Low Velocity Impact Analysis With NASTRAN

Daniel A. Trowbridge
Analex Corporation
NASA Lewis Research Center
Fairview Park, Ohio

and

Joseph E. Grady and Robert A. Aiello
Lewis Research Center
Cleveland, Ohio

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Daniel A. Trowbridge
Analex Corporation
Fairview Park, Ohio 44126

Joseph E. Grady and Robert A. Aiello*
NASA Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

A nonlinear elastic force--displacement relationship is used to calculate the transient impact force and local deformation at the point of contact between impactor and target. The nonlinear analysis and transfer function capabilities of NASTRAN are utilized to define a finite element model that behaves globally linearly elastic, and locally nonlinear elastic to model the local contact behavior. Results are presented for two different structures: a uniform cylindrical rod impacted longitudinally; and an orthotropic plate impacted transversely. Calculated impact force and transient structural response of the targets are shown to compare well with results measured in experimental tests.

INTRODUCTION

Aerospace structures are subjected to impact loading from a variety of sources, including dropped tools, runway debris, and munitions. In advanced composite materials, impact loading can cause significant internal structural damage, and a resulting loss of stiffness and strength. Therefore, the development of an accurate means of calculating structural response due to impact loading is of critical importance in the analysis and design of advanced aerospace structures. In this paper, a computational technique is developed to predict the dynamic response of a structure to a low velocity elastic impact.

Current work in this area has led to a variety of methods for calculating the response of a structure subject to impact loading. Lal [1] used 3 equivalent springs in series to model impact of a composite plate. The springs represented the indentation, and flexural and shear stiffnesses. The calculated displacement results compared well with his test measurements for low velocity impact. Schonberg, et al. [2] developed a closed form solution for the transverse impact of beams and plates by superimposing a static layer solution with elementary plate theory. His results showed that the impact force, maximum displacement and duration of contact were directly proportional to the target's mass, although the analysis could not be readily applied to inhomogeneous materials. Graves [3] wrote a FORTRAN computer program to model the impact of composite plates. He predicted damage contours in the target using a maximum shear stress criterion. Lee, et al. [4] developed a specialty finite element code and used a triangular pulse load to simulate the impact force applied to a composite laminate. They showed that the higher modes of the target's frequency response are more pronounced near the point of contact with the impactor. Sun [5] proposed a modified nonlinear elastic contact law for orthotropic materials that accounted for permanent indentation during unloading. The new contact law was used in a finite element program to calculate the dynamic response of composite beams under transverse impact loading. Tan [6] extended this approach to laminated plates. He used a nine-node isoparametric plate element to model the target, and his

* Currently: Senior Structural Analyst, Analex Corporation, Fairview Park, Ohio.
calculated strain and impact force histories showed good agreement with test data. Sun [7] then included the effects of in-plane prestress on the plate. The calculated transient strain response in the target laminate showed good agreement with their experimental measurements. Aggour [8] extended this work to include the effects of transverse shear and rotary inertia. Strain response from their analysis compared well with experimental measurements obtained by Takeda [9-10]. Takamatsu, et al [11] authored a three dimensional finite element code to predict impact damage in composite laminates. A maximum stress failure criterion, as developed by Tsai [12], was used as a basis for progressively altering the stiffness of the composite target. Humphreys [13] developed a finite element code that also included the effects of material damping, shear deformation, and delamination. He used a stress-based failure criterion. Wu and Springer [14] developed a 3 dimensional transient finite element code that predicts delamination damage based on a novel application of dimensional analysis. Calculated delamination contours were shown to compare favorably with ultrasonic c-scans of impacted test specimens.

The most common approach to low velocity impact analysis is to develop a specialty finite element program [4-14] that can account for the nonlinear elastic force-displacement behavior in the contact region. Often a triangular of half-sine waveform is assumed [1, 4] to represent the transient impact force history. In the work presented here, a more straightforward method of impact modeling is presented, using the general purpose structural analysis program NASTRAN. A nonlinear elastic contact law is used to model the local contact behavior, and the resulting impact force is calculated based on the relative displacement between impactor and target.

**BACKGROUND**

Structural damage due to impact invariably initiates in the immediate vicinity of the impact. Therefore, it is important that the local stress field in the region of contact be calculated accurately. The Hertzian contact law [15] is as an elasticity-based force-displacement relationship that describes contact between two elastic bodies. The Hertzian contact law is given by:

\[ F = K \alpha^n \]  \hspace{1cm} (1)

where

- \( F \) = elastic contact force
- \( K \) = contact stiffness
- \( n \) = exponent

and

\[ \alpha = \text{relative displacement (indentation) between impactor and target} \]
\[ = u_i - u_t \quad (i = \text{impactor}, t = \text{target}) \]

The exponent \( n \) was shown in reference [15] to have the value of \( 3/2 \). In this application, force, displacement and indentation change with time.
During low velocity impact, structural damage to the target is negligible, and areas of the structure remote from the impact deform in a linear elastic manner. An efficient finite element model, therefore, would combine a linear elastic model of the global structure with the prescribed non-linear elastic behavior at the point of contact with the projectile. The nonlinear force-displacement relationship in equation (1) is incorporated into a linear elastic finite element model (MSC/NASTRAN transient solution 27, COSMIC/NASTRAN transient solution 9) by using a NASTRAN transfer function definition and nonlinear analysis capability. In the following section, Hertz' contact law is discussed. The next section describes a straightforward method of incorporating this contact law into NASTRAN. Impact loading of two different structures is then analyzed. The first problem considered is a one-dimensional rod of uniform cross section impacted longitudinally. The second is an orthotropic plate under transverse impact. Calculated results are compared with experimental test data for both cases.

CONTACT LAW

In reference [16], the derivation of the Hertzian force-displacement relationship is given for two spherical isotropic elastic bodies of radius \( r_1 \) and \( r_2 \) in contact. The resulting equation is given by:

\[
F = K \alpha^{3/2}
\]  

(2)

where

\[
K = \frac{4}{3} \frac{r_1 r_2}{r_1 + r_2} \left[ \frac{k_1 k_2}{k_1 + k_2} \right]
\]  

(3)

is the contact stiffness and

\[
k_j = \frac{E_j}{1 - \nu_j^2} \quad j = 1,2
\]  

(4)

where \( E_j \) and \( \nu_j \) are the elastic modulus and Poisson ratio, respectively, and the subscripts 1 and 2 refer to each of the spheres. When a spherical impactor contacts a flat target, equation (3) simplifies to

\[
K = \frac{4}{3} \sqrt{r_1} \left[ \frac{k_i k_t}{k_i + k_t} \right]
\]  

(5)

where \( i \) and \( t \) represent the impactor and target, respectively, and \( k_i \) and \( k_t \) are given by:

\[
K_i = \frac{4}{3} \frac{r_i}{r_i + r_t} \left[ \frac{k_i k_t}{k_i + k_t} \right]
\]  

(6)

and

\[
K_t = \frac{4}{3} \frac{r_t}{r_i + r_t} \left[ \frac{k_i k_t}{k_i + k_t} \right]
\]  

(7)

Experimental test data for both contact cases are compared with calculated results.
In equation (2), $\alpha$ is the local indentation at the contact point, shown schematically in figure 1. We have:

$$\alpha = u_i - u_t$$

where $\alpha$ is the relative local displacement between impactor and target at the point of contact.

**NASTRAN Implementation**

NASTRAN allows for easy implementation of the contact law and is readily available to many users. This makes NASTRAN an excellent finite element program for incorporating this method of impact modeling. A unique feature of this impact analysis is that the impact force is not a pre-defined input parameter, but rather is calculated during the analysis, based on the relative displacement between impactor and target, and using the prescribed force-displacement relationship. This methodology allows for the unloading of the structure when the target and the impactor are no longer in contact and for secondary impacts in the event that they would again come into contact.

The contact law was incorporated into the NASTRAN finite element model as follows:

The impactor is modeled as a lumped mass just touching the target at time $t=0$ and with an initial velocity toward the target. The difference between the displacement of this lumped mass and the displacement of the target is the indentation, $\alpha$. The contact law is prescribed with the transfer function (TF) card, and the nonlinear force (NOLIN3) card. The TF card acts as a dynamic multipoint constraint, relating the displacement, velocity and acceleration of several independent degrees of freedom to a single dependent degree of freedom. In this case, only displacement relationships were needed. The dependent degree of freedom is calculated on the TF card as follows: [17].

$$\sum_{j=1}^{n} (B_0 + B_1 p + B_2 p^2) u_{dep} + \sum_{j=1}^{n} (A_0^j + A_1^j p + A_2^j p^2) u_{ind} = 0$$

where

$B_0, B_1, B_2 =$ the coefficients for the dependent degree of freedom

$A_0^j, A_1^j, A_2^j =$ the coefficients for the independent degrees of freedom
\( u_{\text{dep}} \) = the displacement of the dependent degree of freedom

\( u_{\text{ind}} \) = the displacements of the independent degrees of freedom

\( n \) = the number of independent degrees of freedom

\( p \) = the differential operator, \( \frac{\partial}{\partial t} \); and \( p^2 = \frac{\partial^2}{\partial t^2} \)

For this analysis, the equation would appear:

\[
(1.0) u_{\text{extra point}} + \left[ (-1.0) u_{\text{impactor}} + (1.0) u_{\text{target}} \right] = 0 \tag{10}
\]

that is

\[
n = 2
\]

\[
B_1, B_2, A_1^j, A_2^j = 0.0 \quad (j = 1, n)
\]

\[
B_0 = 1.0
\]

\[
A_0^1 = -1.0
\]

\[
A_0^2 = 1.0
\]

The resulting equation defines the dependent degree of freedom as the indentation at each time step of the transient analysis. The value of the indentation is assigned to an EPOINT. The EPOINT, or extra point, is a nonstructural variable that is used to store the value of the indentation. The EPOINT is provided as input to the NOLIN3 card.

The NOLIN3 card is the means of applying the time-dependent nonlinear load based on the indentation. The NOLIN3 card has the form:

\[
P(t) = \begin{cases} 
S[x(t)]^A, & x(t) > 0 \\
0, & x(t) \leq 0 
\end{cases} \tag{11}
\]

where

\[
P(t) = \text{the resulting nonlinear force}
\]

\[
S = \text{a scale factor}
\]

\[
x(t) = \text{the displacement or velocity of a degree of freedom}
\]

\[
A = \text{an amplification factor}
\]
In modeling of the impact, we define $x(t)$ to be the displacement of the EPOINT, $S$ to be the Hertzian stiffness, and $A$ to be $\sqrt{3}/2$, as given in equation (2). Recall that the displacement of the EPOINT is really the indentation as obtained from the TF card. The resulting function then has the form:

$$F(t) = \begin{cases} K[\alpha(t)]^{3/2}, & \alpha(t) > 0 \\ 0, & \alpha(t) \leq 0 \end{cases}$$

(12)

Note that when $\alpha$ is less than or equal to zero (target and the impactor are no longer in contact), the contact force is also zero. Two NOLIN3 cards are used, one to apply the impact force to the target and the other to apply the same force to the impactor in the opposite direction of its initial velocity. This methodology allows the impactor to slow with increasing impact force and eventually to unload the target as the impactor begins to travel in the opposite direction, away from the target. Since the forcing function is not an input parameter, changing the properties of the target or the impactor does not require assuming a new force–time relationship but only the recalculation of the contact stiffness, $K$, and changing the appropriate material property cards.

RESULTS

One Dimensional Rod

The first problem analyzed is the longitudinal impact of a steel ball on a long aluminum rod of constant cross section. Geometry and material properties of the impactor and target are given in figure 1. The problem was modeled using 144 1–dimensional rod elements with each grid point having a single longitudinal degree of freedom. Two additional degrees of freedom were used to model impactor and the extra point, resulting in a total of 147 degrees of freedom. A single lumped mass with an initial velocity was used to represent the impactor. The Hertzian force–displacement relationship in equation (1) was prescribed using the NASTRAN NOLIN3 card, as shown in the example input deck in the appendix.

The impact force history obtained from the analysis compares well with experimentally determined values [18], as shown in figure 2. The calculated strain response at the midpoint of the target bar is compared with measured values in figure 3. The sign reversal of the second pulse is caused by the reflected tensile stress wave generated by the incident compressive wave reaching the free end of the bar [19].

Some insight into the timing and the location of the impact–induced structural failure can be gained by tracking the distribution of energy in the impactor and the target, as shown in figure 4. The energy balance can be expressed as:

$$U_{\text{tot}} = KE_i + SE_i + KE_t + SE_t$$

(13)

where
The total energy in the system, $U_{tot}$, is divided between the kinetic energy and the strain energy of the target and the impactor in a time-varying manner. Because damping effects are not considered, the total system energy is constant and equal to the initial kinetic energy of the impactor. The strain energy of the impactor is non-zero only during the contact interval ($0 < t_c/L < 0.4$, where $t$ is time, $L$ is the length of the bar, and $c$ is the wave speed in the bar) and peaks when the contact force is greatest, approximately halfway through the contact interval. The kinetic energy of the impactor decreases rapidly as the impactor slows during contact with the target. Eventually, at $t_c/L \approx 0.25$, the impactor velocity (and therefore its kinetic energy) decreases to zero and the elastic rebound begins. The kinetic energy of the impactor never returns to its initial level, however, because approximately 80% of the energy has been transferred to the target in the form of strain energy and kinetic energy. The strain and kinetic energies in the target both increase rapidly during the contact with the impactor and remain constant after contact has ended ($t_c/L > 0.4$). Both strain and kinetic energies maintain equal and constant values until the compressive stress wave generated by the impact reaches the far end of the free-free bar ($t_c/L = 1.0$). A tensile stress wave is generated when the compressive pulse reflects from the stress-free boundary [19]. The superposition of the incident and reflected pulses momentarily leaves the bar stress-free, and the strain energy decreases to zero. The kinetic energy simultaneously increases, maintaining a conservation of total energy. The reflection process is repeated at $t_c/L = 2.0$, when the reflected pulse returns to the impacted end of the bar. Similar energy dissipation diagrams may prove useful in analyzing dynamic failure of more complex structures. By tracking the distribution of energy and assuming a strain energy based failure criterion, it maybe possible to locate areas of failure in the target and predict the time of failure of these areas.

**Composite Plate**

The low velocity transverse impact of a composite plate made from Scotchply 1003 prepreg [20] is now analyzed. The problem is depicted schematically in figure 5, and is described in detail in references [9, 10]. A modified Hertzian contact stiffness has been proposed [5] for application to composite materials. Specifically, equation (7) is replaced by

$$k_t \approx E_{33}$$  (18)
where $E_{33}$ is the transverse (out-of-plane) modulus of the composite plate. Plate membrane and bending stiffness material properties were calculated using the COBSTRAN (Composite Blade Structural Analyzer) computer code [21] which calculates elastic moduli of composite materials from known constituent properties and laminate ply orientations.

A uniform square mesh of QUAD4 elements was used to model the 15.24 cm $\times$ 15.24 cm (6 in $\times$ 6 in) target plate. A mesh convergence study was performed to establish the degree of mesh refinement necessary to arrive at a numerically converged solution. Three different meshes were considered, 25 $\times$ 25, 49 $\times$ 49, and 61 $\times$ 61 elements. Of these, the latter two produced essentially the same strain response for a given impact velocity and were therefore considered to be converged solutions. The results presented here were therefore calculated using the 49 $\times$ 49 element model. Five degrees of freedom ($u_x$, $u_y$, $u_z$, $\theta_x$, and $\theta_y$) were used at each nodal point, giving the model a total of 11510 degrees of freedom. The problem was solved on a Cray XMP in 52 CPU minutes.

The impactor used in the tests [9, 10] was a uniform 2.54 cm (1 in) long, blunt-ended steel rod of radius 0.048 cm (1/16 in). In the analysis, a contact radius of 0.047625 cm (3/16 in) was assumed in the Hertzian contact stiffness calculations. The calculated impact force history is shown in figure 6. Although no direct measurement of the impact force was obtained experimentally, the contact time was measured [8] and found to be 204 microseconds. This is in good agreement with the calculated result. Figure 6 also shows that a secondary impact occurs during the latter half of the contact interval ($t = 175 \, \mu\text{sec}$), probably due to the vibration of the target plate during contact with the impactor.

The resulting displacement response of the plate is shown in figure 7, where it has been assumed that no damage occurs in the target during contact with the impactor. This assumption is valid based on the available test data. Ultrasonic C-scans of the specimens after impact indicate that this level of impactor kinetic energy (10 Joules) is very near the threshold energy level required to cause damage [10] in specimens of this layup. As a result, very little damage occurs at this impact velocity. The anisotropic bending stiffness of the target is evident from the elliptical displacement contours, as the flexural disturbance travels faster in the stiffer direction, as shown in figure 7.

The strain response at gage A is compared to the calculated response in figure 8. The two curves are similar in amplitude and duration but the calculated strain appears to lag the measured values by approximately 25 microseconds. This may be due to the difficulty in establishing experimentally the precise time at which contact occurs based on strain gage readings taken at some distance from the point of contact. The comparison shown in figure 9 for gage B likewise shows a time shift of approximately 25 microseconds between the measured and the calculated response. The amplitude and duration of the calculated strain response correlate quite well with the measured signal.
SUMMARY

A simple means of modeling low velocity, non-damaging impact using NASTRAN was demonstrated. A nonlinear elastic contact model was included in the finite element analysis using NASTRAN transfer function definitions and nonlinear analysis capabilities. The same contact law was used to define the force-indentation relationship for two different impactor/target combinations. Results in both cases showed that the impact force and resulting transient structural response of the target compared well with experimentally measured values. Future work will include the effects of damage and the resulting progressive loss of structural stiffness that occurs during higher velocity impact events.

ACKNOWLEDGEMENT

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REFERENCES


[20] "Scotchply Reinforced Plastic Type 1003 Technical Data Sheet", Structural Products, Industrial Specialties Division/3M.

ID TRANS, LOAD
APP DISP
TIME 60
SOL 9
CEND

TITLE = COSMIC: TRANSIENT RESPONSE ANALYSIS: HERTZIAN IMPACT FF
SUBTITLE = 36" AL. ROD 5/8 STEEL BALL V0=62.1 IN/S
LABEL = ROD

$ NONLINEAR = 5 NONLINEAR LOAD
$ INITIAL CONDITIONS SET
IC = 1
TFL=111
SPC = 4
TSTEP = 7

$ OUTPUT STUFF
SET 30 = 1,72,73,999,1001
NLLOAD = 30
STRESS(PRINT) = 30
DISP(PRINT) = 30
BEGIN BULK

$ EXTRA POINT = INDENTATION
EPOINT,1001
GRID,999,-0.3125,0.0,0.0
GRID,1,0.0,0.0,0.0
=(144),*(1),=,*(0.25),=
CROD,1,1,1,2
=(143),*(1),=,*(1),=(1)

$ LUMP MASS OF IMPACTOR
CON2,200,999,0.0,587-5.0,0.0,0.0,0,..,CON2-2
+CON2-2,3.7456,3.7456,3.7456

$ MATERIAL PROPERTIES
PROO,11,0.196,6.14-3.0,25
MAT1,11,0.06,0.33,2.5,4,.,MAT1-1
+MAT1-1,35.06,36.66,27.66

$ BOUNDARY CONDITIONS
SPC1,4,23456,1,THRU,145
$ REMOVE DEGREES OF FREEDOM FROM IMPACTOR
SPC1,4,23456,999
$ TRANSFER FUNCTION TO DEFINE INDENTATION
TF,111,1001,0,1.0,0.0,0.0,+,TF-1
+TF-1,999,1,1.0,0.0,0.0,+,TF-2
+TF-2,1,1.0,0.0,0.0

$ TIMING
TSTEP,7,2500.0,2.0-7.25

$ LOAD DEPENDENT ON DISPLACEMENT OF IMPACTOR
NOLIN3,5,1,1,6.24-6,1001,1,1.5
$ SLOW DOWN IMPACTOR
NOLIN3,5,999,1,-6.24-6,1001,1,1.5
$ INITIAL CONDITIONS: IMPACTOR VELOCITY = 62.1 IN/SEC
TIC,1,999,1,0.0,62.1
ENDDATA


BEGIN BULK

$ **** EXTRA POINT TO HOLD INDENTATION ............................

EPOINT,10001

$ **** IMPACTOR *** 3/8 IN DIAMETER .................................

GRID,999,0.0.0.0.-0.1875
CONM2,200,999,0,8.096-5.0,0.0.0.0.,+CON2-2
+CON2-2,7.459-6..7.459-6.,1.423-6

$ * * * * * GRIDS AND COQUAD4 ELEMENTS DEFINING THE PLATE GO HERE ...

$ MATERIAL PROPERTIES... MAT2 CARDS GENERATED BY COBSTRAN

PSHELL,1,101,0.15,201,1.0
MAT2,101,4.3E+06,2.9E+05,-1.7E-03,2.8E+06,-3.4E-02,5.7E+05,1.8E-04,+A101
+A101,5.8E-06,8.9E-06,5.9E-13
MAT2,201,5.7E+06,2.9E+05,-1.9E-04,1.4E+06,-3.8E-03,5.7E+05

$ BOUNDARY CONDITIONS

SPC1,1,123456,101,THRU,150
SPC1,1,123456,0001,THRU,5056
SPC1,1,123456,101
=.=.=.,*100
=4B

SPC1,1,123456,150
=.=.=.,*100
=4B

SPC1,1,12456,999
=.=.=.,*100

GRDSET,......6

$ TIME STEP INFO

TSTEP,1.2000,1.0-7.18

$ LOAD DEPENDENT ON RELATIVE DISPLACEMENT OF IMPACTOR

NOLIN3,5,2525,3.+1.945+5,10001,0,1.5
NOLIN3,5,2526,3.+1.945+5,10001,0,1.5
NOLIN3,5,2625,3.+1.945+5,10001,0,1.5
NOLIN3,5,2626,3.+1.945+5,10001,0,1.5

$ SLOW DOWN IMPACTOR

NOLIN3,5,999,3,-7.779+5,10001,0,1.5

$ TRANSFER FUNCTION TO CALCULATE INDENTATION

TF,111,10001,0,+1.0,0.0.0.0.0.,+TF-1
+TF-1,999,3,-1.0,0.0.0.0.0.,+TF-2
+TF-2,2525,3,+0.25,0.0.0.0.0.,+TF-3
+TF-3,2526,3,+0.25,0.0.0.0.0.,+TF-4
+TF-4,2625,3,+0.25,0.0.0.0.0.,+TF-5
+TF-5,2626,3,+0.25,0.0.0.0.0.

$ INITIAL CONDITIONS: IMPACTOR VELOCITY = 1470 IN/SEC (122.5 FT/SEC)

TIC,3,999,3,0.0.1478,0

ENDATA
1.59 cm (0.625 in.) DIAMETER
STEEL BALL

1.27 cm (0.5 in.) DIAMETER ALUMINUM ROD

$V_0 = 158 \text{ cm/s (52.1 in./s)}$

\[ \sigma = u_1 - u_1 \]

**FIGURE 1.** LONGITUDINAL BAR IMPACT PROBLEM.

**FIGURE 2.** IMPACT FORCE FOR BAR PROBLEM.

**FIGURE 3.** STRAIN RESPONSE AT MIDPOINT OF IMPACTED BAR.

**FIGURE 4.** ENERGY DISTRIBUTION FOR LONGITUDINAL BAR IMPACT PROBLEM.
FIGURE 5. - COMPOSITE PLATE IMPACT SPECIMEN CONFIGURATION (REF. 9, 10).

FIGURE 6. - CALCULATED FORCE HISTORY FOR TRANSVERSE IMPACT OF COMPOSITE PLATE.

FIGURE 7. - CALCULATED DISPLACEMENT RESPONSE FOR COMPOSITE PLATE.

FIGURE 8. - STRAIN RESPONSE AT GAGE LOCATION "A".

FIGURE 9. - STRAIN RESPONSE AT GAGE LOCATION "B".
A nonlinear elastic force-displacement relationship is used to calculate the transient impact force and local deformation at the point of contact between impactor and target. The nonlinear analysis and transfer function capabilities of NASTRAN are used to define a finite element model that behaves globally linearly elastic, and locally nonlinear elastic to model the local contact behavior. Results are presented for two different structures: a uniform cylindrical rod impacted longitudinally; and an orthotropic plate impacted transversely. Calculated impact force and transient structural response of the targets are shown to compare well with results measured in experimental tests.