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Acoustic Waves Superimposed on Incompressible Flows

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Introducton. The use of incompressible approximations in deriving solutions to the Lighthill wave equation was investigated for problems where an analytical solution could be found. A particular model problem involves the determination of the sound field of a sphereical oscillating bubble in an ideal fluid. It is found that use of incompressible boundary conditions leads to good approximations in the important region of high acoustic wave number.

Background. The Navier Stokes equations that govern the motion of Newtonian fluids also govern the propagation of acoustic waves. In tailoring the Navier Stokes equations for acoustic calculations, Lighthill was able to rewrite the Navier Stokes forms of the momentum and mass conservation equations in the form of wave equations with inhomogeneous right hand sides. This was an exact result without any of the linearization that leads to the classical wave equation. In particular, the equation for the pressure is

$$\frac{\partial^2 p}{\partial t^2} - c_0^2 \nabla^2 p = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}. \tag{1}$$

The *Lighthill stress tensor* T_{ij} on the righthand side is a function of all the unknown flow quantities. Technically, therefore, nothing has been accomplished in garnering a solution; however, this shift of viewpoint has been enormously productive in the theoretical investigation of jet noise [Lighthill]. Especially important is the fact that the Lighthill equations are homogeneous in regions away from turbulent flow.

The assumption of incompressibility leads to the equation

$$\nabla^2 P = -\rho_0 \frac{\partial^2 U_i U_j}{\partial x_i \partial x_j}.$$

Here we capital P to denote the incompressible pressure and U the incompressible velocity. As above, little p will denote compressible (exact) pressure.

The idea is to find ϵ_p such that $P+\epsilon_p$ well approximates the compressible solution. Ideally $p = P + \epsilon_p$, (the approximation is exact!) but this would happen only with "difficult" properly juxtaposed boundary conditions and inhomogenities corresponding to the exact Lighthill equation. For the model problem we instead approximated these terms with compressible boundary conditions and inhomogenities leading to a problem where the right hand side T_{ij} of (1.1) is known.

Pulsating bubble problem. We illustrate this procedure for the boundary value problem resulting from a pulsating bubble in an ideal fluid. In this situation the Lighthill tensor T_{ij} is zero and our concern will be with approximate boundary conditions.

Consider a bubble of radius a pulsating with radial velocity $\bar{U}(a(t), t)$. This may be approximated with fixed velocity $U(t)$ at $r = a$ [Temken]. For brevity, we ignore the mathematical statements of the governing equations and all important boundary conditions and go right to the solutions.

The compressible solution (normally hard) is

$$p = \frac{\rho_0}{\rho} c_0 a [U(\bar{t}) - \frac{c_0}{a} \int_{-\infty}^{\bar{t}} U(\tau) e^{-c_0/a(\bar{t}-\tau)} d\tau.]$$

The incompressible solution (normally easier) is

$$P = P_0 + \rho_0 \dot{U} \frac{a^2}{r} - \frac{1}{2} \rho_0 U^2 \frac{a^4}{r^4}.$$

The last term is negligible for small Mach number.

The composite solution $P + \epsilon_p$ with incompressible boundary conditions is given with

$$\epsilon_p = \frac{\rho_0 a^2}{\rho} \dot{U} \left(t - \frac{(r-a)}{c} \right) - \rho_0 \dot{U}(t) \frac{a^2}{\rho}.$$

Inserting a Fourier mode $U = exp(i\omega t)$ results in a comparison of $i\omega$ with

$$\frac{-\omega k a}{1 + i k a} e^{ika} e^{-i\omega / c_0}.$$

The comparisons are very favorable for $k = \omega / c_0$ large which indicates accuracy for important high frequencies. Other methods of approximation reveal different local similarities.

Continued Investigation. Other boundary value problems corresponding to dipole and quadrupole sources are now receiving attention. This should determine whether this methodology will be useful for formulating stable problems using incompressible information.

References

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