will suppose there is an external acceleration on the instrument A along this direction and that there is some additional acceleration $\delta$ at the upper position. Moreover, we have a difference in the scale factors of the two accelerometers, given by $\epsilon$. The output volts per input acceleration is off by $\epsilon$ in the second case. The output of an instrument like this is the difference between the two accelerometers. Multiplying this out, and throwing out the second order term, you get the expression in the middle of Figure 2, which is a gradient, $H$, multiplied by the baseline that separates them. So you have a contaminating error $\epsilon A$ which is due to the acceleration field that doesn't really belong in there. This is what is called a scale factor error in the inertial instrument game.

Suppose you had a certain tolerance, $H_{\text{max}}$, for this kind of error. Then it is easy to compare these terms and decide in order to keep the overall error less than $H_{\text{max}}$, you would have to keep this $\epsilon$ below the $\epsilon_{\text{max}}$ in Figure 2. I have computed some numbers that are based on what you could reasonably do. If you are in the laboratory, 10 m/sec$^2$ is the applied $A$, and I'm going to put the $H$ in units we talk in this business which are Eötvös units and 1 E is $10^{-9}$ m/sec$^2$ the natural MKS unit of gradient. The reason I have put 1 E at the head of the table is because today's instruments actually deliver numbers of that order, or almost that, in the laboratory. So I put the tolerance and the field into this expression, and find out that I have to match to two parts in $10^{11}$, which is really dramatic. The people who manage to bring off numbers of this order are really to be admired. I wouldn't care to believe that I could do it. But if we go to space, we find out that the acceleration is less. The numbers we have been talking about today are typically on the order of $10^{-4}$ m/sec$^2$. We need to lower this tolerance, because the reason for going to space in the first place is to get rid of this error, so I'll drop $H_{\text{max}}$ 3 orders of magnitude, A five orders of
\[ \Delta A = (1 + \varepsilon)(A + \delta) - A = \delta + \varepsilon A = \delta H \]

<table>
<thead>
<tr>
<th>Case</th>
<th>( H_{\text{max}} )</th>
<th>( \varepsilon_{\text{max}} )</th>
<th>( E )</th>
<th>( m/s^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lab (1g)</td>
<td>1</td>
<td>10</td>
<td>( 2 \times 10^{-11} )</td>
<td></td>
</tr>
<tr>
<td>STS</td>
<td>( 10^{-3} )</td>
<td>( 2 \times 10^{-9} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Float</td>
<td>( 10^{-4} )</td>
<td>( 2 \times 10^{-10} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 2.**
magnitude, and we only need to match to parts in $10^9$. This is matching over the entire range of the input acceleration. Even so, matching to parts in $10^9$ is not an easy matter.

The last thing that we do to this instrument is to completely float it. In other words, if we remove all mechanical constraint so that there is no way of applying an acceleration or vibration, then we are getting numbers on the order of $10^{-10}$ m/sec$^2$ due to remnant effects like stray electrical charge on the instrument. We could then tighten our tolerance a little further, and we still only have to match parts in $10^4$. That I can promise to do. I may be rash, but that is far easier. So this is the reason for going to space, the reason why we have to go to an actual floatation system.

The second problem we have to deal with is vibration, which is a real nuisance. Suppose I take an inline diagonal instrument, such as shown in Figure 3, and attach it to my rigid rod and shake it by grabbing it in the middle, as I must support it somehow. I grab it in the middle, and shake it up and down. The problem with that is the rigid bar that I supported the accelerometers with does not exist. We are forced to use real materials, and so it bends somewhat because these accelerometers have some mass. You can go through an elementary beam analysis on this and determine what happens. It is really quite peculiar. If you shake it up and down, the sensitive axes are bending in opposite directions and you measure a component of the shake. When you subtract these two you have something left over. Moreover, when the shake reverses, so do the scopes, so you get a result which is off in the same direction, whether up or down. This is a rectification process, a common instrument problem. Note that we get an output, even if the vibration frequency is outside the accelerometer bandwidth.

When elementary beam theory is used, the bias is as shown in Figure 3. Here, $J$ is the amplitude of the applied vibration at the applied frequency $\omega$, $\omega_y$ is the resonant frequency of the beam for this type of excitation. If you have a certain tolerance for this kind of
TRANSVERSE VIBRATION

\[ \Delta H_{xx, bias} = -3 \left( \frac{J_\omega^2}{b_\omega_y} \right)^2 \]

\[ J_\omega^2 < b_\omega_y \sqrt{H_{\text{max}}/3} \]

For: \( b = 0.2 \text{ m}; \ m = 1 \text{ kg}; \ H_{\text{max}} = 10^{-4} \)

0.1 m Diameter Tube; \( m_{\text{beam}} = 0.4 \text{ kg} \)

<table>
<thead>
<tr>
<th>Material</th>
<th>( \omega_y ) (rad/s)</th>
<th>( (J_\omega^2)_{\text{max}} ) (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>9500</td>
<td>3.5x10^{-4}</td>
</tr>
<tr>
<td>Beryllium</td>
<td>24000</td>
<td>8.8x10^{-4}</td>
</tr>
</tbody>
</table>

FIGURE 3.
error, $H_{\text{max}}$, you can solve the equation for $Ju^2$ which is the vibration acceleration. Again, I put 20 centimeters in as the distance between the accelerometers, a kilogram for each one of the basic accelerometers, and the $H_{\text{max}}$ at $10^{-4}$ E.

For standard accelerometers, which are hardly any bigger than your fist, many smaller, we can't make the diameter of the beam much larger than about 10 centimeters so I used that. The larger we make that the stiffer its going to be. Then, for the mass of the beam, I allowed 20 percent of the total accelerometer mass on the grounds that, if I made it more than this, elementry beam theory would no longer apply. That is not horrendously optimistic because then the beam mass itself would tend to lower the frequency so we would gain only a little. For 10 times that mass, we might get a factor of 2 increase in the resonant frequency.

If we use aluminum for the tube material, we get a resonant frequency of 9500 radians per second; and you have to admit that is a pretty stiff beam. Still, the allowable acceleration that turns out to be $3.5 \times 10^{-4}$ m/sec$^2$ or $3.5 \times 10^{-5}$ g's that everyone uses here. That number is much lower than the numbers quoted today for the Shuttle, or what is likely to be achieved on the Space Station. When I tried new materials it turned out that steel and titanium give almost identical results to the aluminum. Beryllium is really the only stuff that is significantly better and it leads to the answers shown in Figure 3, which are about a factor of 2.5 improvement. That's nice, but still nowhere near the actual vibration levels; so there is nothing for it, we can't permit the process to occur.

Our approach to both scale factor problems and vibration is total isolation. You turn the instrument fully loose and let it float. When your vehicle is about to collide with it, we apply a force using magnetic eddy currents. We have a set of coils that form a cage around the gradiometer and we force the thing back when it gets too close, but most of the time the forcing current is off, so it is drag free for the
periods between forces. The curve, Figure 4, has been presented today in different forms. You let something float freely long enough, and sooner or later it runs into whatever boundaries we establish. If you allow a gap between the experiment and the coils, eventually they get in the way. Reasonable numbers are toward the center of the chart, showing the spacecraft acceleration due to applied drag from all sources. You are then allowed the indicated number of seconds of free time between impulses. I would guess 10 to 20 seconds in the shuttle, maybe a little longer in the space station because it is still larger. Quite practical.

We have done a feasibility study on whether that would work with eddy currents. I won't bore you with the details, but there is a published paper on the subject.\(^1\) I will say that at JPL we put together a small test facility to try this out in one axis in the laboratory, and this is shown in Figure 5. It is a torsion pendulum supported by a wire that comes from the top down through the center of gravity of the floated assembly. The assembly consists of a cross beam connecting an empty aluminum box and a counterweight. Not much lateral motion is allowed. When the box gets too close, we turn on the coils and push it away. Typical frequencies run around 50 kHz in order to make sure of totally expelling the field from the box. The thinner you make the box walls, the higher the frequency needed in order to do that. There are also position sensors shown that tie the sensor back to the amplifiers that drive the coils. We are beginning to work on that now.

We have what we believe is a pretty good position for the theory for this kind of eddy current work. Anybody who has tried to calculate eddy currents knows it is a horrendous mess so I put it out on contract with Arizona State University which has turned in terrific stuff; and I expect to have a solid report in a matter of months. There are papers on most of the subjects I've covered here in front of you. Any questions?

Bob Naumann, Marshall Space Flight Center: When you supply the restoring force using the eddy current link, wouldn't you excite vibrations in the gradiometer and would you not have to wait for them to die out to make measurements, because if you have something that stiff doesn't it take a long time for them to die out?

Sonabend: We might have to put a little damping in for that; but we have not tried that yet, Bob. I would like to get as close to doing that as possible. We should be able to see it in this test facility. This is a pretty stiff box, I don't think its going to be easy to excite it.

ANSWER: Yes, the larger the package the worse that problem will get.

Ed Bergmann, C.S. Draper Laboratory: When those jets fired, that is a very abruptly applied acceleration and its not clear to me that your magnetic suspension is going to be stiff enough to accommodate that sudden an impulse.

Sonabend: In this mode, as shown, the magnetic suspension is off and all that happens is that the vehicle approaches the floating package by some amount and you have to leave enough clearance for the largest excursion due to these impulses.

Bergmann: That means you must leave a significant amount of the clearance that you have trying to keep the wall far enough away to deal with that.

Sonabend: If your package is a meter in diameter as we expect it to be, several 10's of centimeters is available to you. So I don't think you will have any problem with that. I haven't seen any amplitudes like that today. At the absolute worst, if you turn on really ferocious thrusters, there is nothing to stop you from caging this thing. We have to have some mechanism for doing that anyway. If you had to do that once a month I don't think anybody would complain.

Fred Henderson, Teledyne Brown Engineering: When it comes to accelerations other than the thrusters, we have a model using this MSL data, that shows all you would need is a thousandth of an inch.
Sonnabend: Yes, probably worse than thrusters is crew motion and we calculated crew motion. If I remember it right, it ran a millimeter or two for the shuttle and probably less with the space station. But that is just an arbitrary calculation I think we can easily allow for all of this.